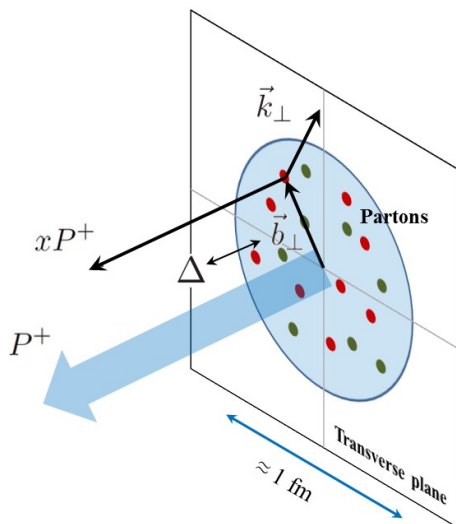


Developments in TMDs, GPDs, Wigner functions

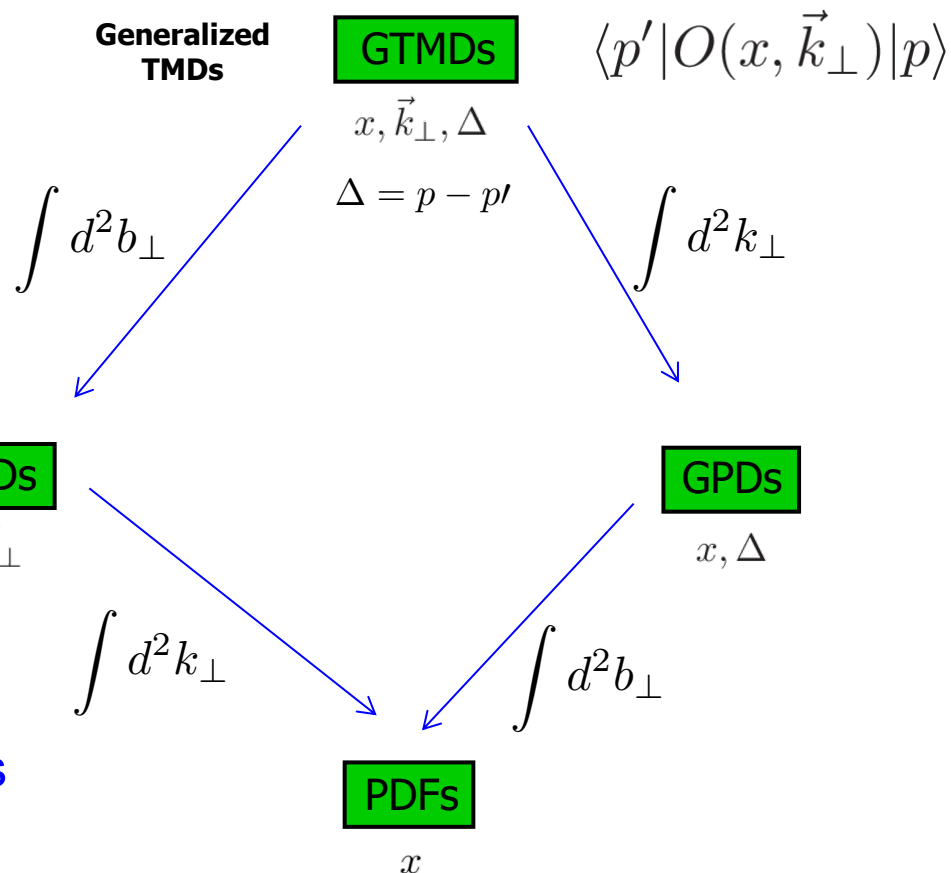
Cyrille Marquet

Centre de Physique Théorique
Ecole Polytechnique & CNRS

Contents



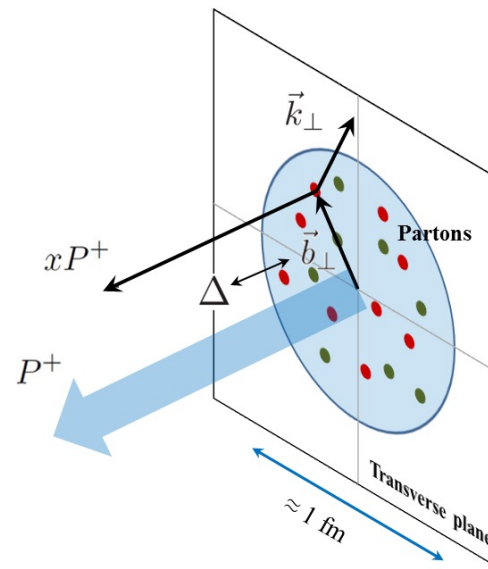
- 0) PDFs
- 1) TMDs
- 2) GDPs
- 3) GTMDs/Wigner functions
- 4) 2-particle distributions



(0+1)D: PDFs, nuclear PDFs

PDFs = Parton Distribution Functions

$$\int d^2 b_{\perp} \int d^2 k_{\perp}$$

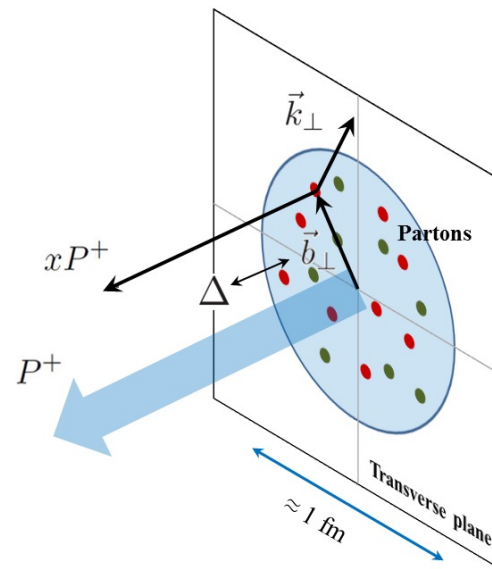


not covered in this talk, see P. Zurita on Friday

(0+3)D: TMDs

TMDs = Transverse Momentum Dependent pdfs

$$\int d^2b_{\perp}$$



Spin physics and TMDs

TMDs are crucial to describe hard processes in polarized collisions
(e.g. Drell-Yan and semi-inclusive DIS)

8 leading-twist TMDs







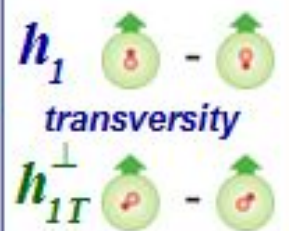
Sivers function

correlation between transverse spin of the nucleon and transverse momentum of the quark

Boer-Mulders function

correlation between transverse spin and transverse momentum of the quark in unpolarized nucleon

nucleon polarization

	U	L	T
U	f_1 number density q 		f_{1T}^\perp Sivers 
L		g_1 helicity Δq 	g_{1T} 
T	h_1^\perp Boer Mulders 	h_{1L}^\perp 	h_1 transversity h_{1T}^\perp 

quark polarization

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quark polarization

I will only discuss unpolarized beams

I will only discuss gluon TMDs, which are more important for the initial stages of HIC

