NLO calculations in the CGC

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Motivation to go from LO to NLO in the CGC

CGC is the effective theory that describes the high energy scattering in the Regge-Gribov limit ($x \to 0$)

Leading Order in α_s CGC calculations:

(pro) : CGC-based theoretical calculations are in qualitative agreement with the experimental data from all types of collisions

(con): LO CGC lacks precision in order to determine unambiguously whether saturation is exhibited by the experimental data.

Need for theory predictions at NLO in α_s in order to perform precise quantitative studies.

• Life gets complicated at NLO, we need to deal with various types of divergences.

There has been a lot activity to provide expressions of observables at NLO.

eA collisions

• dipole factorization

• structure functions/ dijets

pA collisions

• hybrid factorization

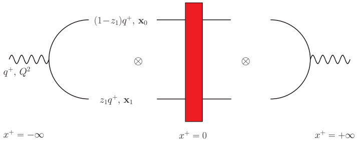
• single inclusive hadron/jet

⋆ NLO corrections to the rapidity evolution equations have been computed as well.

– NLO BK [Balitsky, Chirilli - arXiv:0710.4330 / 1309.7644]

– NLO JIMWLK [Kovner, Lublinsky, Mulian - arXiv:1310.0378] [Lublinsky, Mulian - arXiv:1610.03453]

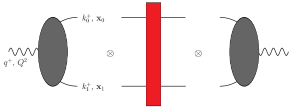
Dipole factorization at LO:

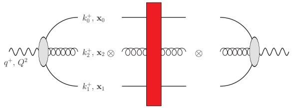


$$\boxed{\sigma^{\gamma A \to X}_{T,L}(x_{Bj}, Q^2) \propto \int_{\mathbf{x}_0, \mathbf{x}_1} \int_0^1 dz_1 \Phi^{q\bar{q}, LO}_{T,L}(\mathbf{x}_{01}, z_1, Q^2) \Big[1 - \langle s_{01} \rangle\Big]}$$

s_{01} is the dipole operator

NLO impact factor inclusive DIS with massless quarks:
[Balitsky, Chirilli - arXiv:1009.4729 /1207.3844], [Beuf - arXiv:1112.4501 /1606.00777 / 1708.06557]
[Hannien, Lappi, Paatelainen - arXiv:1711.08207]





$$\boxed{\sigma_{T,L}(x_{Bj}, Q^2) = \sum_{q\bar{q}\ st.} \left|\Psi^{\gamma^*_{T,L}}_{q\bar{q}}\right|^2 \big[1 - \langle s_{01} \rangle_0\big] + \sum_{q\bar{q}g\ st.} \left|\Psi^{\gamma^*_{T,L}}_{q\bar{q}g}\right|^2 \big[1 - \langle s_{012} \rangle_0\big]}$$

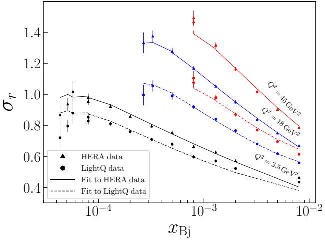
- perturbatively calculable $\Psi^{\gamma^*_{T,L}}_{q\bar{q}}$ LFWF at one lone loop, $\Psi^{\gamma^*_{T,L}}_{q\bar{q}g}$ at tree level
- UV divergences cancelled between $q\bar{q}$ and $q\bar{q}g$
- low-x resummation performed at the end

Fits to HERA data with NLO impact factor

[Beuf, Hanninen, Lappi, Matysaari - arXiv:2007.01645]

Fit of dipole amplitude on HERA data for reduced cross section

$$\boxed{\sigma_r(x_{Bj}, y, Q^2) = F_2(x_{Bj}, Q^2) - \frac{y^2}{1+(1-y)^2} F_L(x_{Bj}, Q^2)}$$



⋆ HERA data ⇒ charm and bottom quarks provide sizeable contribution to DIS on a proton.

- Massless quark calculations are not sufficient for precise quantitative predictions.

