

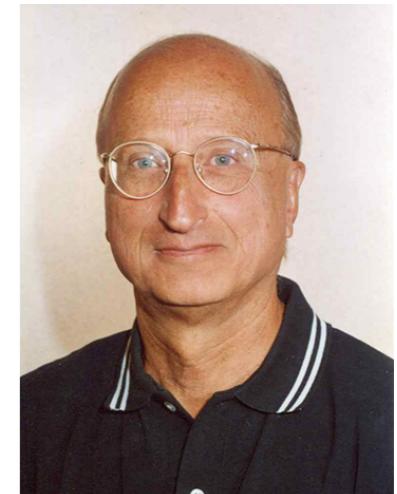


Peter Schuck
(1940-2022)

Intersection of nuclear structure and heavy ion collisions across energy scales: perspective from low-energy nuclear structure

*“Ab-initio”
Equation of Motion
(EOM) framework*

*Phenomenological
Nuclear Field Theory(NFT)
by A. Bohr, B. Mottelson,
R. Broglia, P.-F. Bortignon et al.
NBI Copenhagen*



Ricardo Broglia
(1939-2022)

Elena Litvinova

Western Michigan University

MICHIGAN STATE
UNIVERSITY



Collaborators: Peter Schuck, Peter Ring, Yinu Zhang, Herlik Wibowo, Manqoba Hlatshwayo

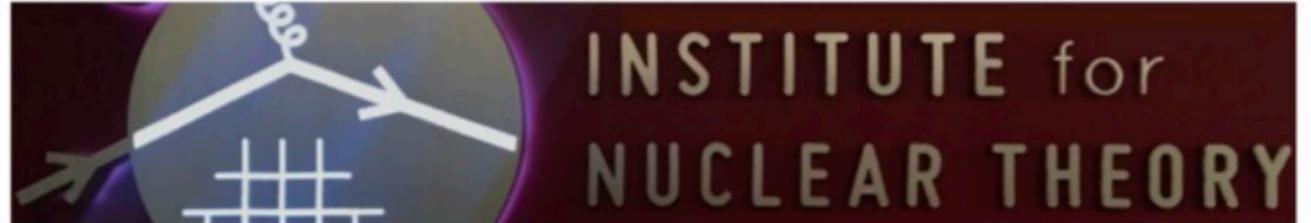
Outline: (Potential) intersections

Intersection of nuclear structure and high-energy nuclear collisions

Organizers:

Jiangyong Jia (Stony Brook & BNL)
Giuliano Giacalone (ITP Heidelberg)
Jacquelyn Noronha-Hostler (Urbana-Champaign)
Dean Lee (Michigan State & FRIB)
Matt Luzum (São Paulo)
Fuqiang Wang (Purdue)

Jan 23rd - Feb 24th 2023



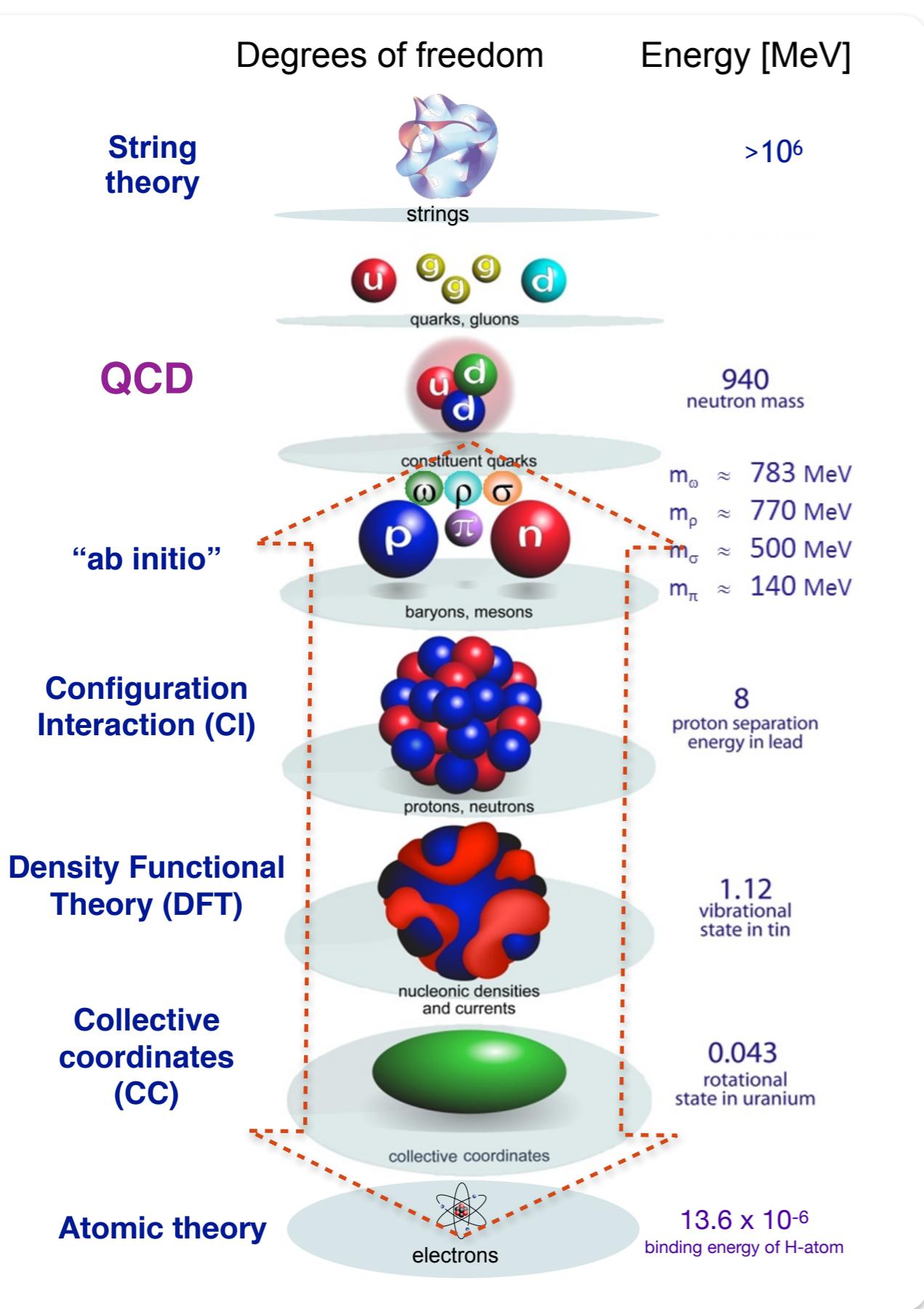
[G. Giacalone Wed@9:30]



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- Many-body theory: a general framework across the energy scales
- Understanding correlations / interactions
- The role of nuclear shapes in RHIC [G. Giacalone Wed@9:30]
- Giant resonances in cold and hot nuclei (peripheral RHIC) [D. Brandenburg]
- Transport theory, emergent hydrodynamics and (linear) response [X. Du Mon@16:30, Q. Wang Tue@11:30 etc.]
- Quantum computation [J. Barata Tue@9:30]

Hierarchy of energy scales and nuclear many-body problem



- **The major conflict:**

Separation of energy scales => effective field theories

vs

The physics on a certain scale is governed by the next higher-energy scale

Hamiltonian:

$$H = K + V$$

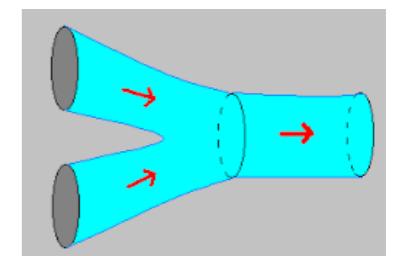
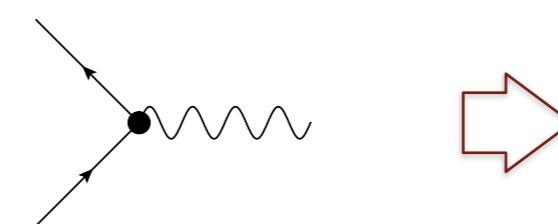
center of mass

internal degrees of freedom:
next energy scale

Standard Model:

free propagation and interaction, singularities & renormalizations

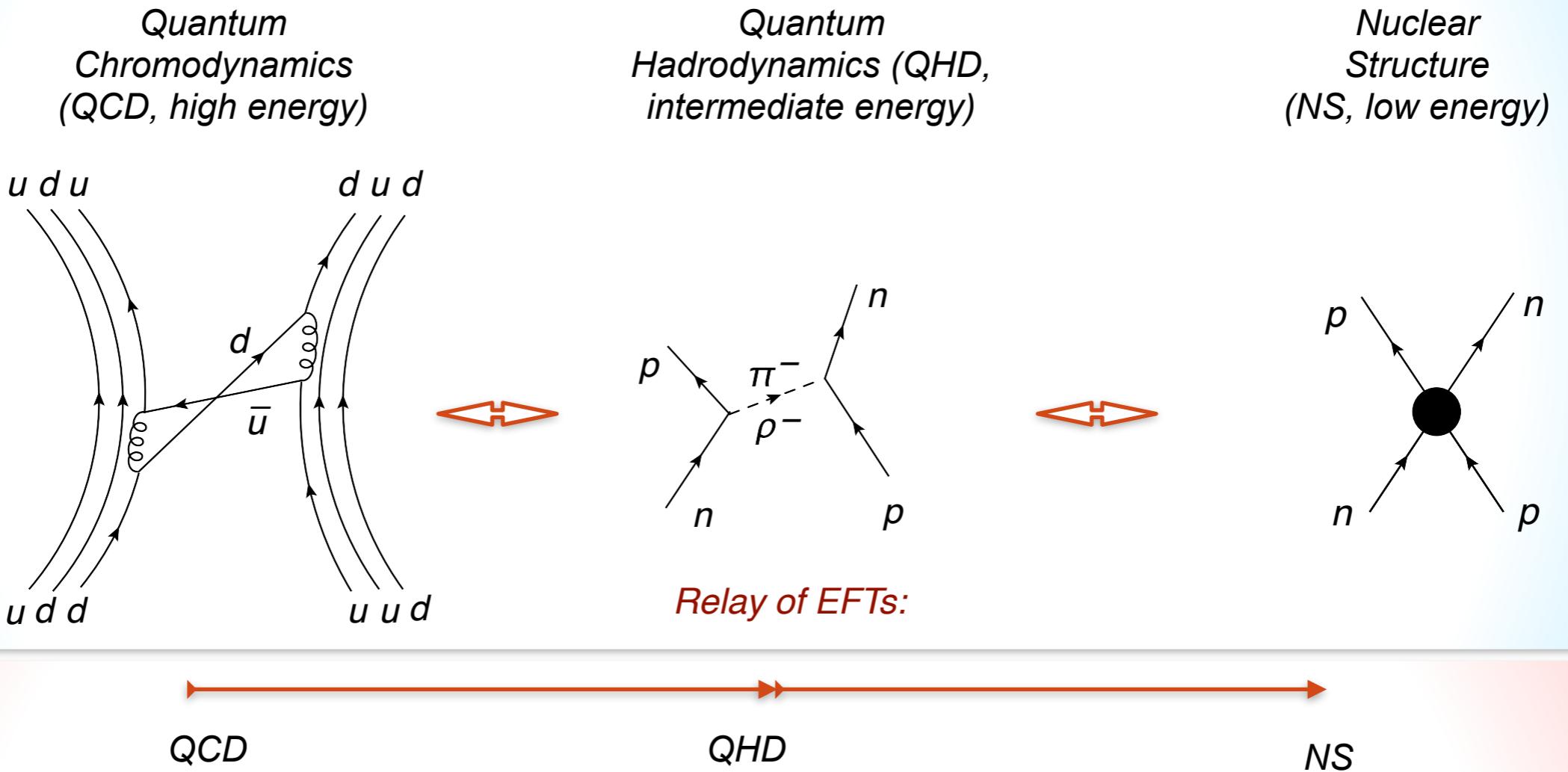
String theory:
merging strings
NO "Interaction"



- **Possible solution:**

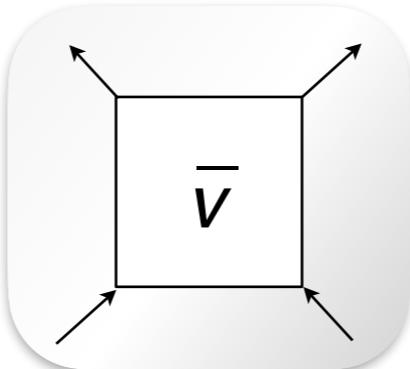
- Keep/establish connections between the scales via emergent phenomena
- A universal approach to the strongly-coupled QMBP?

The underlying mechanism of NN-interaction:



Formalism:

- Generic bare “interaction”: model-independent, all channels included
- Higher-orders are treated via **in-medium propagators**
- No perturbation theory



In implementations:

- Meson-exchange (ME) at leading order
- Effective coupling constants/masses (adjusted on the mean-field (MF) level, NL3(*)) + subtraction of qPVC
- Bare ME + subtraction of MF artifacts (in progress)

A strongly-correlated many body system: single-fermion propagator, particle-hole propagator and related observables

$$H = \sum_{12} \bar{\psi}_1 (-i\gamma \cdot \nabla + M)_{12} \psi_2 + \frac{1}{4} \sum_{1234} \bar{\psi}_1 \bar{\psi}_2 \bar{\psi}_{1234} \psi_4 \psi_3 = T + V^{(2)}$$

Hamiltonian,
extendable to 3B forces
(3BFs are minimized in covariant theories)

$$G_{11'}(t - t') = -i \langle T \psi(1) \bar{\psi}(1') \rangle$$

$$1 = \{\xi_1, t\}$$

Single-particle propagator

$$G_{11'}(\varepsilon) = \sum_n \frac{\eta_1^n \bar{\eta}_{1'}^{n*}}{\varepsilon - \varepsilon_n^+ + i\delta} + \sum_m \frac{\chi_1^m \bar{\chi}_{1'}^{m*}}{\varepsilon + \varepsilon_m^- - i\delta}$$

Fourier transform:
Spectral expansion

$$\eta_1^n = \langle 0^{(N)} | \psi_1 | n^{(N+1)} \rangle$$

$$\chi_1^m = \langle m^{(N-1)} | \psi_1 | 0^{(N)} \rangle$$

Ground state of N particles

(Excited) state of $(N+1)$ particles

Residues - spectroscopic (occupation) factors

Poles - single-particle energies

$$R_{12,1'2'}(t - t') = -i \langle T (\bar{\psi}_1 \psi_2)(t) (\bar{\psi}_{2'} \psi_{1'})(t') \rangle$$

Particle-hole response function

Fourier transform: Spectral expansion

$$R_{12,1'2'}(\omega) = \sum_{\nu>0} \left[\frac{\rho_{21}^\nu \bar{\rho}_{2'1'}^{\nu*}}{\omega - \omega_\nu + i\delta} - \frac{\bar{\rho}_{12}^{\nu*} \rho_{1'2'}^\nu}{\omega + \omega_\nu - i\delta} \right]$$

Excitation energies

$$\rho_{12}^\nu = \langle 0 | \bar{\psi}_2 \psi_1 | \nu \rangle$$

Residues - transition densities

Poles - excitation energies

Exact equations of motion (EOM) for binary interactions: one-body problem

One-fermion propagator

$$G_{11'}(t - t') = -i\langle T\psi(1)\bar{\psi}(1') \rangle$$

EOM: Dyson Eq.

$$G(\omega) = G^{(0)}(\omega) + G^{(0)}(\omega)\Sigma(\omega)G(\omega) \quad (*) \quad \Sigma(\omega) = \Sigma^{(0)} + \Sigma^{(r)}(\omega)$$

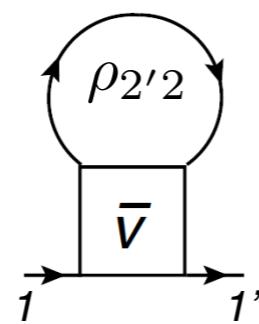
Irreducible kernel (Self-energy, exact):

Instantaneous term (Hartree-Fock incl. “tadpole”)

Short-range correlations

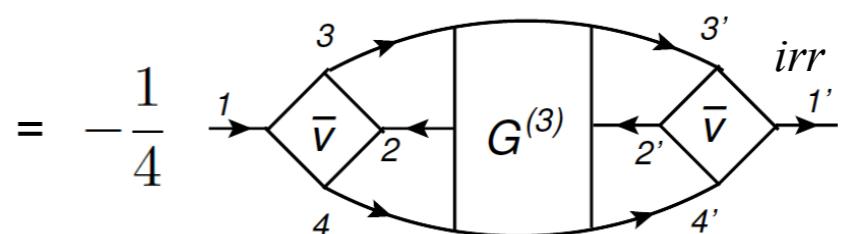
$$\Sigma_{11'}^{(0)} = -\langle \gamma^0 \{ [V, \psi_1], \bar{\psi}_{1'} \} \gamma^0 \rangle$$

$$= \sum_{22'} \bar{v}_{121'2'} \langle \bar{\psi}_2 \psi_{2'} \rangle$$



t-dependent (dynamical) term (symmetric version):
Long-range correlations: connecting scales
Symmetric form

$$\begin{aligned} \Sigma_{11'}^{(r)} &= i\langle T\gamma^0[V, \psi_1](t)[V, \bar{\psi}_{1'}](t')\gamma^0 \rangle^{irr} \\ &= -\frac{1}{4} \sum_{234} \sum_{2'3'4'} \bar{v}_{1234} G_{432', 23'4'}^{(3)irr}(t - t') \bar{v}_{4'3'2'1'} \end{aligned}$$



$$\rho_{11'} = -i \lim_{t=t'-0} G_{11'}(t - t')$$

is the full solution of (*):
includes the dynamical term!

Koltun-Migdal-Galitsky sum rule: **the binding energy**

“Ab-initio DFT”:

$$E_0 = \frac{1}{2\pi} \int_{-\infty}^{\varepsilon_F^-} d\varepsilon \sum_{12} (T_{12} + \varepsilon \delta_{12}) \text{Im}G_{21}(\varepsilon)$$

Equation of motion (EOM) for the particle-hole response

Particle-hole propagator
(response function):

$$R_{12,1'2'}(t - t') = -i \langle T(\bar{\psi}_1 \psi_2)(t)(\bar{\psi}_{2'} \psi_{1'})(t') \rangle$$

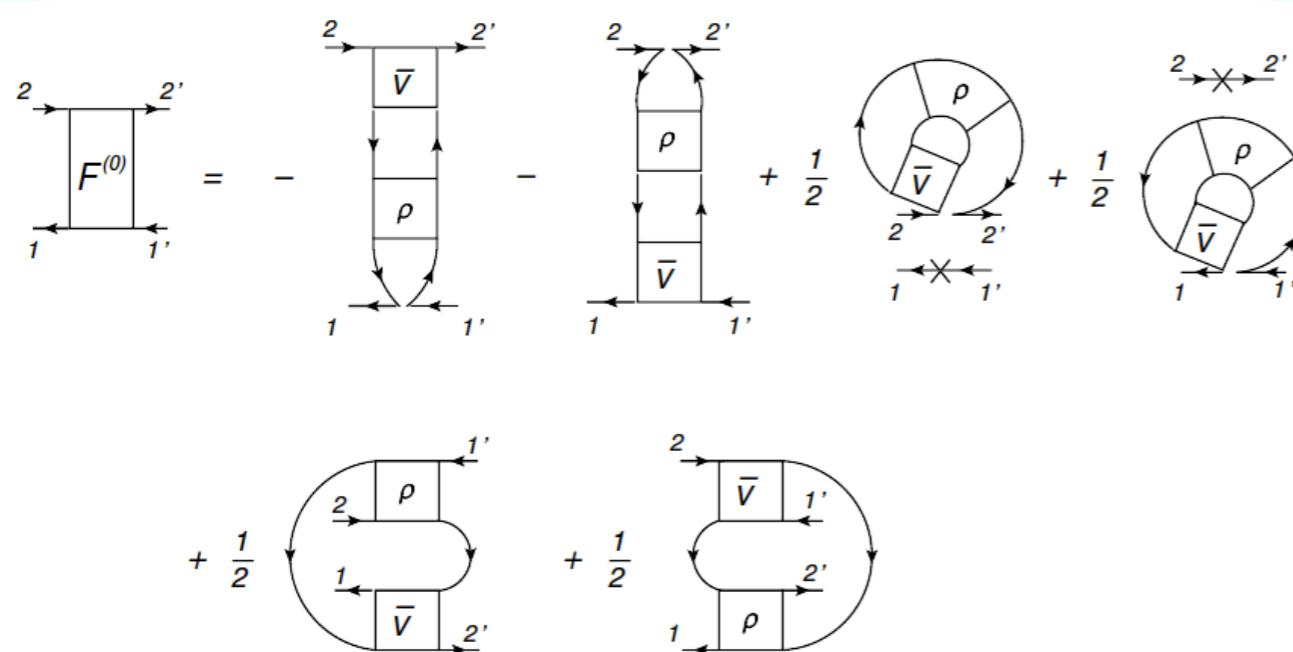
spectra of excitations,
masses, decays, ...

EOM: Bethe-Salpeter-Dyson Eq.

$$R(\omega) = R^{(0)}(\omega) + R^{(0)}(\omega)F(\omega)R(\omega) \quad (**)$$

Irreducible kernel (exact):

Instantaneous term (“bosonic” mean field):
Short-range correlations

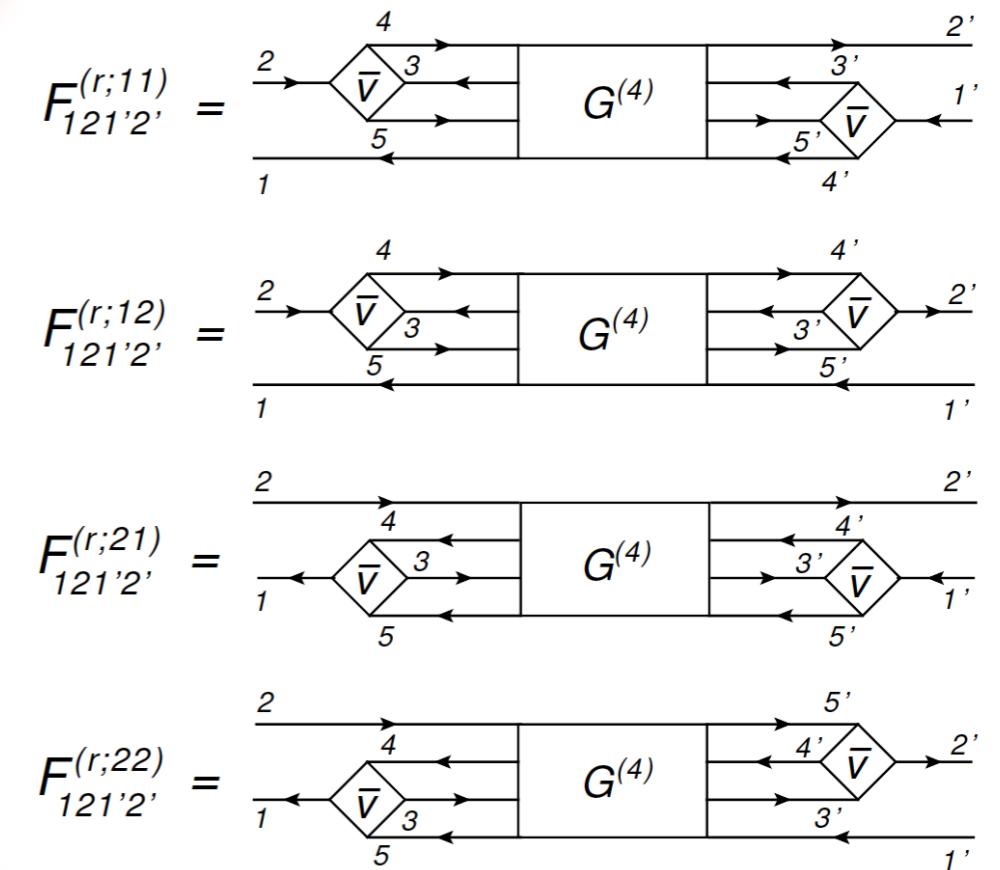


Self-consistent mean field $F^{(0)}$, where

$$\rho_{12,1'2'} = \delta_{22'} \rho_{11'} - i \lim_{t' \rightarrow t+0} R_{2'1,21'}(t - t')$$

contains the full solution of (**) including the dynamical term!

t-dependent (dynamical) term:
Long-range correlations



$$F_{12,1'2'}^{(r)}(t - t') = \sum_{ij} F_{12,1'2'}^{(r;ij)}(t - t')$$

Dynamical kernels: bridging the scales

- **Quantum many-body problem in a nutshell:** Direct EOM for $G^{(n)}$ generates $G^{(n+2)}$ in the (symmetric) dynamical kernels and further high-rank correlation functions (CFs); an equivalent of the BBGKY hierarchy or Schwinger-Dyson equations. $N_{\text{Equations}} = N_{\text{Particles}} \& \text{ Coupled}$ 😱 !!!

