Spin polarization: from kinetic theory to hydrodynamics

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The 7th International Conference on the Initial Stages in High-Energy Nuclear Collisions, June 19-23, 2023, Copenhagen

Outline

- **Introduction to polarization phenomena in HIC**
- **Spin distributions from Wigner functions**
- **Spin dynamics for vector mesons in quantum kinetic theory**
- **Ideal spin hydrodynamics from Wigner functions**
- **Summary**

Barnet effect and Einstein-de Haas effect

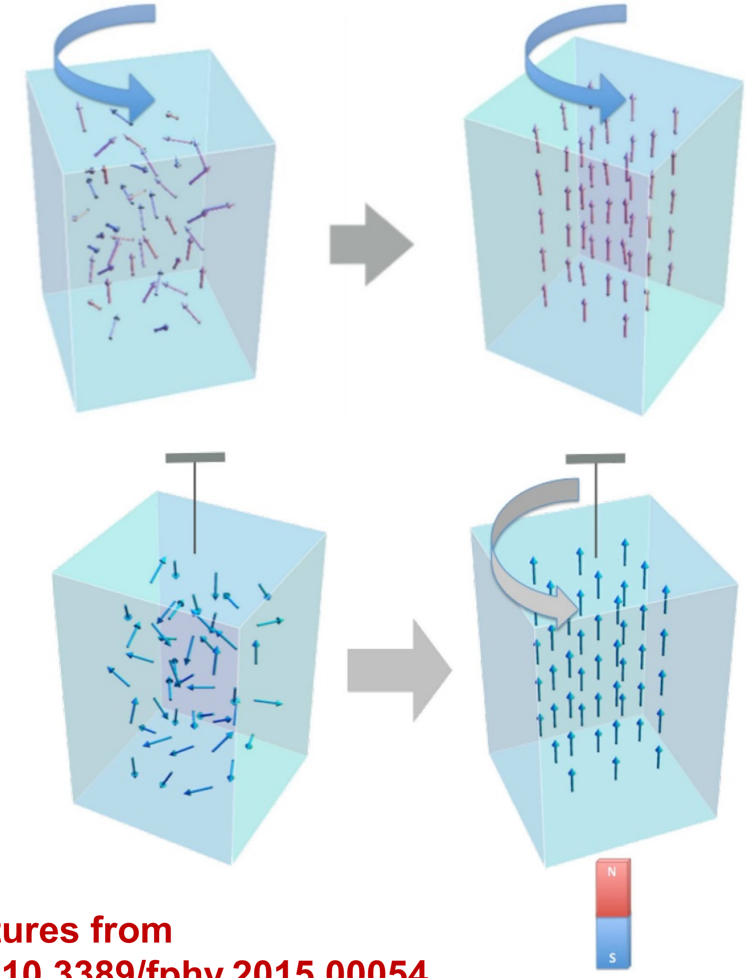
Barnett effect:

Barnett, Magnetization by rotation, Phys Rev. 6, 239-270 (1915).

Spin-orbit (LS) coupling!

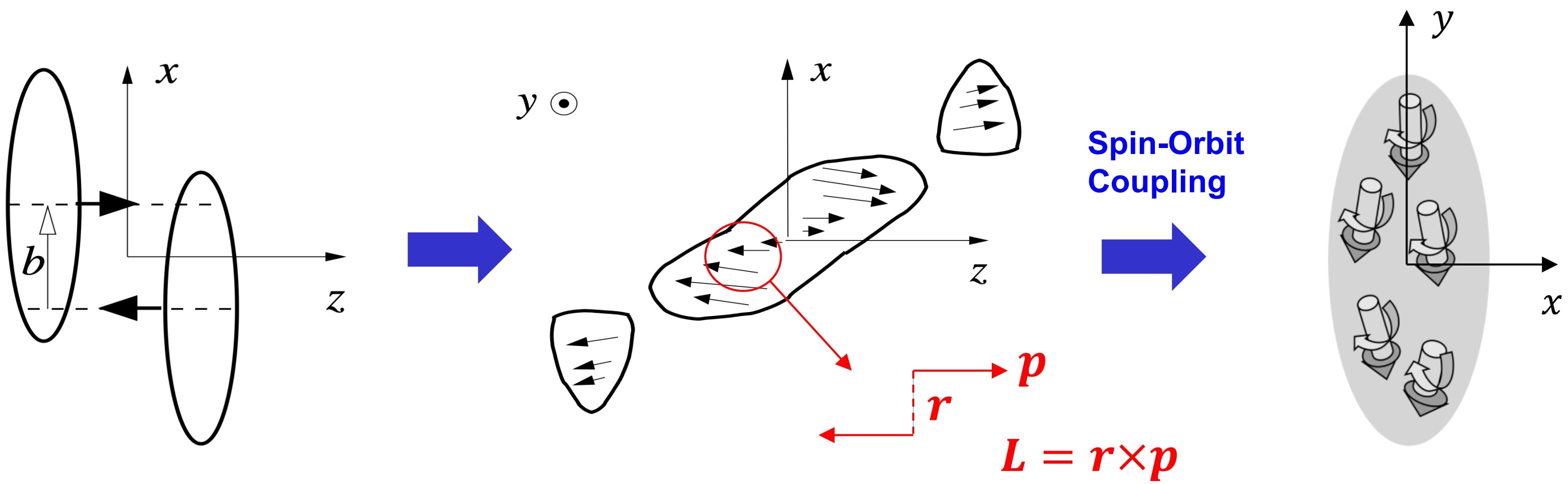
Einstein-de Haas effect:

Einstein, de Haas, Experimental proof of the existence of Ampere's molecular currents, Verhandl. Deut. Phys. Ges. 17, 152–170 (1915).



Pictures from doi:10.3389/fphy.2015.00054

Global OAM and polarization in HIC

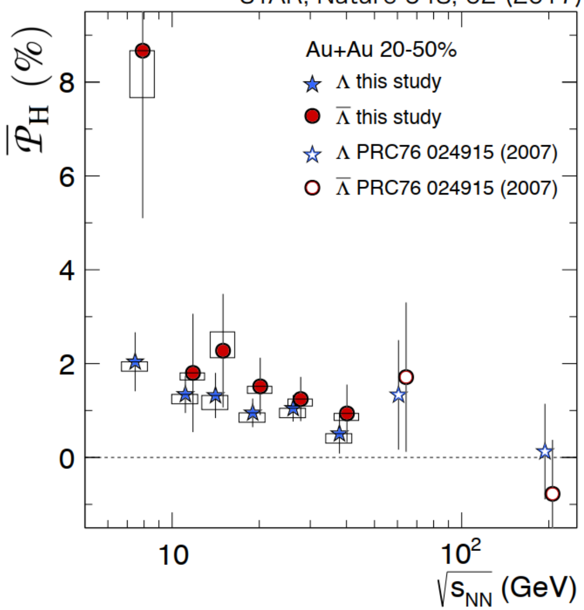


Global OAM leads to global polarization of Λ hyperons through spin-orbit coupling