Inclusive DIS at NLO with massive quarks
[Beuf, Lappi, Paatelainen - arXiv:2103.14549 / 2112.03158 / 2204.02486]
Numerical predictions for heavy quark cross-section
[Hanninen, Matysaari, Paatelainen, Penttala - arXiv:2211.03504]

Exclusive and diffractive production in DIS at NLO

A lot of activity over the years:

• Diffractive structure functions:
[Beuf, Hanninen, Lappi, Mulian, Mantysaari - arXiv:2206.13161] $\rightarrow$ partial NLO ($q\bar{q}g$ contr.)
[Beuf, Lappi, Mantysaari, Paatelainen, Penttala - in progress] $\rightarrow$ full result

• NLO diffractive dijet production:
[Boussarie, Grabovsky, Szymanowski, Wallon - arXiv:1606.00419]

• Diffractive (hard) dijets +(soft) jet – (partial NLO analysis - enhanced NLO contributions)
[Iancu, Mueller, Triantafyllopoulos - arXiv:2112.06353],
[Iancu, Mueller, Triantafyllopoulos, Wei - arXiv:2207.06268]

• Diffractive dihadron production:
[Fucila, Grabovsky, Li, Szymanowski, Wallon - arXiv:2211.05774]

• Exclusive light vector meson production at NLO:
[Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon - arXiv:1612.08026]
[Mantysaari, Penttala - arXiv:2203.16911]

• Exclusive heavy vector meson production at NLO:
[Mantysaari, Penttala - arXiv:2104.02349]
[Mantysaari, Penttala - arXiv:2204.14031]

Inclusive production in DIS at NLO

• Single inclusive and dihadron production at NLO (using helicity methods):
[Bergabo, Jalilian-Marian - arXiv:2207.03606]
[Bergabo, Jalilian-Marian - arXiv:2210.03208]

• Real NLO corrections to dihadron production
[Iancu, Mulian - arXiv:2211.04837]

• NLO impact factor for photon+dijet production in full momentum space
[Roy, Venugopalan - arXiv:1911.04530]

Fixed order NLO DIS dijet production cross section obtained:

• DIS dijet production at NLO (using covariant perturbation theory):
[Caucal, Salazar, Venugopalan - arXiv:2108.06347]
[Caucal, Salazar, Schenke, Venugopalan - arXiv:2208.13872]
[Caucal, Salazar, Schenke, Stebel, Venugopalan - arXiv:2304.03304]

• Photoproduction of dijets at NLO ($Q^2 \to 0$) (using LCPT):
[Taels, TA, Beuf, Marquet - arXiv:2204.11650]

$\Rightarrow$ results of dijet production at NLO are consistent.

Inclusive dijet production at NLO

[Dominguez, Marquet, Xiao, Yuan - arXiv:1101.0715]

Dijet production at LO: Two typical transverse scales:

$k_t = p_1 + p_2$: total momentum
$P_T = z_2 p_1 - z_1 p_2$: relative momentum

back-to-back limit ($k_t \ll P_T$)
dipole/quadrupole operators $\rightarrow$ WW gluon TMDs

- Does one get TMD factorization from CGC calculations at NLO in the back-to-back limit?
- What about large logs of P_T/k_t (Sudakov logs)?

[Taels, TA, Beuf, Marquet - arXiv:2204.11650]

- back-to-back limit is studied.

- Sudakov double logs obtained with the wrong sign when naive low-x leading log resummation is performed.

- Correct sign is obtained if collinearly improved low-x resummation is performed.

[Caucal, Salazar, Schenke, Stebel, Venugopalan - arXiv:2304.03304]

(using the collinearly improved low-x resummation)

- first full CGC calculation of dijets that shows TMD factorization at NLO

- impact factor = Soft factor $\otimes$ Coeff. function

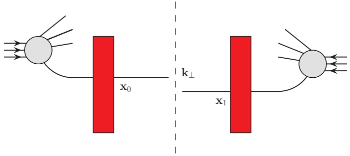
where Soft factor resums both double and single Sudakov logs!