Gluon TMDs and gauge links

- the operator definition:

gauge link $\mathcal{P} \exp \left[-ig \int_{\alpha}^{\beta} d\eta^{\mu} A^a(\eta) T^a \right]$

$$\mathcal{F}_{g/h}(x, k_{\perp}) = 2 \int \frac{d\xi^{-} d^2 \boldsymbol{\xi}_t}{(2\pi)^3 p_h^{+}} e^{i x p_h^{+} \xi^{-} - i k_{\perp} \cdot \boldsymbol{\xi}_t} \langle h | \text{Tr} [F^{+i}(\xi^{-}, \boldsymbol{\xi}_t) U_{[\xi, 0]} F^{+i}(0)] | h \rangle$$

gauge link $U_{[\alpha, \beta]}$ renders gluon TMD gauge invariant

different processes require a different gauge-link structure,
implying in turn different gluon TMDs

Gluon TMDs and gauge links

- the operator definition:

Dominguez, CM, Xiao and Yuan (2011)

$$\mathcal{F}_{g/h}(x, k_{\perp}) = 2 \int \frac{d\xi^- d^2 \xi_t}{(2\pi)^3 p_h^+} e^{i x p_h^+ \xi^- - i k_{\perp} \cdot \xi_t} \langle h | \text{Tr} [F^{+i}(\xi^-, \xi_t) U_{[\xi, 0]} F^{+i}(0)] | h \rangle$$

several paths are possible for the gauge links

examples :



- in the large k_t limit, the process dependence of the gauge links disappears (like for the integrated gluon distribution), and a single gluon distribution is sufficient

Gluon TMDs and gauge links

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several paths are possible for the gauge links

examples :



- in the large k_t limit, the process dependence of the gauge links disappears (like for the integrated gluon distribution), and a single gluon distribution is sufficient

in the small-x limit, one can translate these formal operator definitions into a language that is more familiar to the heavy-ion community

Gluon TMDs at small x

- in the small-x limit, all the gluon TMDs share a universal perturbative tail Kotko, Kutak, CM, Petreska, Sapeta, van Hameren (2015)

$$\mathcal{F}_{g/h}(x, k_{\perp}) = UGD(x, k_{\perp}) + \mathcal{O}(Q_s^2/k_{\perp}^2)$$



so-called unintegrated gluon distribution

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so-called unintegrated gluon distribution

- this is also the case for the polarized gluon TMDs:

$$2 \int \frac{d\xi^- d^2 \xi_t}{(2\pi)^3 p_h^+} e^{ixp_h^+ \xi^- - ik_{\perp} \cdot \xi_t} \langle h | \text{Tr} [F^{+i}(\xi^-, \xi_t) U_{[\xi, 0]} F^{+j}(0)] | h \rangle$$

$$= \frac{\delta_{ij}}{2} \mathcal{F}(x, k_{\perp}) + \left(\frac{k_i k_j}{k_{\perp}^2} - \frac{\delta_{ij}}{2} \right) \mathcal{H}(x, k_{\perp})$$



unpolarized gluon TMD



linearly-polarized gluon TMD

at small x, $\mathcal{F} = \mathcal{H}$ if one ignores saturation effects (linear regime):

$$\mathcal{F}_{g/h}(x, k_{\perp}) - \mathcal{H}_{g/h}(x, k_{\perp}) = \mathcal{O}(Q_s^2/k_{\perp}^2)$$

Gluon TMDs in the CGC

- one can compute the gluon TMDs at small x , using the Color Glass Condensate effective theory

$$\frac{\langle h | \cdot | h \rangle}{\langle h | h \rangle} \rightarrow \langle \cdot \rangle_x = \int DA^+ |\phi_x[A^+]|^2 .$$

which describes of the dense parton content of the hadron/nucleus wave function, in terms of the large gluon field A^+

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- then, the evolution with decreasing x can be computed from the so-called JIMWLK equation

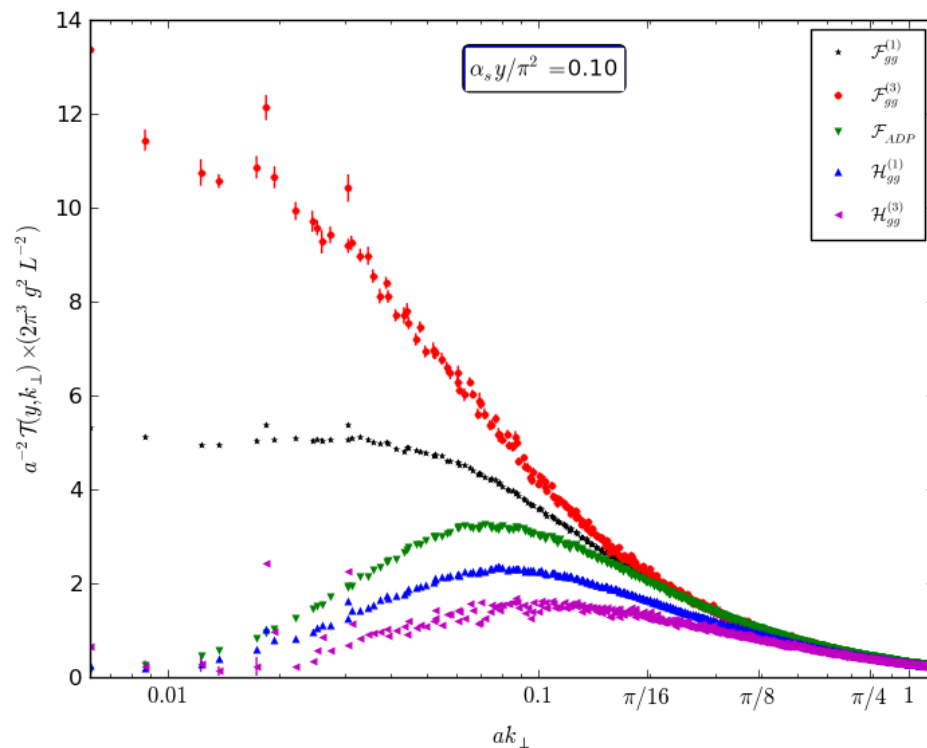
$$\frac{d}{d \ln(1/x)} \langle O \rangle_x = \langle H_{JIMWLK} O \rangle_x$$

Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

JIMWLK numerical results

initial condition at $y=0$: MV model
evolution: JIMWLK at leading log

CM, Petreska, Roiesnel (2016)
CM, Roiesnel, Tael (2017)



saturation effects impact the various gluon TMDs in very different ways

TMDs in heavy-ion collisions

the CGC provides a framework which can translate (some of the) hadron structure lore into a language that is more familiar to the heavy-ion community

in particular, the phenomenology of the early times dynamics
(energy density, $dN/d\eta$, ...) can be rephrased in terms TMD distributions

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goal: realistic model of the nuclear color charge distributions whose fluctuations are connected to TMDs:

2D spatial structure
imposed by MC Glauber

$$\langle \rho(\mathbf{x}) \rho(\mathbf{y}) \rangle_x = F \left(\underbrace{x, \mathbf{x} - \mathbf{y} \leftrightarrow k_\perp}_{\text{3D momentum structure related to } x \text{ and } k_t \text{ dependence of TMDs}}, \overbrace{\mathbf{b} = \frac{\mathbf{x} + \mathbf{y}}{2}}^{\text{2D spatial structure imposed by MC Glauber}} \right)$$

3D momentum structure related
to x and k_t dependence of TMDs

it started with IP Glasma and the rcBK Monte Carlo, still progressing towards this goal

Schenke, Tribedy and Venugopalan (2012)