- **Non-perturbative solutions:**

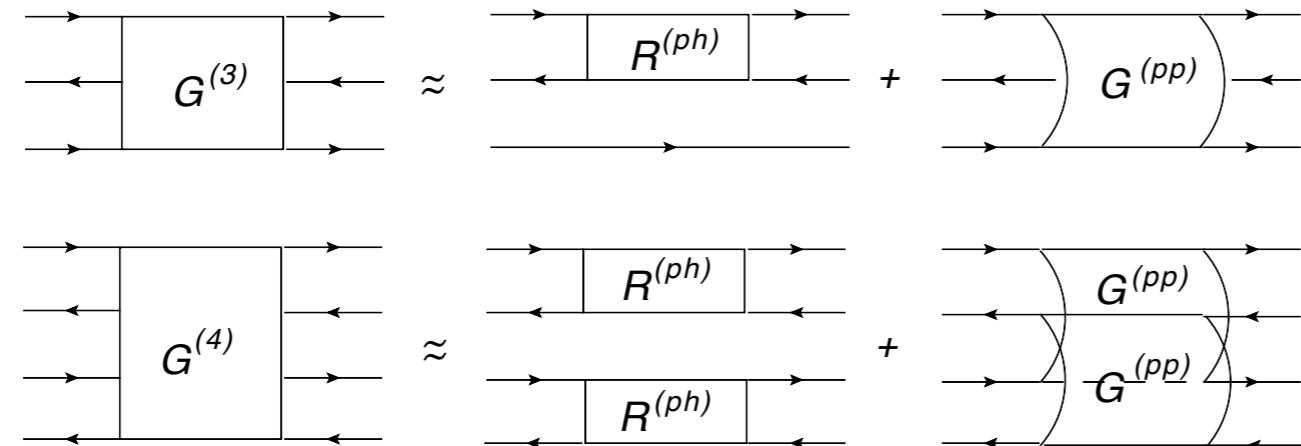
Cluster decomposition

Leading at:

$$\begin{aligned} \blacklozenge G^{(3)} &= \boxed{G^{(1)} G^{(1)} G^{(1)}} + \boxed{G^{(2)} G^{(1)}} + \boxed{\Xi^{(3)}} \\ \blacklozenge G^{(4)} &= \boxed{G^{(1)} G^{(1)} G^{(1)} G^{(1)}} + \boxed{G^{(2)} G^{(2)}} + \boxed{G^{(3)} G^{(1)}} + \boxed{\Xi^{(4)}} \end{aligned}$$

weak self-consistent GFs second RPA etc.	intermediate phonon coupling <i>this work</i>	strong Faddeev + future work	coupling
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Beyond weak coupling:

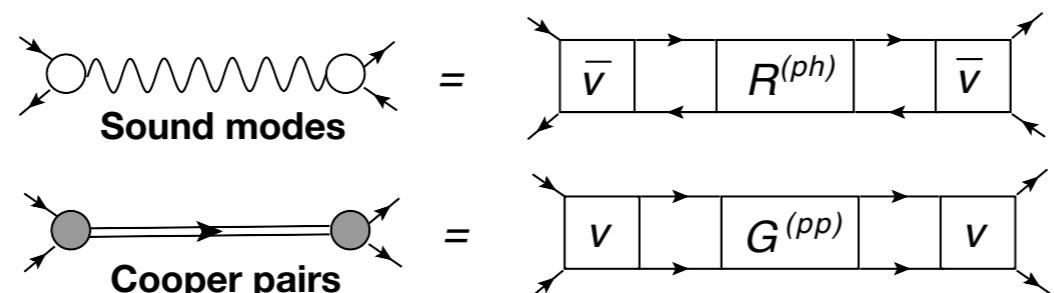


- P. C. Martin and J. S. Schwinger, Phys. Rev. 115, 1342 (1959).
- N. Vinh Mau, Trieste Lectures 1069, 931 (1970)
- P. Danielewicz and P. Schuck, Nucl. Phys. A567, 78 (1994)
- ...

Emergence of effective “bosons” (phonons, vibrations):

Emergence of superfluidity:

Exact mapping: particle-hole ($2q$) quasibound states



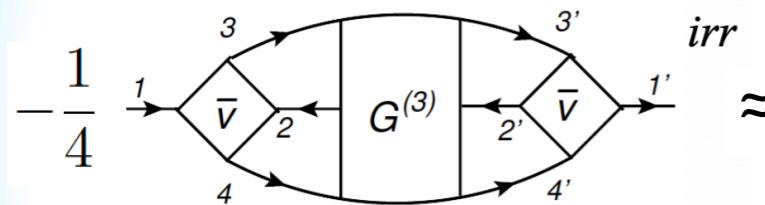
Quasiparticle-vibration coupling (qPVC) in nuclei. Cf. NFT

Diquarks in hadrons

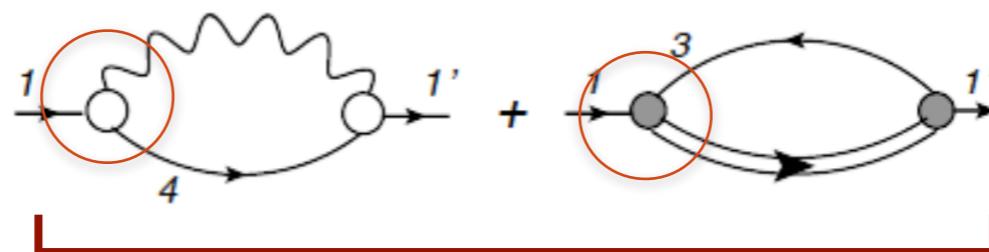
Cf. C. Popovici, P. Watson, and H. Reinhardt [PRD83, 025013 (2011)]

Emergence of effective degrees of freedom at intermediate coupling

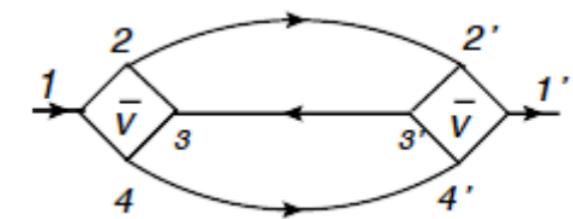
Dynamical self-energy $\Sigma^{(r)}$:



“Radiative-correction”



“Second-order”



Can be treated in a unified way (see below):
qPVC in nuclei, ? in hadrons / matter

Emergent phonon vertices and propagators: **calculable from the underlying H** , which does not contain phonon degrees of freedom

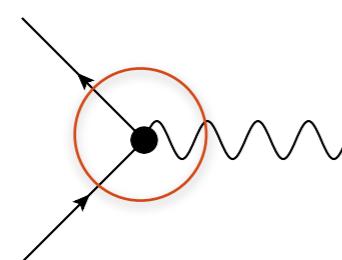
$$H = \sum_{12} h_{12} \psi_1^\dagger \psi_2 + \frac{1}{4} \sum_{1234} \bar{v}_{1234} \psi_1^\dagger \psi_2^\dagger \psi_4 \psi_3$$

“Ab-initio”

$$H = \sum_{12} \tilde{h}_{12} \psi_1^\dagger \psi_2 + \sum_{\lambda \lambda'} \mathcal{W}_{\lambda \lambda'} Q_\lambda^\dagger Q_{\lambda'} + \sum_{12\lambda} \left[\Theta_{12}^\lambda \psi_1^\dagger Q_\lambda^\dagger \psi_2 + h.c. \right]$$

Effective qPVC

Cf.: The Standard Model elementary interaction vertices: boson-exchange interaction is the **input**:



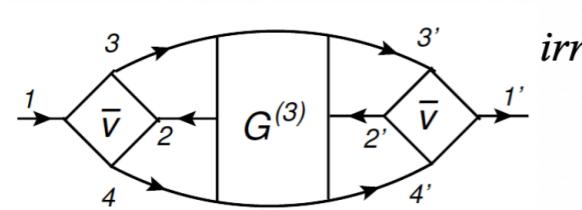
γ, g, W^\pm, Z^0

Possibly derivable?

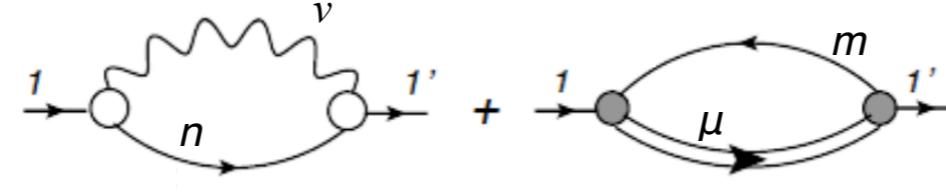
E.L., P. Schuck, PRC 100, 064320 (2019)
E.L., Y. Zhang, PRC 104, 044303 (2021)

Problems with approximate treatments: poles “mismatch”, (non)-positivity and optical theorem

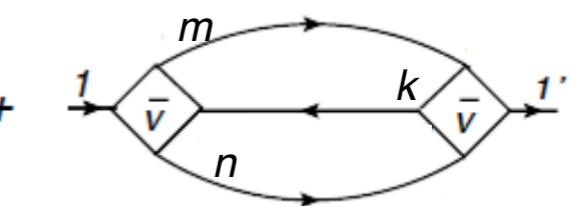
Dynamical self-energy:



“Radiative-correction”



“Second-order”



Approximate:

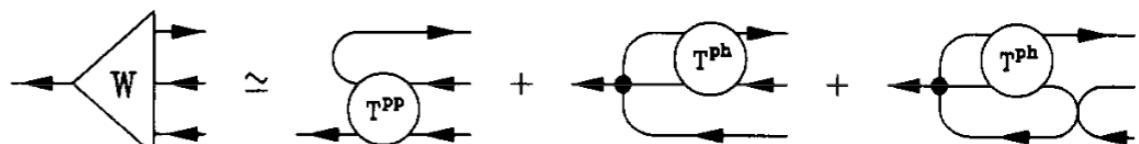
$$\Sigma_{11'}^{(r)+}(\omega) = \sum_{33'} \sum_{\nu n} \frac{\eta_3^n g_{13}^\nu g_{1'3'}^{\nu*} \eta_{3'}^{n*}}{\omega - \omega_\nu - \varepsilon_n^{(+)} + i\delta} + \sum_{22'} \sum_{\mu m} \frac{\chi_2^{m*} \gamma_{12}^{\mu(+)} \gamma_{1'2'}^{\mu(+)*} \chi_{2'}^m}{\omega - \omega_\mu^{(++)} - \varepsilon_m^{(-)} + i\delta} - \sum_{mnk} \frac{w_1^{mnk} w_{1'}^{mnk*}}{\omega - \varepsilon_m^+ - \varepsilon_n^+ - \varepsilon_k^- + i\delta}$$

Exact:

$$\Sigma_{11'}^{(r)+}(\omega) \sim \langle v G^{(3)+}(\omega) v \rangle_{11'}$$

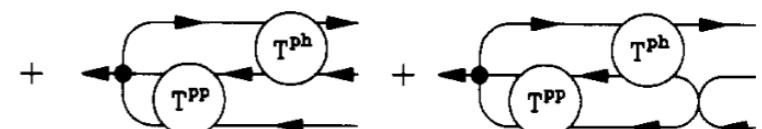
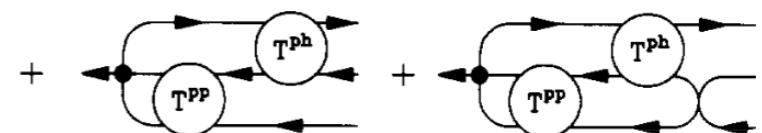
Watson-Faddeev series:
P. Danielewicz and P. Schuck, Nucl. Phys. A567, 78 (1994):

$$G_{432', 23'4'}^{(3)+}(\omega) = \sum_{\pi} \frac{\langle 0 | \bar{\psi}_2 \psi_4 \psi_3 | \pi \rangle \langle \pi | \bar{\psi}_{3'} \bar{\psi}_{4'} \psi_{2'} | 0 \rangle}{\omega - \omega_\pi + i\delta}$$



2p1h-RPA

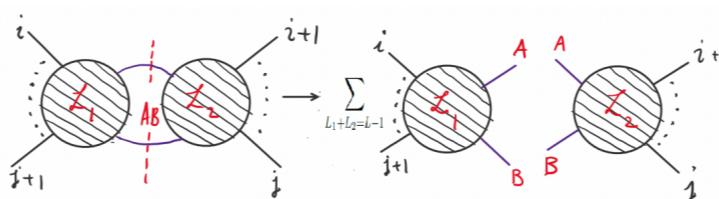
P. Schuck, F. Villars and P. Ring
Nucl. Phys. A208, 302 (1973)



Scattering amplitudes in particle physics

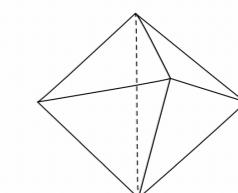
(Yang-Mills theories etc.):

- Positivity preserved when eliminating all the virtual particles
- Amplitudes \Leftrightarrow Polyhedron “living” in the kinematic space
- Emergent unitarity



N. Arkani-Hamed, J. Trnka,
A. Hodges et al.

Amplitude is a volume of polyhedron



Each face labeled by $\langle abcd \rangle$

$$\frac{\langle 1345 \rangle^3}{\langle 1234 \rangle \langle 1245 \rangle \langle 2345 \rangle \langle 1235 \rangle} \quad \frac{\langle 1356 \rangle^3}{\langle 1235 \rangle \langle 1256 \rangle \langle 2356 \rangle \langle 1236 \rangle}$$

“Ab-initio” qPVC in superfluid systems

Superfluid dynamical kernel: adding particle-number violating contributions

Mapping on the qPVC in the canonical basis

	=	
	=	
	=	
	=	
	=	
	=	

*Quasiparticle dynamical self-energy (matrix):
normal and pairing phonons are unified*

$$\hat{\Sigma}^r = \left(\begin{array}{c} \text{Feynman diagrams for } \hat{\Sigma}^r \\ \text{involving } \text{wavy lines and } \text{solid lines} \end{array} \right)$$



*Bogoliubov
transformation*

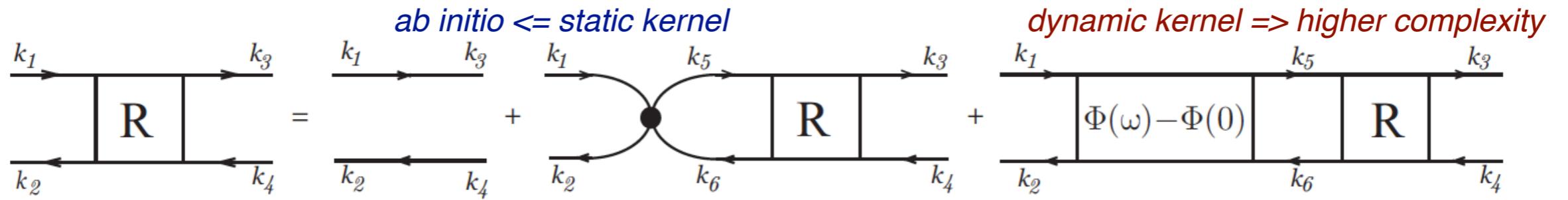
**Compact form,
(almost) as simple as
non-superfluid**

E.L., Y. Zhang, PRC 104, 044303 (2021)
Y. Zhang et al., PRC 105, 044326 (2022)

*Cf.: Quasiparticle static
self-energy (matrix) in HFB*

$$\hat{\Sigma}^0 = \begin{pmatrix} \tilde{\Sigma}_{11'} & \Delta_{11'} \\ -\Delta_{11'}^* & -\tilde{\Sigma}_{11'}^T \end{pmatrix}$$

Nuclear response: toward a complete theory



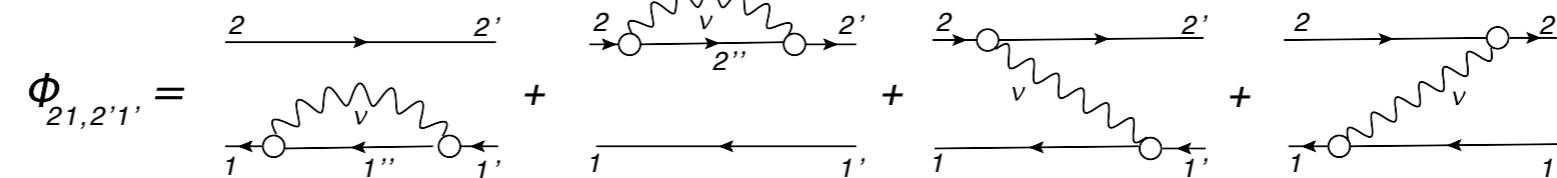
Dyson-Bethe-Salpeter Equation:

Conventional NFT: derived as the leading approximation

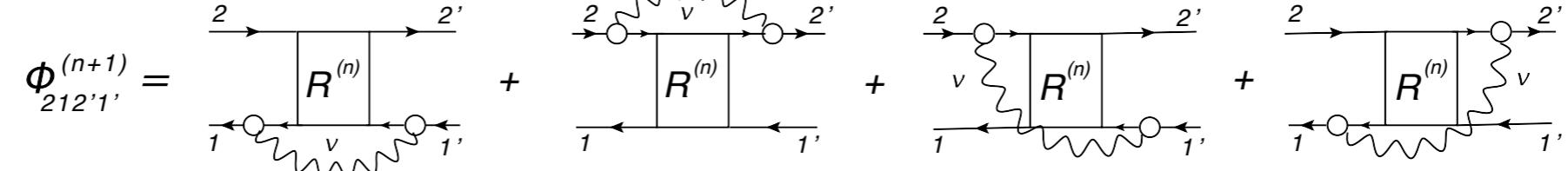
Cf. P.-F. Bortignon,
G. Colò, E. Vigezzi,
G. Potel, F. Barranco
&
V. Tselyaev (t-blocking)

$$R(\omega) = R^0(\omega) + R^0(\omega) [V + \Phi(\omega) - \Phi(0)] R(\omega)$$

Subtraction for effective interactions (Tselyaev 2013)



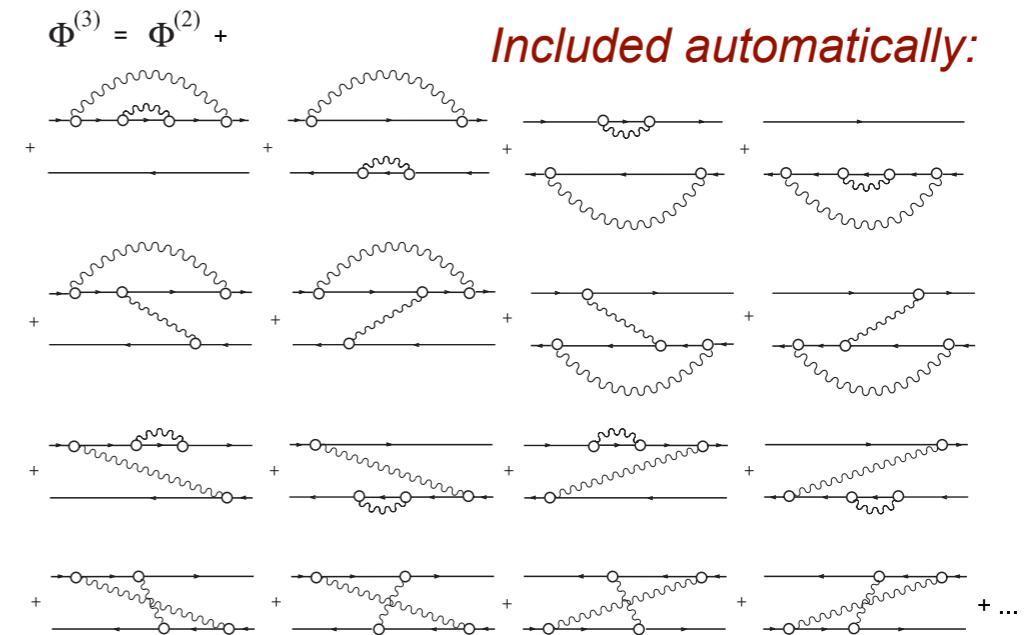
Extended NFT:



Generalized approach for the correlated propagators

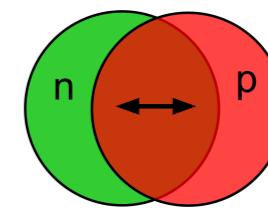
n -th order: E.L. PRC 91, 034332 (2015)

Ab-initio formulation,
 $\Phi^{(3)}$ implementation; 2q+2phonon correlations:
E.L., P. Schuck, PRC 100, 064320 (2019)