Forward hadron production in pA collisions

[Dumitru, Hayashigaki, Jalilian-Marian - hep-ph/0506308]

State-of-the-art calculation framework for forward production in pA collisions: Hybrid factorization

- The wave function of the projectile proton is treated in the spirit of collinear factorization (an assembly of partons with zero intrinsic transverse momenta).
- Perturbative corrections to this wave function are provided by the usual QCD perturbative splitting processes.
- Target is treated as distribution of strong color fields which during the scattering event transfer transverse momentum to the propagating partonic configuration. (CGC like treatment)



$$\frac{d\sigma^{q\to H}}{d^2k\, d\eta} = \int_{x_F}^{1} \frac{d\zeta}{\zeta^2} D^q_{\mu_0^2}(\zeta) \frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right) \int e^{ik(x_0-x_1)} \langle s(x_0,x_1)\rangle$$

high transverse momentum in the produced hadron is acquired from the interaction with the target.

Does LO "Hybrid" formula take into account all contributions at high $k_\perp$?

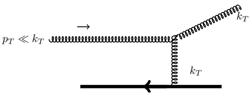
[TA, Kovner - arXiv:1102.5327]

For $k_\perp \gg Q_s$:

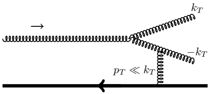
$$\frac{d\sigma}{d^2kd\eta} \propto \left[\frac{d\sigma}{d^2kd\eta}\right]_{el.} + \left[\frac{d\sigma}{d^2kd\eta}\right]_{inel.}$$

Real contributions at NLO.

"Elastic Scattering" (LO)



"Inelastic Scattering" (NLO)

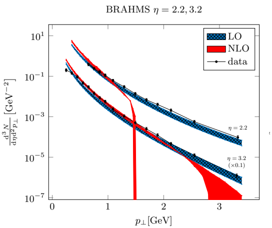


[Chirilli, Xiao, Yuan - arXiv:1112.1061 / arXiv:1203.6139] $\to$ Full NLO computation.

Collinear divergences: absorbed into DGLAP evolution of PDFs and FFs.

Rapidity divergences: absorbed into evolution of the target.

[Stasto, Xiao, Zaslavsky - arXiv:1307.4057] $\to$ Numerical studies of full NLO result.



cross sections turn out to be negative at large transverse momentum!

Several solutions proposed to fix the problem:

- kinematical constraints
- different choice of rapidity scales
- threshold/ Sudakov resummations

Revisiting NLO hybrid formula - kinematical constraints

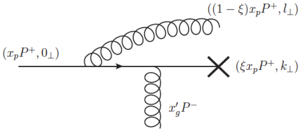
[TA, Armesto, Beuf, Kovner, Lublinsky - arXiv:1411.2869]

- choice of frame:

(i) target moves fast and carries almost all the energy.
(ii) projectile moves fast enough: accommodates partons with momentum fraction x_p but does not develop a large low-x tail.
(iii) target is evolved to s from initial s_0 via BK.

- Ioffe time restriction: only pairs whose coherence time is greater than the propagation time through the target can be resolved.

$$\boxed{\text{coherent scattering} \rightarrow \frac{(1-\xi)\xi x_p}{l_\perp^2} > \frac{1}{s_0}}$$



New BK-like terms arise due to Ioffe time restriction.

[Watanabe, Xiao, Yuan, Zaslavsky - arXiv:1505.05183] $\rightarrow$ exact kinematical constraint.