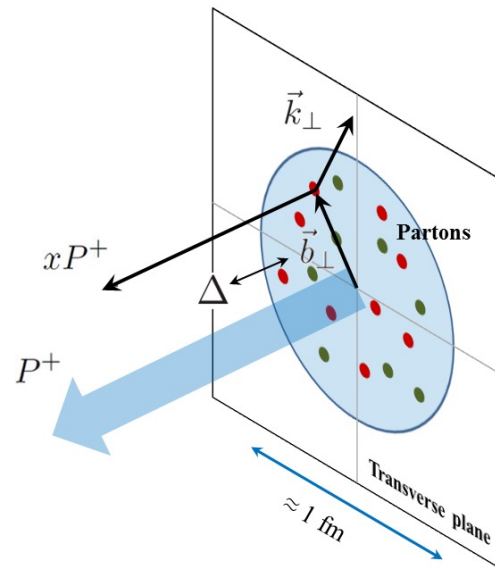
Albacete, Dumitru, Fujii and Nara (2013)

Schlichting and Singh (2021)

(3+1)D: GPDs

GPDs = Generalized Parton Distributions

$$\int d^2 k_{\perp}$$



$$\Delta = P - P' \quad \left\{ \begin{array}{l} \Delta_{\perp} \leftrightarrow b_{\perp} \\ \xi = \frac{P^+ - P'^+}{P^+ + P'^+} \end{array} \right.$$

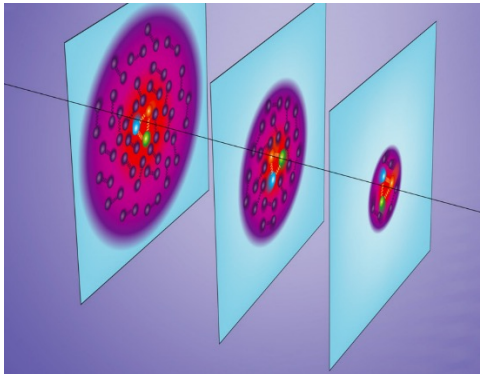
GPDs and transverse imaging

- accessible in exclusive processes

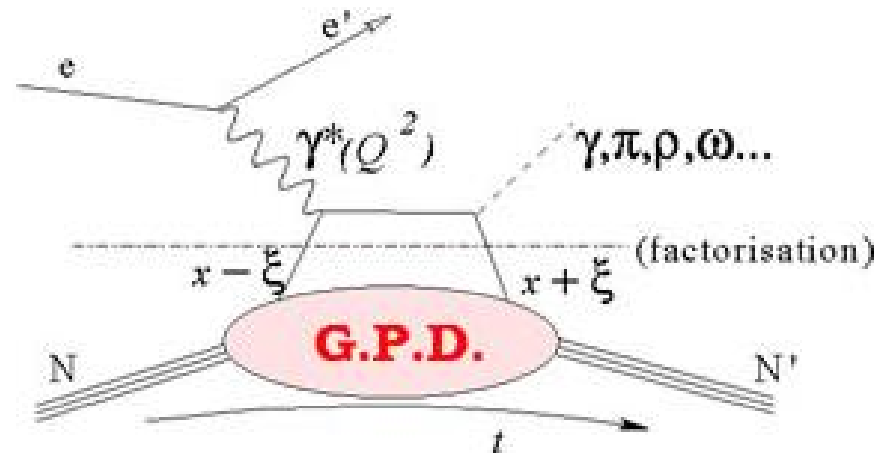
Diehl (review in 2003)

~ FTs of impact-parameter dependent pdfs

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H(x, \xi = 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$



deeply-virtual Compton scattering (DVCS)



- main open questions:

- deconvolution problem to extract GPDs from Compton form factors
- NLO and power corrections, access to transversity GPDs
- nuclear GPDs (exclusive vs dissociative process, light vs heavy nuclei, ...)

GPDs at small x

- gluon GPDs most important

$$xH_{g/h}(x, \Delta) = 2 \int \frac{d\xi^-}{2\pi p_h^+} e^{ixp_h^+ \xi^-} \langle p + \Delta | \text{Tr} [F^{+i}(\xi^-) U_{[\xi^-, 0]} F^{+i}(0)] | p \rangle$$

- contains non-flip (H) and helicity-flip (E) GPDs
- even more structure with transverse Lorentz index $i \neq j$

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$$xH_{g/h}(x, \Delta) = 2 \int \frac{d\xi^-}{2\pi p_h^+} e^{ixp_h^+ \xi^-} \langle p + \Delta | \text{Tr} [F^{+i}(\xi^-) U_{[\xi^-, 0]} F^{+i}(0)] | p \rangle$$

- contains non-flip (H) and helicity-flip (E) GPDs
- even more structure with transverse Lorentz index $i \neq j$

- GPDs in the CGC :

Hatta, Xiao and Yuan (2017)

can be addressed in the $\xi = x \rightarrow 0$ limit,

with the additional assumption that the non-forward matrix element can be ignored:

$$\frac{\langle p + \Delta | \cdot | p \rangle}{\langle p | p \rangle} \rightarrow \langle \cdot \rangle_x = \int DA^+ |\phi_x[A^+]|^2 .$$

then, GPDs naturally emerge as moments to Generalized TMDs/Wigner functions:

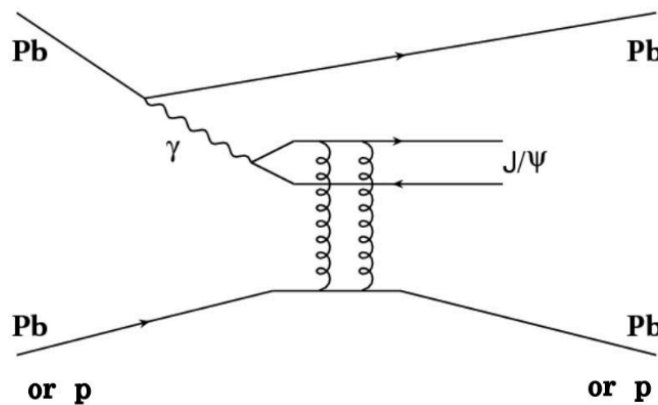
$$xH_{g/h}(x, \Delta_\perp) \propto \int d^2 k_\perp k_\perp^2 \times \text{GTMD}(x, k_\perp, \Delta_\perp)$$

GPDs in heavy-ion collisions

GPDs should be discussed in the context ultra-peripheral collisions (UPCs)

however, they are rarely mentioned in motivation slides:

Photoproduction at high-energy hadron colliders



Heavy-ion collision:

large flux of quasi-real photons

Enables photoproduction studies

in analogy to deep-inelastic scattering facilities including nuclei as target

Focus on J/ψ production:

Hard scale amenable to perturbative QCD, high precision data

- Some results also on Upsilon (LHCb, pp, and CMS, pA)