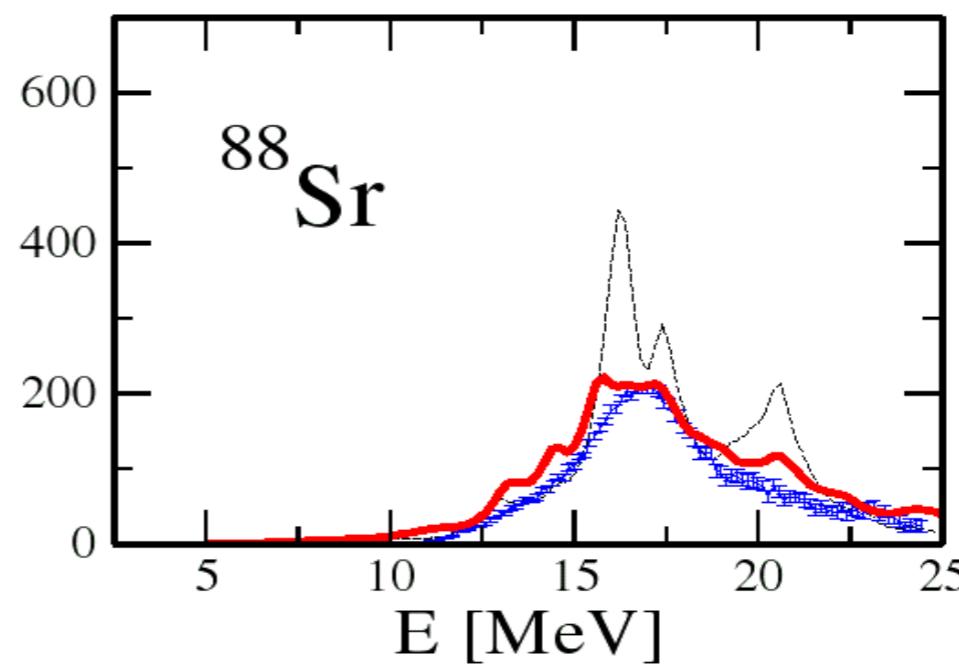
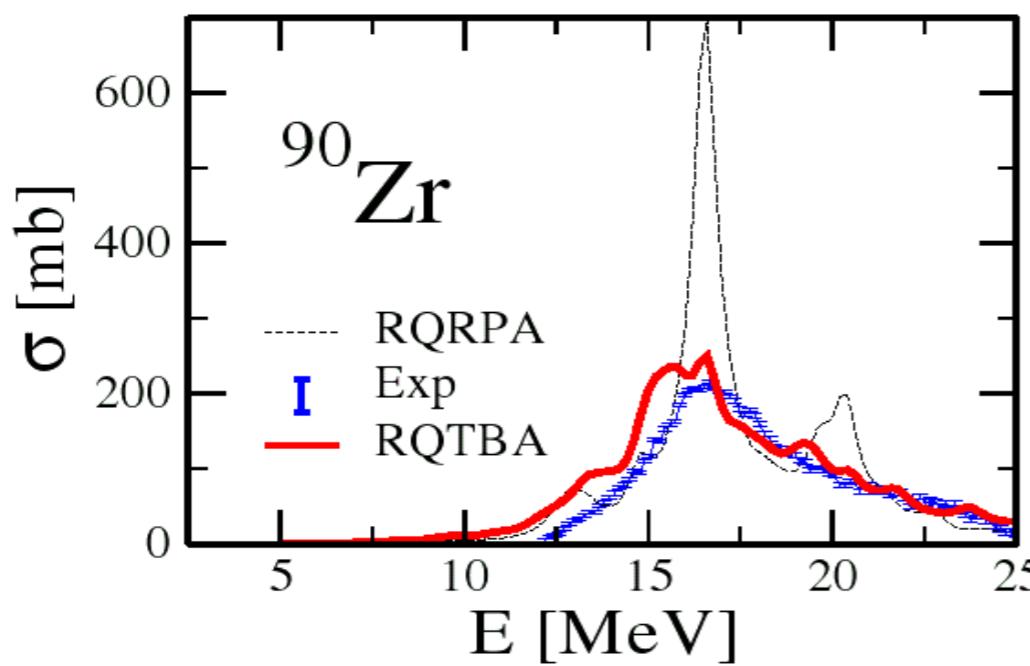
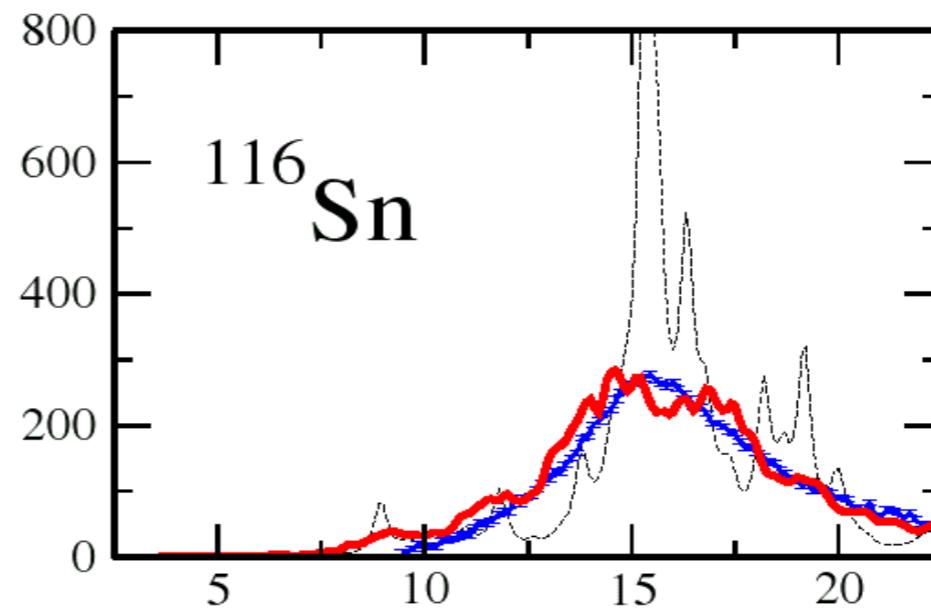
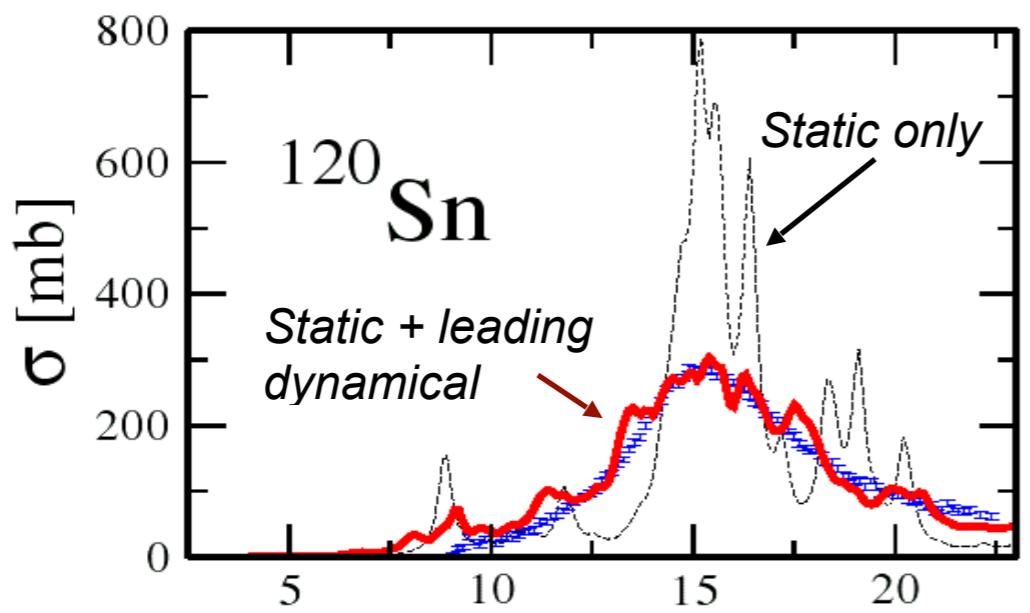
Giant Dipole Resonance within Relativistic Quasiparticle Time Blocking Approximation (RQTBA)

Response to the operator: $P = \sum_{i=1}^A \left(\tau_z^{(i)} - \frac{N - Z}{2A} \right) r_i Y_{1M}(\hat{\vec{r}}_i)$

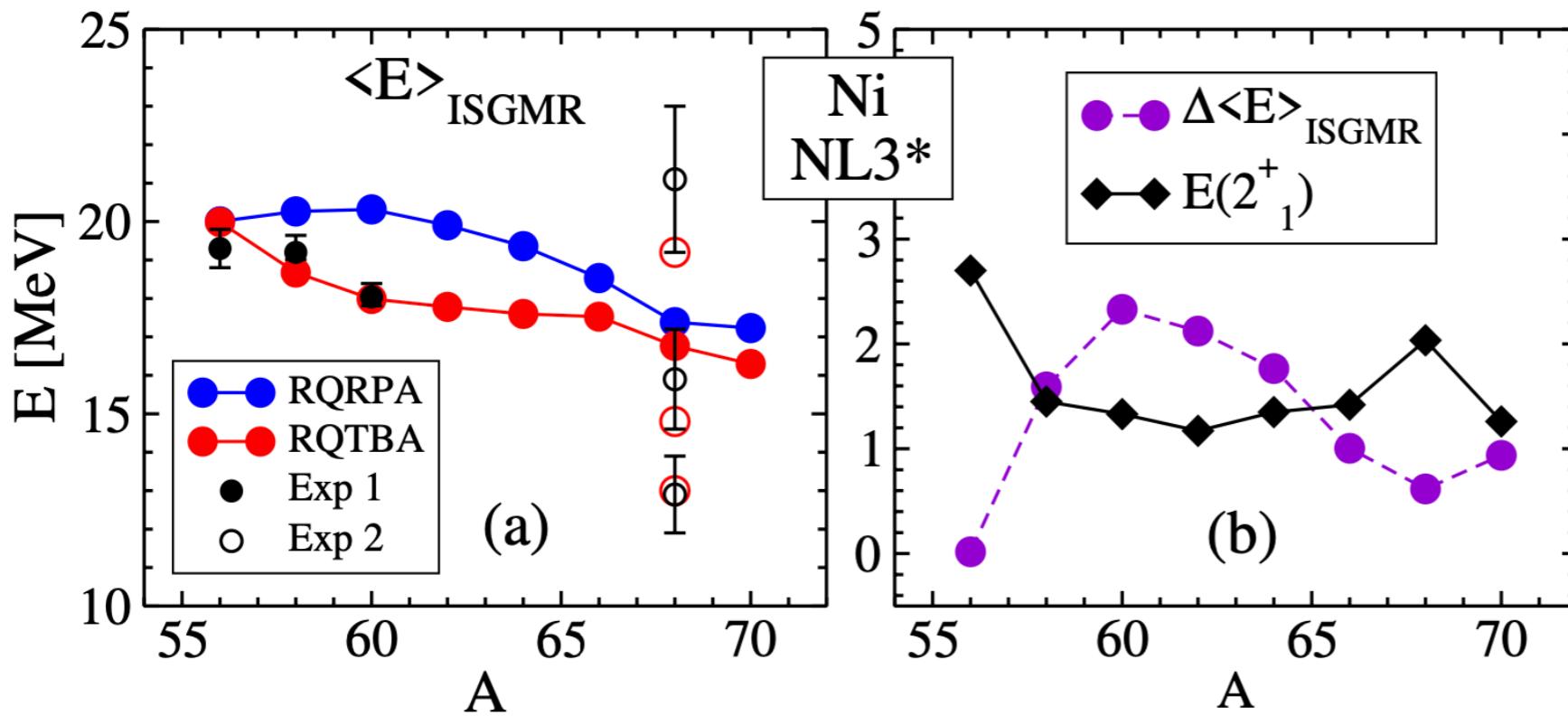
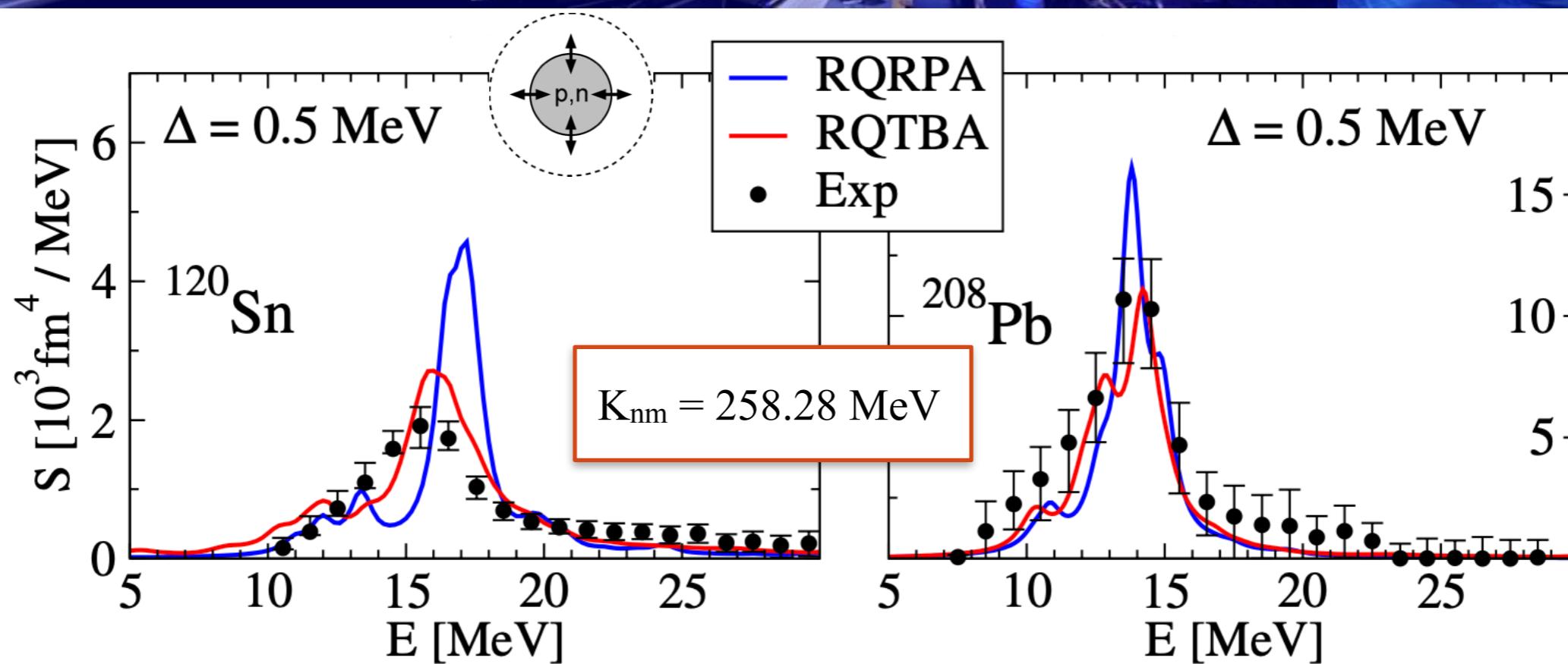


$$\begin{aligned}\Delta L &= 1 \\ \Delta T &= 1 \\ \Delta S &= 0\end{aligned}$$

Accurate GDR description is important for the analysis of ultra peripheral RHIC [D. Brandenburg]



Isoscalar giant monopole resonance (ISGMR): The “fluffiness” puzzle and nuclear incompressibility



“Softness” increases:

- with the neutron number
- with superfluidity
- with correlations beyond QRPA (q)PVC

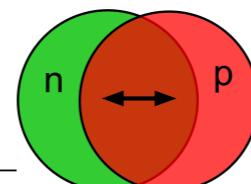
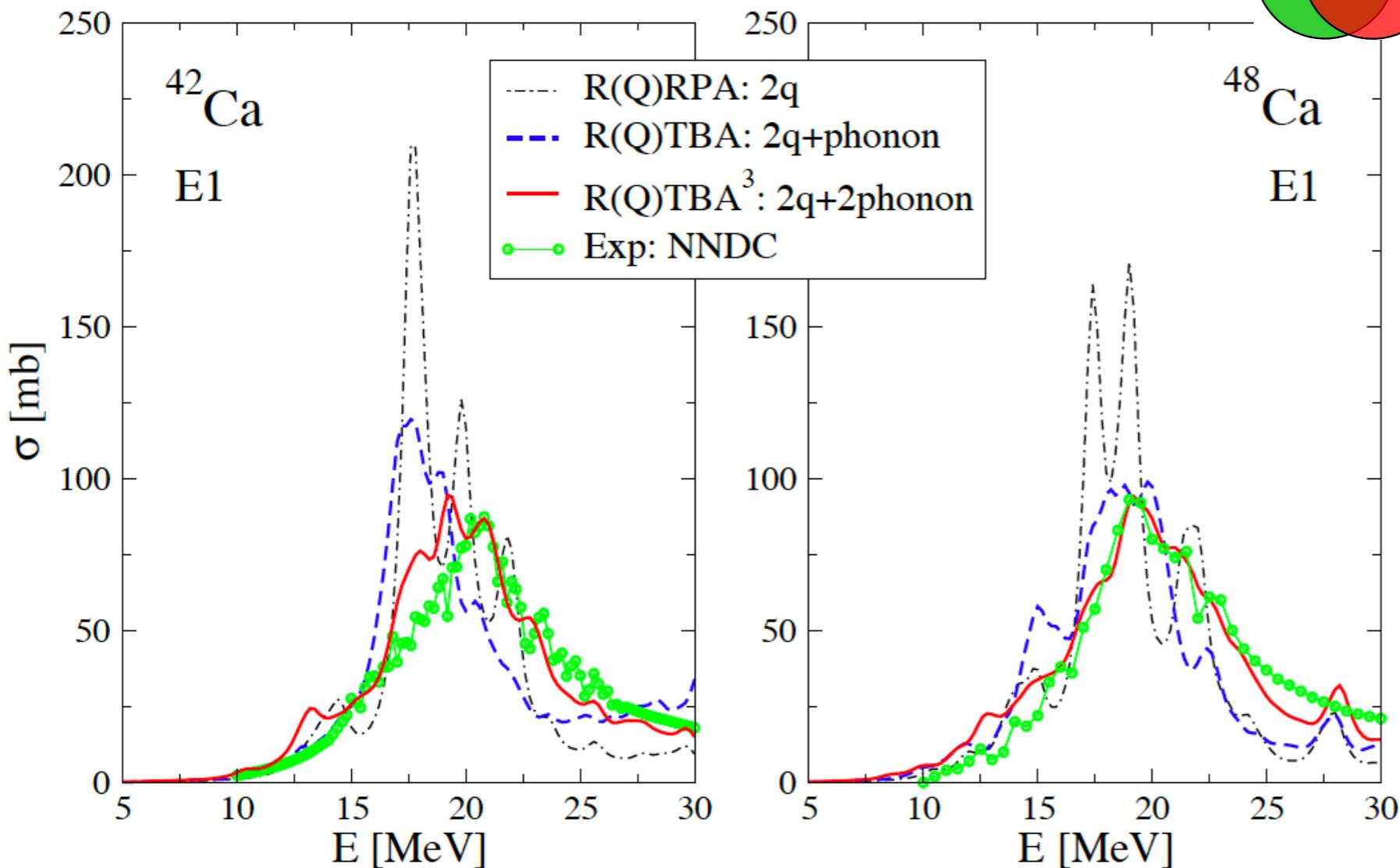
Stiffer EOS can be used

Cf.: Skyrme HFB+ q PVC

G. Colò & Y. Niu

Toward complete theory: correlated $3p3h$ configurations $2q+2\text{phonon}$

Giant Dipole Resonance in Ca isotopes

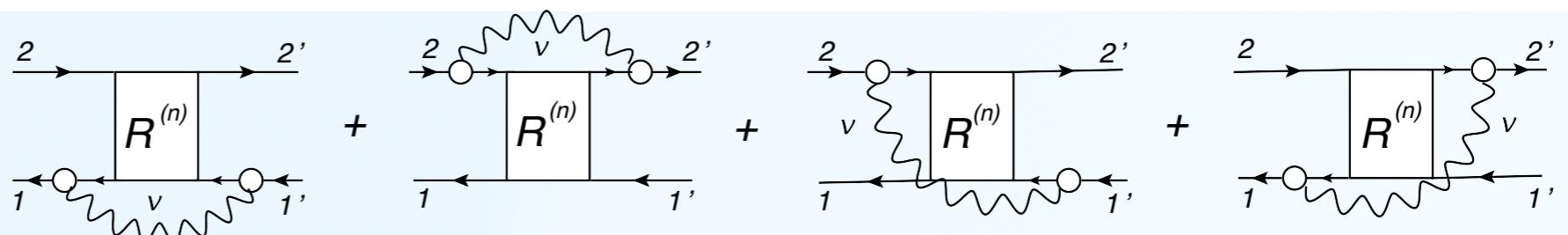


- The new complex configurations $2q+2\text{phonon}$ included for the first time enforce fragmentation and spreading toward higher and lower energies, thus, modifying both giant and pygmy dipole resonances;
- Exp. Data: V.A. Erokhova et al., Bull. Rus. Acad. Phys. 67, 1636 (2003)
- RQTBA³ demonstrates an overall systematic improvement of the description of nuclear excited states heading toward spectroscopic accuracy without strong limitations on masses and excitation energies.

E.L., P. Schuck,
PRC 100, 064320 (2019)

A hierarchy of the dynamical kernels:

$n = 0$ (no dynamical)
 $n = 1$
 $n = 2$



Beyond-MF “dynamical” nuclear shapes: ^{96}Zr

I. First (reference) approximation: Mean-field (MF) theory

(covariant density functional theory, CDFT)

$$\Delta T \Delta E \sim 1$$

^{96}Zr MF ground state :

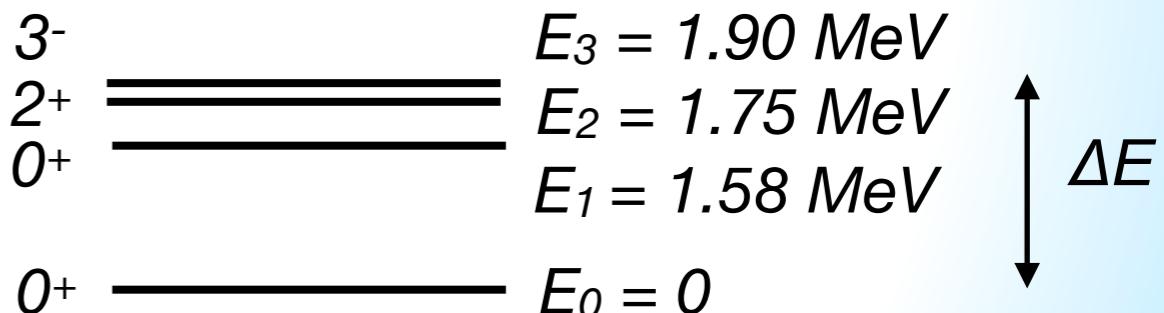
$$J^\pi = 0^+, \beta_2 = \beta_3 = 0$$



[G. Giacalone
Wed@9:30]

Low-energy levels

(difficult to resolve
by the NS theory): **Mixing phenomenon**



- MF solutions (with geometrical and/or other constraints) may violate symmetries
- Beyond-MF extensions can restore symmetries

II. Beyond MF: Shape vibrations around the static density: (quantum fluctuations)

Time-dependent density:

$$\rho(\mathbf{r}, t) = \rho_0(\mathbf{r}) + \sum_n \sum_{\lambda\mu} \delta\rho_{\lambda\mu}^n(\mathbf{r}) e^{iE_n t}$$

Retaining only low-energy vibrations

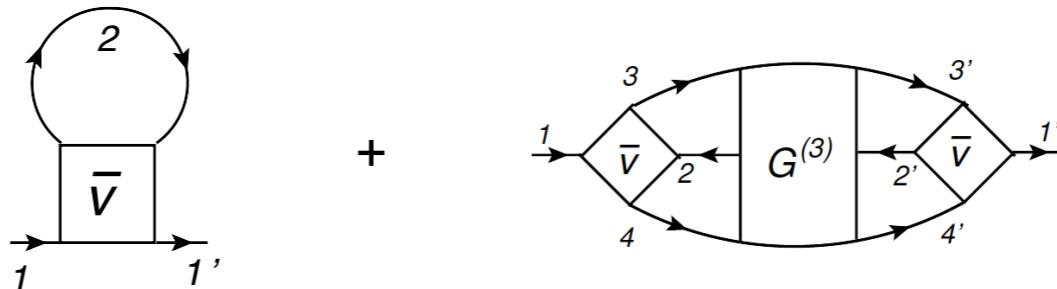
$$|E_n - E_0| \leq \Delta E$$

may look like a nearly static shape

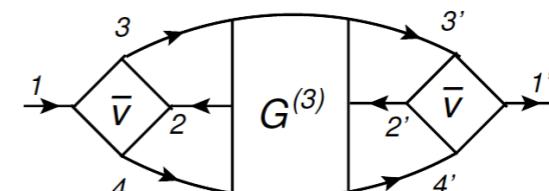
Mean field approximation and beyond

Exact “ab-initio” self-energy :

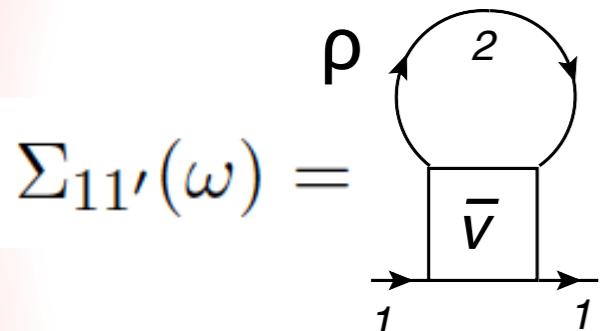
$$\Sigma_{11'}(\omega) = \Sigma_{11'}^{(0)} + \Sigma_{11'}^{(r)}(\omega)$$



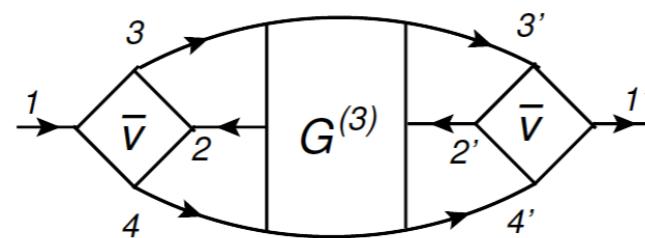
+



Mean field approximation (density functional theory, DFT)

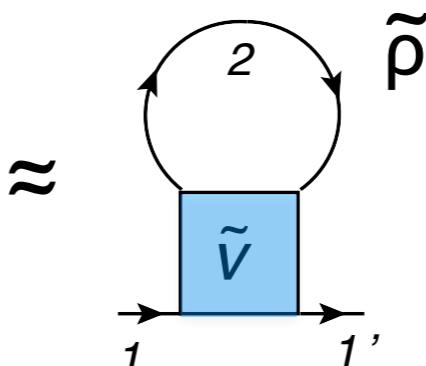


+



static

ω -dependent

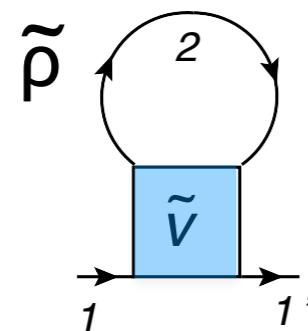


DFT: static [$\omega = (?)$]

Adjusted to nuclear ground states
=> absorbs qPVC dynamics in a static “approximation” in the parameters