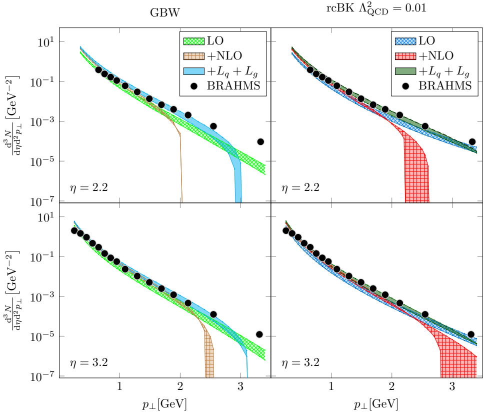
$$\boxed{\int_0^{1-\frac{l_\perp^2}{x_p s}} \frac{d\xi}{(1-\xi)} = \ln\frac{1}{x_g} + \ln\frac{k_\perp^2}{l_\perp^2}}$$

New terms $(L_q + L_g)$ arise from $l_\perp^2 < (1-\xi)k_\perp^2$

The new terms in both works are consistent and equivalent.

Revisiting NLO hybrid formula - kinematical constraints

[Watanabe, Xiao, Yuan, Zaslavsky - arXiv:1505.05183]



BRAHMS data with $\sqrt{s_{NN}} = 200$ *GeV.*

The negativity problem is shifted to higher transverse momentum but not cured!

Revisiting NLO hybrid formula

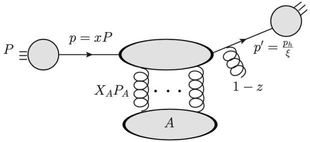
[Liu, Ma, Chao - arXiv:1909.02370]

- a new method to regularize rapidity divergence in the region $\xi \to 1$.

$$\boxed{(1-\xi)^{-1+\eta} = \frac{\delta(1-\xi)}{\eta} + \frac{1}{(1-\eta)_+} + O(\eta)}$$

[Kang, Liu - arXiv:1910.10166], [Liu, Kang, Liu - arXiv:2004.11990]

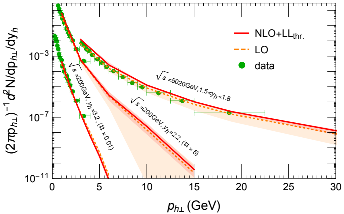
Soft-Collinear Effective Theory (SCET), threshold resummation



$$\boxed{\sigma^{(n)} \propto \sum_{k=0}^{n-1} \left(\frac{\ln^k(1-z)}{1-z} \right)_+}$$

$1-z$ is the energy fraction carried by the soft radiation.

In the forward region $z \to 1$ very quickly $\Rightarrow$ logs need to be resummed.



charged hadron production $p + Pb$ at LHC and hadron productions at $d + Au$ at RHIC.

Revisiting NLO hybrid formula

[Xiao, Yuan - arXiv:1806.0352], [Shi, Wang, Wei, Xiao - arXiv:2112.06975]

- extra logs from the kinematical constraint written in coordinate space

$$\boxed{\ln\frac{k_\perp^2}{\mu_r^2}\quad,\quad \ln\frac{\mu^2}{\mu_r^2}\quad,\quad \ln^2\frac{k_\perp^2}{\mu_r^2}}$$

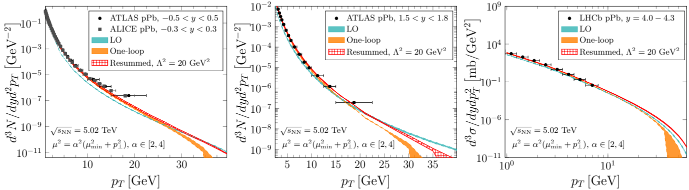
with $\mu_r = 2e^{-\gamma_E}/r_\perp$. In the threshold region ($k_\perp$ or $p_\perp \gg \mu_r$) logs become large and needs to be resummed.