Liang and Wang, PRL 94,102301(2005); Betz, Gyulassy, Torrieri, PRC (2007); Becattini, Piccinini, Rizzo, PRC (2008); Gao, Chen, Deng, Liang, QW, Wang, PRC (2008)

STAR: global polarization of Λ hyperon

STAR, Nature 548, 62 (2017)



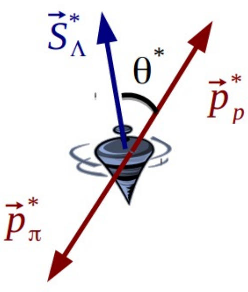
parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

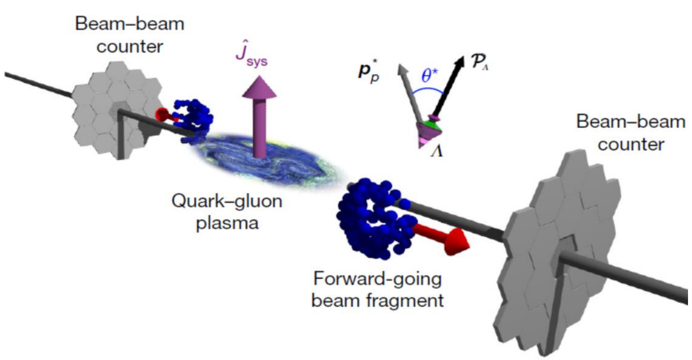
$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P}_{\mathbf{\Lambda}} \cdot \mathbf{p}_{\mathbf{p}}^*)$$

α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame

Updated by BES III, PRL129, 131801 (2022)



$\Lambda \rightarrow p + \pi^+$
(BR: 63.9%, cτ~7.9 cm)

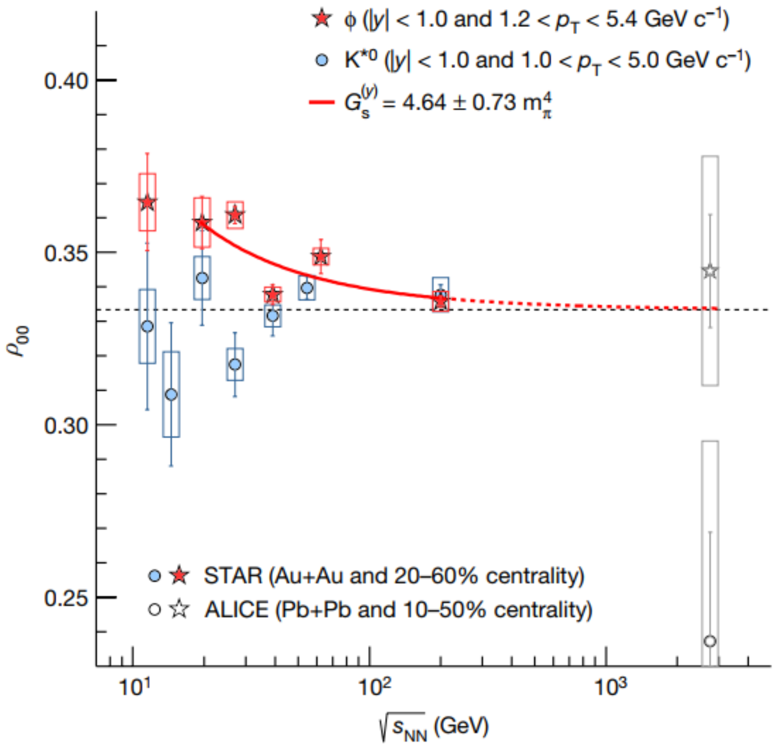


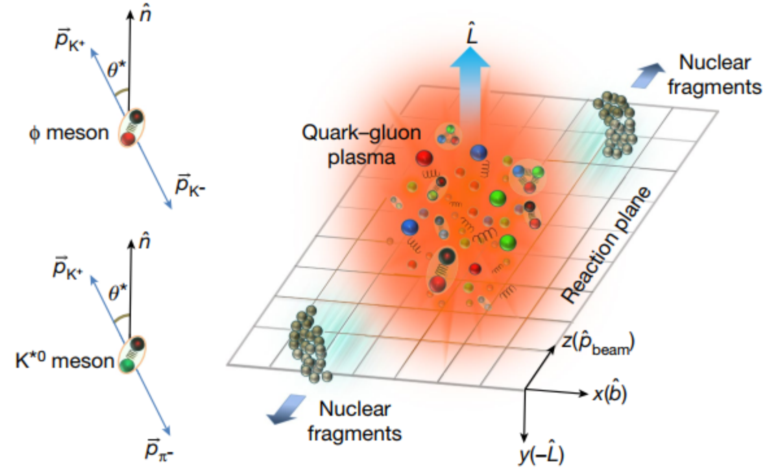
ω = (9 ± 1)x10^{21}/s, the largest angular velocity that has ever been observed in any system

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Gao et al., PRC (2008)

STAR: global spin alignments of vector mesons

STAR, Nature 614, 244 (2023);





Theory prediction:
Sheng, Oliva, QW (2020);
Sheng, Oliva, et al., (2022).

Implication of correlation or fluctuation of strong force fields

Single-particle distribution function in classical theory (no spin)

- **Single particle distribution function in phase space $\boldsymbol{f(t,x,p)}$**

$$f(t,\mathbf{x},\mathbf{p})d^3xd^3p$$

particle number in phase space volume $\boldsymbol{d^3xd^3p}$

- **The evolution of $\boldsymbol{f(t,x,p)}$ is given by the classical Boltzmann equation**

$$\begin{aligned} \frac{d}{dt}f(t,\mathbf{x},\mathbf{p}) &= \left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{E_p}\cdot\nabla+\mathbf{F}\cdot\nabla_{\mathbf{p}}\right)f(t,\mathbf{x},\mathbf{p})=\mathcal{C}[f] \\ \mathcal{C}[f] &= \int_{124} d\widetilde{\Gamma}_{1,2\to p,4}(f_1f_2-f_pf_4) \end{aligned}$$

Classical feature: $\boldsymbol{x}$ and $\boldsymbol{p}$ of the particle can be determined at the same time !