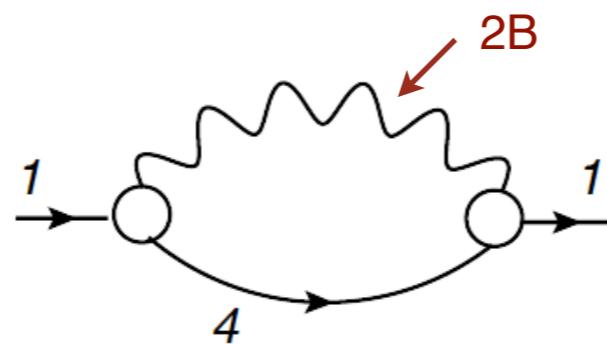
Beyond mean field: quasiparticle-vibration coupling (qPVC), leading approximation:

$$\Sigma_{11'}(\omega) =$$



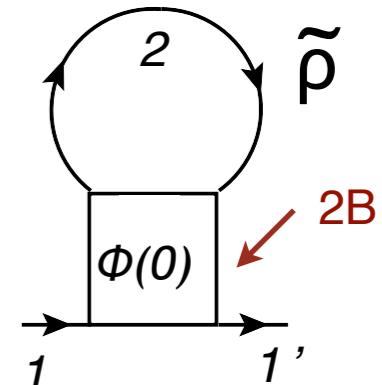
DFT: static, basis

+



qPVC ω -dependent

-

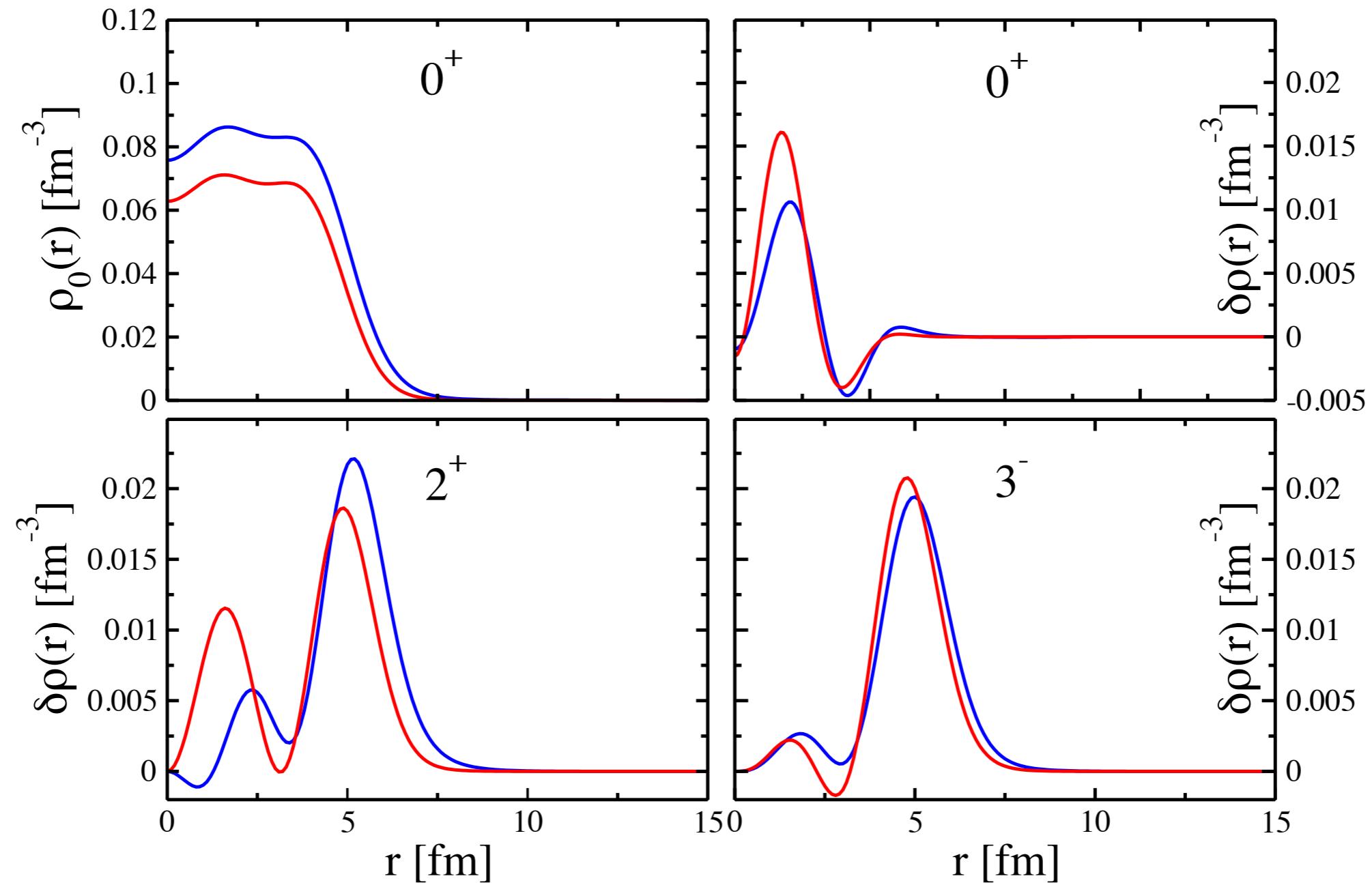


qPVC double counting removal

$\Phi(0)$

Beyond-MF “dynamical” nuclear shapes: ^{96}Zr

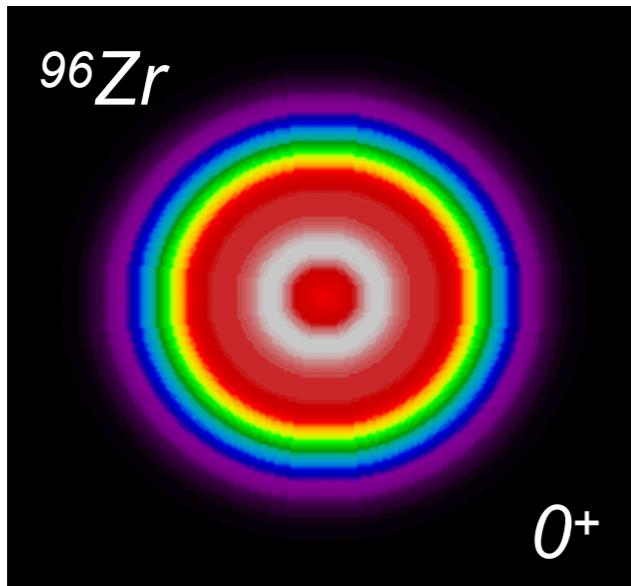
Ground-state and transition densities: the radial dependence



Adding angular dependence:

$$\delta\rho_{\lambda\mu}^n(\mathbf{r}) = \delta\rho^n(r) Y_{\lambda\mu}(\Omega)$$

Beyond-MF “dynamical” nuclear shapes: ^{96}Zr

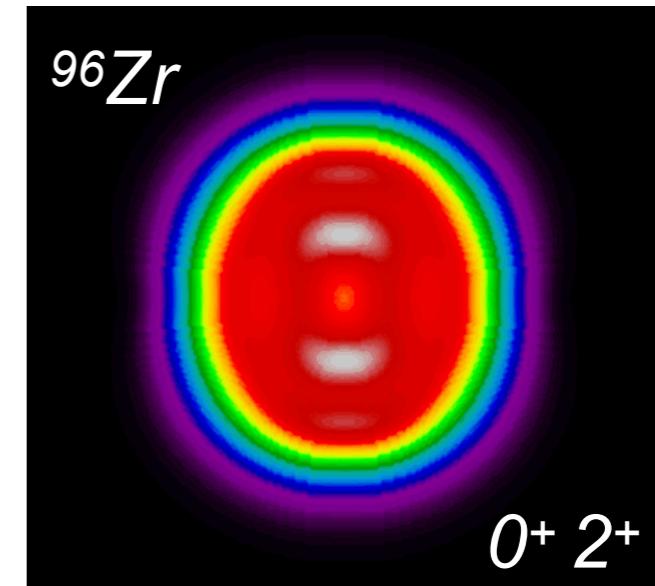


$$\rho(\mathbf{r}, t) = \rho_0(\mathbf{r})$$

Neutrons

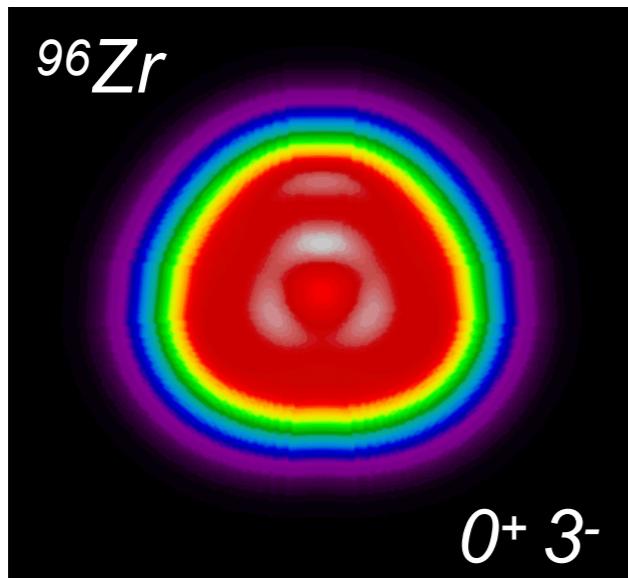
Constraints:

- E₁ ≈ E₂ ≈ E₃
- Oscillations are in phase
- Only $\mu = 0$ components



$$\rho(\mathbf{r}, t) = \rho_0(\mathbf{r}) + \delta\rho_{20}^{(2)}(\mathbf{r})e^{iE_2 t}$$

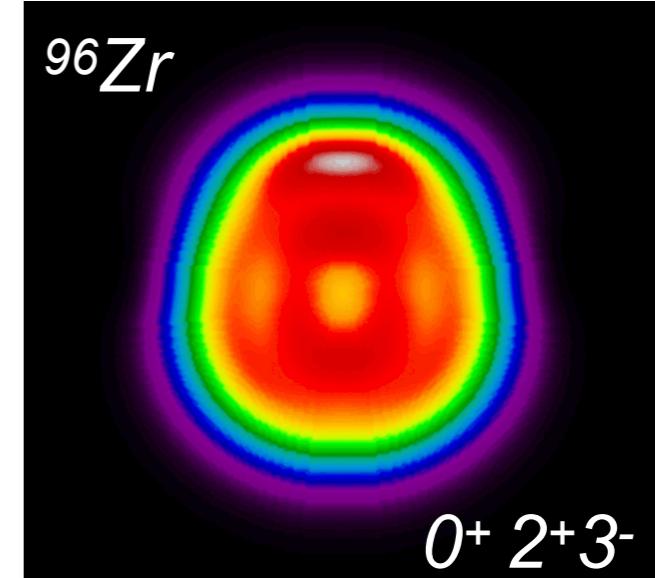
Further Refining:



$$\rho(\mathbf{r}, t) = \rho_0(\mathbf{r}) + \delta\rho_{30}^{(3)}(\mathbf{r})e^{iE_3 t}$$

- Relaxing the constraints
- Including more correlations in $\delta\rho(\mathbf{r}, t)$
- Extracting deformation parameters

Similar calculations can be done for deformed MF ground states



$$\rho(\mathbf{r}, t) = \rho_0(\mathbf{r}) + \delta\rho_{20}^{(2)}(\mathbf{r})e^{iE_2 t} + \delta\rho_{30}^{(3)}(\mathbf{r})e^{iE_3 t}$$

Nuclei at the limits of existence: Finite-temperature response with the ph+phonon dynamical kernel

$$R_{12,1'2'}(t - t') = -i\langle \mathcal{T}(\bar{\psi}_1\psi_2)(t)(\bar{\psi}_{2'}\psi_{1'})(t') \rangle \rightarrow -i\langle \mathcal{T}(\bar{\psi}_1\psi_2)(t)(\bar{\psi}_{2'}\psi_{1'})(t') \rangle_T$$

$$\langle \dots \rangle \equiv \langle 0 | \dots | 0 \rangle \rightarrow \langle \dots \rangle_T \equiv \sum_n \exp\left(\frac{\Omega - E_n - \mu N}{T}\right) \langle n | \dots | n \rangle$$

averages



thermal averages

**Method: EOM
for Matsubara
Green's functions**

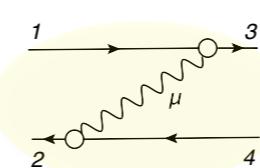
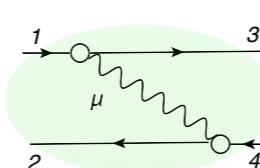
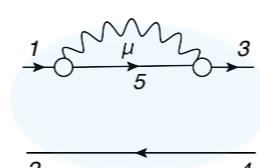
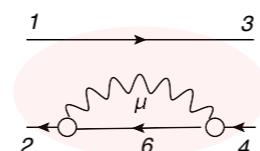


$$\begin{aligned} \mathcal{R}_{14,23}(\omega, T) &= \tilde{\mathcal{R}}_{14,23}^0(\omega, T) + \\ &+ \sum_{1'2'3'4'} \tilde{\mathcal{R}}_{12',21'}^0(\omega, T) [\tilde{V}_{1'4',2'3'}(T) + \delta\Phi_{1'4',2'3'}(\omega, T)] \mathcal{R}_{3'4,4'3}(\omega, T) \\ \delta\Phi_{1'4',2'3'}(\omega, T) &= \Phi_{1'4',2'3'}(\omega, T) - \Phi_{1'4',2'3'}(0, T) \end{aligned}$$

$T > 0$:

$$\begin{aligned} \Phi_{14,23}^{(ph)}(\omega, T) &= \frac{1}{n_{43}(T)} \sum_{\mu, \eta_\mu=\pm 1} \eta_\mu \left[\delta_{13} \sum_6 \gamma_{\mu;62}^{\eta_\mu} \gamma_{\mu;64}^{\eta_\mu*} \times \right. \\ &\times \frac{(N(\eta_\mu \Omega_\mu) + n_6(T)) (n(\varepsilon_6 - \eta_\mu \Omega_\mu, T) - n_1(T))}{\omega - \varepsilon_1 + \varepsilon_6 - \eta_\mu \Omega_\mu} + \\ &+ \delta_{24} \sum_5 \gamma_{\mu;15}^{\eta_\mu} \gamma_{\mu;35}^{\eta_\mu*} \times \\ &\times \frac{(N(\eta_\mu \Omega_\mu) + n_2(T)) (n(\varepsilon_2 - \eta_\mu \Omega_\mu, T) - n_5(T))}{\omega - \varepsilon_5 + \varepsilon_2 - \eta_\mu \Omega_\mu} - \\ &- \gamma_{\mu;13}^{\eta_\mu} \gamma_{\mu;24}^{\eta_\mu*} \times \\ &\times \frac{(N(\eta_\mu \Omega_\mu) + n_2(T)) (n(\varepsilon_2 - \eta_\mu \Omega_\mu, T) - n_3(T))}{\omega - \varepsilon_3 + \varepsilon_2 - \eta_\mu \Omega_\mu} - \\ &- \gamma_{\mu;31}^{\eta_\mu*} \gamma_{\mu;42}^{\eta_\mu} \times \\ &\times \left. \frac{(N(\eta_\mu \Omega_\mu) + n_4(T)) (n(\varepsilon_4 - \eta_\mu \Omega_\mu, T) - n_1(T))}{\omega - \varepsilon_1 + \varepsilon_4 - \eta_\mu \Omega_\mu} \right], \end{aligned}$$

1p1h+phonon dynamical kernel:

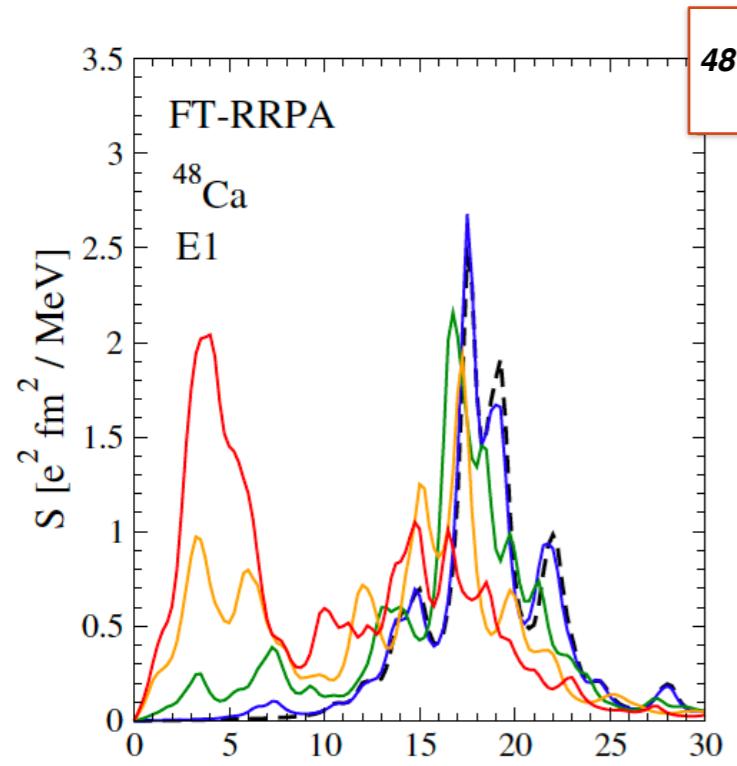


$T = 0$:

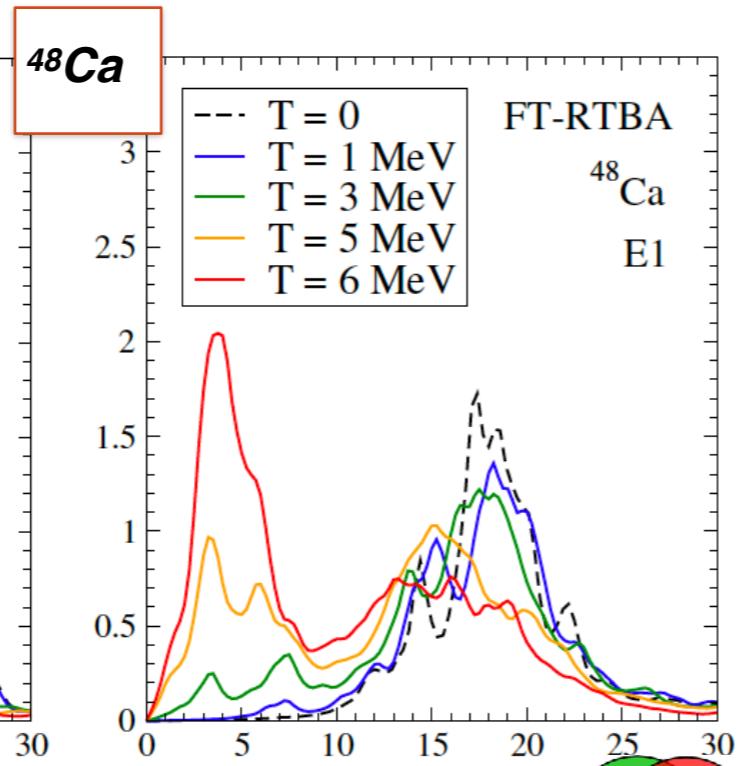
$$\begin{aligned} \Phi_{14,23}^{(ph,ph)}(\omega) &= \sum_{\mu} \times \\ &\times \left[\delta_{13} \sum_6 \frac{\gamma_{62}^{\mu} \gamma_{64}^{\mu*}}{\omega - \varepsilon_1 + \varepsilon_6 - \Omega_\mu} + \right. \\ &+ \delta_{24} \sum_5 \frac{\gamma_{15}^{\mu} \gamma_{35}^{\mu*}}{\omega - \varepsilon_5 + \varepsilon_2 - \Omega_\mu} - \\ &- \frac{\gamma_{13}^{\mu} \gamma_{24}^{\mu*}}{\omega - \varepsilon_3 + \varepsilon_2 - \Omega_\mu} - \\ &\left. - \frac{\gamma_{31}^{\mu*} \gamma_{42}^{\mu}}{\omega - \varepsilon_1 + \varepsilon_4 - \Omega_\mu} \right] \end{aligned}$$

Dipole Strength at $T>0$: ^{48}Ca and ^{132}Sn

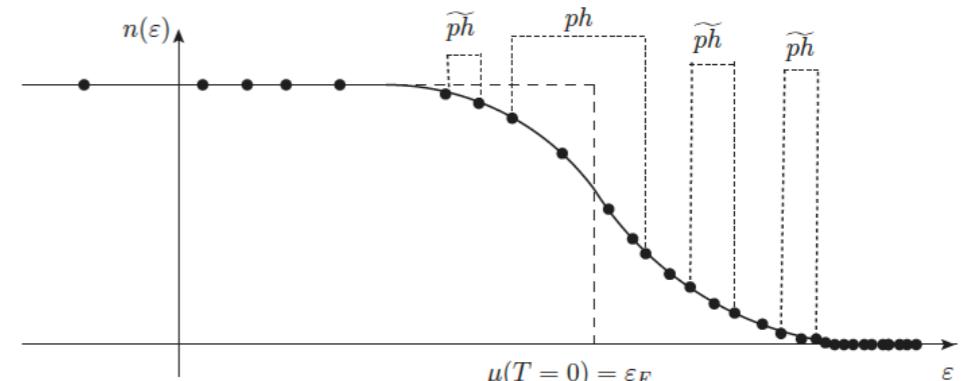
Static only (FT-RRPA)



Static + dynamic (FT-RTBA)



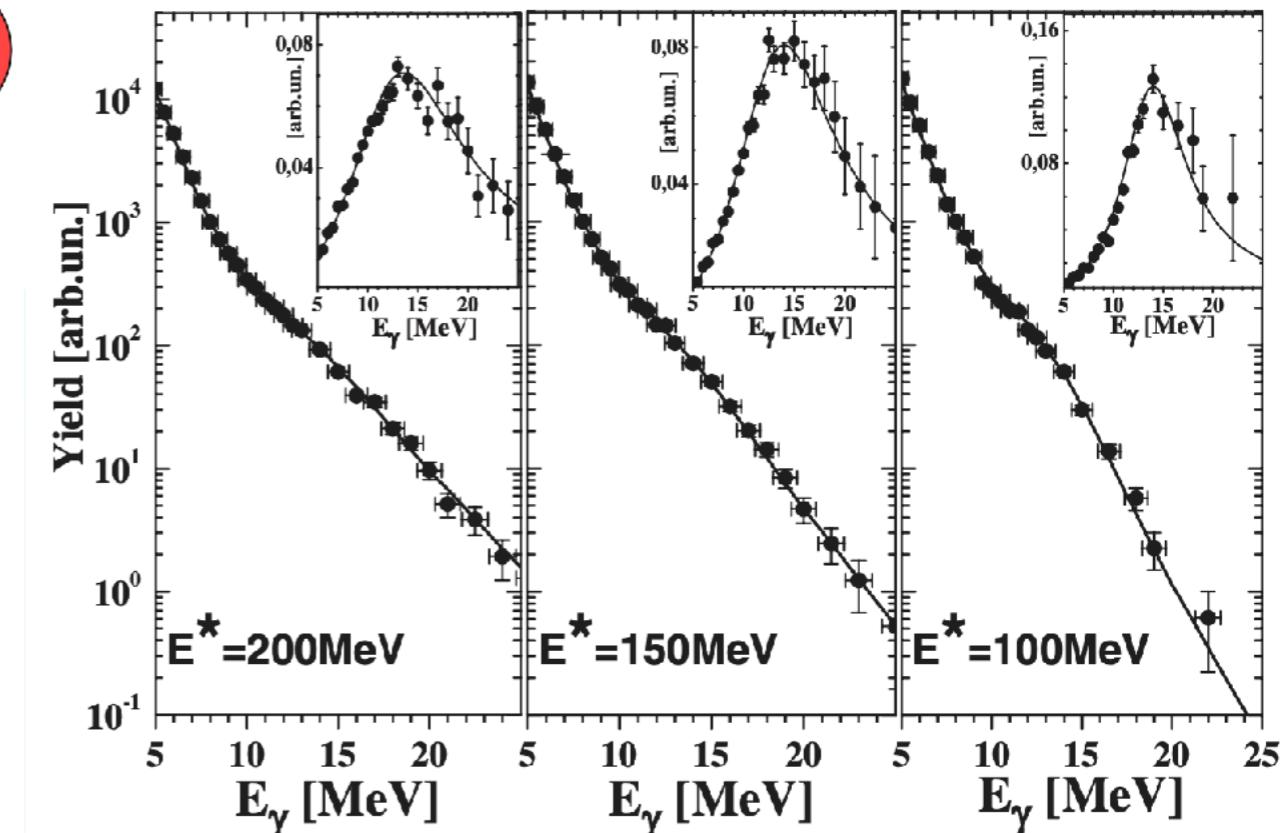
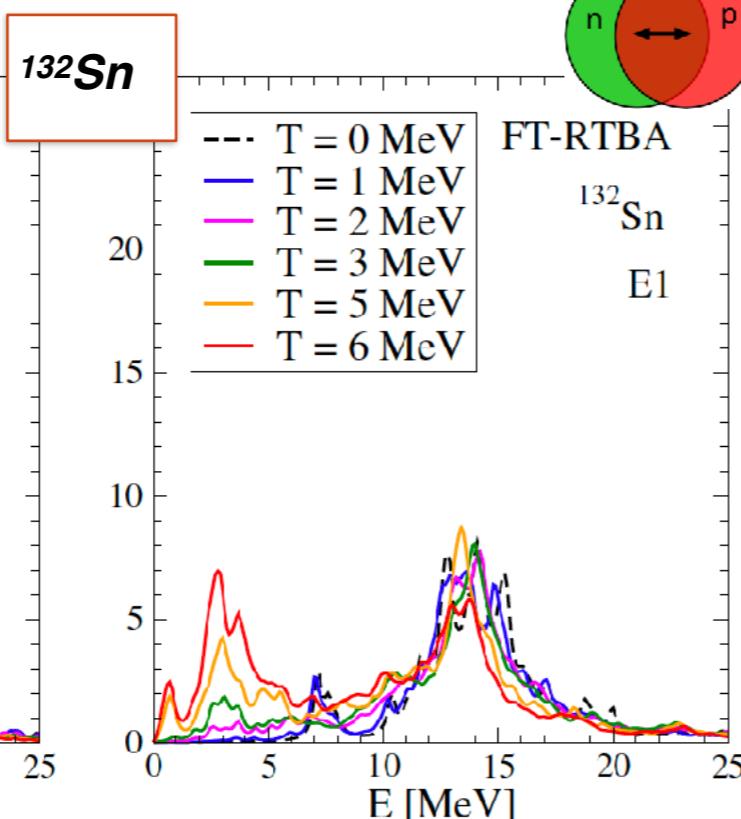
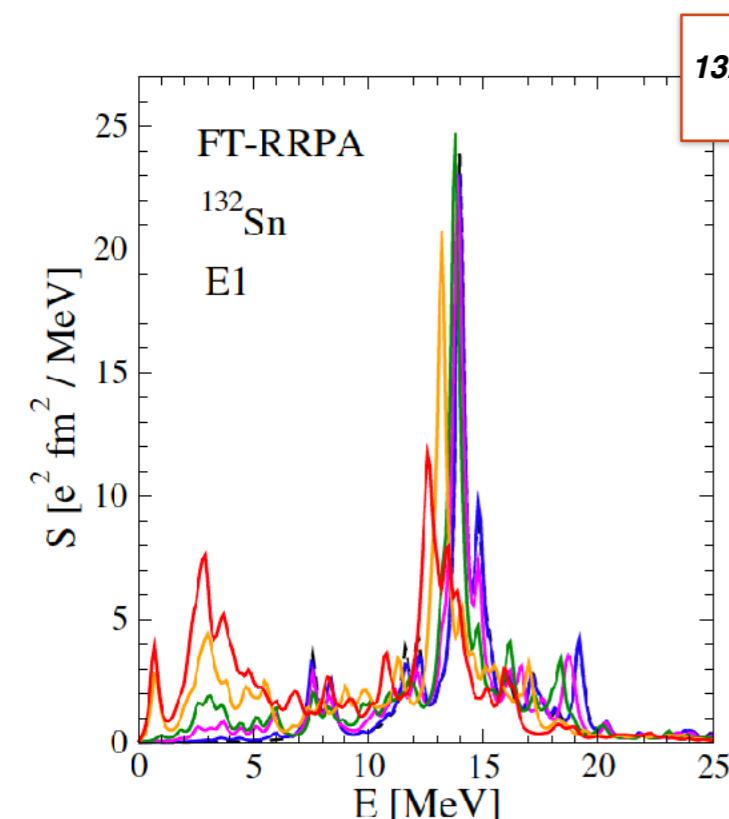
Thermal unblocking:



*0th approximation:
Uncorrelated propagator*

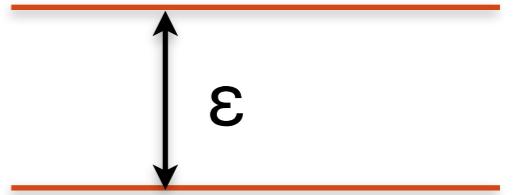
$$\tilde{R}_{14,23}^0(\omega) = \delta_{13}\delta_{24} \frac{n_2 - n_1}{\omega - \varepsilon_1 + \varepsilon_2}$$

O. Wieland et al., PRL 97, 012501 (2006):
GDR in ^{132}Ce



Model Hamiltonians on quantum computer: The Lipkin model

Two-level Lipkin (Meshkov-Glick), LMG, Hamiltonian:

$$\hat{H} = \epsilon \hat{J}_z - \frac{v}{2} \left(\hat{J}_+^2 + \hat{J}_-^2 \right) - \frac{w}{2} \left(\hat{J}_+ \hat{J}_- + \hat{J}_- \hat{J}_+ \right)$$


Quasipin operators:

$$\hat{J}_z = \frac{1}{2} \sum_{p=1}^N \left(\hat{a}_{p,+}^\dagger \hat{a}_{p,+} - \hat{a}_{p,-}^\dagger \hat{a}_{p,-} \right), \quad N = 2j + 1$$

$$\hat{J}_+ = \sum_{p=1}^N \hat{a}_{p,+}^\dagger \hat{a}_{p,-} \text{ and } \hat{J}_- = (\hat{J}_+)^{\dagger}$$

Excitation operator:

$$\hat{O}_n^\dagger = \sum_\alpha \sum_{\mu_\alpha} \left[X_{\mu_\alpha}^\alpha(n) \hat{K}_{\mu_\alpha}^\alpha - Y_{\mu_\alpha}^\alpha(n) \left(\hat{K}_{\mu_\alpha}^\alpha \right)^\dagger \right]$$

Configuration complexity:

High complexity:
Important at intermediate
and strong couplings

$$\hat{K}_{\mu_1}^1 = a_i^\dagger a_{j'}$$

$$\hat{K}_{\mu_2}^2 = a_i^\dagger a_j^\dagger a_{j'} a_{i'}$$

...

Lipkin Hamiltonian on quantum computer (QC)

The algorithm: Variational Quantum Eigensolver (VQE) + quantum EOM (qEOM)

- VQE: a minimal encoding scheme is found (“J-scheme”) and implemented, based on the symmetry of the LMG Hamiltonian. Yields an accurate ground state $|0\rangle$.
- qEOM generates efficiently the EOM matrix [P. Ollitrault et al., Phys. Rev. Res. **2**, 043140 (2020)].

$$\begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B}^* & \mathcal{A}^* \end{bmatrix} \begin{bmatrix} X^n \\ Y^n \end{bmatrix} = E_{0n} \begin{bmatrix} \mathcal{C} & \mathcal{D} \\ -\mathcal{D}^* & -\mathcal{C}^* \end{bmatrix} \begin{bmatrix} X^n \\ Y^n \end{bmatrix}$$

**Generalized
Eigenvalue
Equation (GEE)**

- Measured on QC:

$$\mathcal{A}_{\mu_\alpha \nu_\beta} = \langle 0 | \left[\left(\hat{K}_{\mu_\alpha}^\alpha \right)^\dagger, \left[\hat{H}, \hat{K}_{\nu_\beta}^\beta \right] \right] | 0 \rangle$$

$$\mathcal{B}_{\mu_\alpha \nu_\beta} = - \langle 0 | \left[\left(\hat{K}_{\mu_\alpha}^\alpha \right)^\dagger, \left[\hat{H}, \left(\hat{K}_{\nu_\beta}^\beta \right)^\dagger \right] \right] | 0 \rangle$$

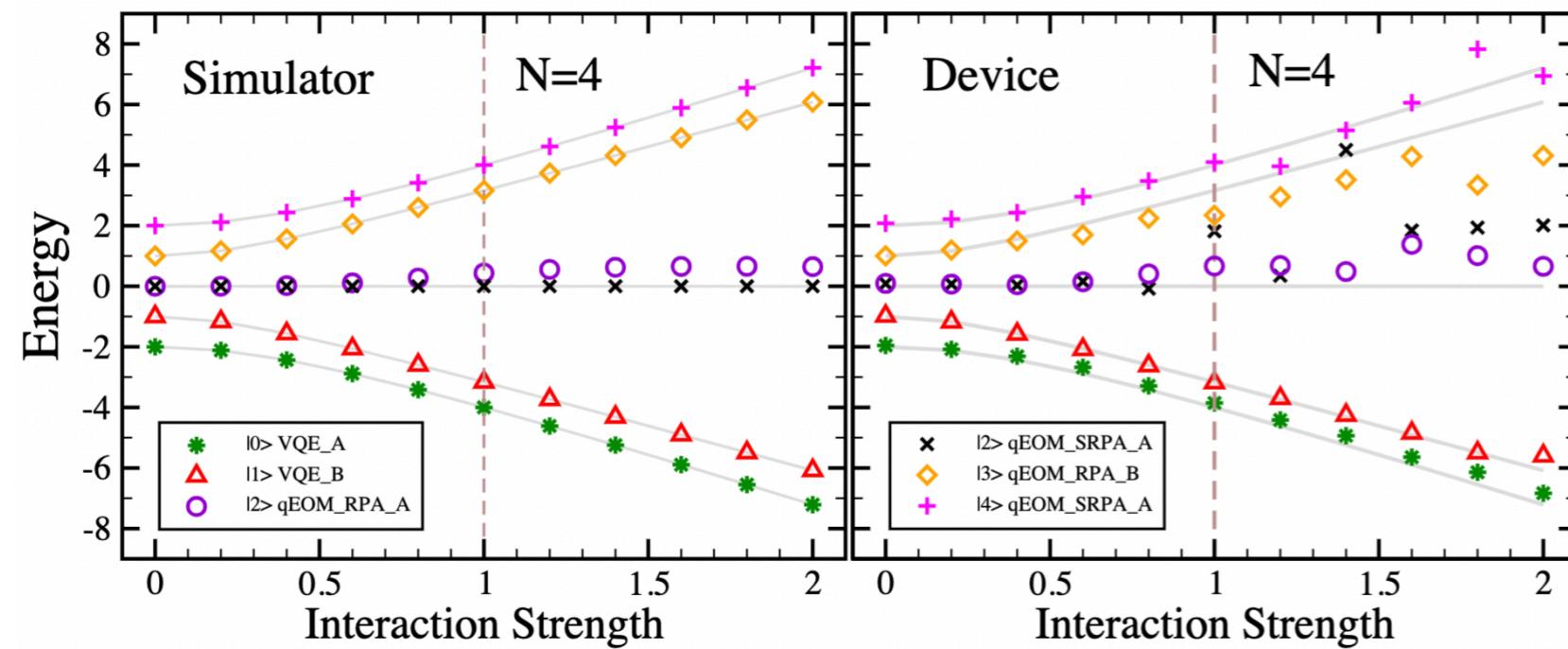
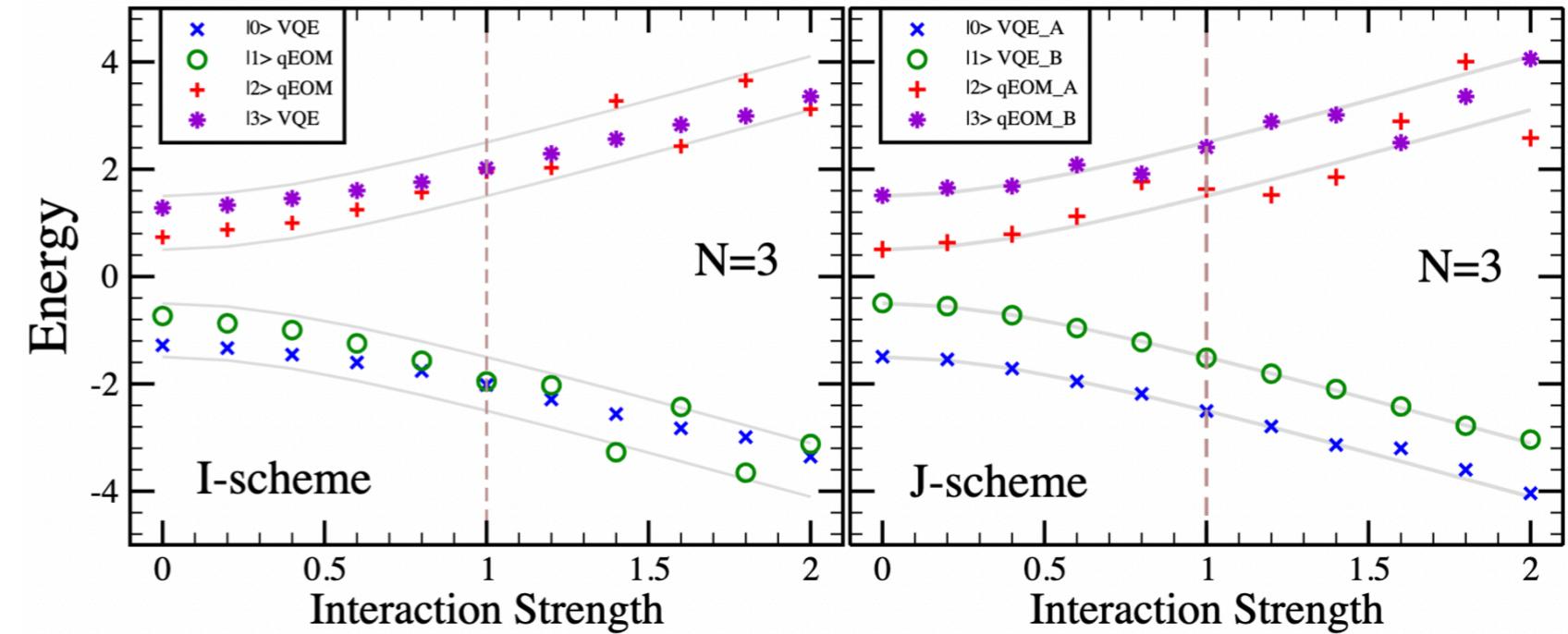
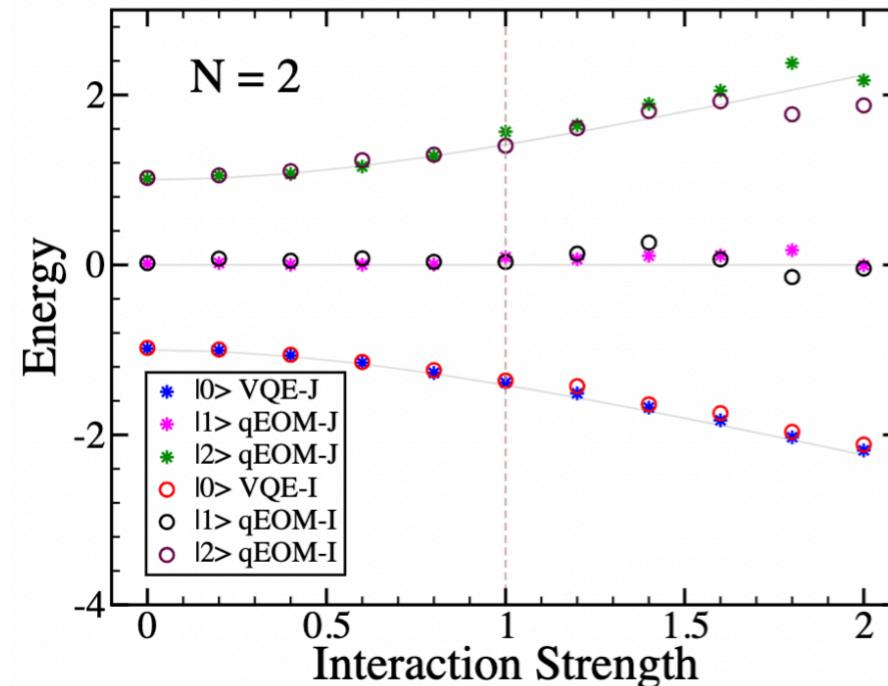
$$\mathcal{C}_{\mu_\alpha \nu_\beta} = \langle 0 | \left[\left(\hat{K}_{\mu_\alpha}^\alpha \right)^\dagger, \hat{K}_{\nu_\beta}^\beta \right] | 0 \rangle$$

$$\mathcal{D}_{\mu_\alpha \nu_\beta} = - \langle 0 | \left[\left(\hat{K}_{\mu_\alpha}^\alpha \right)^\dagger, \left(\hat{K}_{\nu_\beta}^\beta \right)^\dagger \right] | 0 \rangle.$$

Quantum advantage:

Number of measurements depends on the q -number, (almost) not growing with configuration complexity

Lipkin Hamiltonian on quantum computer: hardware results



Conventions:

- n_q = number of states
- N = number of particles
- $\nu = v/\varepsilon$ effective interaction strength

I-scheme: individual spin basis, $n_q = 2^N$

J-scheme: total spin basis (coupled form), symmetry: $n_q = N/2 + 1$

Observations:

- Higher-rank excitation \sim higher accuracy
- Stronger coupling \sim lower accuracy
- More particles \sim lower accuracy
- Less qubits \sim higher accuracy



Summary:

- The dynamical interaction kernels of the EOMs have the potential to bridge the gaps between the scales via emergent collectivity, solving burning nuclear structure issues.
- The emergent collective effects renormalize interactions in correlated media, underly the spectral fragmentation mechanisms, affect superfluidity and weak decay rates.
- The response theory with the dynamical kernels can provide information on nuclear shape oscillations.
- Configurations of growing complexity can be efficiently treated by quantum algorithms.

The “upbend” puzzle: understanding the Oslo data

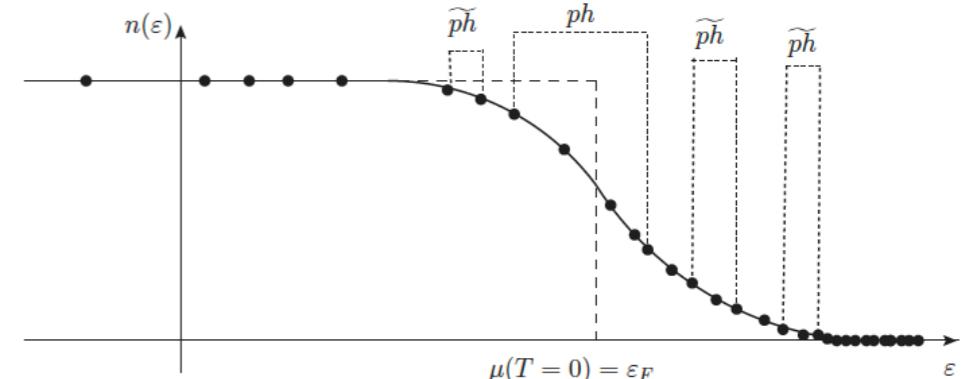
$$S(E, T) = -\frac{1}{\pi} \lim_{\Delta \rightarrow +0} \text{Im} \langle V^{0\dagger} \mathcal{R}(E + i\Delta, T) V^0 \rangle$$

The final strength function at $T > 0$:

$$\lim_{E \rightarrow 0} S(E, T) = 0$$

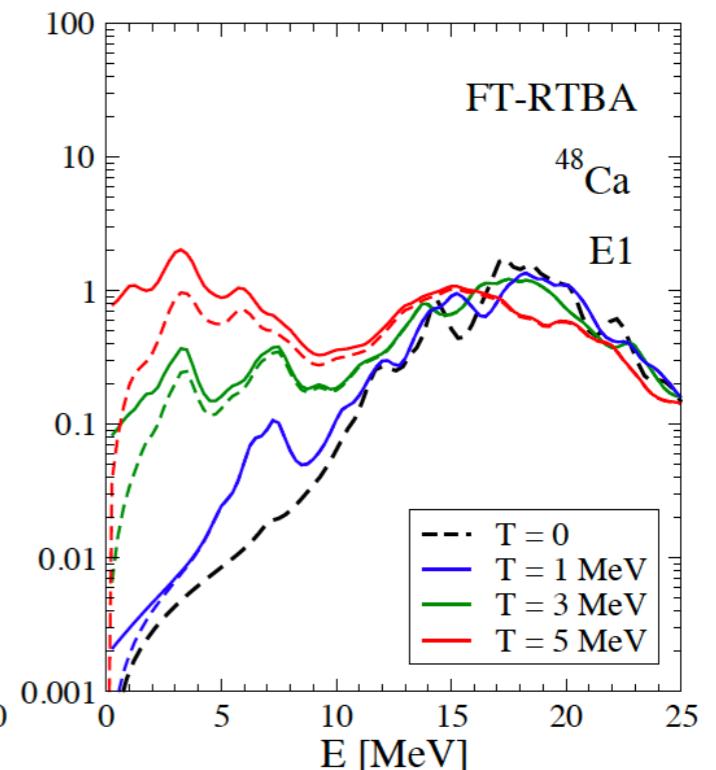
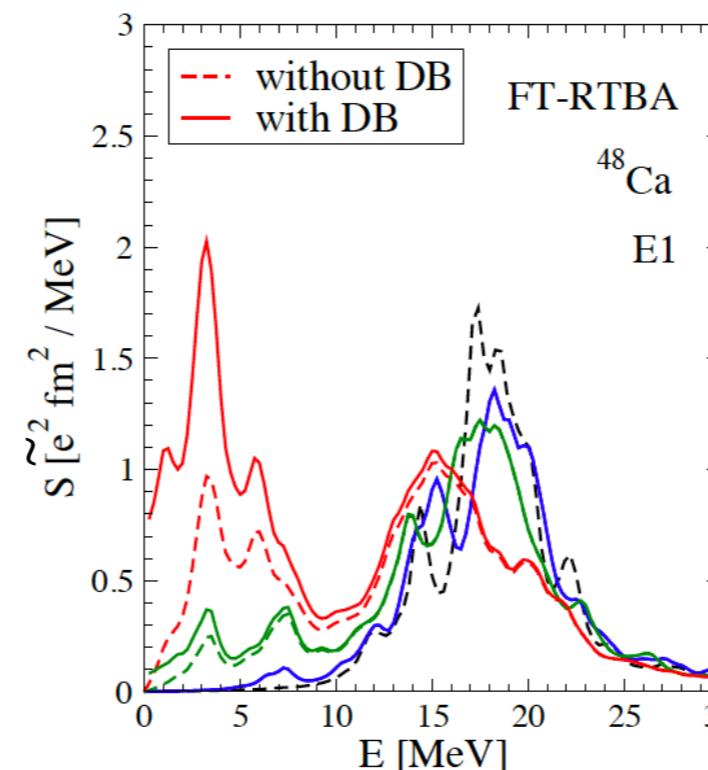
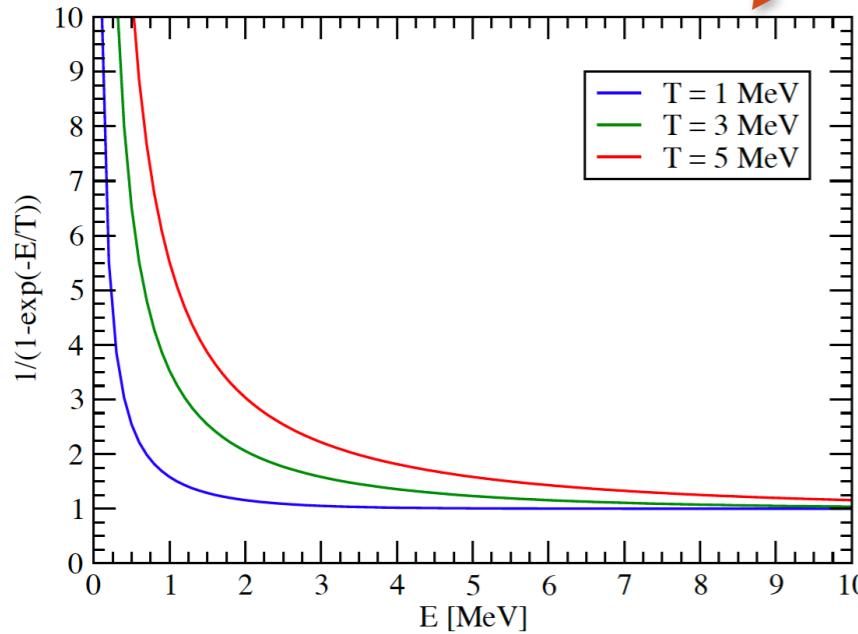
$$\tilde{S}(E) = \frac{1}{1 - e^{-E/T}} S(E)$$

Thermal unblocking:



Dipole strength: absorption at $T > 0$:

The generic exponential factor:



- The exponential factor brings an additional enhancement in $E < T$ energy region and provides the finite zero-energy limit of the strength (regardless its spin-parity)

Exact equations of motion (EOM) for binary interactions: one-body problem

One-fermion propagator

$$G_{11'}(t - t') = -i\langle T\psi(1)\bar{\psi}(1') \rangle$$

EOM: Dyson Eq.

$$G(\omega) = G^{(0)}(\omega) + G^{(0)}(\omega)\Sigma(\omega)G(\omega) \quad (*) \quad \Sigma(\omega) = \Sigma^{(0)} + \Sigma^{(r)}(\omega)$$

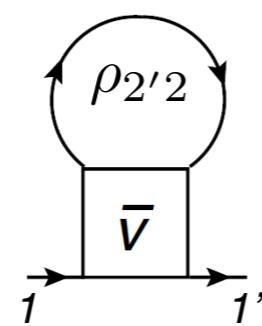
Irreducible kernel (Self-energy, exact):

Instantaneous term (Hartree-Fock incl. “tadpole”)

Short-range correlations

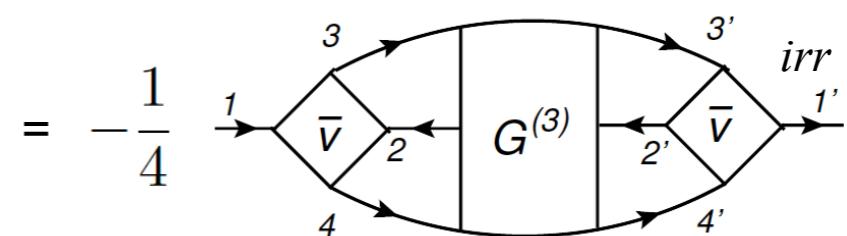
$$\Sigma_{11'}^{(0)} = -\langle \gamma^0 \{ [V, \psi_1], \bar{\psi}_{1'} \} \gamma^0 \rangle$$

$$= \sum_{22'} \bar{v}_{121'2'} \langle \bar{\psi}_2 \psi_{2'} \rangle$$



t-dependent (dynamical) term (symmetric version):
Long-range correlations

$$\begin{aligned} \Sigma_{11'}^{(r)} &= i\langle T\gamma^0[V, \psi_1](t)[V, \bar{\psi}_{1'}](t')\gamma^0 \rangle^{irr} \\ &= -\frac{1}{4} \sum_{234} \sum_{2'3'4'} \bar{v}_{1234} G_{432', 23'4'}^{(3)irr}(t - t') \bar{v}_{4'3'2'1'} \end{aligned}$$



$$\rho_{11'} = -i \lim_{t=t'-0} G_{11'}(t - t')$$

is the full solution of (*):
includes the dynamical term!

