System size dependence of pre-equilibrium and applicability of hydrodynamics in heavy-ion collisions

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1 Introduction

- hydrodynamics often is central part of simulation frameworks of hadronic collisions
  but: applicability to early times or small systems is questionable
- Kinetic theory is more accurate in both of these limits

⇒ Our aim: employ simplified kinetic theory description of transverse flow observables; compare to hydrodynamics to assess regime of applicability [1, 2]

2 Kinetic theory setup

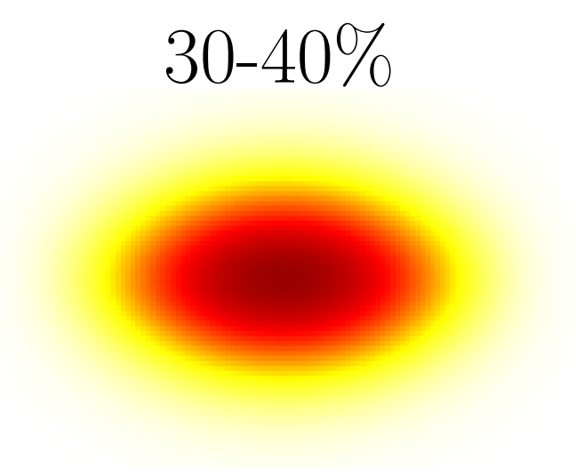
- we describe the system by a phase space distribution f of massless on-shell bosons
  - boost invariance; initially vanishing longitudinal momentum and transverse anisotropy
- time evolution described by Boltzmann equation in conformal relaxation time approximation

$$p^\mu \partial_\mu f = C_{\mathrm{RTA}}[f] = -\frac{p^\mu u_\mu}{\tau_R}(f - f_{\mathrm{eq}}) \ , \quad \tau_R = 5\frac{\eta}{s}T^{-1} \tag{1}$$

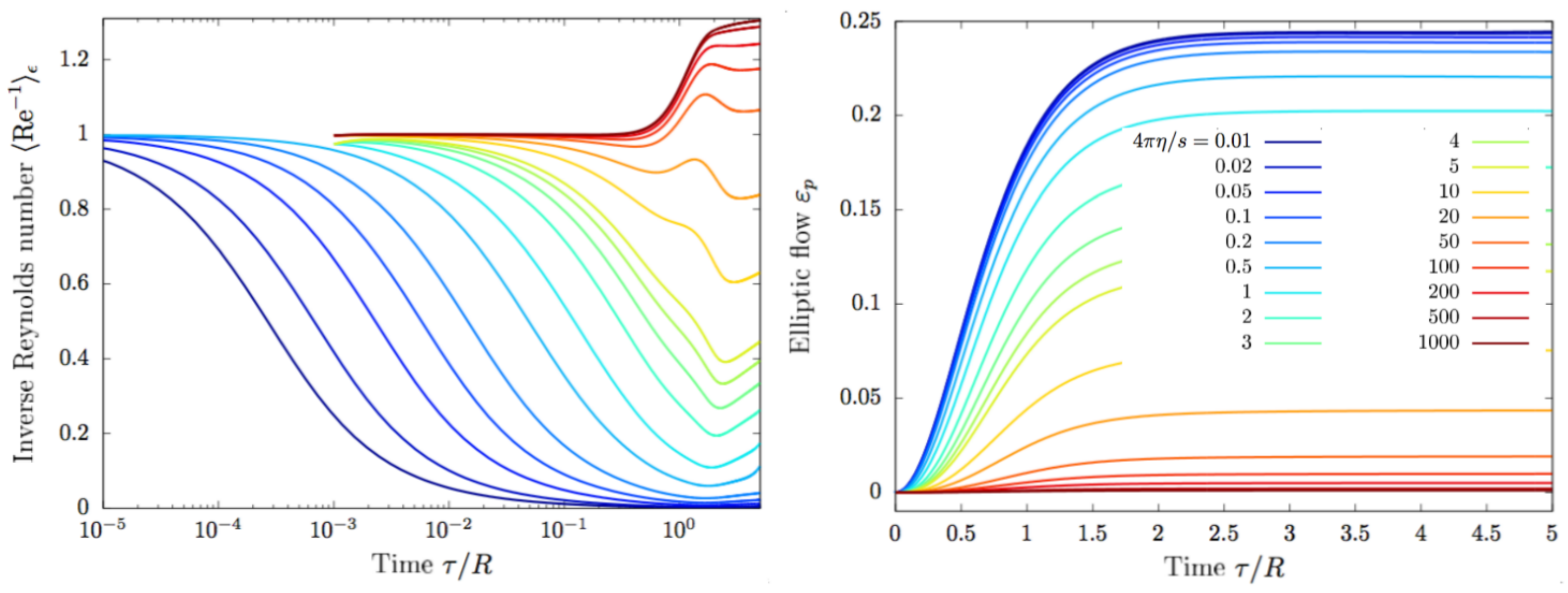
- advantage: results depend only on transverse initial state geometry and a single dimensionless parameter [3]: opacity $\hat{\gamma} \sim$ ”total interaction rate”
  - collects dependencies on viscosity, transverse size and energy scale

