Unequal Rapidity Correlators from a Numerical Implementation of JIMWLK

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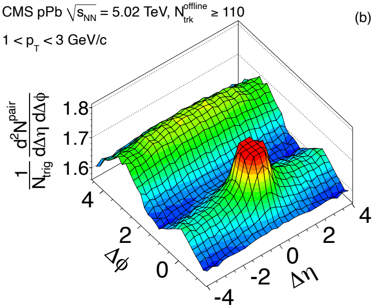
20 June 2023
Initial Stages, Copenhagen, Denmark

Outline

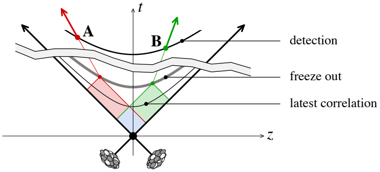
- Motivation
- Colour Glass Condensate formalism
- Equal rapidity correlators: example 1 – DIS
- Equal rapidity correlators: example 2 – qg production
- qg production at unequal rapidities
- Preliminary numerical results

Motivation

- Origin of structure of azimuthal correlations in small systems (pp, pA) not fully understood
- Azimuthal correlations extend far in rapidity
- Such correlations must originate early in collision



CMS, Phys. Lett. B 718 (2013) 795



Dumitru, Gelis, McLerran, Venugopalan, Nucl. Phys. A810 (2008) 91

Colour Glass Condensate

- Colour Glass Condensate (CGC) is suitable framework for studying correlations with large rapidity separations ($\Delta Y > 1/\alpha_{\mathrm{s}}$) $\rightarrow$ *"unequal rapidity correlators"*. In practical calculations: renormalisation group equations, Wilson lines, etc. evolve in rapidity
- Most common phenomenological applications consider correlations with small rapidity separations ($\Delta Y \leq 1/\alpha_{\mathrm{s}}$) $\rightarrow$ *"equal rapidity correlators"*. E.g. DIS, single inclusive particle production, multi-particle production
- Recent work on extending calculations to include large rapidity separations:
  - Dense–dense formalism

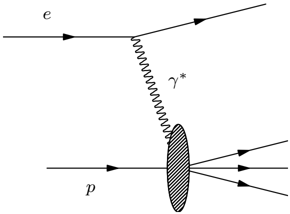
    Gelis, Lappi, Venugopalan, Phys. Rev. D79 (2008) 094017,
    Lappi, Acta Phys. Polon. B40 (2009) 1997 & Nucl. Phys. A910-911 (2013) 518
  - Phenomenological applications

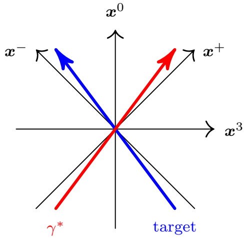
    Schenke, Schlichting, Phys. Rev. C94 (2016) 044907
  - Multi-particle correlations in small collision systems

Dusling, Gelis, Lappi, Venugopalan, Nucl. Phys. A836 (2010) 159
Kovner, Lublinsky, Phys. Rev. D83 (2011) 034017 & Phys. Rev. D84 (2011) 094011
Dumitru, Dusling, Gelis, Jalilian-Marian, Lappi, Venugopalan, Phys. Lett. B697 (2011) 21
Dusling, Venugopalan, Phys. Rev. Lett. 108 (2012) 262001 & Phys. Rev. D87 (2013) 051502 & Phys. Rev. D87 (2013) 054014

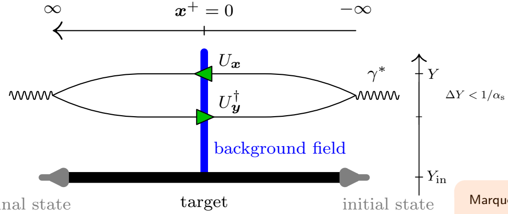
Equal Rapidity Example 1: DIS

- Consider ep collision:





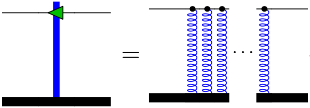
- QCD interaction between projectile and target through $q\bar{q}$ dipole



Marquet, Weigert, Nucl. Phys. A843 (2010) 68-97

DIS Example Continued

- Wilson lines colour-rotate projectile – basic building blocks of CGC calculations



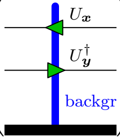
$$= U_{\boldsymbol{v}} := \mathrm{P}\exp\left\{-ig\int_{-\infty}^{\infty} \mathrm{d}x^{+} A^{-}(x^{+}, \boldsymbol{v}, 0)\right\}$$

- DIS cross section

$$\sigma^{\gamma^{*}p}(x, Q^{2}) = \int_{\boldsymbol{r}}\int_{0}^{1} \mathrm{d}\alpha \left|\psi^{\gamma^{*}\to q\bar{q}}(\alpha, \boldsymbol{r}^{2}, Q^{2})\right|^{2} 2\int_{\boldsymbol{b}} \mathrm{Re}\left\langle 1 - S^{(2)}_{\boldsymbol{xy}}\right\rangle_{Y}$$

  - Dipole operator

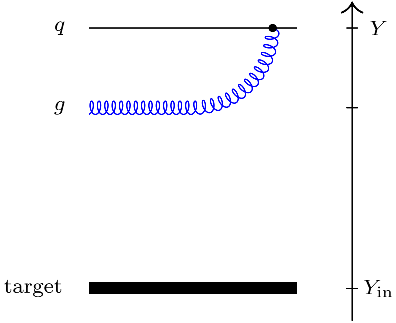
$$S^{(2)}_{\boldsymbol{xy}} := \frac{1}{N_{\mathrm{c}}}\mathrm{tr}\left(\quad \right) = \frac{\mathrm{tr}\left(U_{\boldsymbol{x}}U^{\dagger}_{\boldsymbol{y}}\right)}{N_{\mathrm{c}}}$$



  - Expectation value $\langle\rangle$ is average over background field configurations – contains all information about target

Equal Rapidity Example 2: qg Production 1/2

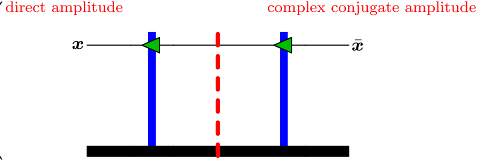
- Partonic level process: qg production in any nuclear–nuclear collision



Altinoluk, Kovner, Phys. Rev. D 83 (2011) 105004
Chirilli, Xiao, Yuan, Phys. Rev. D 86 (2012) 054005

- First deal with produced q: $\frac{\mathrm{d}\sigma_{\mathrm{q}}}{\mathrm{d}\eta_p \mathrm{d}^2 p} = xq(x) \frac{1}{(2\pi)^2} \int_{\boldsymbol{x}\bar{\boldsymbol{x}}} e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\bar{\boldsymbol{x}})} \left\langle S^{(2)}_{\boldsymbol{x}\bar{\boldsymbol{x}}} \Big|_{\bar{U}=U} \right\rangle_Y$

$$S^{(2)}_{\boldsymbol{x}\bar{\boldsymbol{x}}} = \frac{1}{N_{\mathrm{c}}} \mathrm{tr}\left(U_{\boldsymbol{x}} \bar{U}^{\dagger}_{\bar{\boldsymbol{x}}}\right) = \frac{1}{N_{\mathrm{c}}} \mathrm{tr}\left(\quad \right)$$



qg Production Example Continued

Hentschinski, Weigert, Schafer, Phys. Rev. D 73 (2006) 051501
Kovner, Lublinsky, Weigert, Phys. Rev. D 74 (2006) 114023
Kovner, Lublinsky, JHEP 11 (2006) 083

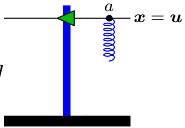
- Next, deal with produced g:

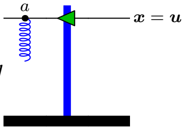
$$\frac{\mathrm{d}\sigma_{\mathrm{qg}}}{\mathrm{d}\eta_p \mathrm{d}^2 p \mathrm{d}\eta_k \mathrm{d}^2 k} = \frac{1}{16\pi^4} \int_{\boldsymbol{x}\bar{\boldsymbol{x}}} e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\bar{\boldsymbol{x}})} \left\langle H_{\mathrm{prod}}(\boldsymbol{k}) S^{(2)}_{\boldsymbol{x}\bar{\boldsymbol{x}}} \Big|_{\bar{U}=U} \right\rangle_Y$$

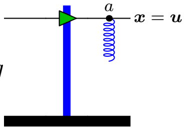
- Production Hamiltonian $H_{\mathrm{prod}}(\boldsymbol{k})$

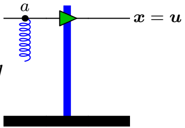
$$:= \frac{1}{4\pi^3} \int_{\boldsymbol{y}\bar{\boldsymbol{y}}} e^{-i\boldsymbol{k}\cdot(\boldsymbol{y}-\bar{\boldsymbol{y}})} \int_{\boldsymbol{u}\bar{\boldsymbol{u}}} \mathcal{K}^i_{\boldsymbol{y}\boldsymbol{u}} \mathcal{K}^i_{\bar{\boldsymbol{y}}\bar{\boldsymbol{u}}} \left(L^a_{\boldsymbol{u}} - \tilde{U}^{\dagger ab}_{\boldsymbol{y}} R^b_{\boldsymbol{u}} \right) \left(\bar{L}^a_{\bar{\boldsymbol{u}}} - \bar{\tilde{U}}^{\dagger ac}_{\bar{\boldsymbol{y}}} \bar{R}^c_{\bar{\boldsymbol{u}}} \right)$$

- $L^a_{\boldsymbol{u}}$ and $R^b_{\boldsymbol{u}}$ are Lie derivatives

$L^a_{\boldsymbol{u}} U_{\boldsymbol{x}} = -ig$ 

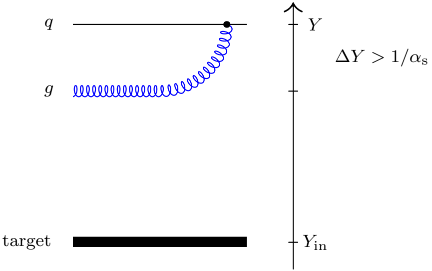
, $R^a_{\boldsymbol{u}} U_{\boldsymbol{x}} = -ig$ 

$L^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x}} = ig$ 

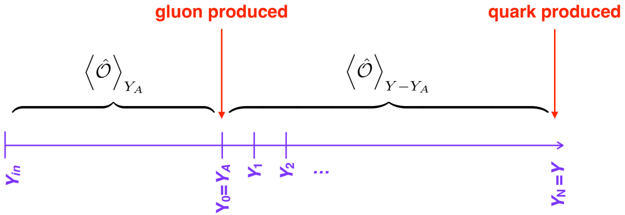
, $R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x}} = -ig$ 