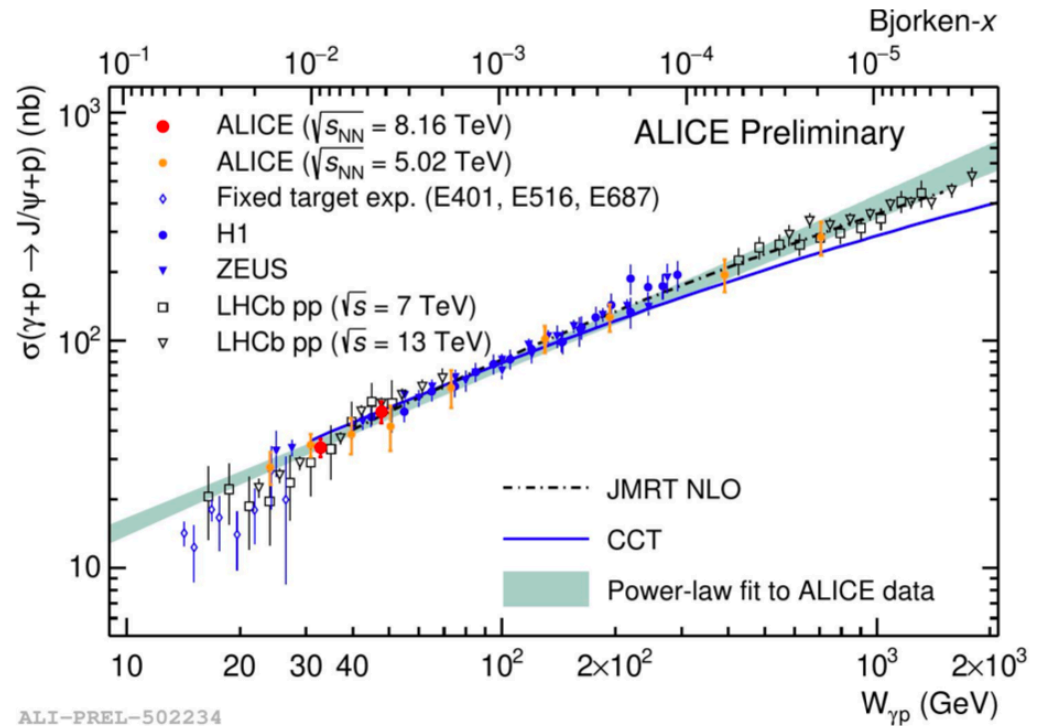
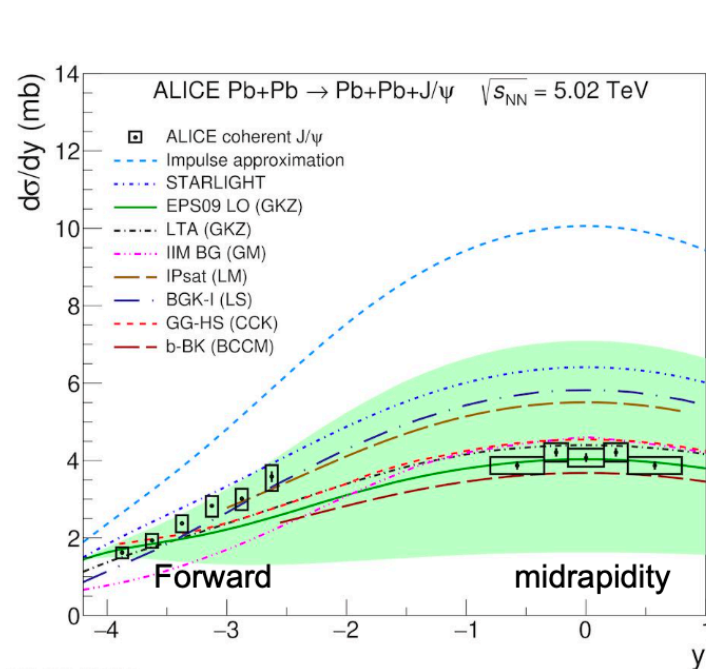
For small $q\bar{q}$ at leading twist, leading $\alpha(s)$, $t \rightarrow 0$: $\sigma \propto (\text{gluon PDF})^2$

Brodsky et al.: [PRD50 \(1994\) 3134-3144](#)

exclusive vector mesons do probe gluons as opposed to quark in DVCS,
but the relevant object is the gluon GPD, not the PDF squared

UPCs and parton distributions

so, what about having a GPD curve on these plots ?



hadron structure theorists are used to fixed target energies, but they have started to work on this, and one could see progress soon for $\gamma + p$ UPCs

for $\gamma + Pb$ UPCs, it's further down the road

(3+3)D: Wigner functions

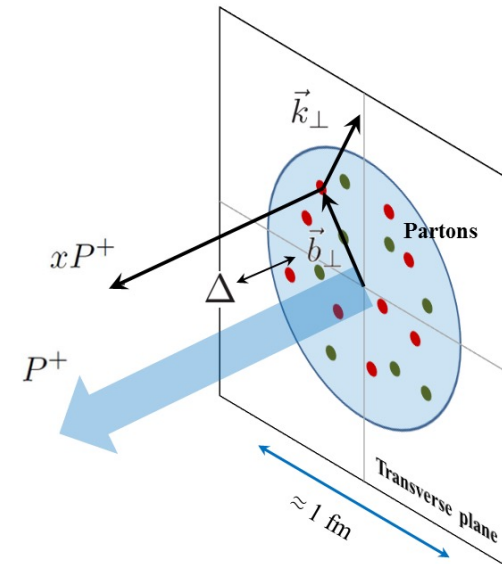
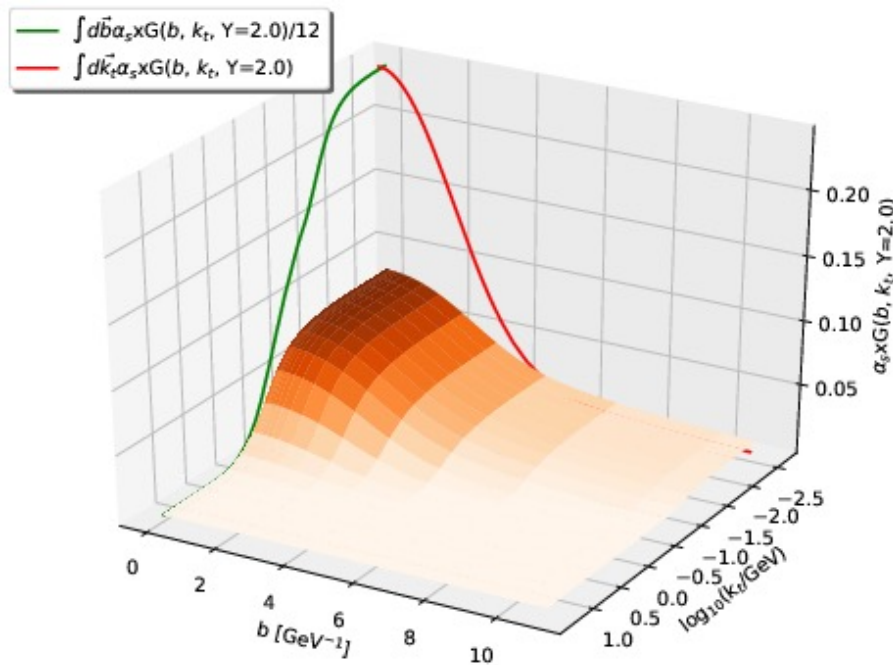


figure from Cepila, Contreras and Matas (2019)

GTMDs and Wigner functions

- add non-zero momentum transfer to the TMD definition,
e.g. for the gluon :

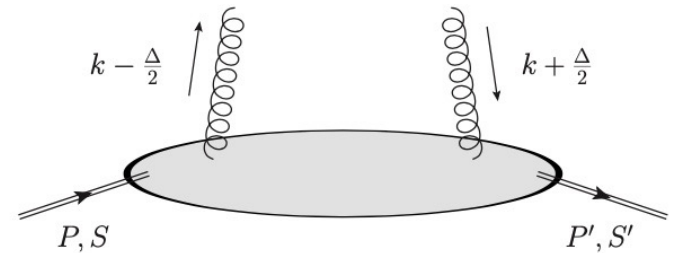
Belitsky, Ji and Yuan (2003)

$$\text{GTMD}_{g/h}(x, k_{\perp}, \Delta) = 2 \int \frac{d\xi^- d^2 \xi_t}{(2\pi)^3 p_h^+} e^{ix p_h^+ \xi^- - i k_{\perp} \cdot \xi_t} \langle p + \Delta | \text{Tr} [F^{+i}(\xi^-, \xi_t) U_{[\xi, 0]} F^{+i}(0)] | p \rangle$$

+ full decomposition into a tensor structure
made of k^i , Δ^i , ...