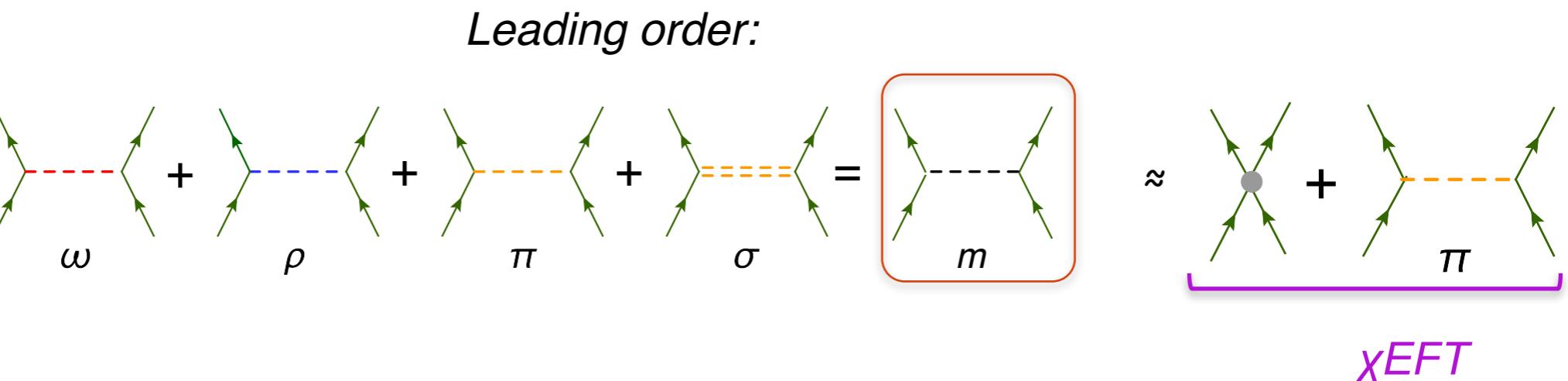
Koltun-Migdal-Galitsky sum rule: **the binding energy**

“Ab-initio DFT”:

$$E_0 = \frac{1}{2\pi} \int_{-\infty}^{\varepsilon_F^-} d\varepsilon \sum_{12} (T_{12} + \varepsilon \delta_{12}) \text{Im}G_{21}(\varepsilon)$$

Comparison to the “ab-initio” chiral PT

Quantum
Hadrodynamics
(QHD)



Relativistic Nuclear Field theory:
non-PT, in-medium

$F^{(0)} = -\bar{V} - p + \frac{1}{2}(\bar{V} + p) + \frac{1}{2}(\bar{V} - p)$

$+ \frac{1}{2}(\bar{V} + p) + \frac{1}{2}(\bar{V} - p)$

$F_{121'2'}^{(r;11)} = \bar{V}(1,2,3,4) G^{(4)}(3',4',5,5') \bar{V}(5,4',3',2')$

$F_{121'2'}^{(r;12)} = \bar{V}(1,2,3,4) G^{(4)}(3',4',5,5') \bar{V}(5,4',3',1')$

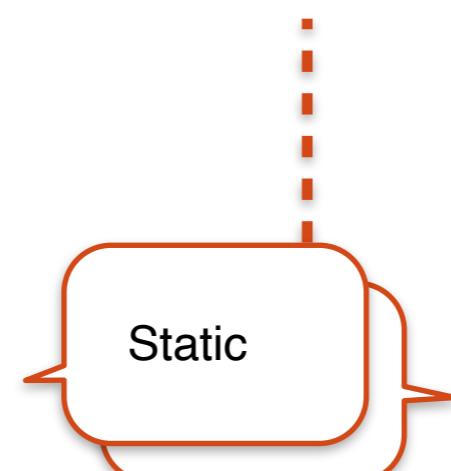
$F_{121'2'}^{(r;21)} = \bar{V}(1,2,3,4) G^{(4)}(3',4',5,5') \bar{V}(5,4',3',2')$

$F_{121'2'}^{(r;22)} = \bar{V}(1,2,3,4) G^{(4)}(3',4',5,5') \bar{V}(5,4',3',1')$

+ Dynamical

Beyond the leading order:

xEFT: PT in the vacuum



	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)	X H	-	-
NLO (Q^2)	X H K N D	-	-
$N^2LO (Q^3)$	H K	H I X *	-
$N^3LO (Q^4)$	X H K N ...	I H D X ...	I H I K ...
$N^4LO (Q^5)$	I D K H ...	I H K *	I H I X ...

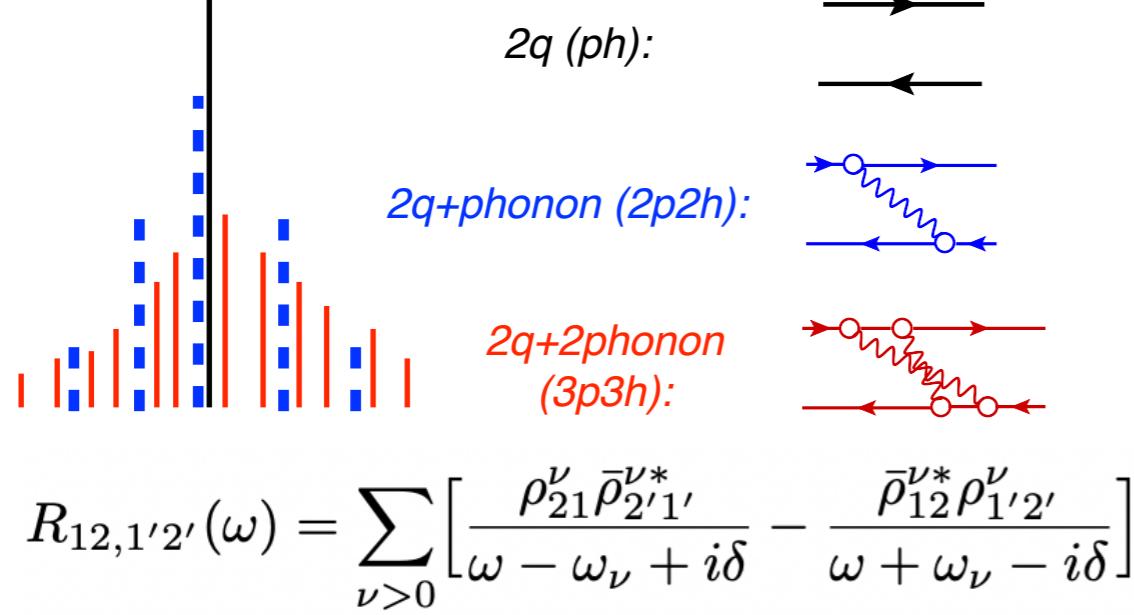
E. Epelbaum et al., Front. Phys. 8, 98 (2020)

+ Dynamical

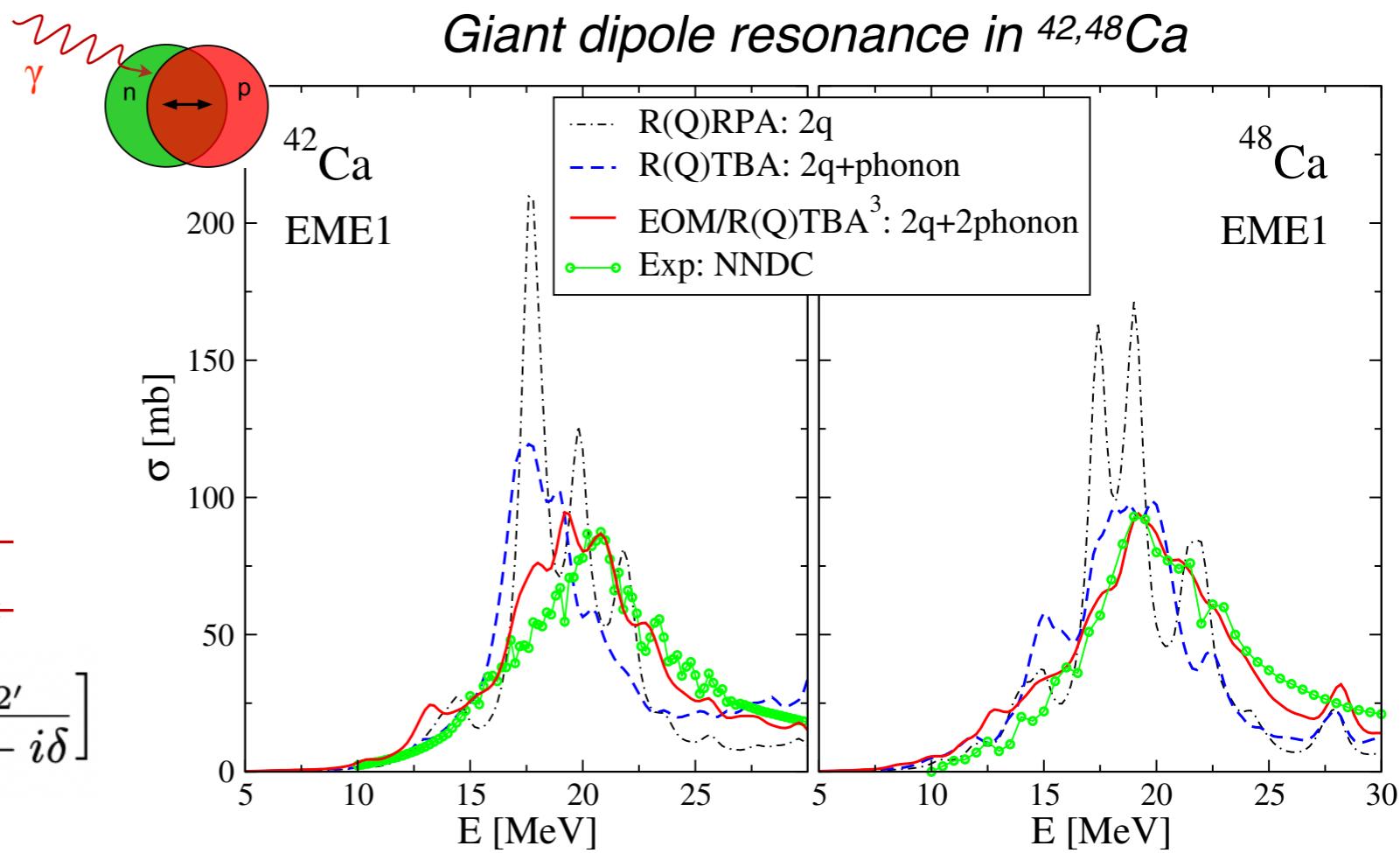
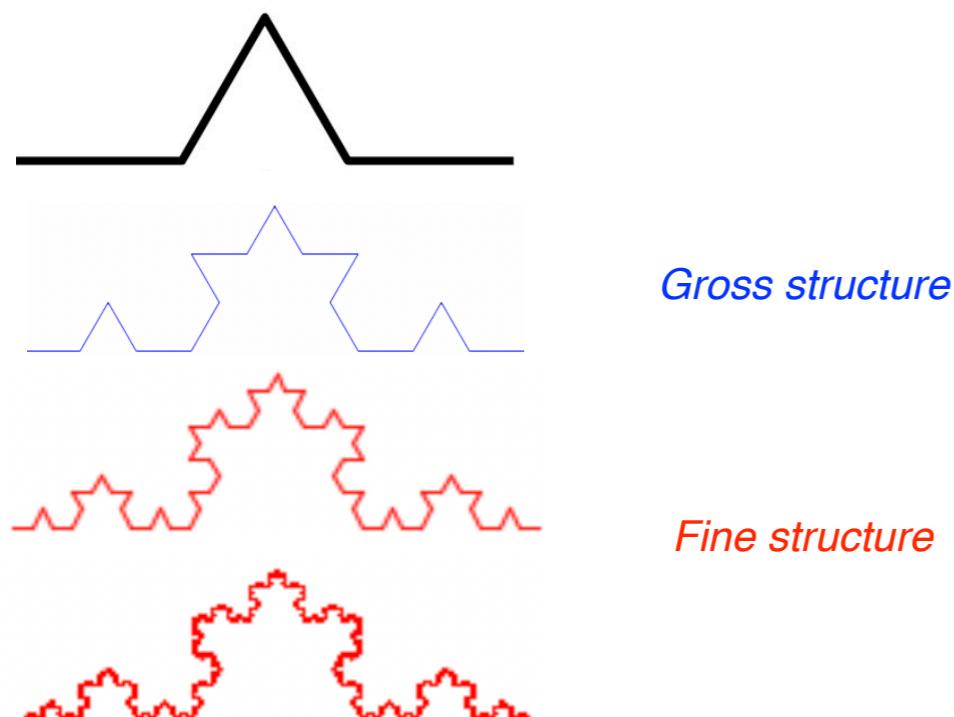
“Standard” many-body methods?
Problematic for medium-heavy nuclei.

Leading NFT (q)PVC is insufficient: the “3p3h” configurations

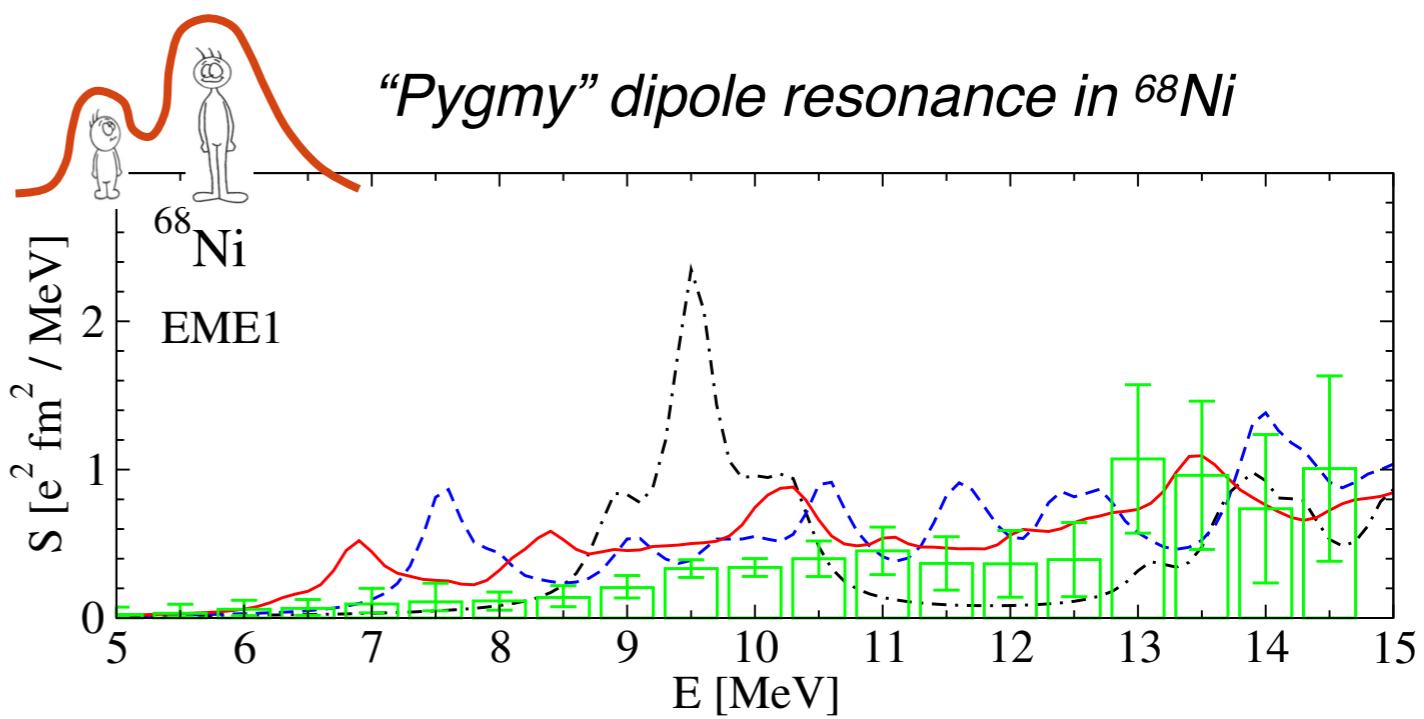
Fragmentation mechanism



Fractals: Koch curve



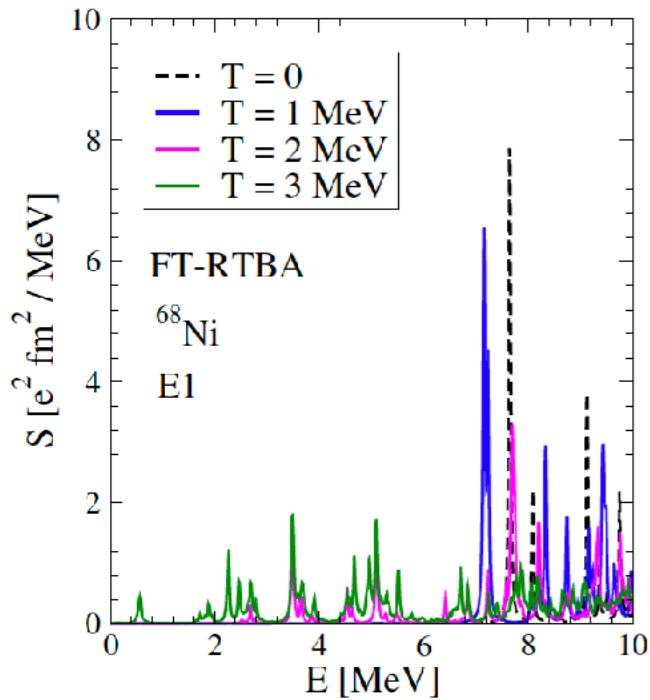
“Pygmy” dipole resonance in ^{68}Ni



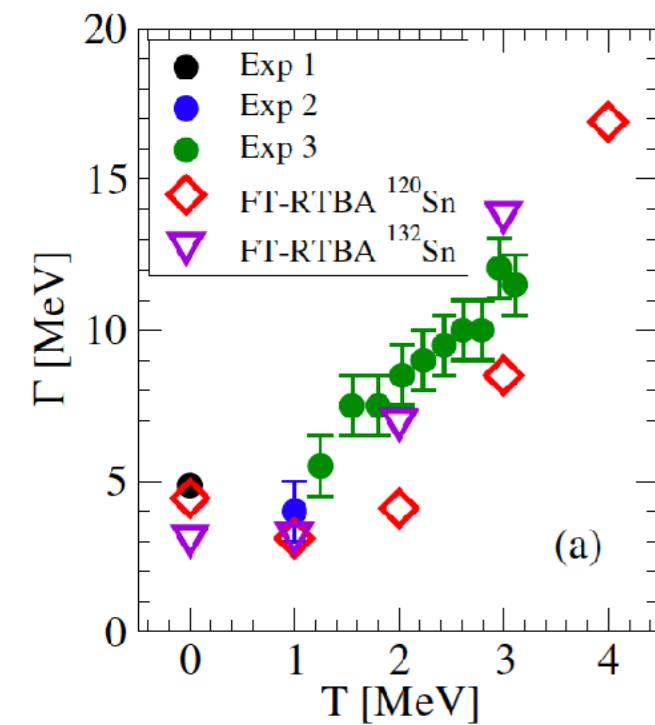
Data: O. Wieland et al., Phys. Rev. C 98, 064313 (2018)

Evolution of the pygmy dipole resonance (PDR) at $T > 0$

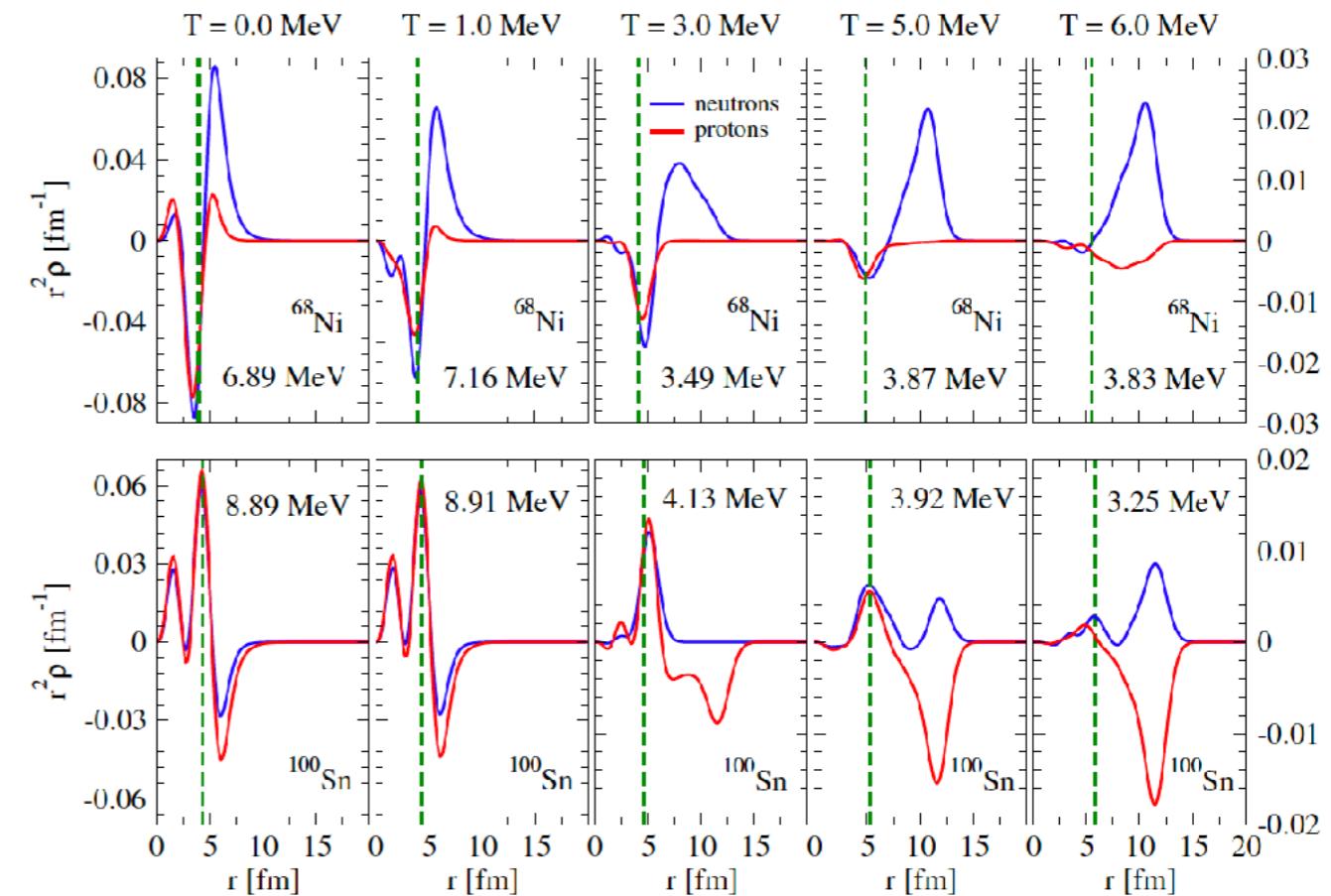
Low-energy strength distribution in ^{68}Ni