Large Rapidity Separations

- Still qg production, but now $\Delta Y > 1/\alpha_s$

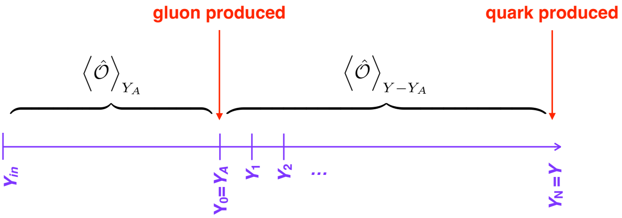


- Separate rapidity range into two segments & discretise ΔY range



Modified Cross Section for qg Production

Iancu, Triantafyllopoulos, JHEP 1311 (2013) 067



- New cross section

$$\frac{\mathrm{d}\sigma_{qg}}{\mathrm{d}Y\,\mathrm{d}^2p\mathrm{d}Y_\mathrm{A}\mathrm{d}^2k_\mathrm{A}} = \frac{1}{16\pi^4}\int_{\boldsymbol{x}\bar{\boldsymbol{x}}} e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\bar{\boldsymbol{x}})} \left\langle H_\mathrm{prod}(\boldsymbol{k}_\mathrm{A}) \left\langle S^{(2)}_{\boldsymbol{x}\bar{\boldsymbol{x}},N} \right\rangle_\nu \Big|_{\bar{U}_\mathrm{A}=U_\mathrm{A}} \right\rangle_{Y_\mathrm{A}}$$

- Evaluation of nested correlators $\left\langle H_\mathrm{prod}(\boldsymbol{k}_\mathrm{A}) \left\langle S^{(2)}_{\boldsymbol{x}\bar{\boldsymbol{x}},N} \right\rangle_\nu \Big|_{\bar{U}_\mathrm{A}=U_\mathrm{A}} \right\rangle_{Y_\mathrm{A}}$:
  - Evolve $S^{(2)}_{\boldsymbol{x}\bar{\boldsymbol{x}}}$ from $Y_A = Y_0$ to $Y = Y_N$
  - Set all barred quantities to barred ones (no more distinguishing between direct and complex conjugate amplitudes)
  - Act with $H_\mathrm{prod}(\boldsymbol{k}_\mathrm{A})$ at Y_A
  - Evolve from target rapidity Y_in to Y_A

Wilson Line Evolution Equation

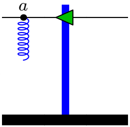
Blaizot, Iancu, Weigert, Nucl.Phys. A713 (2003) 441-469

- Langevin JIMWLK evolution equation for Wilson line

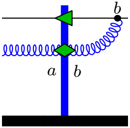
$$U^\dagger_{\boldsymbol{x},n+1} = \exp\left\{i\epsilon g \alpha^{\mathrm{L}}_{\boldsymbol{x},n}\right\} U^\dagger_{\boldsymbol{x},n} \exp\left\{-i\epsilon g \alpha^{\mathrm{R}}_{\boldsymbol{x},n}\right\}$$

- Fields colour rotate Wilson lines (cf. Lie derivatives)

$$U_{\boldsymbol{x},0}\alpha^{\mathrm{R}}_{\boldsymbol{x},0} = \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \nu^{a,i}_{\boldsymbol{z},0}$$



$$\alpha^{\mathrm{L}}_{\boldsymbol{x},0} U_{\boldsymbol{x},0} = \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \nu^{a,i}_{\boldsymbol{z},0}$$



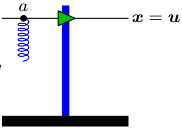
Bilocal Evolution Equations

- $H_{\rm prod}(\boldsymbol{k}_{\rm A}) \left\langle S^{(2)}_{\boldsymbol{x}\bar{\boldsymbol{x}},N} \right\rangle \implies$ need to evolve $LU, RU, LU^\dagger, RU^\dagger$ (only one of four evolution equations needed – other three can be derived from taking Hermitian conjugate and/or using relation $L^a_{\boldsymbol{u}} = \tilde{U}^{\dagger ab}_{\boldsymbol{u}} R^b_{\boldsymbol{u}}$)
- E.g.

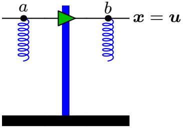
$$R^a_{\boldsymbol{u},0} U^\dagger_{\boldsymbol{x},n+1} = e^{i\epsilon g \alpha^{\rm R}_{\boldsymbol{x},n}} R^a_{\boldsymbol{u},0} U^\dagger_{\boldsymbol{x},n} e^{-i\epsilon g \alpha^{\rm L}_{\boldsymbol{x},n}}$$
$$- \frac{i\epsilon g}{\sqrt{4\pi^3}} e^{i\epsilon g \alpha^{\rm R}_{\boldsymbol{x},n}} U^\dagger_{\boldsymbol{x},n} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{x}\boldsymbol{z}} \left[U_{\boldsymbol{z},n} \nu^i_{\boldsymbol{z},n} U^\dagger_{\boldsymbol{z},n}, U_{\boldsymbol{z},n} R^a_{\boldsymbol{u},0} U^\dagger_{\boldsymbol{z},n} \right] + \mathcal{O}\left(\nu\epsilon^2\right)$$