QKT for massive fermions in Wigner functions

- **Wigner function (4x4 matrix) for spin 1/2 massive fermions**

$$W_{\alpha\beta}(x,p) = \int d^4y \exp\left(\frac{i}{\hbar}p\cdot y\right)\left\langle \overline{\psi}_\beta\left(x-\frac{y}{2}\right)\psi_\alpha\left(x+\frac{y}{2}\right)\right\rangle$$

Heinz, PRL 51, 351 (1983);
Vasak-Gyulassy-Elze, Ann. Phys. 173, 462 (1987)

- **Wigner function decomposition in 16 generators of Clifford algebra**

spin 4-vector

$$W = \frac{1}{4}\left[\mathscr{F} + i\gamma^5\mathscr{P} + \gamma^\mu\mathscr{V}_\mu + \gamma^5\gamma^\mu\mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right]$$

scalar p-scalar vector axial-vector tensor

$$j^\mu = \int d^4p\mathscr{V}^\mu, \qquad j_5^\mu = \int d^4p\mathscr{A}^\mu, \qquad T^{\mu\nu} = \int d^4p p^\mu\mathscr{V}^\nu$$

Recent reviews:
Hidaka-Pu-QW-Yang, PPNP (2022)
Gao-Liang-QW, IJMPA (2021)

Vasak-Gyulassy-Elze, Ann. Phys. 173, 462 (1987);
Elze-Gyulassy-Vasak, Nucl. Phys. B 276, 706 (1986);

Continuous spin variable in quantum kinetic theory (with collisions)

- **Extended phase space:** $(\boldsymbol{x}, \boldsymbol{p}) \Longrightarrow (\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{s})$

$$f(x,p,\mathfrak{s})\delta(p^2-m^2)d^4p\,dS(p)$$

$$dS(p) = \frac{1}{\pi}\sqrt{\frac{p^2}{3}}d^4\mathfrak{s}\,\underline{\delta(\mathfrak{s}^2-3)\delta(p\cdot\mathfrak{s})}$$

continuous spin variable s is space-like, which is normalized as $\boldsymbol{s}^2 = -3$ and normal to momentum $\boldsymbol{p}\cdot\boldsymbol{s} = 0$ ($\boldsymbol{p}$ is time-like)

Weickgenannt, Speranza, Sheng, QW, Rischke (2021)

- **Boltzmann equation**

$$p\cdot\partial f = \underline{\mathfrak{C}[f]}$$

collision term

Spin DOF as Grassmann variables in WL formalism (w/o collisions): Mueller, Venugopalan (2019)

$$\underline{\mathfrak{C}[f]} = \int \underline{d\Gamma_1 d\Gamma_2 d\Gamma'}\,\mathcal{W}\,[f(x+\underline{\Delta_1},p_1,\mathfrak{s}_1)f(x+\underline{\Delta_2},p_2,\mathfrak{s}_2) - f(x+\underline{\Delta},p,\mathfrak{s})f(x+\underline{\Delta'},p',\mathfrak{s}')]$$

$$d\Gamma \equiv d^4p\,\delta(p^2-m^2)dS(p)$$

Phase space measure

Space-time shift: "side-jump" [Chen, Son, Stephanov (2015)]

Spin DOF: Matrix Valued Spin Dependent Distributions (MVSD)

Relativistic MVSD for fermion in QFT

$$p^\mu \equiv \frac{1}{2}(p_1^\mu + p_2^\mu) \qquad q^\mu \equiv p_1^\mu - p_2^\mu$$

$$f_{rs}(x,p) \equiv \int \frac{d^4q}{2(2\pi)^3} \exp\left(-\frac{i}{\hbar} q \cdot x\right) \delta(\underline{p \cdot q}) \left\langle a^\dagger(\underline{s, \mathbf{p}_2}) a(\underline{r, \mathbf{p}_1}) \right\rangle$$

Relativistic MVSD can be parameterized in terms un-polarized distributions and Spin Density Matrix (polarization part)

$$f_{rs}^{(+)}(x,\mathbf{p}) = \frac{1}{2} \underline{f_q(x,\mathbf{p})} \left[\delta_{rs} - \underline{P_\mu^q(x,\mathbf{p})} \underline{n_j^{(+)\mu}(\mathbf{p})} \tau_{rs}^j\right],$$

$$f_{rs}^{(-)}(x,-\mathbf{p}) = \frac{1}{2} \underline{f_{\overline{q}}(x,-\mathbf{p})} \left[\delta_{rs} - \underline{P_\mu^{\overline{q}}(x,-\mathbf{p})} \underline{n_j^{(-)\mu}(\mathbf{p})} \tau_{rs}^j\right],$$

Pauli matrices in spin space (rs-space)

Un-polarized dist.

Spin polarization dist.

Four-vectors of three basis directions in rest frame of q and $\overline{q}$ (one is the spin quantization direction)

MVSD:
Sheng, Weickgenannt, et al. (2021); Sheng, QW, Rischke (2022)

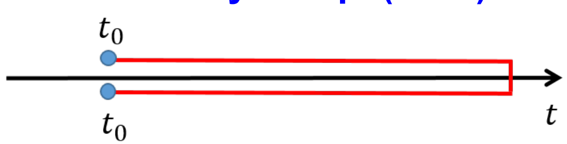
Spin Boltzmann equation for massive fermions

- **For massive fermions, spin is independent degree of freedom. We use Closed-Time-Path (CTP) or Schwinger-Keldysh formalism.**

$$\left(\begin{array}{cc} G^{++}(x_1,x_2) & G^{+-}(x_1,x_2) \\ G^{-+}(x_1,x_2) & G^{--}(x_1,x_2) \end{array} \right) = \left(\begin{array}{cc} G^F(x_1,x_2) & \underline{G^<(x_1,x_2)} \\ \underline{G^>(x_1,x_2)} & G^{\overline{F}}(x_1,x_2) \end{array} \right)$$

Chou, Su, Hao, Yu, Phys. Rep. (1985); Blaizot, Iancu, Phys. Rep. (2002)

$$G^<_{\alpha\beta}(x,p) \equiv -\int d^4y e^{ip\cdot y/\hbar} \left\langle \bar{\psi}_\beta\left(x-\frac{y}{2}\right)\psi_\alpha\left(x+\frac{y}{2}\right)\right\rangle$$



Wigner transformation for spin-1/2 fermions

- **Wigner function in terms of MVSD at leading and next-to-leading order**

$$\begin{aligned} G^{<,(0)}_{\alpha\beta}(x,p) \quad = \quad & -2\pi\hbar\,\theta(p_0)\delta\left(p^2-m^2\right)\sum_{r,s} u_\alpha\left(r,p\right)\overline{u}_\beta\left(s,p\right)\underline{f^{(+,0)}_{rs}\left(x,p\right)} \\ & -2\pi\hbar\,\theta(-p_0)\delta\left(p^2-m^2\right)\sum_{r,s} v_\alpha\left(s,\overline{p}\right)\overline{v}_\beta\left(r,\overline{p}\right)\left[\delta_{rs}-\underline{f^{(-,0)}_{rs}\left(x,\overline{p}\right)}\right] \end{aligned}$$

$$\overline{\boldsymbol{p}} = (\boldsymbol{E_p}, -\boldsymbol{p})$$

$$G^{<,(0)}_{\alpha\beta}[\underline{f^{(+,0)}_{rs} \to f^{(+,1)}_{rs}}, \underline{\delta_{rs} - f^{(-,0)}_{rs} \to -f^{(-,1)}_{rs}}] \Longrightarrow G^{<,(1)}_{\alpha\beta}(x,p)$$

Sheng, Weickgenannt, et al. (2021)