- rewritten in momentum space

$$\boxed{\ln\frac{k_\perp^2}{\Lambda^2} + I_1(\Lambda)\quad,\quad \ln\frac{\mu^2}{\Lambda^2} + I_1(\Lambda)\quad,\quad \ln^2\frac{k_\perp^2}{\Lambda^2} + I_2(\Lambda)}$$

Λ is an auxilary scale in momentum space , $\Lambda \gg \Lambda_{QCD}$

- soft gluon emission $\rightarrow \ln\frac{k_\perp^2}{\Lambda^2}$ and $\ln^2\frac{k_\perp^2}{\Lambda^2}$ resummed into Sudakov factor
- collinear logs $\rightarrow \ln\frac{\mu^2}{\Lambda^2} \rightarrow$ threshold resummation (DGLAP of PDFs and FFs)



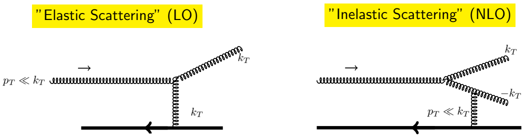
A new approach to forward pA scatterings

Common assumption in all these works: *large logs can be resummed within the collinear factorization.*

[TA, Armesto, Beuf, Kovner, Lublinsky - arXiv:2307.xxxxx]

TMD-factorized framework is a natural choice to resum all large logs.

in [arXiv:1102.5327], the mechanisms that give rise to high transverse momentum hadrons:



– It is more natural to think the inelastic contribution in the TMD framework:
produced high k_T quark coming directly from quark TMD PDF.

$\star$ another potential source to producing high transverse momentum hadron:

low k_T parton scatters softly, but subsequently fragments into a high transverse momentum hadron.

-*Hadron arising from TMD FF.*

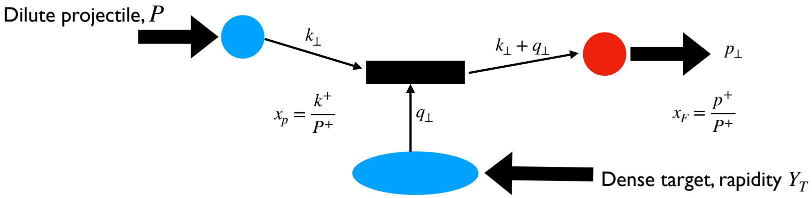
$\star$ soft logs – we follow [arXiv:1411.2869]

The setup of the problem

[TA, Armesto, Beuf, Kovner, Lublinsky - arXiv:2307.XXXXX]

TMD-factorized parton model expression:

$$\boxed{\frac{d\sigma^{LO+NLO}}{d^2p_\perp d\eta} \propto \int \frac{d\zeta}{\zeta^2}\int_{k_\perp q_\perp} \mathcal{T}(x_F/\zeta, k_\perp; \mu_T^2) P(k_\perp, q_\perp) \mathcal{F}(\zeta, p_\perp, (k_\perp + q_\perp); \mu_F^2) + \text{Gen. NLO}}$$



$\mathcal{T}(x_F/\zeta, k_\perp; \mu_T^2) \to$ initial TMD PDF $\qquad$ $\mathcal{F}(\zeta, p_\perp, (k_\perp + q_\perp)) \to$ TMD FF

$P(k_\perp, q_\perp) \to$ differential probability to produce a parton with momentum $(k_\perp + q_\perp)$ from a parton with momentum $k_\perp$

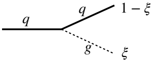
The factorization scales:

$$\mu_T^2 = \max\left\{k_\perp^2, q_\perp^2, Q_s^2\right\} \approx \max\left\{(k_\perp + q_\perp)^2, Q_s^2\right\}, \; \mu_F^2 = \left((q_\perp + k_\perp) - p_\perp/\zeta)^2\right) \approx \max\left\{(q_\perp + k_\perp)^2, (p_\perp/\zeta)^2\right\}$$

TMD distributions

- TMD PDFs are generated from the collinear ones (large k)

$$\boxed{x\mathcal{T}_q(x,k^2,k^2;\xi_0) = \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^{1} d\xi \frac{1+(1-\xi)^2}{\xi}\frac{x}{1-\xi} f^q_{k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2}}$$



– The soft divergence of the gluon emission is regulated by the cut off ξ_0.
– Partons with high longitudinal momentum are produced from partons with lower longitudinal momentum by DGLAP splitting.
– Transverse resolution scale in these splittings is equal to the transverse momentum of the parton.