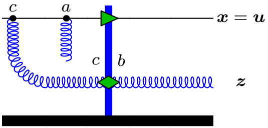
Iancu, Triantafyllopoulos, JHEP 1311 (2013) 067

- One step:

$R^a_{\boldsymbol{u},0} U^\dagger_{\boldsymbol{x},1} = ig$ 

Lappi, Ramnath, Phys. Rev. D 100, 054003 (2019)

$+ig \frac{i\epsilon g}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{x}\boldsymbol{z}} \nu^{i,b}_{\boldsymbol{z},0}$ { 

$-$ 

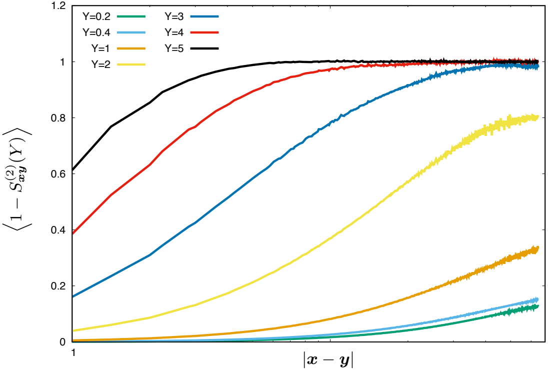
} $+ \cdots$

Preliminary Numerical Results: Dipole Correlator

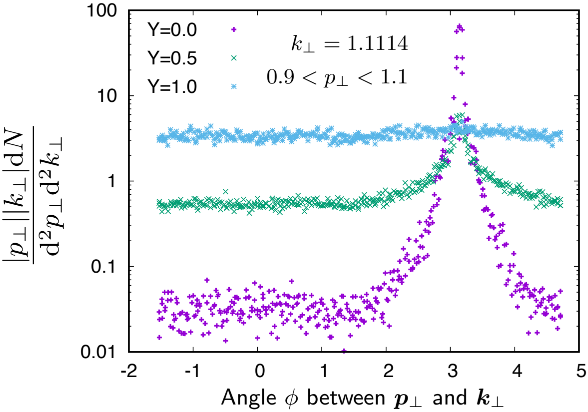
- Study evolution equations for $U^\dagger_{\boldsymbol{x},n}$ and $R^a_{\boldsymbol{u},0}U^\dagger_{\boldsymbol{x},n+1}$ on lattice, fixed coupling, MV model initial condition
- Look at inclusive qg production cross section

$$\frac{\mathrm{d}\sigma_{qg}}{\mathrm{d}Y\mathrm{d}^2p\mathrm{d}Y_\mathrm{A}\mathrm{d}^2k_\mathrm{A}} \sim \left\langle H_\mathrm{prod}(\boldsymbol{k}_\mathrm{A}) \left\langle S^{(2)}_{\boldsymbol{x}\bar{\boldsymbol{x}},N} \right\rangle_\nu \Big|_{\bar{U}_\mathrm{A}=U_\mathrm{A}} \right\rangle_{Y_\mathrm{A}}$$

- Dipole correlator as function of transverse coordinate separation at various rapidities



- Two-particle correlation as function of ϕ ($\boldsymbol{p}$ is quark moment, $\boldsymbol{k}$ is gluon momentum)



Summary

- Want to understand particle correlations over large rapidity separations
- CGC effective theory is useful
- CGC formalism with DIS example
- Inclusive qg production at equal rapidities
- Extension of inclusive qg production to unequal rapidities
- Preliminary numerical results