Spin Boltzmann equation for massive fermions (with collisions)

- **Kadanoff-Baym's equation in terms of on-shell two-point function**

Mrowczynski-Heinz (1994);
Schonhofen-Cubero-Friman-Norenberg-Wolf (1994);
……

$$
\begin{aligned}
& \left(i\frac{1}{2}\hbar\gamma_\mu\partial_x^\mu + \gamma_\mu p^\mu - m\right) G^<(x,p) \\
= \quad & -i\frac{1}{2}\hbar\left[\Sigma^<(x,p)G^>(x,p) - \Sigma^>(x,p)G^<(x,p)\right] \\
& -\frac{1}{4}\hbar^2\left[\left\{\Sigma^<(x,p), G^>(x,p)\right\}_{\mathrm{PB}} - \left\{\Sigma^>(x,p), G^<(x,p)\right\}_{\mathrm{PB}}\right]
\end{aligned}
$$

- **With two-point functions being expressed in terms of MVSDs, the Boltzmann equation with spin DOF can be derived from Kadanoff-Baym equation**

$$
\Sigma^{\lessgtr} \Rightarrow G^{\lessgtr} \Rightarrow f_{rs}^{(\pm)}(x,p)
$$

Spin Boltzmann equation for massive fermions

- **At leading order spin Boltzmann equation (SBE) with local collision terms**

$$\frac{1}{E_p} p\cdot\partial_x \mathrm{tr}\left[f^{(0)}(x,p)\right] = \mathscr{C}_{\mathrm{scalar}}\left[f^{(0)}\right]$$
$$\frac{1}{E_p} p\cdot\partial_x \mathrm{tr}\left[n_j^{(+)\mu}\tau_j f^{(0)}(x,p)\right] = \mathscr{C}_{\mathrm{pol}}\left[f^{(0)}\right] \Longrightarrow \boldsymbol{f^{(0)}(x,p)}$$

- **At next-to-leading order, SBE describes how $\boldsymbol{f^{(1)}(x,p)}$ evolves for given $\boldsymbol{f^{(0)}(x,p)}$ with space-time derivatives of $\boldsymbol{f^{(0)}(x,p)}$**

$$\frac{1}{E_p} p\cdot\partial_x \mathrm{tr}\left[f^{(1)}(x,p)\right] = \mathscr{C}_{\mathrm{scalar}}\left[\underline{f^{(0)},\partial_x f^{(0)}}, f^{(1)}\right]$$
$$\frac{1}{E_p} p\cdot\partial_x \mathrm{tr}\left[n_j^{(+)\mu}\tau_j f^{(1)}(x,p)\right] = \mathscr{C}_{\mathrm{pol}}\left[\underline{f^{(0)},\partial_x f^{(0)}}, f^{(1)}\right]$$

determined by leading order SBE

Convenient for simulation !

Sheng, Speranza, Rischke, QW, Weickgenannt (2021)

spin transport for massive fermions from KB equation was also studied in:

Yang, Hattori, Hidaka (2020); Wang, Zhuang (2021)

Polarization from different sources in QKT with Wigner functions (without collisions)

- **Axial vector component of WF (spin vector) has many contributions**

$$\mathcal{J}_5^\mu = \mathcal{J}^\mu_{\text{thermal}} + \mathcal{J}^\mu_{\text{shear}} + \mathcal{J}^\mu_{\text{accT}} + \mathcal{J}^\mu_{\text{chemical}} + \mathcal{J}^\mu_{\text{EB}},$$

Fu, Liu, et al., (2021);
Becattini, et al, (2021);

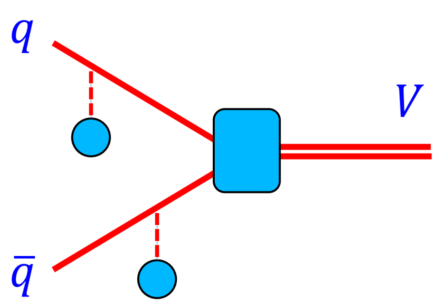
- **Thermal vorticity**
$$\mathcal{J}^\mu_{\text{thermal}} = a\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}p_\nu\partial_\alpha\frac{u_\beta}{T},$$
- **Shear viscous tensor**
$$\mathcal{J}^\mu_{\text{shear}} = -a\frac{1}{(u\cdot p)T}\underline{\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta p^\sigma\partial_{<\sigma}u_{\nu>}}$$
- **Fluid acceleration**
$$\mathcal{J}^\mu_{\text{accT}} = -a\frac{1}{2T}\epsilon^{\mu\nu\alpha\beta}p_\nu u_\alpha\left(Du_\beta - \frac{1}{T}\partial_\beta T\right).$$
- **Gradient of chemical potential**
$$\mathcal{J}^\mu_{\text{chemical}} = a\frac{1}{(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta\partial_\nu\frac{\mu}{T},$$
- **Electromagnetic fields**
$$\mathcal{J}^\mu_{\text{EB}} = a\frac{1}{(u\cdot p)T}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta E_\nu + a\frac{B^\mu}{T},$$

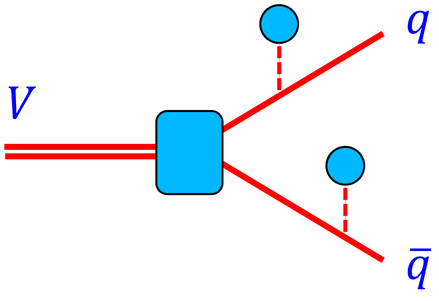
Hidaka, Pu, Yang (2018); Yi, Pu, Yang (2021)

Relativistic Spin Dynamics based on Spin Kinetic Equation (SKE) with MVSDs for vector mesons

Sheng, Oliva, Liang, QW, et al., 2206.05868, 2205.15689

Review on QKE and SKE based on Wigner functions:
Hidaka, Pu, QW, Yang, Prog. Part. Nucl. Phys. 127 (2022) 103989





RSBE in MVSD for vector meson: fusion and dissociation process in

- **Relativistic MVSD for vector meson in QFT**

$$f^V_{\lambda_1\lambda_2} = \int \frac{d^4q}{2(2\pi\hbar)^3} \exp\left(-i\frac{q\cdot x}{\hbar}\right)\delta(p\cdot q)\left\langle a^\dagger_V(\lambda_2,\mathbf{p}_2)\,a_V(\lambda_1,\mathbf{p}_1)\right\rangle$$

- **RSBE for fusion (coalescence) and dissociation process $q\bar{q} \leftrightarrow V$ can be simplified as**

Coalescence collision kernel

$$k\cdot\partial_x f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = \frac{1}{8}\left[\underline{\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})}\,\underline{\mathcal{C}^{\mu\nu}_{\text{coal}}(x,\mathbf{k})}\right.$$
$$\left.-\underline{\mathcal{C}_{\text{diss}}(\mathbf{k})}\,f^V_{\lambda_1\lambda_2}(x,\mathbf{k})\right],$$