- then FT gives the Wigner distributions

$$\Delta_{\perp} \leftrightarrow b_{\perp}$$



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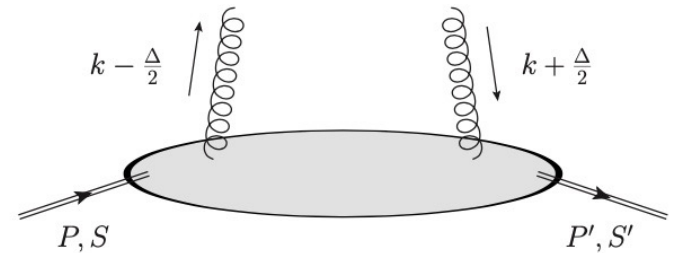
$$\Delta_{\perp} \leftrightarrow b_{\perp}$$

- again, at small-x (i.e. $\xi = x \rightarrow 0$), simple connection with the dipole scattering amplitude:

Hatta, Xiao and Yuan (2016)

$$xW(x, \vec{q}_{\perp}, \vec{b}_{\perp}) \approx \frac{2N_c}{\alpha_s} \int \frac{d^2 \vec{r}_{\perp}}{(2\pi)^2} e^{i \vec{q}_{\perp} \cdot \vec{r}_{\perp}} \left(\frac{1}{4} \vec{\nabla}_b^2 - \vec{\nabla}_r^2 \right) S_x(\vec{b}_{\perp}, \vec{r}_{\perp})$$

$$S_x(\vec{b}_{\perp}, \vec{r}_{\perp}) = \left\langle \frac{1}{N_c} \text{Tr} U \left(\vec{b}_{\perp} - \frac{\vec{r}_{\perp}}{2} \right) U^{\dagger} \left(\vec{b}_{\perp} + \frac{\vec{r}_{\perp}}{2} \right) \right\rangle_x$$



Diffractive processes

extraction of the Wigner function envisioned using diffractive processes

Enberg, Ingelman and Pasechnik (2010)

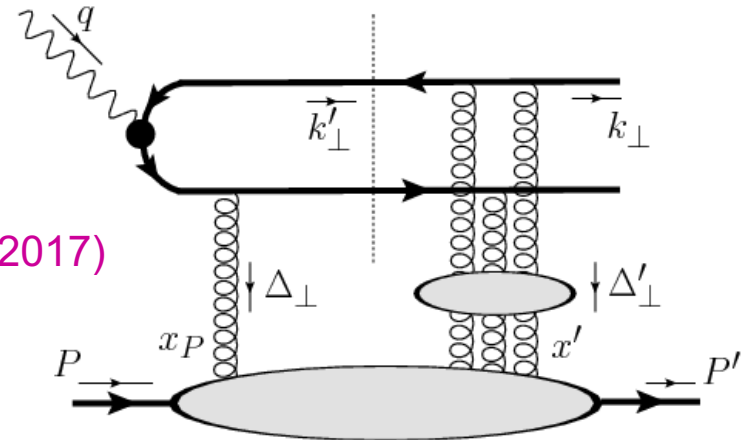
revived recently in the CGC context

Altinoluk, Armesto, Beuf, Rezaeian (2016)

Hatta, Xiao, Yuan (2016)

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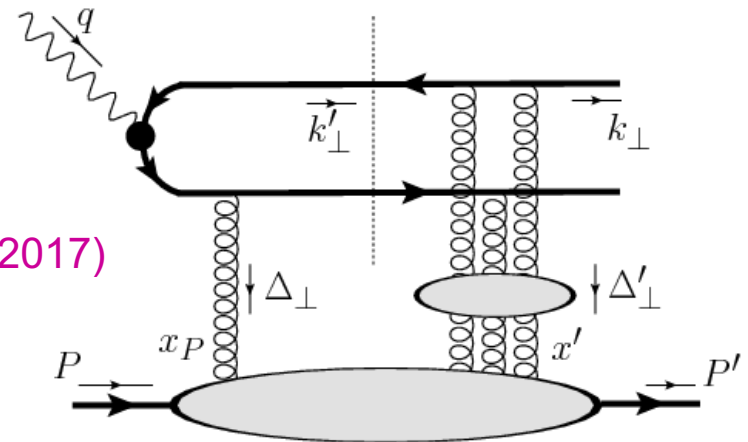
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however, higher-order corrections blur the connection with the Wigner function

soft gluon emissions add complications Hatta, Xiao, Yuan and Zhou (2021)

this process is power suppressed compared to the NLO correction

with a hard gluon emission Iancu, Mueller, Triantafyllopoulos, Wei (2022)

Onto diffractive TMDs

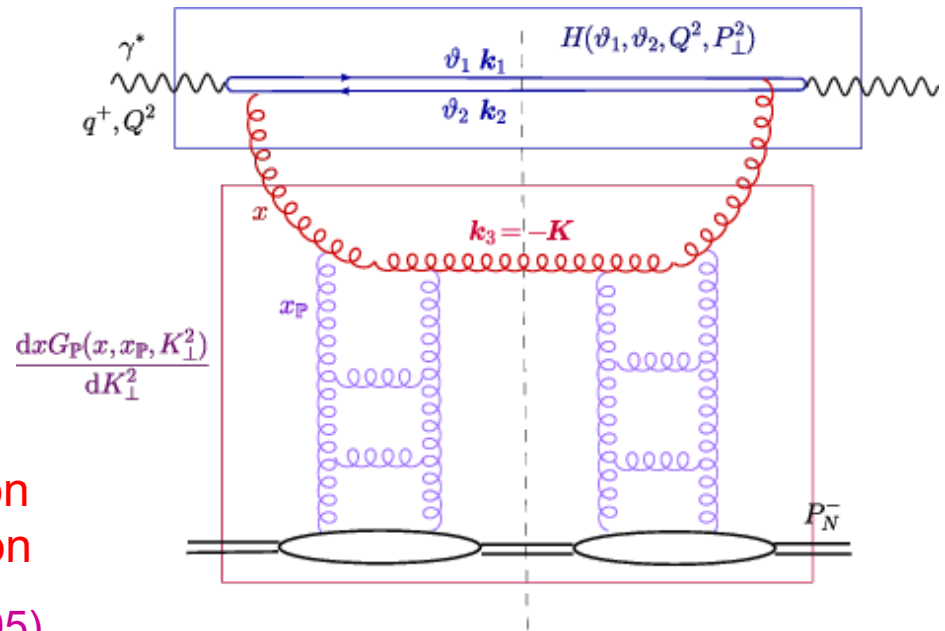
next-to-leading order corrections dominant at high p_t

$$\frac{d\sigma^{\text{LO}}}{d^2 P_\perp} \propto \frac{1}{P_\perp^6}$$

$$\frac{d\sigma^{\text{NLO}}}{d^2 P_\perp} \propto \frac{\alpha_s}{P_\perp^4}$$

dominant, and sensitivity to saturation physics much bigger when hard gluon is emitted