- Evolution of the TMDs

$$\boxed{x\mathcal{T}_q(x,k^2;\mu^2;\xi_0) = \theta(\mu^2-k^2)\Big[x\mathcal{T}_q(x,k^2;k^2;\xi_0) - \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^{1} d\xi \frac{1+(1-\xi)}{\xi} x\mathcal{T}_q(x,k^2;l^2;\xi_0)\Big]}$$

Increasing the transverse resolution $\Rightarrow$ number of q at a fixed transverse momentum decreases due to DGLAP splittings into qg pair with higher long. momentum given by the resolution scale.

With these definitions, collinear q-PDF andf TMD q-PDF are related via

$$\boxed{xf^q_{\mu^2}(x) = \int_0^{\mu^2}\pi dk^2 x\mathcal{T}_q(x,k^2;\mu^2;\xi_0)}$$

And it satisfies the DGLAP evolution equations...

Forward pA - quark channel

– start from the expressions obtained in LCPT (with Ioffe time restriction) in [arXiv:1411.2869] (no collinear subtraction and no + prescription)

– projectile contains quarks with transverse momentum smaller than μ_0, target sits at some rapidity with no need of further evolution.

– assumptions: large N_c, factorization of the dipoles, and translationally invariant dipoles.

After Including the fragmentation and FT to momentum space:

$$\frac{d\sigma^{q\to q\to H}}{d^2p d\eta} = \frac{d\sigma_0^{q\to q\to H}}{d^2p d\eta} + \frac{d\sigma_{1,\mathrm{r}}^{q\to q\to H}}{d^2p d\eta} + \frac{d\sigma_{1\mathrm{v.}}^{q\to q\to H}}{d^2p d\eta}$$

LO term

$$\boxed{\frac{d\sigma_0^{q\to q\to H}}{d^2p d\eta} = S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{\mu_0^2}(\zeta) \frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right) s(p/\zeta)}$$

$$\frac{d\sigma}{d^2p\, d\eta}\bigg|^{\mathrm{q\to q}}_{\mathrm{NLO,r}} = \frac{g^2}{(2\pi)^3} S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{\mu_0^2}(\zeta) \int_{k^2,q^2>\mu_0^2} \int_{\xi_0} d\xi \frac{x_F}{\zeta(1-\xi)} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta(1-\xi)}\right) \frac{N_c}{2}\left[\frac{1+(1-\xi)^2}{\xi}\right] s(k)s(q)$$
$$\times \left\{\frac{1}{2}\frac{(q-k)^2}{(p/\zeta-k)^2(p/\zeta-q)^2} + \frac{1}{2}\frac{(1-\xi)^2(q-k)^2}{[p/\zeta-(1-\xi)k]^2[p/\zeta-(1-\xi)q]^2}\right\} + (\mathrm{Gen.\,NLO})_1$$

NLO real terms

The first term can be cast into

$$\boxed{\frac{1}{2}\int_{k,q} s(k)s(q)\frac{(q-k)^2}{(p/\zeta-k)^2(p/\zeta-q)^2} = \int_{k,q}\frac{1}{k^2}s(-k+p/\zeta)\left[1-\frac{k\cdot q}{q^2}\right]s(-q+p/\zeta)}$$

Second term (after rescaling $\zeta(1-\xi)\to\zeta'$) can be acts into the same form.
Using the definition of TMD PDF (analogously TMD FF)

$$\boxed{x\mathcal{T}_q(x,k^2,k^2;\xi_0) = \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^{1} d\xi\frac{1+(1-\xi)^2}{\xi}\frac{x}{1-\xi}f^q_{k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2}}$$