Dissociation collision kernel

$$\underline{\epsilon^\mu(\lambda,\mathbf{k})} = \left(\frac{\mathbf{k}\cdot\epsilon_\lambda}{m_V},\ \epsilon_\lambda + \frac{\mathbf{k}\cdot\epsilon_\lambda}{m_V(E^V_{\mathbf{k}}+m_V)}\mathbf{k}\right) \Longrightarrow k_\mu\epsilon^\mu(\lambda,\mathbf{k}) = 0$$

polarization vector of vector meson

In rest frame of vector meson: ϵ_λ is polarization 3-vector and n_x, n_y, n_z are three basis directions

$$\boxed{\begin{aligned}\epsilon_0 &= \mathbf{n}_y\\ \epsilon_{+1} &= -\frac{1}{\sqrt{2}}(\mathbf{n}_z + i\mathbf{n}_x)\\ \epsilon_{-1} &= \frac{1}{\sqrt{2}}(\mathbf{n}_z - i\mathbf{n}_x)\end{aligned}}$$

MVSD or spin density matrix element for vector mesons

Forml solution to MVSD (spin density matrix) for vector mesons

$$f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) \sim \frac{1}{\mathcal{C}_{\text{diss}}(\mathbf{k})}\left[1-e^{-\mathcal{C}_{\text{diss}}(\mathbf{k})\Delta t}\right] \times \epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}(x,\mathbf{k})$$

Sheng, Lucia, Liang, QW, et al, 2205.15689, 2206.05868

where the coalescence collision kernel $C^{\mu\nu}_{coal}$ is given by

$$\begin{aligned}\mathcal{C}^{\mu\nu}_{\text{coal}}(x,\mathbf{k}) = &\int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^2}\frac{1}{E^{\overline{q}}_{\mathbf{p}'}E^{q}_{\mathbf{k}-\mathbf{p}'}}\delta\left(E^V_{\mathbf{k}}-E^{\overline{q}}_{\mathbf{p}'}-E^{q}_{\mathbf{k}-\mathbf{p}'}\right)\\ &\times \mathrm{Tr}\left\{\underline{\Gamma^\nu}\left(p'\cdot\gamma-m_{\overline{q}}\right)\left[1+\gamma_5\gamma\cdot \underline{P^{\overline{q}}(x,\mathbf{p}')}\right]\right.\\ &\left.\times\underline{\Gamma^\mu}\left[(k-p')\cdot\gamma+m_q\right]\left[1+\gamma_5\gamma\cdot \underline{P^{q}(x,\mathbf{k}-\mathbf{p}')}\right]\right\}\\ &\times \underline{f_{\overline{q}}(x,\mathbf{p}')f_q(x,\mathbf{k}-\mathbf{p}')},\end{aligned}$$

BS wave function for VM [Roberts et al (2019, 2021)]

$$\boxed{\Gamma^\alpha \approx g_V B(\mathbf{k}-\mathbf{p}',\mathbf{p}')\gamma^\alpha}$$

Covariant polarization phase space distributions for q and $\overline{q}$

un-polarized distributions for q and $\overline{q}$

Sheng, Lucia, Liang, QW, et al, 2205.15689, 2206.05868

Spin density matrix element for vector mesons

Spin density matrix (normalized MVSD) for vector mesons

$$f^V_{\lambda_1\lambda_2} \propto \rho^V_{\lambda_1\lambda_2} = \frac{\epsilon^*_\mu(\lambda_1, \mathbf{k})\epsilon_\nu(\lambda_2, \mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}}{\sum_{\lambda=0,\pm 1} \epsilon^*_\mu(\lambda, \mathbf{k})\epsilon_\nu(\lambda, \mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}}$$

For ϕ meson, covariant polarization phase space distributions for s and $\bar{s}$ appearing in $C^{\mu\nu}_{coal}$ have the form

$$P^\mu_s(x, \mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} + \frac{g_\phi}{(u\cdot p)T_{\text{eff}}}\underline{F^\phi_{\rho\sigma}}\right)p_\nu$$

$$P^\mu_{\bar{s}}(x, \mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} - \frac{g_\phi}{(u\cdot p)T_{\text{eff}}}\underline{F^\phi_{\rho\sigma}}\right)p_\nu$$

field strength tensor of ϕ field

Spin density matrix element for vector mesons

The fusion (coalescence) collision kernel $C_{coal}^{\mu\nu}$ can be evaluated in the rest frame of ϕ meson, which gives ρ_{00}^{ϕ}

$$
\begin{aligned}
\rho_{00}(x,\underline{\mathbf{0}}) \approx & \frac{1}{3} + C_1 \left[\frac{1}{3}\boldsymbol{\omega}' \cdot \boldsymbol{\omega}' - (\boldsymbol{\epsilon}_0 \cdot \boldsymbol{\omega}')^2\right] \\
& + C_2 \left[\frac{1}{3}\boldsymbol{\varepsilon}' \cdot \boldsymbol{\varepsilon}' - (\boldsymbol{\epsilon}_0 \cdot \boldsymbol{\varepsilon}')^2\right] \\
& - \frac{4g_\phi^2}{m_\phi^2 T_{\mathrm{eff}}^2} C_1 \left[\frac{1}{3}\mathbf{B}'_\phi \cdot \mathbf{B}'_\phi - \underline{(\boldsymbol{\epsilon}_0} \cdot \mathbf{B}'_\phi)^2\right] \\
& - \frac{4g_\phi^2}{m_\phi^2 T_{\mathrm{eff}}^2} C_2 \left[\frac{1}{3}\mathbf{E}'_\phi \cdot \mathbf{E}'_\phi - \underline{(\boldsymbol{\epsilon}_0} \cdot \mathbf{E}'_\phi)^2\right],
\end{aligned}
$$

rest frame of ϕ meson

$$
C_1 = \frac{8m_s^4 + 16m_s^2 m_\phi^2 + 3m_\phi^4}{120 m_s^2 (m_\phi^2 + 2m_s^2)},
$$

$$
C_2 = \frac{8m_s^4 - 14m_s^2 m_\phi^2 + 3m_\phi^4}{120 m_s^2 (m_\phi^2 + 2m_s^2)}.
$$

All fields with prime are defined in the rest frame of ϕ meson

spin quantization direction

Features: (1) Perfect factorization of x and p dependence; (2) Perfect cancellation for mixing terms (protected by symmetry): all fields appear in squares, i.e. ρ_{00}^{ϕ} measures fluctuations of fields. Surprising results!