$$\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1} \left(\frac{1}{a\pi} R \frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}\right)^{1/4} \tag{2}$$

- Our initial conditions: averages of the centrality classes of Pb+Pb at 5.02 TeV [4]
  - fixes R and $\frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}$, so we vary $\hat{\gamma}$ by changing $\frac{\eta}{s}$
    but: this is equivalent to varying system size!



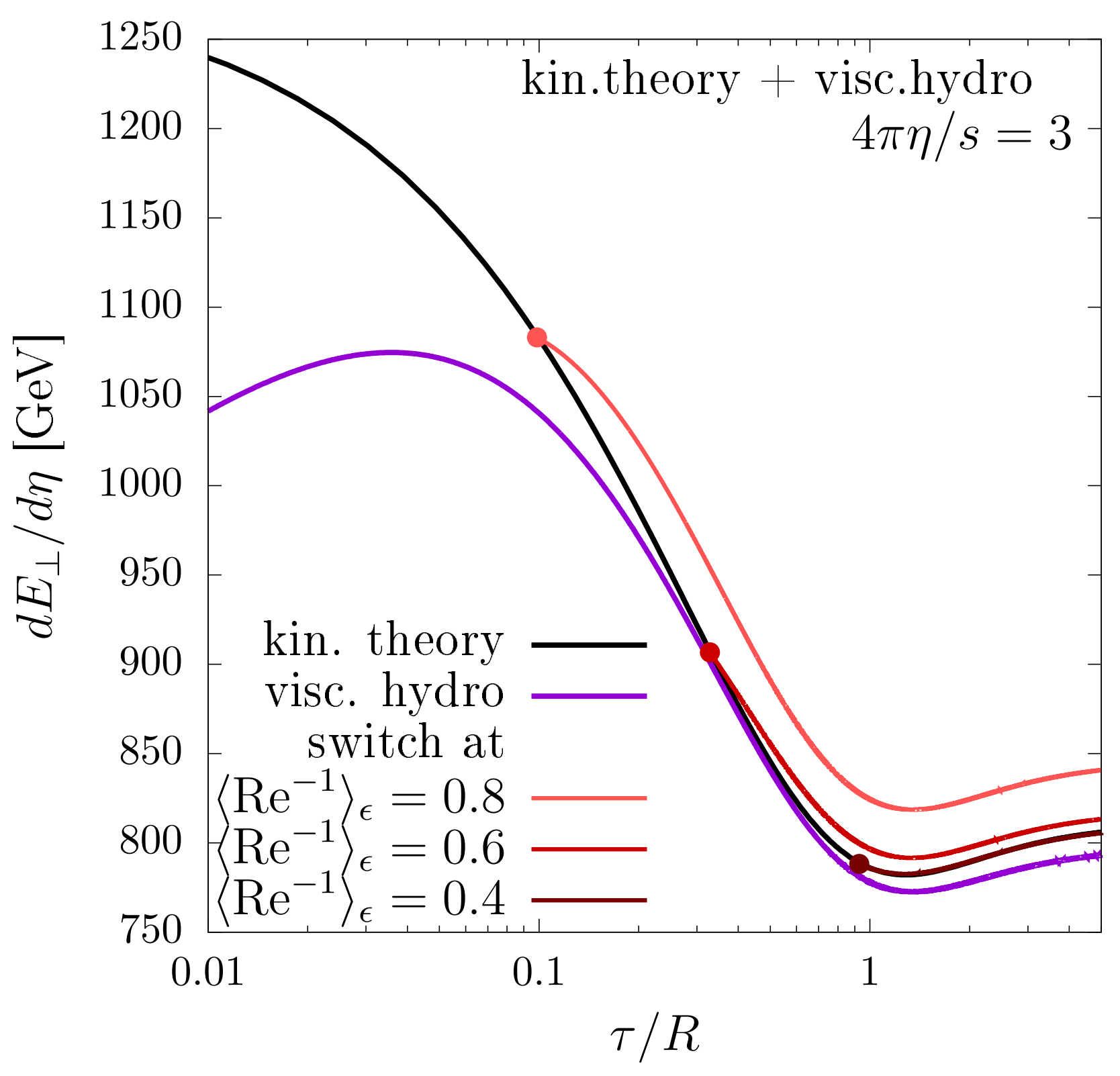
3 Equilibration & development of transverse flow