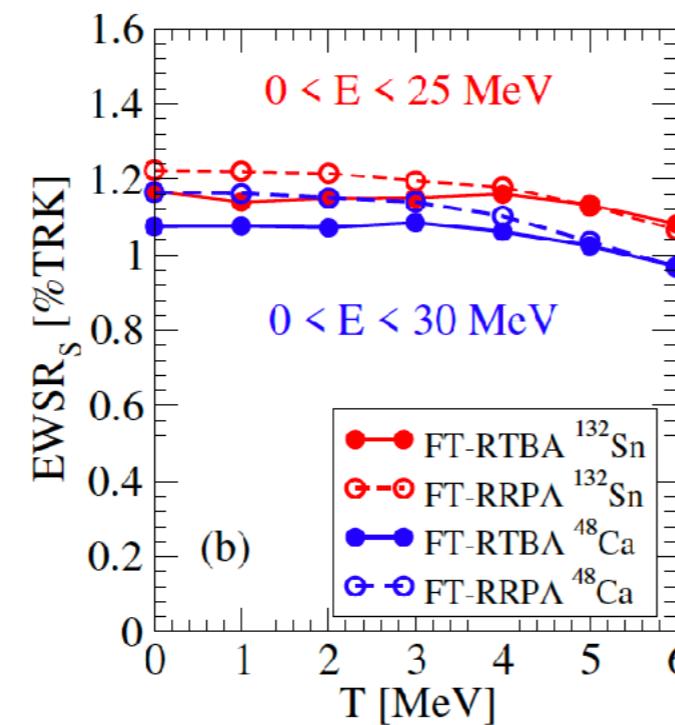
GDR's width



Transition density for the low-energy peak in ^{68}Ni , ^{100}Sn



Energy-weighted sum rule

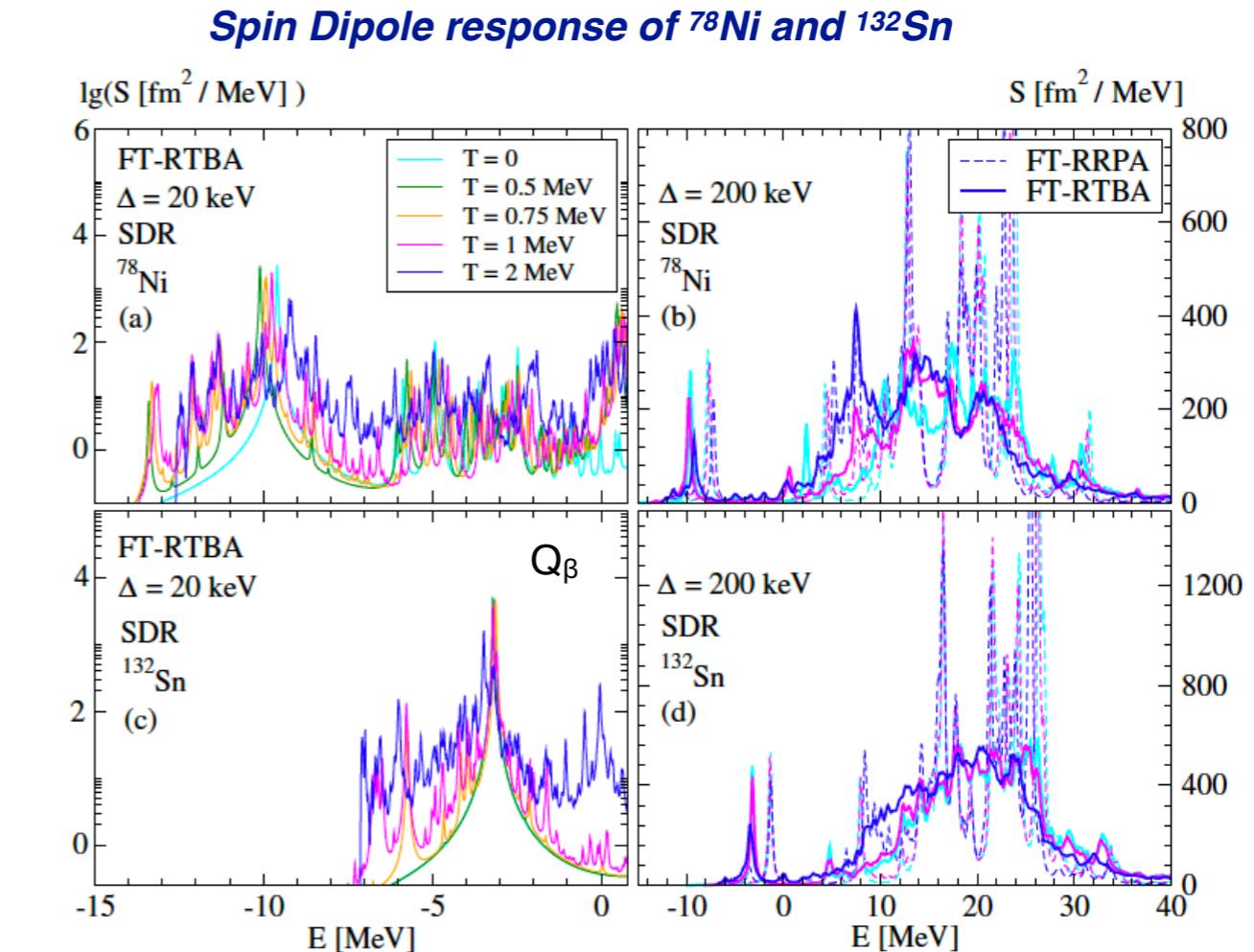
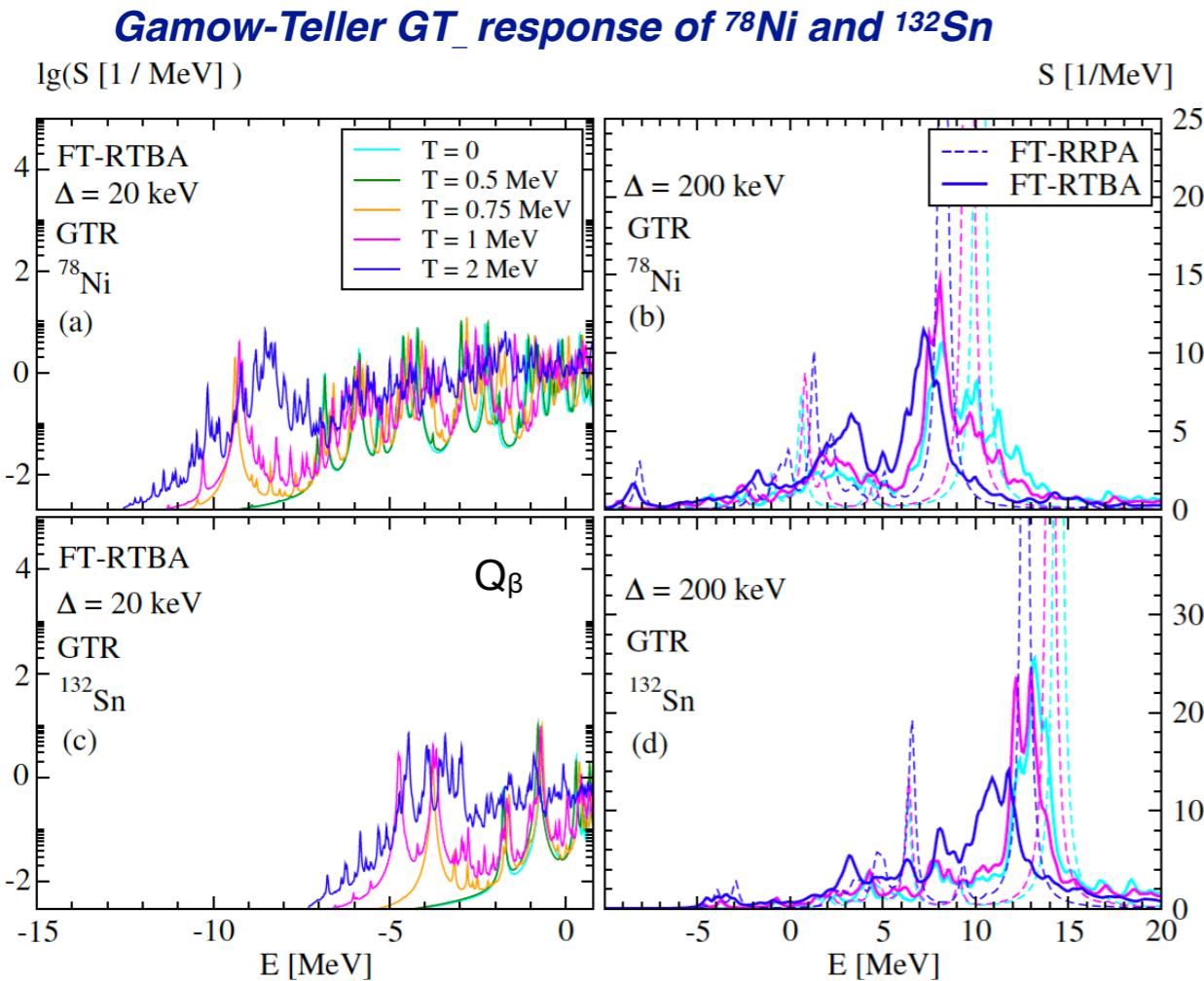


- The low-energy peak (PDR) gains the strength from the GDR with the temperature growth: $EWSR \sim \text{const}$
- The total width $\Gamma \sim T^2$ (as in the Landau theory); shape fluctuations are missing for $T \sim 2-3$ MeV
- The PDR develops a new type of collectivity originated from the thermal unblocking
- The same happens with other low-lying modes (2^+ , 3^- , ...) \Rightarrow strong PVC \Rightarrow “destruction” of the GDR at high temperatures

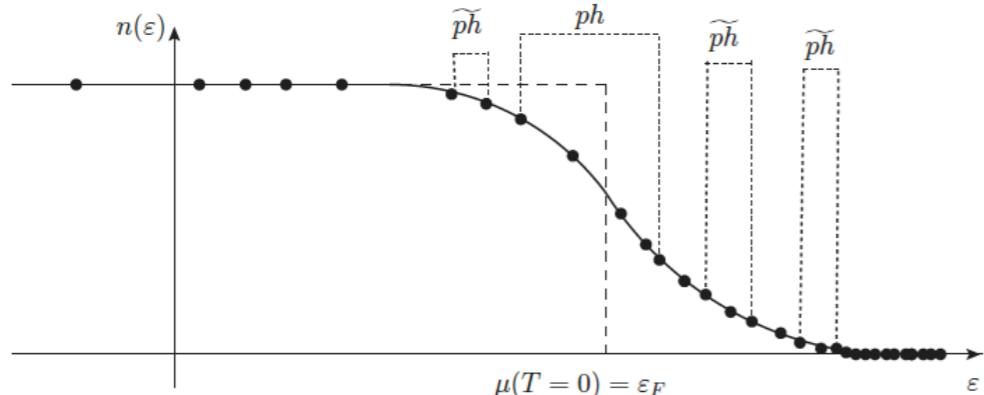
E.L., H. Wibowo, Phys. Rev. Lett. 121, 082501 (2018).

H. Wibowo, E.L., Phys. Rev. C 100, 024307 (2019).

Spin-Isospin response and beta decay in hot stellar environments



Thermal unblocking mechanism:

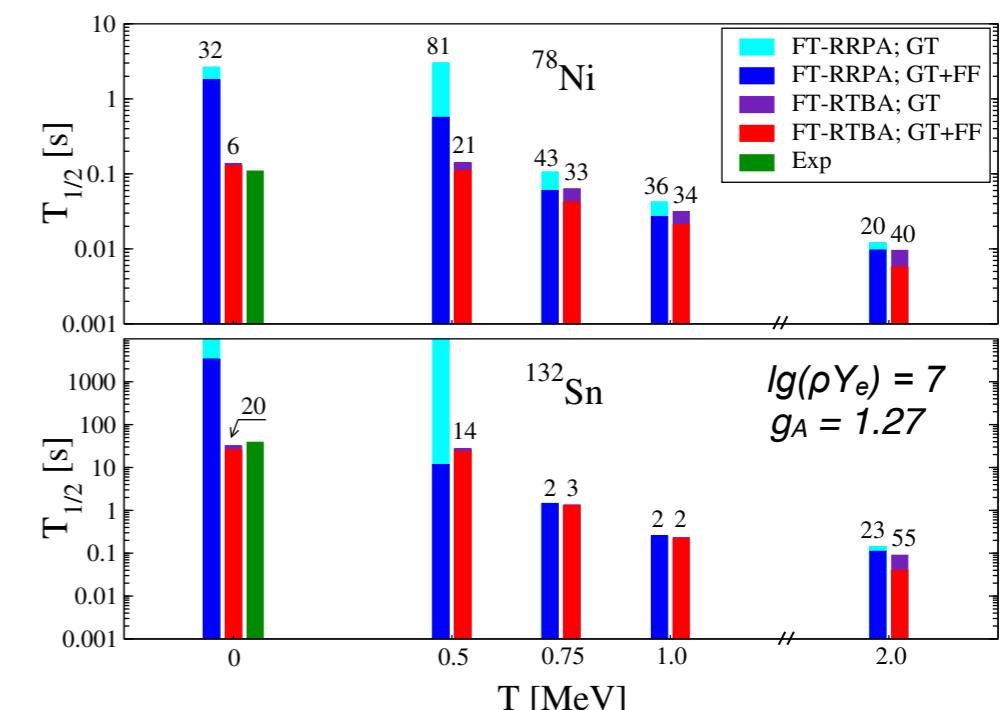


E. Litvinova, H. Wibowo, Phys. Rev. Lett. 121, 082501 (2018)

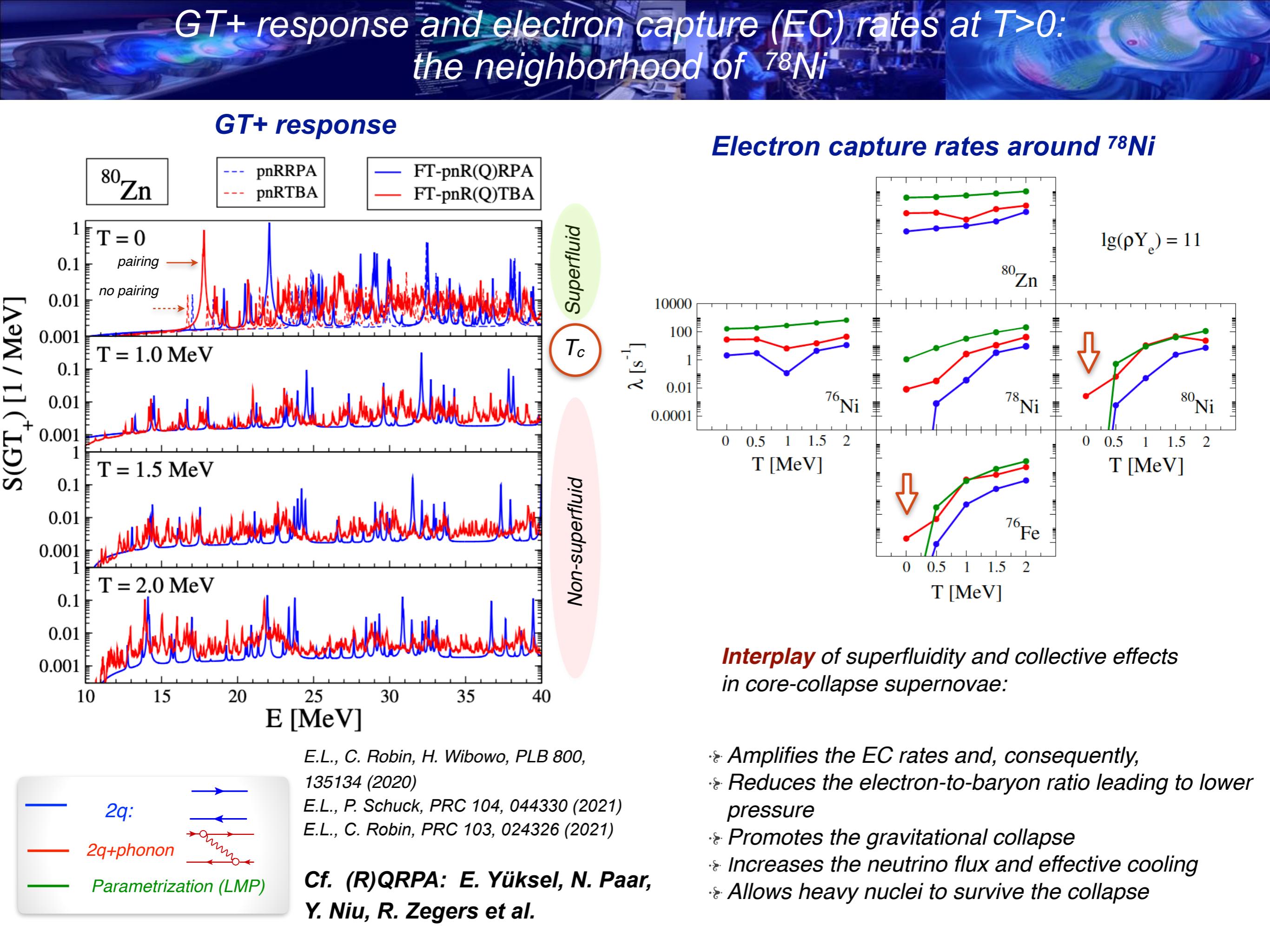
H. Wibowo, E. Litvinova, Phys. Rev. C 100, 024307 (2019)

E. Litvinova, C. Robin, H. Wibowo, Phys. Lett. B 800, 135134 (2020)

Beta decay half-lives in a hot stellar environment



GT+ response and electron capture (EC) rates at $T>0$: the neighborhood of ^{78}Ni



Towards complete formalism at $T>0$: the pairing channel

Averages redefined:

$$G_{12,1'2'}(t - t') = -i\langle \mathcal{T}(\psi_1\psi_2)(t)(\bar{\psi}_{2'}\bar{\psi}_{1'})(t') \rangle \rightarrow -i\langle \mathcal{T}(\psi_1\psi_2)(t)(\bar{\psi}_{2'}\bar{\psi}_{1'})(t') \rangle_T$$

Grand Canonical average: $\langle \dots \rangle \equiv \langle 0| \dots |0 \rangle \rightarrow \langle \dots \rangle_T \equiv \sum_n \exp\left(\frac{\Omega - E_n - \mu N}{T}\right) \langle n| \dots |n \rangle$

Matsubara imaginary-time formalism: temperature-dependent dynamical kernel

Direct:

$$\begin{aligned} \mathcal{K}_{121'2'}^{(r;11)}(\omega_n) &= - \sum_{\nu'\nu''} w_{\nu'} w_{\nu''} \\ &\times \left[\sum_{\nu\mu} \frac{\Theta_{121'2'}^{\mu\nu;\nu'\nu''}(+)}{i\omega_n - \omega_{\nu\nu'} - \omega_{\mu\nu''}^{(++)}} (e^{-(\omega_{\nu\nu'} + \omega_{\mu\nu''}^{(++)})/T} - 1) \right. \\ &\left. - \sum_{\nu\mu} \frac{\Theta_{121'2'}^{\nu\nu;\nu'\nu''}(-)}{i\omega_n + \omega_{\nu\nu'} + \omega_{\nu\nu''}^{(--)}} (e^{-(\omega_{\nu\nu'} + \omega_{\nu\nu''}^{(--})/T} - 1) \right] \end{aligned}$$

Exchange:

$$\begin{aligned} \mathcal{K}_{121'2'}^{(r;12)}(\omega_n) &= \sum_{\nu'\nu''} w_{\nu'} w_{\nu''} \\ &\times \left[\sum_{\nu\mu} \frac{\Sigma_{121'2'}^{\mu\nu;\nu'\nu''}(+)}{i\omega_n - \omega_{\nu\nu'} - \omega_{\mu\nu''}^{(++)}} (e^{-(\omega_{\nu\nu'} + \omega_{\mu\nu''}^{(++)})/T} - 1) \right. \\ &\left. - \sum_{\nu\mu} \frac{\Sigma_{121'2'}^{\nu\nu;\nu'\nu''}(-)}{i\omega_n + \omega_{\nu\nu'} + \omega_{\nu\nu''}^{(--)}} (e^{-(\omega_{\nu\nu'} + \omega_{\nu\nu''}^{(--})/T} - 1) \right], \end{aligned}$$

E.L., P.Schuck, Phys. Rev. C 104, 044330 (2021)

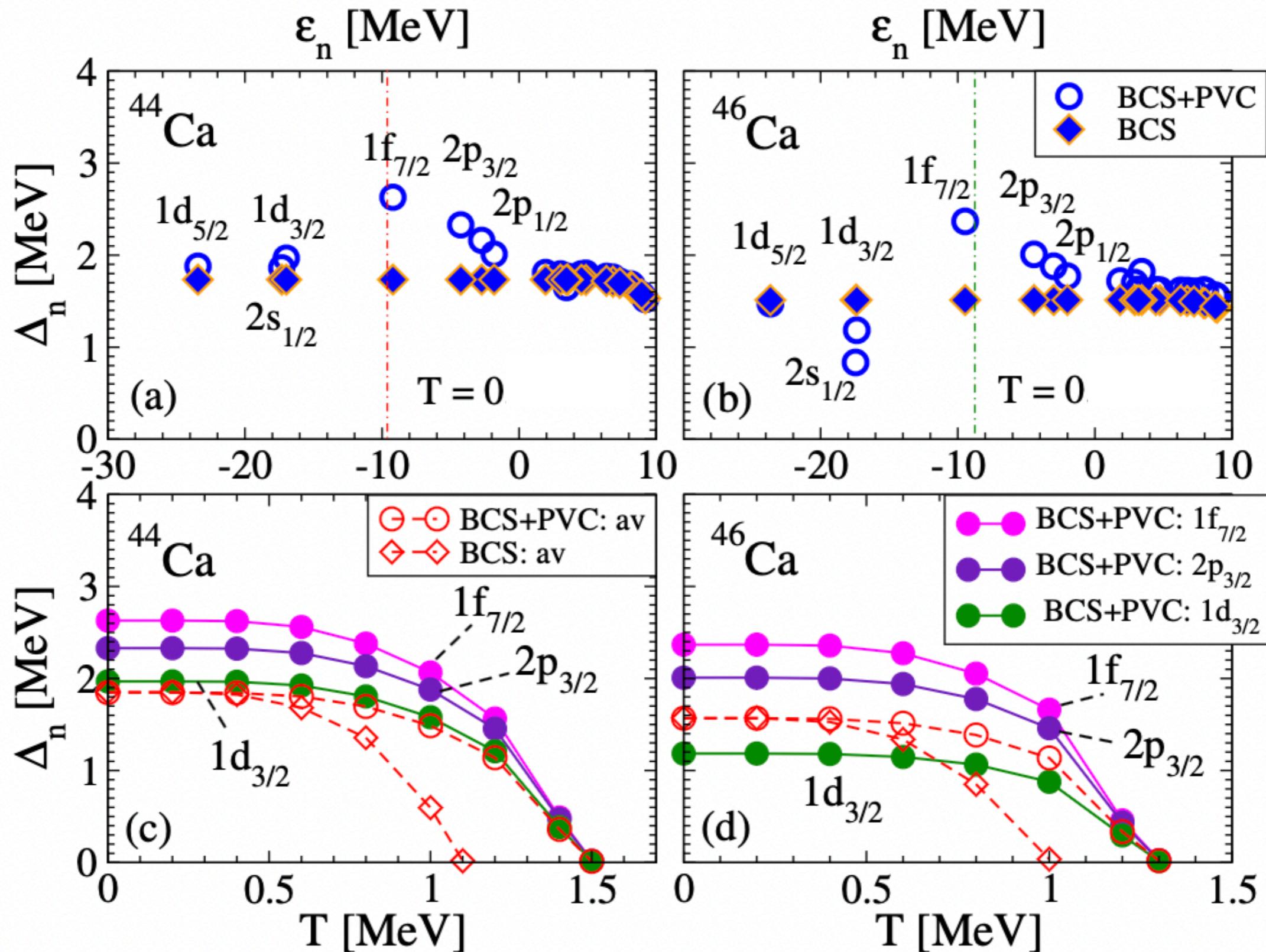
BCS-like gap Eq., but with non-trivial T -dependence in $K^{(r)}$:

$$\Delta_1(T) = - \sum_2 \mathcal{V}_{1\bar{1}2\bar{2}} \frac{\Delta_2(T)(1 - 2f_2(T))}{2E_2}$$