Lorentz transformation for ϕ fields

We can express ρ_{00}^{ϕ} in terms of ϕ fields in the lab frame and obtain the dependence on momenta of ϕ mesons through Lorentz transformation

$$\mathbf{B}'_{\phi} = \gamma\mathbf{B}_{\phi} - \gamma\mathbf{v}\times\mathbf{E}_{\phi} + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{B}_{\phi}}{v^2}\mathbf{v},$$
$$\mathbf{E}'_{\phi} = \gamma\mathbf{E}_{\phi} + \gamma\mathbf{v}\times\mathbf{B}_{\phi} + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{E}_{\phi}}{v^2}\mathbf{v},$$

where $\gamma = E_{\mathbf{k}}^{\phi}/m_{\phi}$ and $\mathbf{v} = \mathbf{k}/E_{\mathbf{k}}^{\phi}$

Then we obtain factorization form $\left\langle \rho_{00}^{\phi} \right\rangle$ in terms of lab-frame fields

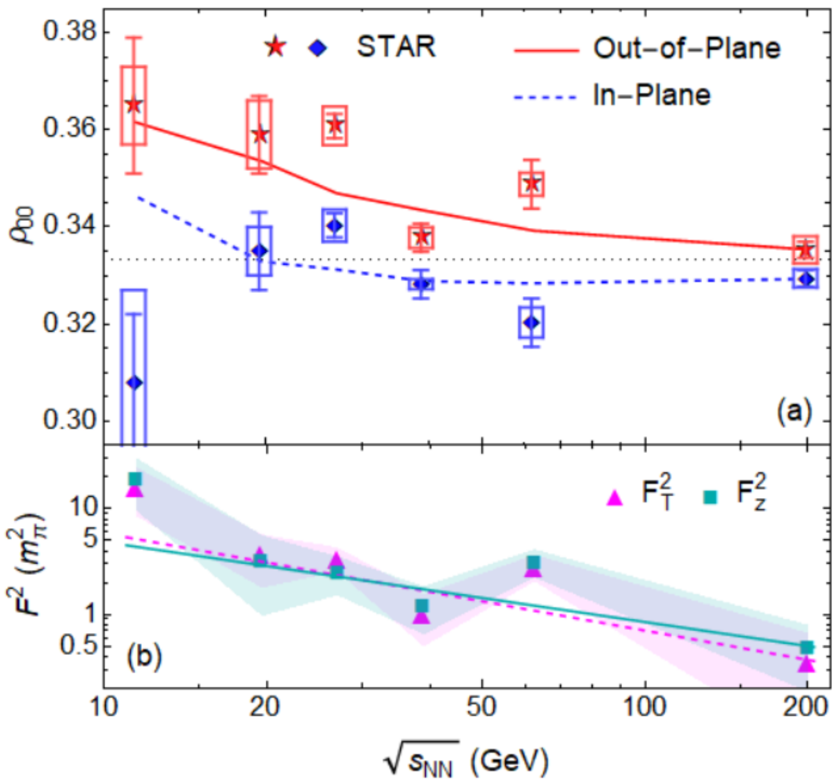
$$\left\langle \overline{\rho}_{00}^{\phi}(x,\mathbf{p}) \right\rangle_{x,\mathbf{p}} \approx \frac{1}{3} + \frac{1}{3}\sum_{i=1,2,3} \langle I_{B,i}(\mathbf{p})\rangle \frac{1}{m_{\phi}^2}\left[\langle \boldsymbol{\omega}_i^2\rangle - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\text{eff}}^2}\left\langle (\mathbf{B}_i^{\phi})^2 \right\rangle\right] + \frac{1}{3}\sum_{i=1,2,3} \langle I_{E,i}(\mathbf{p})\rangle \frac{1}{m_{\phi}^2}\left[\langle \boldsymbol{\varepsilon}_i^2\rangle - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\text{eff}}^2}\left\langle (\mathbf{E}_i^{\phi})^2 \right\rangle\right]$$

space-time average

momentum average

three basis directions in lab frame

Spin density matrix element for vector mesons

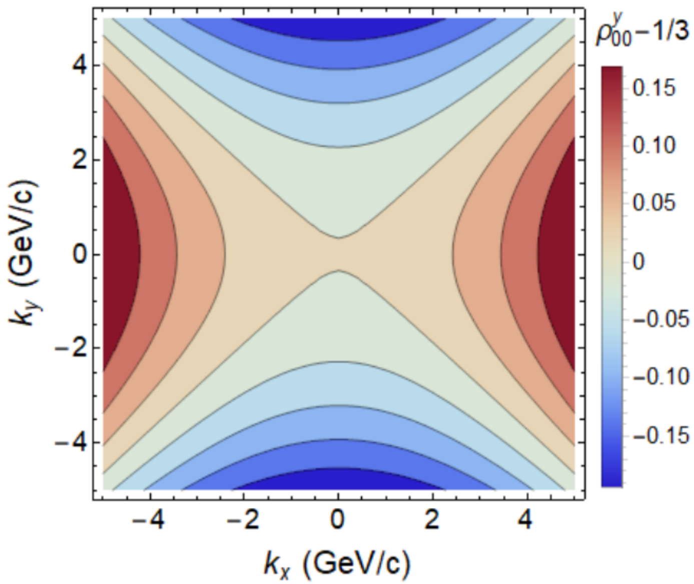


Sheng, Lucia, Liang, QW, et al, 2205.15689, 2206.05868

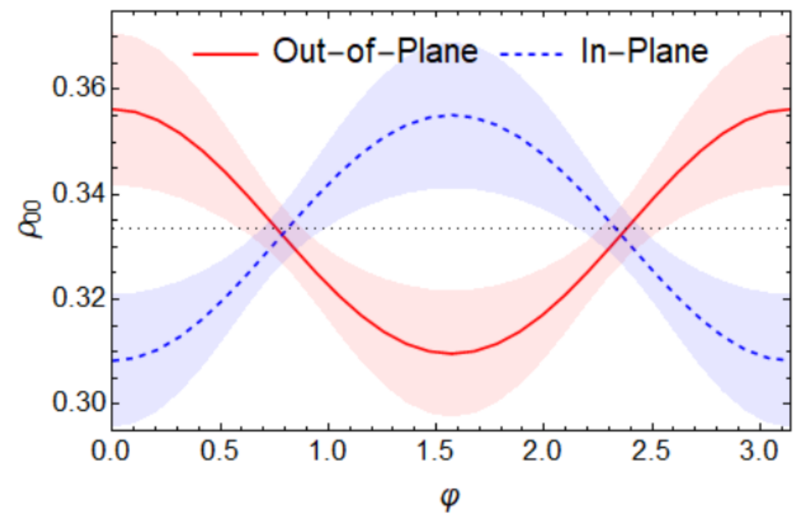
(a) The STAR's data on phi meson's ρ_{00}^{y} (out-of-plane, red stars) and ρ_{00}^{x} (in-plane, blue diamonds) in 0-80% Au+Au collisions as functions of collision energies. The red-solid line and blue-dashed line are calculated with values of F_T^2 and F_z^2 from fitted curves in (b).

(b) Values of F_T^2 (magenta triangles) and F_z^2 (cyan squares) with shaded error bands extracted from the STAR's data on the phi meson's ρ_{00}^{y} and ρ_{00}^{x} in (c). The magenta-dashed line (cyan-solid line) is a fit to the extracted F_T^2 (F_z^2) as a function of $\sqrt{s_{NN}}$ (see the text).