The real contribution reads

$$\frac{d\sigma^{q\to q\to H}_{1,\mathrm{r}}}{d^2p d\eta} = S_\perp\int_{x_F}^{1}\frac{d\zeta}{\zeta^2}\int_{k^2>\mu_0^2}\frac{x_F}{\zeta}\left\{D^q_{\mu_0^2}(\zeta)\mathcal{T}_q\left(\frac{x_F}{\zeta},k^2;k^2,\xi_0=\frac{k^2\zeta}{x_F s_0}\right)+f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\mathcal{F}^q\left(\zeta,k^2;k^2,\xi_0=\frac{k^2\zeta}{x_F s_0}\right)\right\}$$
$$\times\int_q s(-k+p/\zeta)\left[1-\frac{k\cdot q}{q^2}\right]s(-q+p/\zeta)+(\mathrm{Gen.\,NLO})$$

– incoming quark with mom. k, scatters with mom exchange $-k+p/\zeta$, outgoing quark with mom. p/ζ collinearly fragments into a hadron with mom. p.

– (shift $k\to -q+p/\zeta$ and $q\to -k+p/\zeta$) incoming quark with vanishing mom., scatters with mom. transfer q, first perturbatively fragments into a quark with mom p/ζ, which then fragments into a hadron with momentum p.

NLO virtual contributions

Starting from the expressions in [arXiv:1411.2869], adopting the same assumptions:

$$\frac{d\sigma_{1,v}^{q\to q\to H}}{d^2p d\eta} = -2\frac{g^2}{(2\pi)^3}S_\perp \frac{N_c}{2}\int_{x_F}^{1}\frac{d\zeta}{\zeta^2}\,D^q_{H,\mu_0^2}(\zeta)\int_{k^2>\mu_0^2}\int_{k^2\zeta/(x_F s_0)}^{1} d\xi\,\frac{x_F}{\zeta}\,f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\frac{1+(1-\xi)^2}{\xi}$$
$$\times\int_q s\left(\frac{p}{\zeta}\right)s(q)\left\{\left[\frac{\frac{p}{\zeta}-q-k}{(\frac{p}{\zeta}-q-k)^2}\frac{k}{k^2}+\frac{1}{k^2}\right]+\left[\frac{\frac{p}{\zeta}(1-\xi)-q-k}{(\frac{p}{\zeta}(1-\xi)-q-k)^2}-\frac{\frac{p}{\zeta}-q-k}{(\frac{p}{\zeta}-q-k)^2}\right]\frac{k}{k^2}\right\}$$

- incoming q $\rightarrow$ qg pair, pair scatters, recombines into q.
- NLO corr. to LO elastic q scattering.

- qg loop that appears either before or after the scattering.
- "proper" virtual diagram

Does not contain any large logs (a Gen. NLO correction)

in the first term one can perform the angular integration over the angle of vector k:

$$\int_{\mu_0^2} d^2k\left[\frac{\frac{p}{\zeta}-q-k}{(\frac{p}{\zeta}-q-k)^2}\frac{k}{k^2}+\frac{1}{k^2}\right]=\int_{\mu_0^2}^{(q-\frac{1}{\zeta}p)^2}\frac{d^2k}{k^2}$$

NLO virtual contributions

The virtual NLO contribution can be split into two intervals

$$-2\frac{g^2}{(2\pi)^3}S_\perp\frac{N_c}{2}\int_{x_F}^1\frac{d\zeta}{\zeta^2}\,D^q_{H,\mu_0^2}(\zeta)\int_q\left[\int_{\mu_0^2}^{\mu^2}+\int_{\mu^2}^{(q-\frac{1}{\zeta}p)^2}\right]\frac{d^2k}{k^2}\int_{k^2\zeta/(x_F s_0)}^1 d\xi\,\frac{x_F}{\zeta}\,f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\,\frac{1+(1-\xi)^2}{\xi}s\left(\frac{p}{\zeta}\right)\,s(q)$$

- the first term combines with LO to evolve the resolution scale of the TMD to μ^2.
- contribution from the pairs of the transverse size close to the resolution scale.