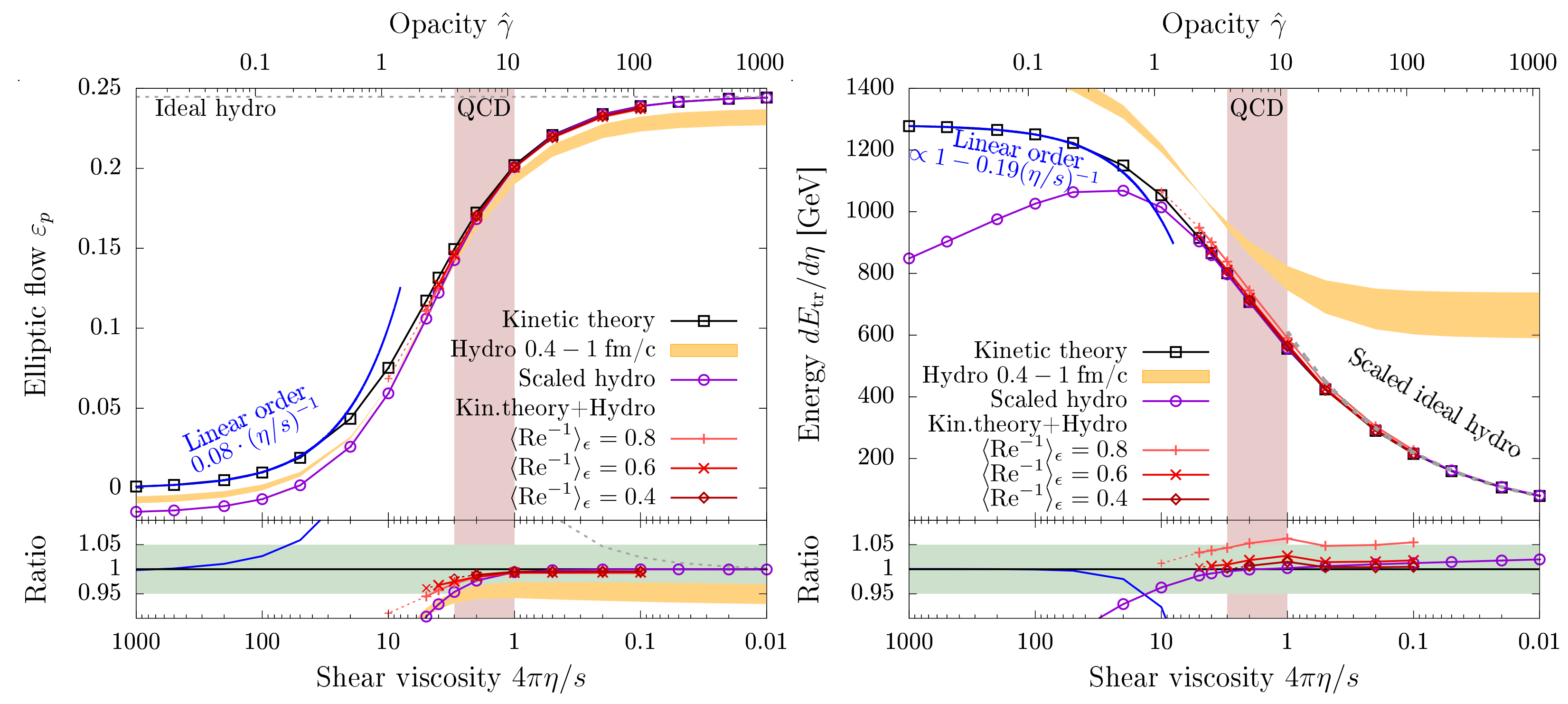
- inverse Reynolds number $\mathrm{Re}^{-1} = \left(\frac{6\pi^{\mu\nu}\pi_{\mu\nu}}{e^2}\right)^{1/2}$ quantifies relative size of non-equilibrium effects in the system
  - timescale of equilibration (⇔ drop of Re^{-1}) depends strongly on system size
  - transverse expansion ($\tau \sim R$) drives system away from equilibrium
    ⇒ small systems never fully equilibrate!
- elliptic flow ε_p builds up on similar timescales $\tau \sim R$ for all system sizes
  - ε_p continuously varies from $\varepsilon_p = 0$ (free-streaming) to a large $\hat{\gamma}$ limit of $\epsilon_p \simeq 0.25$.