Spin density matrix element for vector mesons

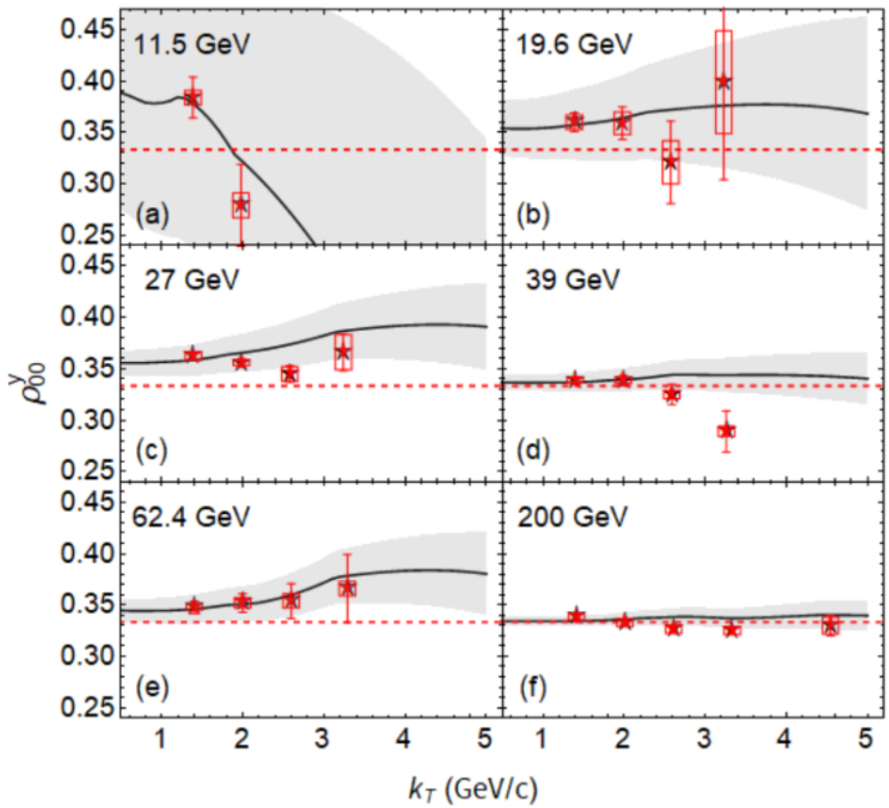


Contour plot of $\rho_{00}^{y} - 1/3$ for ϕ mesons as a function of k_x and k_y in 0-80% Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.



Calculated ρ_{00}^{y} (out-of-plane) and ρ_{00}^{x} (in plane) of ϕ mesons as functions of the azimuthal angle φ in 0-80% Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Shaded error bands are from the extracted parameters F_T^2 and F_z^2.

Spin density matrix element for vector mesons



Sheng, Lucia, Liang, QW, et al, 2205.15689, 2206.05868

Calculated ρ_{00}^{y} (solid line) of ϕ mesons as functions of transverse momenta in 0-80% Au+Au collisions at different colliding energies in comparison with STAR data. Shaded error bands are from the extracted parameters F_T^2 and F_z^2 .

Take-home message and Questions for discussions

Take-home message

- P_Λ **measures the fields (net mean field),** ρ_{00}^{ϕ} **measures field squared (field correlation or fluctuation).**
- **The** ϕ **field is induced by current of pseudo-Goldstone boson during the hadronization**

Questions to be answered in the future:

- **Any connection with QCD sum rules and QCD vacuum properties? Any connection with quark or gluon condensates (trace anomaly)?**
- **Implication for J/Psi polarization (gluon fields)?**
- **Any connection with effects from glasma fields? (Kuma, Mueller, Yang, 2023)**
- **Other contributions from hydro quantities [Li, Liu (2022); Wagner, Weickgenannt, Speranza (2022)]**

Ideal spin hydrodynamics with Wigner functions

H.-H. Peng, J.-J. Zhang, X.-L. Sheng, QW, Chin. Phys. Lett. 38, 116701 (2021)
[Feature: (1) Rigorous power counting scheme; (2) Analytical solution of Wigner function to the 2nd order; (3) Exact evolution equations for spin hydro variables to the 2nd order]

Earlier works:
Florkowski, Friman, Jaiswal, Speranza, Phys.Rev. C97, 041901 (2018)
Florkowski, Friman, Ryblewski, Speranza, Phys.Rev. D97, 116017 (2018)

Review:
Florkowski, Kumar, Ryblewski, Prog.Part.Nucl.Phys. 108, 103709 (2019)

Quantum kinetic equation and Wigner functions

- **The kinetic equation of Wigner function can be derived from the Dirac equation**

$$\left[\gamma_\mu\left(p^\mu+\frac{i}{2}\partial^\mu\right)-m\right]W(x,p)=0$$

- **Power counting**

$$\mathrm{Kn}\sim\left|\frac{\partial_\mu O}{O}\right|\ll 1$$

$$\chi_s\sim\frac{\left|S^{\lambda,\mu\nu}\right|}{n}\sim\frac{\left|M^\mu\right|}{n}\ll 1$$

Wigner function at O(1)

$$\begin{aligned}W_0(x,p)=&\frac{1}{(2\pi)^3}\delta(p^2-m^2)\\&\times\sum_{rs}\left[\theta(p_0)u(r,\mathbf{p})\overline{u}(s,\mathbf{p})f^+_{rs}(x,\mathbf{p})\right.\\&\left.-\theta(-p_0)v(r,-\mathbf{p})\overline{v}(s,-\mathbf{p})f^+_{rs}(x,-\mathbf{p})\right]\end{aligned}$$

Weickgenannt, Sheng, Speranza, QW, Rischke (2019)
Sheng, Weickgenannt, Speranza, Rischke, QW (2021)

The 1st and 2nd solutions to Wigner functions

- The 1st and 2nd order corrections in space-time gradient for the Wigner function can be obtained by solving the kinetic equation

$$\delta W = \frac{i}{4m}\left[\gamma^{\mu}, \partial_{\mu} W_0\right] + \frac{1}{16m^2}(\gamma\cdot\partial)W_0(\gamma\cdot\overleftarrow{\partial}) + \frac{\gamma\cdot p + m}{8m(p^2-m^2)}\partial^2 W_0$$

$$W = W_0 + \delta W$$

Peng, J.-J. Zhang, X.-L. Sheng, QW (2021)

- The appearance of δW is a result of the uncertainty principle for quantum particles with non-local correlation. These corrections include the electric dipole moment induced by an inhomogeneous charge distribution, the magnetization current, and the off-mass-shell correction.