Golec-Biernat, Marquet (2005)



Onto diffractive TMDs

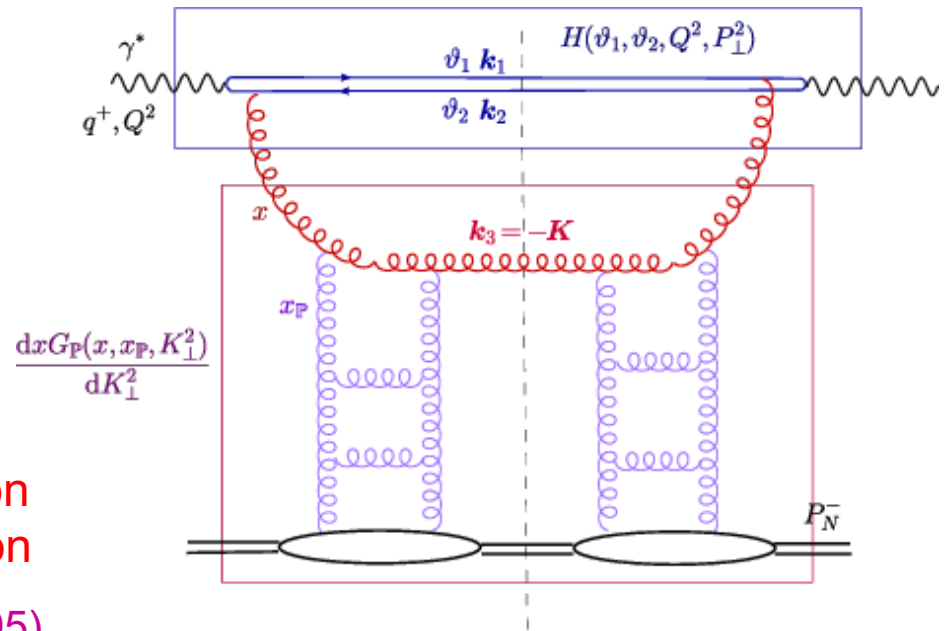
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dominant, and sensitivity to saturation physics much bigger when hard gluon is emitted

Golec-Biernat, Marquet (2005)



instead of the gluon Wigner function, the gluon-emission process factorizes into yet another parton distribution: **the diffractive TMD**

$$\frac{d\sigma_D^{\gamma_{T,L}^* A \rightarrow q\bar{q}gA}}{d\vartheta_1 d\vartheta_2 d^2 \mathbf{P} dY_{\mathbb{P}}} = H_{T,L}(\bar{Q}^2, P_\perp^2) xG_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp^2)$$

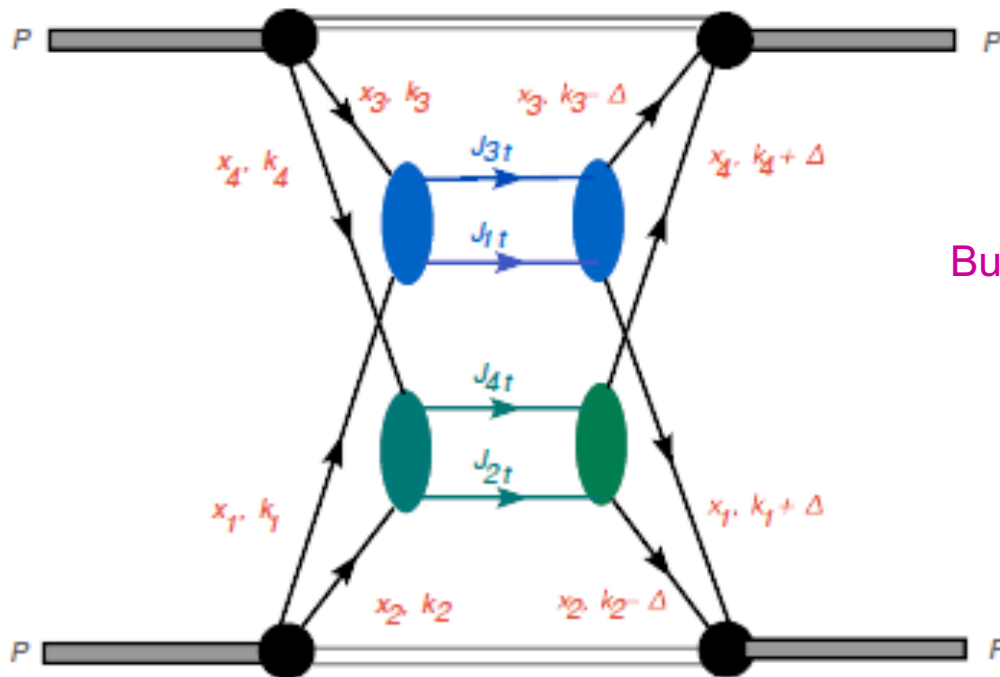
Iancu, Mueller, Triantafyllopoulos, Wei (2022)

Hatta, Xiao, Yuan (2022)

What about two-particle
distributions ?

Double parton scattering

keeping track of both partonic transverse momentum and position is crucial to describe multiple partonic interactions



Buffing, Diehl, Kasemets 2018

a double Wigner distribution is needed to describe such events in QCD

even if in practice, several simplifications are made to involve double PDFs instead

Correlations in small systems

in dilute-dense collisions, two-particle correlations are more sensitive to the quantum fluctuations of the dilute projectile than that of the target

the complexity of the dilute projectile must be taken seriously

Correlations in small systems

in dilute-dense collisions, two-particle correlations are more sensitive to the quantum fluctuations of the dilute projectile than that of the target

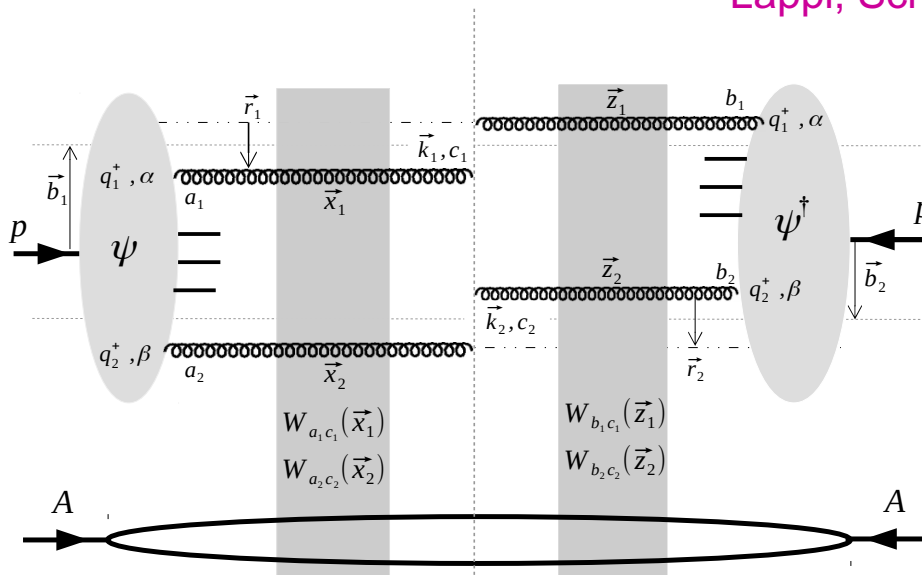
the complexity of the dilute projectile must be taken seriously

the anisotropy harmonics in an initial-state only approach are controlled by a double Wigner distribution:

Lappi, Schenke, Schlichting, Venugopalan (2016)

Kovner and Rezaeian (2017)

Dusling, Mace, Venugopalan (2018)



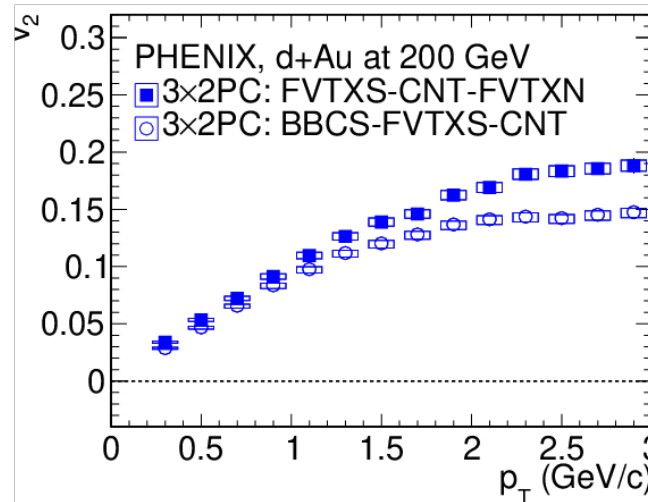
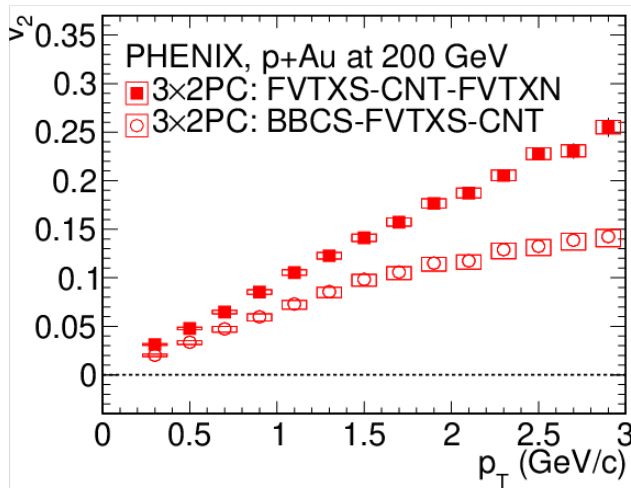
the dynamics of the target is less crucial, it just has to have strong color fields

anisotropies can be generated during the interaction (under control) or come from pre-existing correlation in the wave function (mostly ignored)

Non-flow v_n at forward rapidities

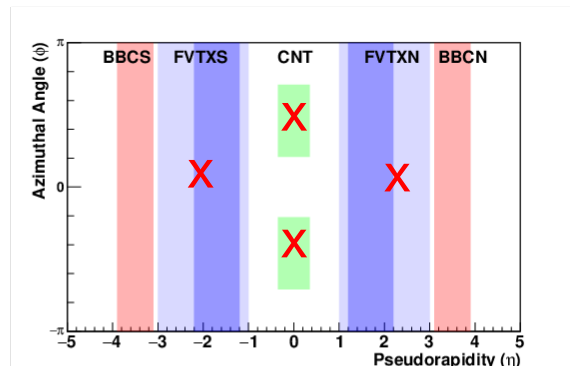
Reverse ordering for v_2

$$v_2(p+Au) > v_2(d+Au)$$



not consistent with hydro/final-state effect

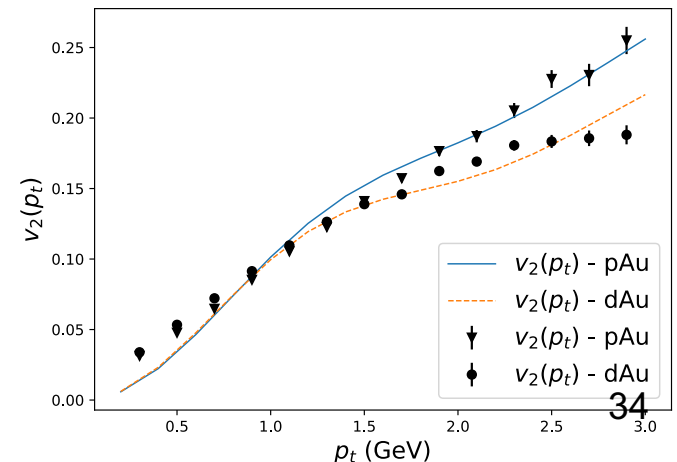
PHENIX collaboration (2022)



PHENIX sub-detectors used

can be reproduced
with simple model

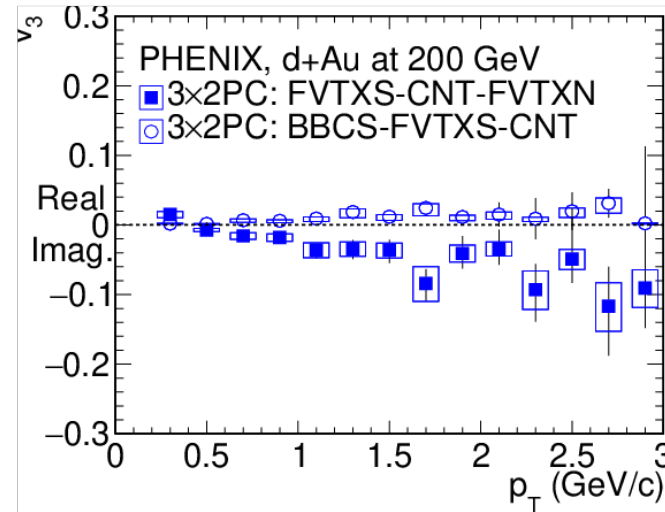
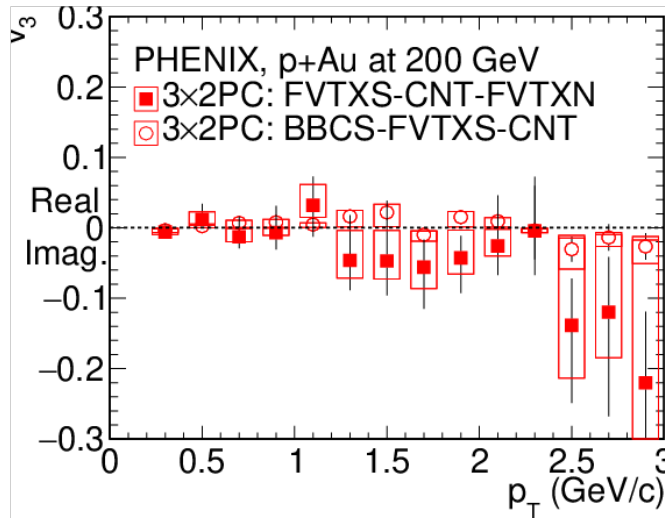
Kohara and CM
to appear