4 Hydrodynamics

- not obvious how to compare hydrodynamics to kinetic theory, because the two descriptions behave differently during pre-equilibrium
  - if initialized in the same way, kinetic theory and hydro will disagree in equilibrium
  - timescale of dynamics depends on local energy density ⇒ inhomogeneous cooling
    ⇒ decrease in eccentricities by differing amounts in different descriptions [5]
- counteract pre-equilibrium differences by applying a local scaling factor to initial condition of hydrodynamics, which was calculated in Bjorken flow
  - different initial conditions, but descriptions come into agreement during equilibration
- hybrid simulations: switching from pre-equilibrium in kinetic theory to hydrodynamics for late times
  - switching times based on Re^{-1}, as it describes degree of equilibration
  - results are more accurate if system closer to equilibrium when switching



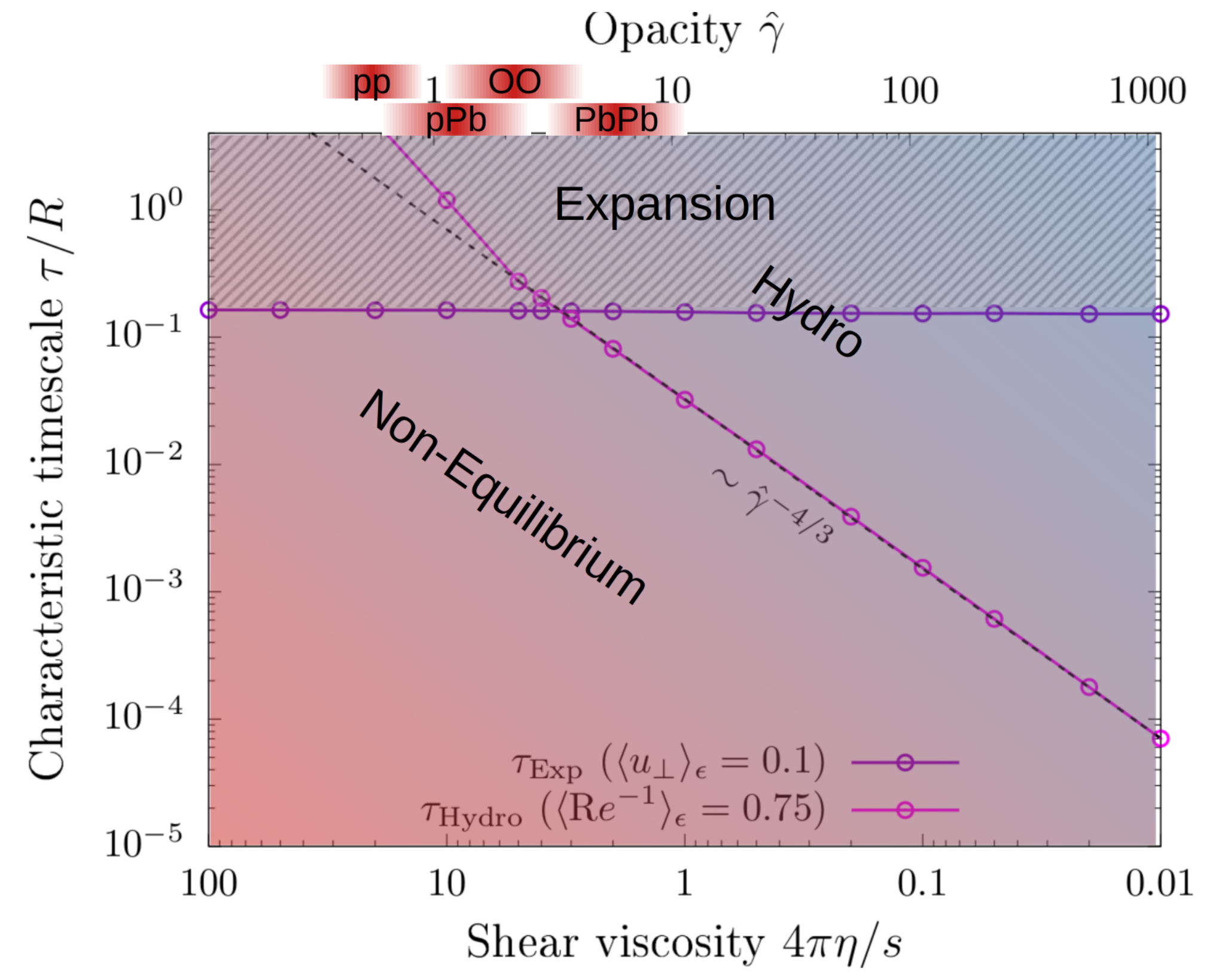
5 Comparison of final state observables



- Naive hydrodynamics initialized at $\tau_0 = 0.4 - 1\,\mathrm{fm}$ underestimates elliptic flow ε_p, vastly overestimates transverse energy $\frac{\mathrm{d}E_{\mathrm{tr}}}{\mathrm{d}\eta}$
- scaled hydro: perfect agreement with kinetic theory at large opacities
  - holds down to $\hat{\gamma} \gtrsim 4$, which includes the physically relevant regime of QCD
- hybrid results in good agreement at large $\hat{\gamma}$; improve on scaled hydro at intermediate $\hat{\gamma}$
  - as seen before: switching at smaller Re^{-1} ⇒ better agreement