MVSDs and conservation law

- **The MVSDs in thermal equilibrium are assumed to be in the form [Becattini, Chandra, Del Zanna, Grossi, Ann. Phys. (2013)]**

$$f^{+}_{\mathrm{eq},rs}(x,\mathbf{p}) = \frac{1}{2m}\overline{u}(r,\mathbf{p})\left(e^{\beta\cdot p-\xi-\underline{\omega_{\mu\nu}\sigma^{\mu\nu}/4}}+1\right)^{-1}u(s,\mathbf{p})$$

$$f^{-}_{\mathrm{eq},rs}(x,-\mathbf{p}) = -\frac{1}{2m}\overline{v}(r,-\mathbf{p})\left(e^{\beta\cdot\overline{p}-\xi-\underline{\omega_{\mu\nu}\sigma^{\mu\nu}/4}}+1\right)^{-1}v(s,-\mathbf{p})$$

$$p = (E_p,\mathbf{p})$$
$$\overline{p} = (E_p,-\mathbf{p})$$

- **The current density, the energy-momentum tensor (density), and the spin tensor (density) can be obtained from vector and axial vector components of WF**

$$J^{\mu}\left[\beta^{\rho},\xi,\omega^{\rho\sigma}\right] = \int d^4p\mathcal{V}^{\mu}(x,p)$$

$$T^{\mu\nu}\left[\beta^{\rho},\xi,\omega^{\rho\sigma}\right] = \int d^4p p^{\nu}\mathcal{V}^{\mu}(x,p)$$

$$S^{\lambda,\mu\nu}\left[\beta^{\rho},\xi,\omega^{\rho\sigma}\right] = -\frac{1}{2}\epsilon^{\lambda\mu\nu\rho}\int d^4pA_{\rho}(x,p)$$

$$\Longrightarrow$$

$$\partial_\mu J^\mu = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\lambda S^{\lambda,\mu\nu} = T^{\mu\nu} - T^{\nu\mu}$$

Evolution equations for hydro variables

- **Constitutive relations for current, energy momentum and spin tensor to second order in Kn and χ_s**

$$J^{\mu}_{\rm eq} = n_{\rm eq} u^{\mu} + \underline{\delta j^{\mu}},$$

$$T^{\mu\nu}_{\rm eq} = \epsilon_{\rm eq} u^{\mu} u^{\nu} - P_{\rm eq}\Delta^{\mu\nu} + \underline{\delta T^{\mu\nu}_{S} + \delta T^{\mu\nu}_{A}}$$

depend on spin potential $\omega_{\mu\nu}$

$$\boxed{S^{\lambda,\mu\nu}_{\rm eq}(x) = \frac{1}{4}\left(u^{\lambda}\omega^{\mu\nu} + 2u^{[\mu}\omega^{\nu]\lambda}\right) K_1 \cosh\xi}$$

spin tensor

- **The equations of motions for β^{μ}, ξ, $\omega_{\mu\nu}$**

$$\dot{\beta} = \frac{K_2 + \beta^{-1} K_1 \cosh^2\xi}{K_1 K_3 \cosh^2\xi - K_2 K_2 \sinh^2\xi} K_1 \theta\,,$$

$$\dot{\xi} = \frac{\left(K_2 + \beta^{-1} K_1\right) K_2 - K_1 K_3}{K_1 K_3 \cosh^2\xi - K_2 K_2 \sinh^2\xi} \theta \sinh\xi \cosh\xi$$

$$\dot{u}^{\mu} = \frac{K_1}{K_1 + \beta K_2} \tanh\xi \nabla^{\mu}\xi - \frac{1}{\beta}\nabla^{\mu}\beta\,,$$

$$\frac{d}{d\tau} \equiv u_{\mu}\partial^{\mu}, \quad \nabla^{\mu} \equiv \Delta^{\mu\nu}\partial_{\nu}$$

$$K_n(\beta) \equiv \frac{8}{(2\pi)^3}\int \frac{d^3\mathbf{p}}{2E_{\mathbf{p}}} E^n_{\mathbf{p}} e^{-\beta E_{\mathbf{p}}}$$

$$\boxed{\dot{\omega}^{\mu\nu} = \Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta}\dot{\omega}^{\alpha\beta} - u^{\mu}\dot{\omega}^{\nu\alpha}u_{\alpha} + u^{\nu}\dot{\omega}^{\mu\alpha}u_{\alpha}}$$

evolution equation for spin potential

MVSDs and conservation law

- **Here the terms in l.f.s. of the evolution equition for $\boldsymbol{\omega^{\mu\nu}}$ are**

$$
\begin{aligned}
\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta}\dot{\omega}^{\alpha\beta} &= C_3\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta}\omega^{\alpha\beta} + C_4\Delta^{[\mu}_{\beta}\sigma_h^{\nu]\rho}\omega^{\beta}{}_{\rho} \\
&\quad -\frac{1}{2}C_4(\nabla^{[\mu}\omega^{\nu]}{}_{\rho})u^{\rho} + C_2C_4u^{\rho}\omega^{[\mu}{}_{\rho}\nabla^{\nu]}\xi \\
\dot{\omega}^{\mu\nu}u_{\nu} &= C_1\omega^{\mu\nu}u_{\nu} + C_2\Delta^{\mu}_{\rho}\omega^{\rho\nu}\nabla_{\nu}\xi \\
&\quad +\sigma_h^{\mu\nu}\omega_{\nu\rho}u^{\rho} + \frac{1}{2}\Delta^{\mu}_{\rho}(\nabla^{\nu}\omega^{\rho}{}_{\nu}),
\end{aligned}
$$

- **where $\boldsymbol{C_i}$ $(\boldsymbol{i} = \mathbf{1,2,3,4})$ are analytical function of hydro variables $(\boldsymbol{\beta},\ \boldsymbol{\xi},\ \boldsymbol{\theta},\ \dot{\boldsymbol{\beta}},\ \dot{\boldsymbol{\xi}})$**

Peng, J.-J. Zhang, X.-L. Sheng, QW (2021)

Viscous spin hydrodynamics

Hattori, Hongo, et al., PLB(2019); Li, Stephanov, Yee, PRL(2021); Fukushima, Pu, PLB (2021); Bhadury, Florkowski, et al., PRD(2021); Weickgenannt, Wagner, et al., PRD(2022); She, Huang, et al., Sci.Bull. (2022); many others ……..

First order viscous spin hydrodynamics

1. Introduce spin potential term $\omega_{\mu\nu}S^{\mu\nu}$ into Gibbs-Duhem relation, assume constitutive relation for spin tensor $S^{\mu\nu}[u^{\alpha},\omega^{\alpha\beta}]$
2. Introduce anti-symmetric term into EM tensor $T^{\mu\nu}_{asym}[q^{\alpha},\phi^{\alpha\beta}]$
3. From entropy principle (divergence of entropy current should be non-negative), one obtains expressions for $q^{\mu}[u^{\alpha},\omega^{\alpha\beta}]$ and $\phi^{\mu\nu}[u^{\alpha},\omega^{\alpha\beta}]$.

Summary

Wigner function approach as quantum kinetic theory in phase space

Spin Boltzmann equation with local and non-local collisions

Spin hydrodynamics
Local and global equilibrium of spin