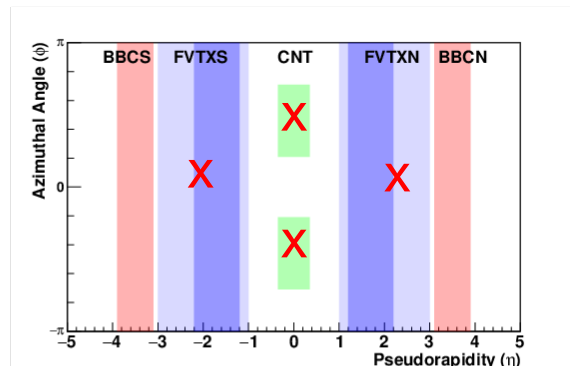
Non-flow v_n at forward rapidities

Negative v_3

not consistent with hydro/final-state effect



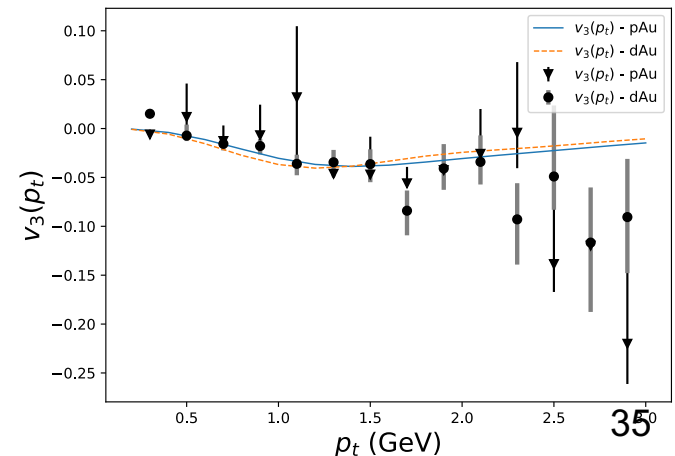
PHENIX collaboration (2022)



can be reproduced
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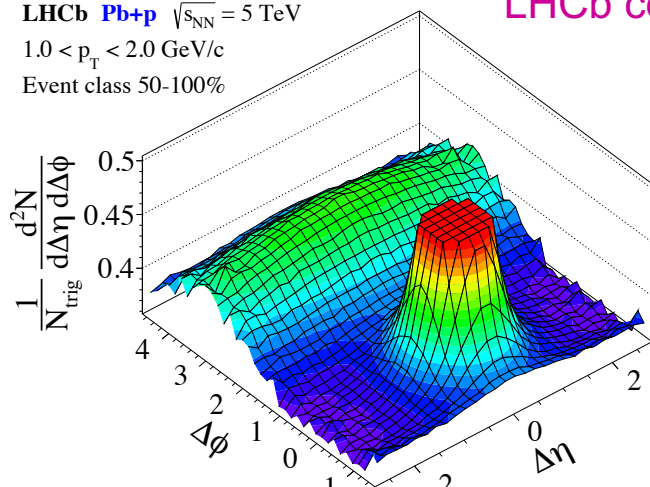
Kohara and CM
to appear

PHENIX sub-detectors used

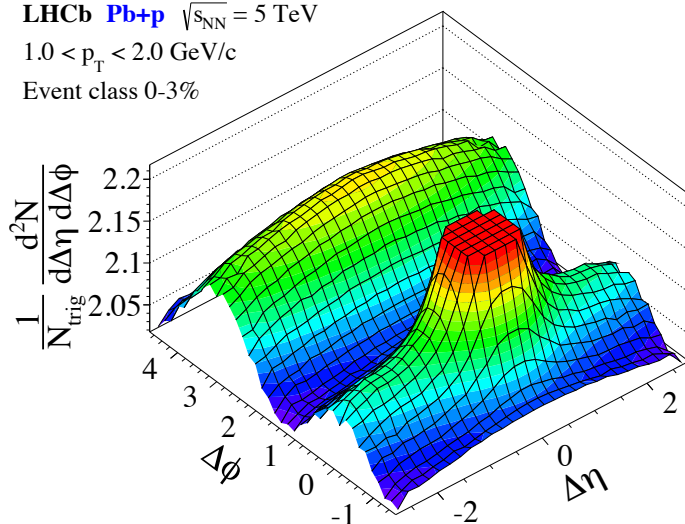


Measurement at LHCb ?

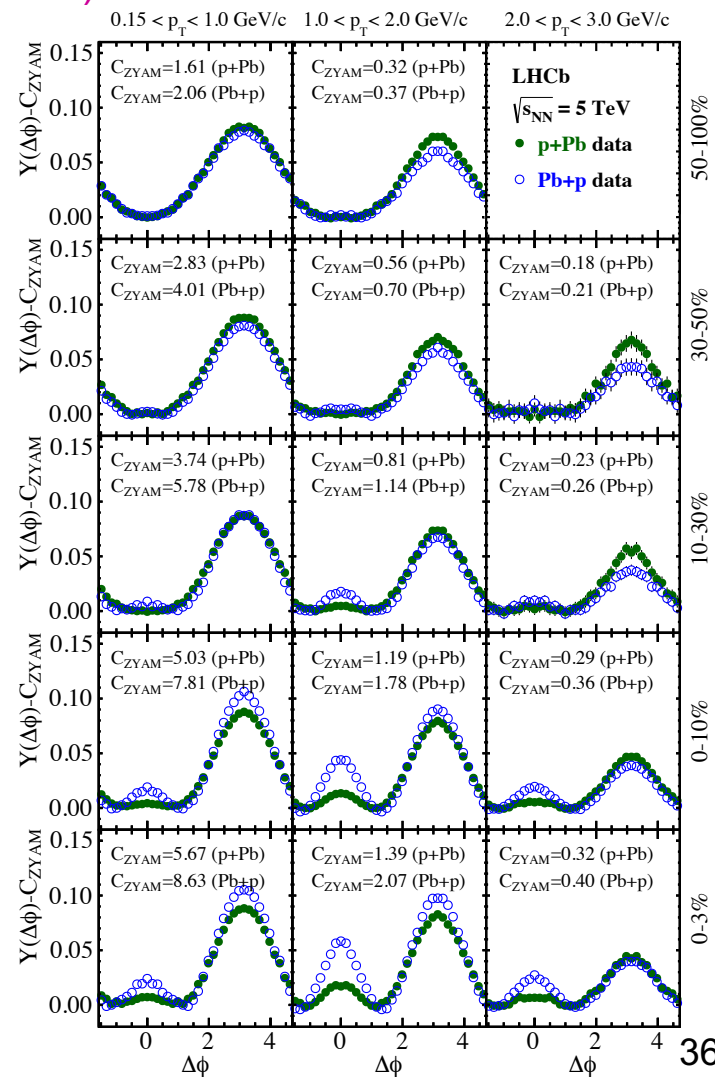
LHCb Pb+p $\sqrt{s_{NN}} = 5$ TeV
 $1.0 < p_T < 2.0$ GeV/c
 Event class 50-100%



LHCb Pb+p $\sqrt{s_{NN}} = 5$ TeV
 $1.0 < p_T < 2.0$ GeV/c
 Event class 0-3%



LHCb collaboration (2016)



v_n not extracted from correlation functions ?

Conclusions

- proton TMDs and GPDs are studied by two rather distinct communities
 - each is a field in itself, with no particular interest in small x
 - also nuclear TMDs and GPDs activities are limited
- the Color Glass Condensate provides a framework to incorporate such hadron structure knowledge into the description of heavy-ion collisions
 - one can study TMDs and GPDs at small x using a common language
 - GTMDs/Wigner functions appear naturally
 - it will help make sure that our assumptions are compatible with what is known from hadron structure, especially for small systems
- our understanding of small systems suffers from the insufficient knowledge of the hadron structure
 - provides a timely context for the QGP and hadron structure communities to get closer, let's not wait for the Electron-Ion Collider