(no large logs & Gen. NLO correction)

LO + NLO virtual:

$$S_\perp\int_{x_F}^1\frac{d\zeta}{\zeta^2}\,D^q_{H,\mu_0^2}(\zeta)\,\frac{x_F}{\zeta}f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)s\left(\frac{p}{\zeta}\right)$$
$$-2\frac{g^2}{(2\pi)^3}S_\perp\frac{N_c}{2}\int_{x_F}^1\frac{d\zeta}{\zeta^2}\,D^q_{H,\mu_0^2}(\zeta)\int_q\int_{\mu_0^2}^{\mu^2}\frac{d^2k}{k^2}\int_{k^2\zeta/(x_F s_0)}^1 d\xi\,\frac{x_F}{\zeta}\,f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\,\frac{1+(1-\xi)^2}{\xi}s\left(\frac{p}{\zeta}\right)$$
$$=S_\perp\int_{x_F}^1\frac{d\zeta}{\zeta^2}\int_0^{\mu_0^2}d^2k\left[D^q_{H,\mu_0^2}(\zeta)\,\frac{x_F}{\zeta}\mathcal{T}_q\left(\frac{x_F}{\zeta},k^2;\mu^2;\xi_0=\frac{\zeta\mu^2}{x_F s_0}\right)+\mathcal{F}^q_H\left(\zeta,k^2;\mu^2;\xi_0=\frac{\zeta\mu^2}{x_F s_0}\right)\,\frac{x_F}{\zeta}f^q_{\mu_0^2}(\frac{x_F}{\zeta})\right]s\left(\frac{p}{\zeta}\right)$$

NLO virtual contributions

- evolve the factorization scale in the collinear PDFs and FFs up to μ^2

$$D^q_{H,\mu_0^2}(\zeta) \to \int_0^{\mu_0^2} d^2l \mathcal{F}^q_H\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right); \quad f^q_{\mu_0^2}(\frac{x_F}{\zeta}) \to \int_0^{\mu_0^2} d^2k \mathcal{T}_q\left(\frac{x_F}{\zeta}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right)$$

this introduces the term at $O(\alpha_s^2)$ therefore legitimate in our $O(\alpha_s)$ calculation.

- alter the scattering amplitude

$$s(\frac{p}{\zeta}) \to \int_q s\left(-(k+l)+\frac{p}{\zeta}\right)\left[1-\frac{(k+l)\cdot q}{q^2}\right] s\left(-q+\frac{p}{\zeta}\right)$$

($|k+l|^2 \lesssim \mu_0^2 \ll p^2/\zeta^2$ & $q^2 \sim \max(Q_s^2, p^2/\zeta^2)$ & $\int_q s(q) = 1$) $\Rightarrow$ this modification only adds subleading power corrections of the order μ_0^2/Q_s^2

LO + NLO virtual:

$$S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2}\int_q \int_0^{\mu_0^2} d^2l \int_0^{\mu_0^2} d^2k$$
$$\times \mathcal{F}^q_H\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right)\frac{x_F}{\zeta}\mathcal{T}_q\left(\frac{x_F}{\zeta}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) s\left(-(k+l)+\frac{p}{\zeta}\right)\left[1-\frac{(k+l)\cdot q}{q^2}\right] s\left(-q+\frac{p}{\zeta}\right)$$

LO+NLO virtual+NLO real: Final TMD factorized expression

$$S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2}\int_q \int d^2l \int d^2k\, \mathcal{F}^q_H\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right)\frac{x_F}{\zeta}\mathcal{T}_q\left(\frac{x_F}{\zeta}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right)$$
$$\times s\left(-(k+l)+\frac{p}{\zeta}\right)\left[1-\frac{(k+l)\cdot q}{q^2}\right] s\left(-q+\frac{p}{\zeta}\right) + (\mathrm{Gen.\,NLO})$$

Summary

• We have covered the recent developments in the NLO calculations in the CGC both for *eA* and *pA* collisions.

• The progress continues in order to provide full NLO results which will provide the necessary precision for quantitive studies to determine whether saturation is exhibited by experimental data.

• A new approach to forward pA scatterings is discussed.

Apologies from the people whose work have not been presented due to constraint on time!