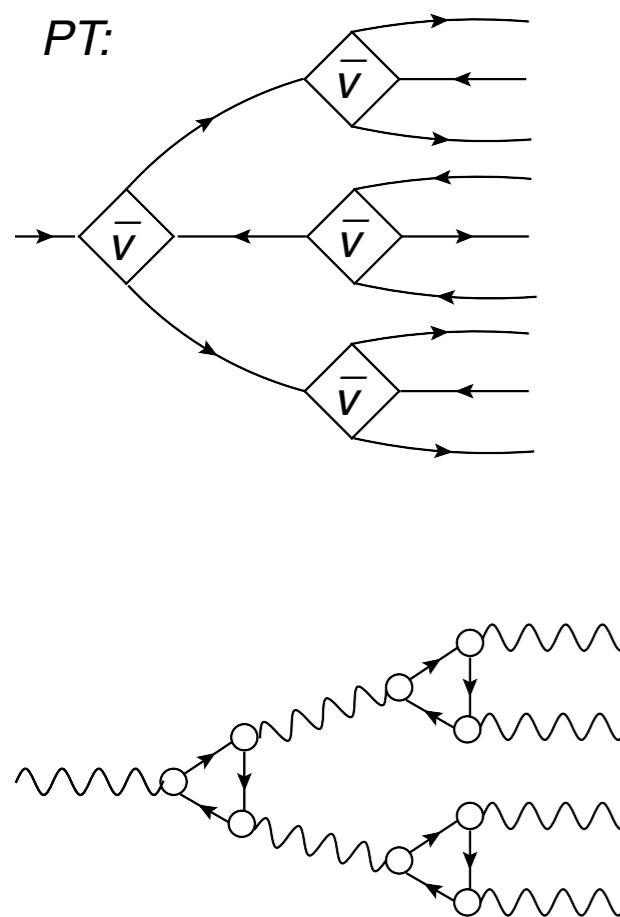
$$f_1(T) = \frac{1}{\exp(E_1/T) + 1}$$

$$\mathcal{V}_{121'2'} = \frac{1}{2} \left(K_{121'2'}^{(0)} + K_{121'2'}^{(r)}(2\lambda) \right)$$

Pairing gap at $T = 0$, $T > 0$ and critical temperature



Effective vs bare forces: NL3* and Bonn potential



σ-meson self-coupling:

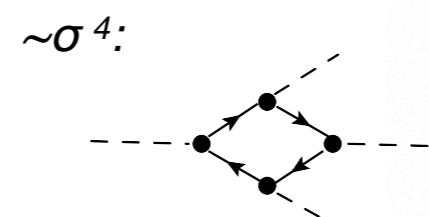
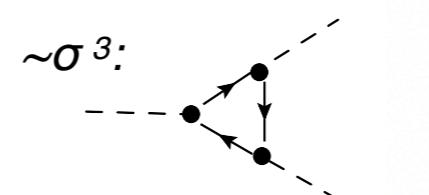
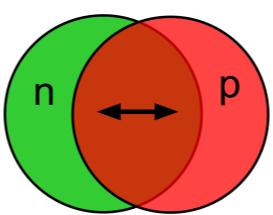
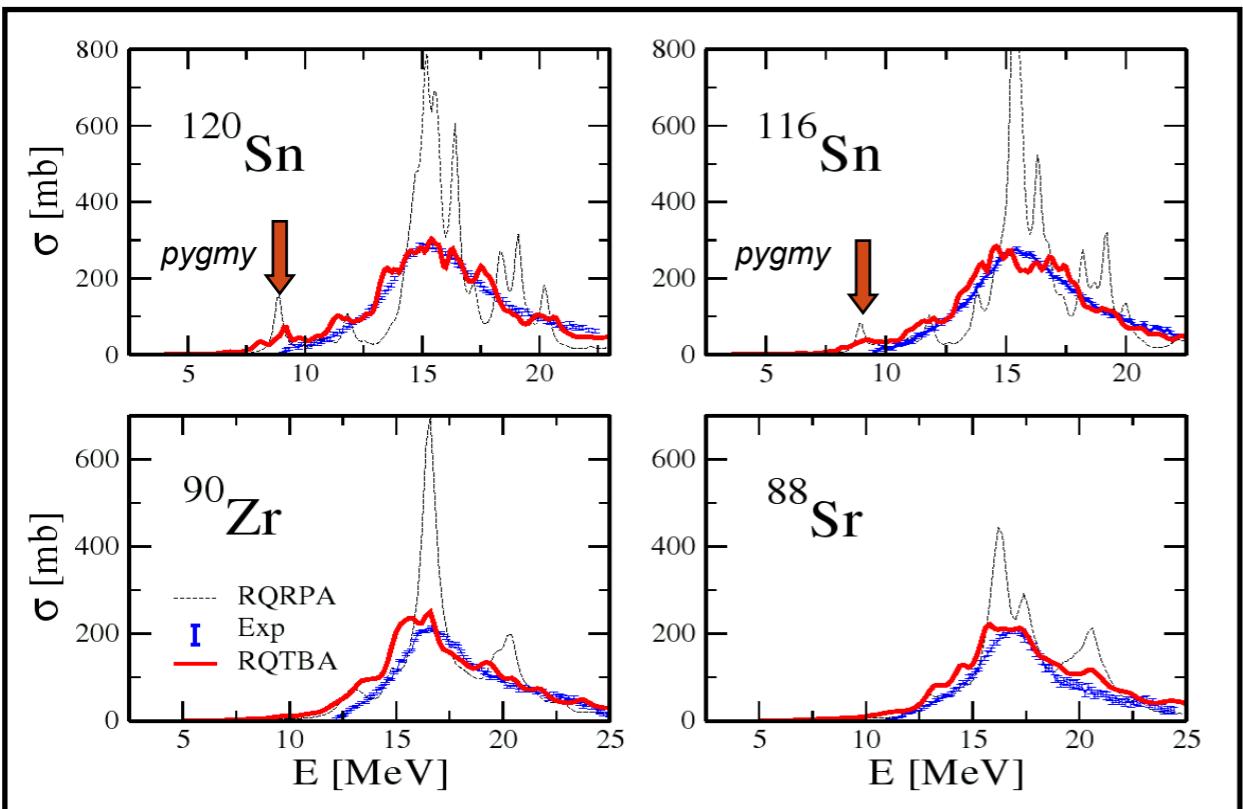


TABLE III: Parameters of the effective interaction NL3* in the RMF theory together with the nuclear matter properties obtained with this effective force. The values of nuclear matter properties obtained with NL3 are shown in parentheses.

Parameters of NL3* <i>Bonn A</i>		<i>Bonn A</i>
$M = 939$ (MeV)		
$m_\sigma = 502.5742$ (MeV)	⁵⁵⁰	$g_\sigma = 10.0944$ ^{10.51}
$m_\omega = 782.600$ (MeV)	^{782.6}	$g_\omega = 12.8065$ ^{17.72}
$m_\rho = 763.000$ (MeV)	⁷⁶⁹	$g_\rho = 4.5748$ ^{3.28}
$g_2 = -10.8093$ (fm $^{-1}$)	⁰	
$g_3 = -30.1486$	⁰	
Nuclear matter properties		
$\rho_0 = 0.150$	(0.148) fm $^{-3}$	
$(E/A)_\infty = 16.31$	(16.30) MeV	
$K = 258.28$	(271.76) MeV	
$J = 38.6$	(37.4) MeV	
$m^*/m = 0.594$	(0.60)	

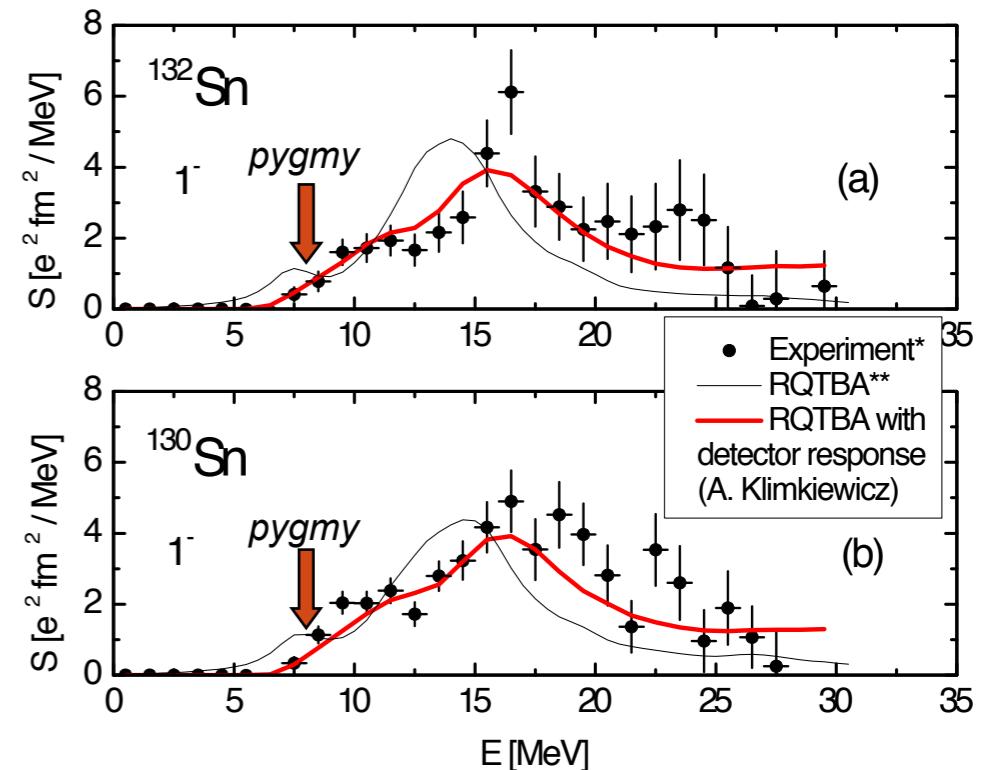
The giant resonance width puzzle: Relativistic Quasiparticle Time Blocking Approximation (RQTBA)

Giant dipole resonance (GDR) in stable nuclei



Giant
&
pygmy
dipole
resonances

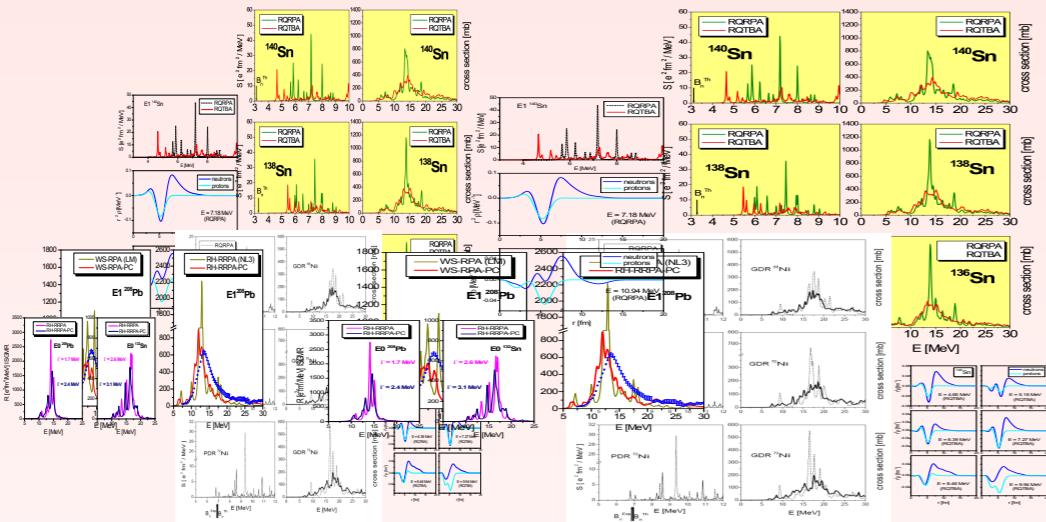
Neutron-rich Sn



* P. Adrich et al.,
PRL 95, 132501 (2005)

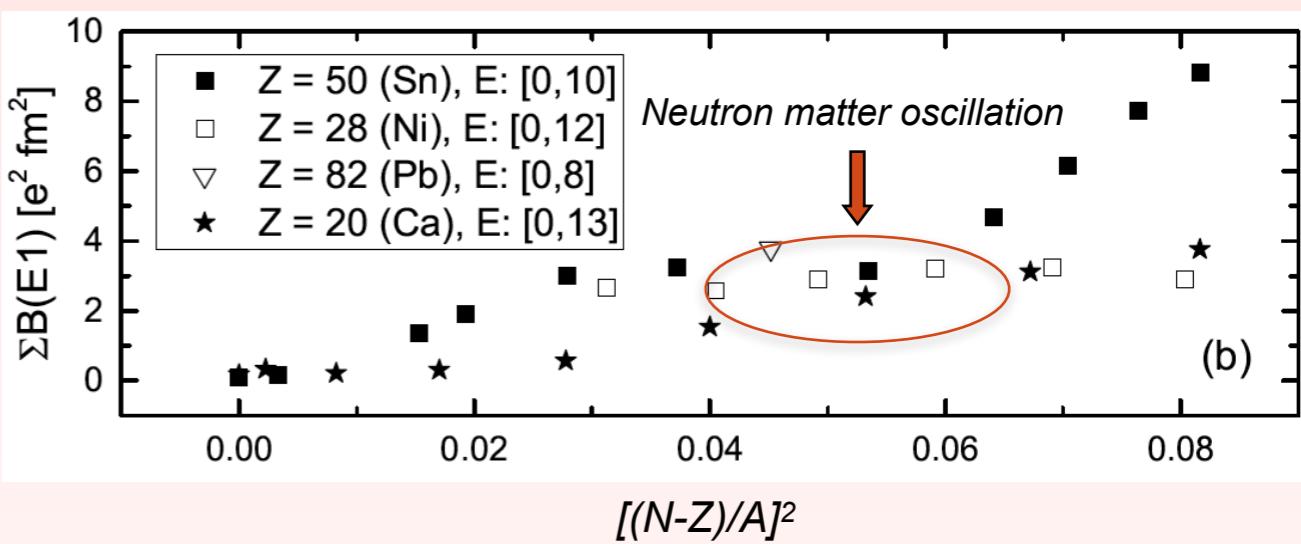
**E. L., P. Ring, and V. Tselyaev,
Phys. Rev. C 78, 014312 (2008)

Systematic GMR calculations (various multipoles)



~50+ works on various GMRs

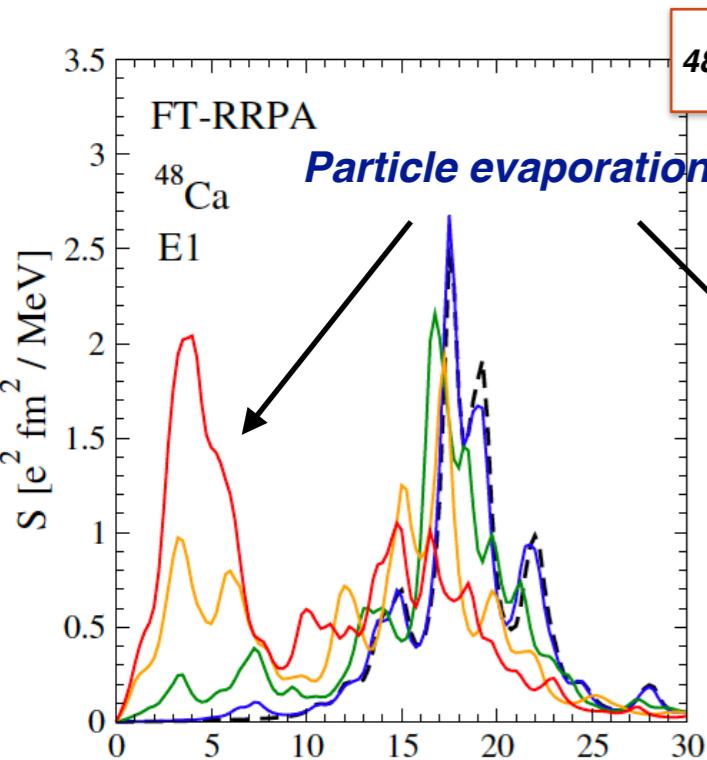
Pygmy dipole strength systematics (important for EOS)



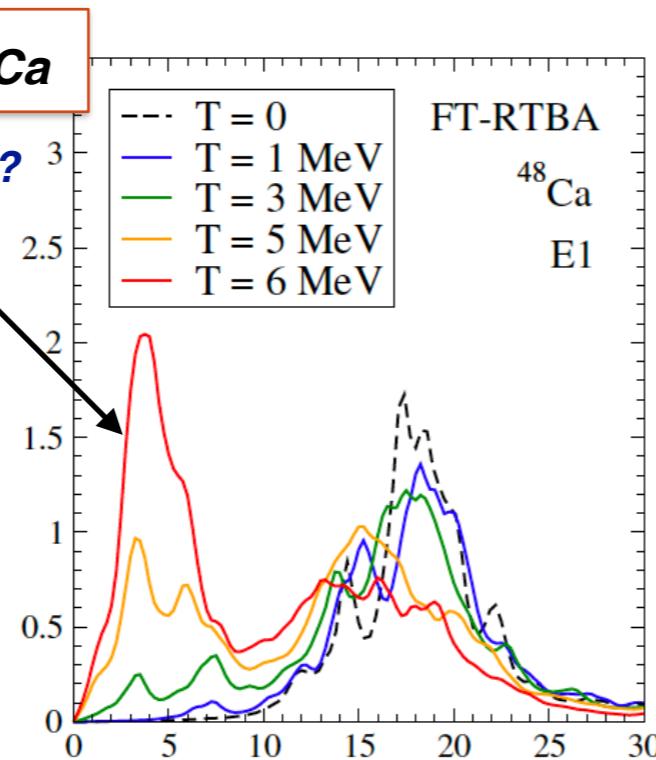
I.A. Egorova, E. Litvinova, Phys. Rev. C 94, 034322 (2016)

Dipole Strength at $T>0$: ^{48}Ca and ^{132}Sn

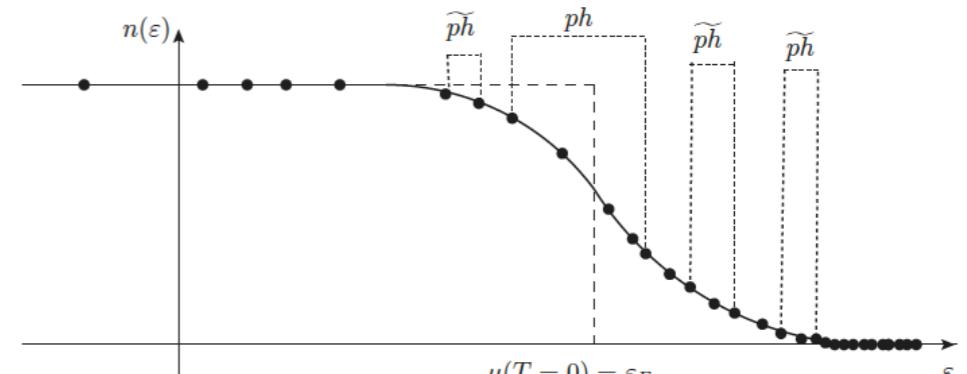
Static only (FT-RRPA)



Static + dynamic (FT-RTBA)

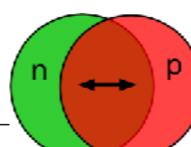
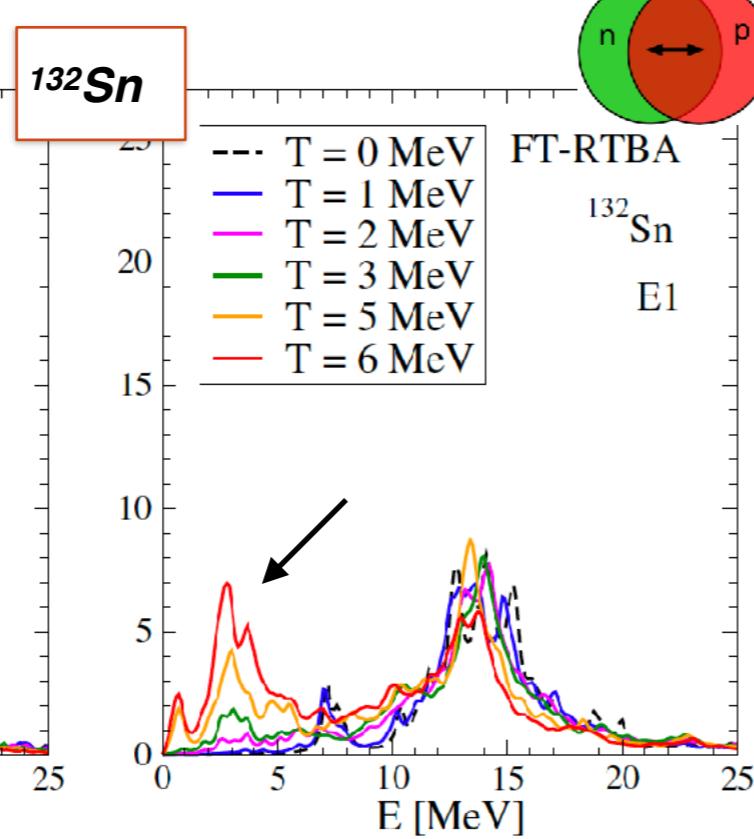
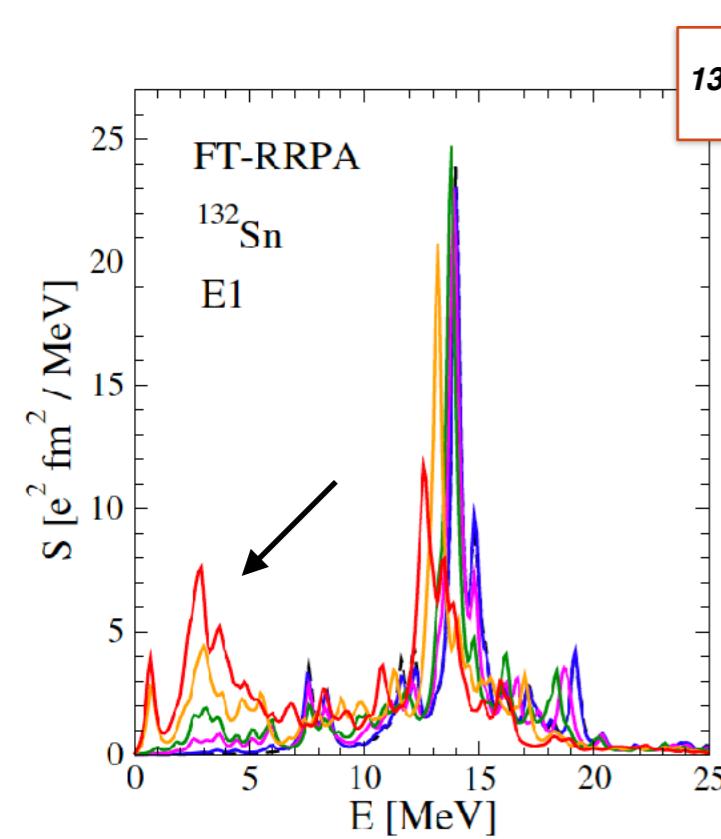


Thermal unblocking:



0th approximation:
Uncorrelated propagator

$$\tilde{R}_{14,23}^0(\omega) = \delta_{13}\delta_{24} \frac{n_2 - n_1}{\omega - \varepsilon_1 + \varepsilon_2}$$



- New transitions due to the thermal unblocking effects
- More collective and non-collective modes contribute to the PVC self-energy (~400 modes at $T=5-6$ MeV, cf.)
- Broadening of the resulting GDR spectrum
- Development of the low-energy part => a feedback to GDR
- The spurious translation mode is properly decoupled as the mean field is modified consistently
- The role of the new terms in the Φ amplitude increases with temperature
- The role of dynamical correlations and fragmentation remain significant in the high-energy part