  - requiring at most 5% disagreement: hydrodynamics applicable for $\langle \mathrm{Re}^{-1}\rangle_\epsilon \lesssim 0.75$.

6 Regime of applicability of hydrodynamics

- timescale of onset of transverse expansion: defined by first time transverse flow velocity reaches the value $\langle u_\perp \rangle_\epsilon = 0.1$
  - mostly independent of opacity; takes values $\tau_{\mathrm{Exp}} \sim 0.2R$
- hydrodynamization timescale defined by drop to $\langle \mathrm{Re}^{-1}\rangle_\epsilon = 0.75$
  - follows a power law before transverse expansion; depends strongly on system size
- timescale ordering reversed at $\hat{\gamma}_{\mathrm{crit}} \sim 3$ ⇒ for $\hat{\gamma} < \hat{\gamma}_{\mathrm{crit}}$, hydrodynamics applicable only for $\tau > \tau_{\mathrm{Exp}}$ (if at all); non-equilibrium description of transverse expansion is required
  - in pp, pPb, most OO collisions: $\hat{\gamma} < \hat{\gamma}_{\mathrm{crit}}$
  - in central OO collisions opacities are slightly above $\hat{\gamma}_{\mathrm{crit}}$



7 Conclusion

- applied kinetic theory to the description of transverse flow on the full range in system size
- pure hydro requires locally scaled initial condition; works only for $\hat{\gamma} \gtrsim 4$
- comparing to hybrid simulations: hydro accurate on the 5% level if $\mathrm{Re}^{-1} \lesssim 0.75$
- not applicable in pp and pPb collisions, but OO collisions cover the transition regime

References

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