In-medium parton showers with overlapping emissions

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Reporting (eventually) on recent work with

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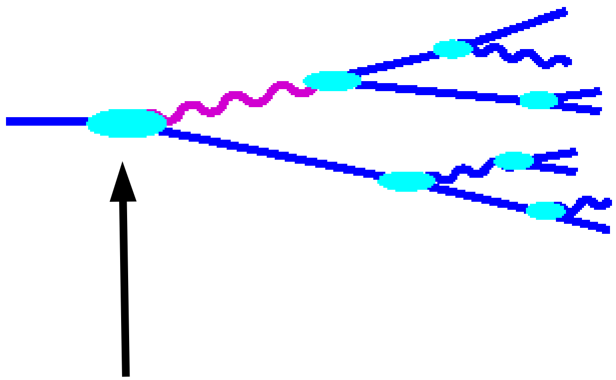
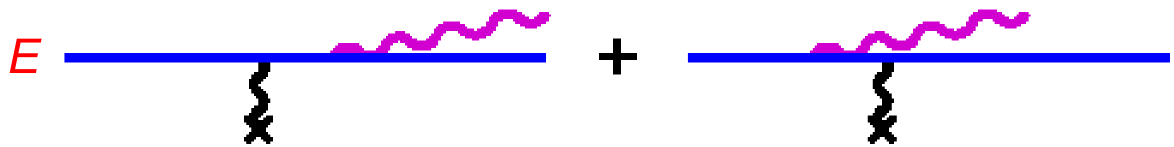
letter: 2212.08086
details: 2302.10215

Medium-induced showering



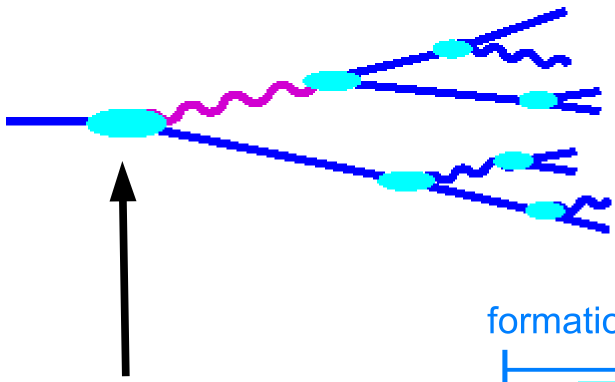
Naively:



Prob. of brem ~ α per collision with medium

(up to logs)

Medium-induced showering



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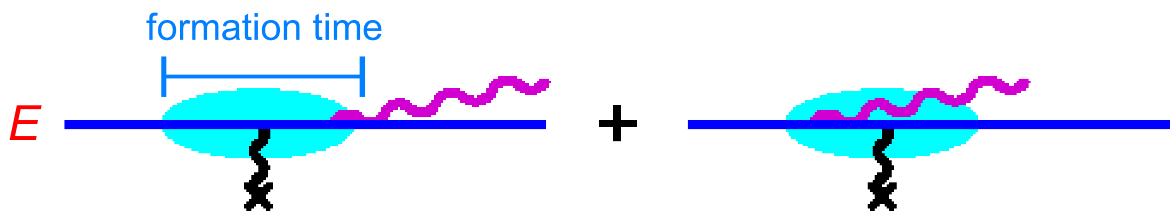
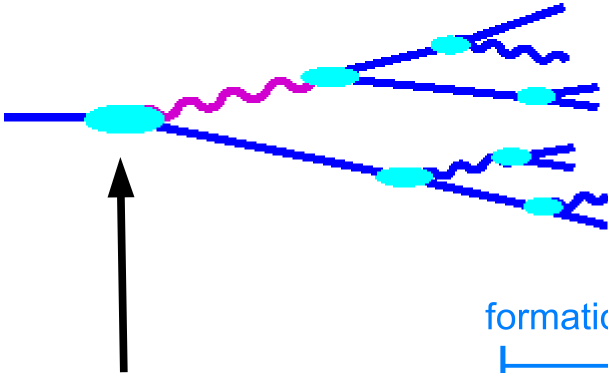


Formation time means quantum <u>duration</u> of splitting process.

Formation time grows with energy *E*.

Medium-induced showering



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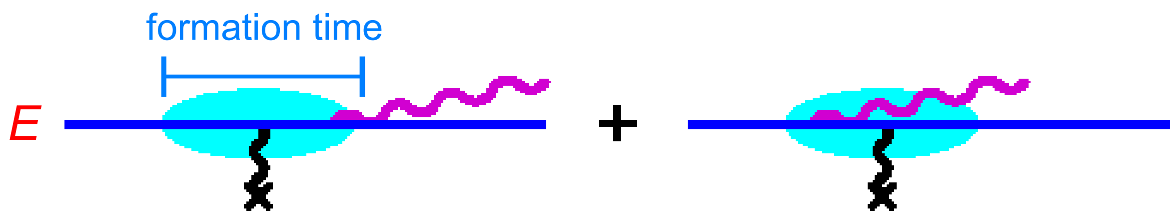
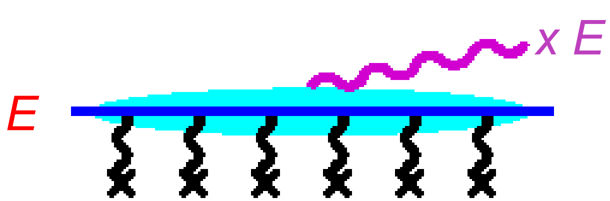


Formation time means quantum <u>duration</u> of splitting process.

Formation time grows with energy *E*.

LPM Effect:

What happens when formation time $\gg$ mean free time between collisions w/ medium?



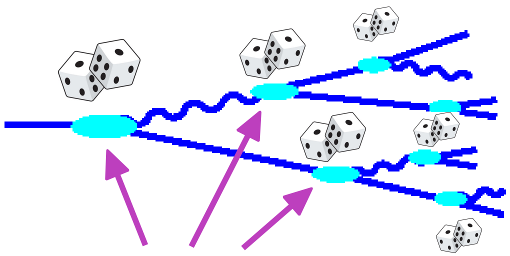
Prob. of brem ~ α per formation time

QED (1950s): LPM [Landau-Pomeranchuk & Migdal]

QCD (1990s): BDMPS-Z + many later variations

→ calculation of splitting rates $\dfrac{d\Gamma^{\,\rm split}}{dx}$

Can we then describe in-medium shower development by

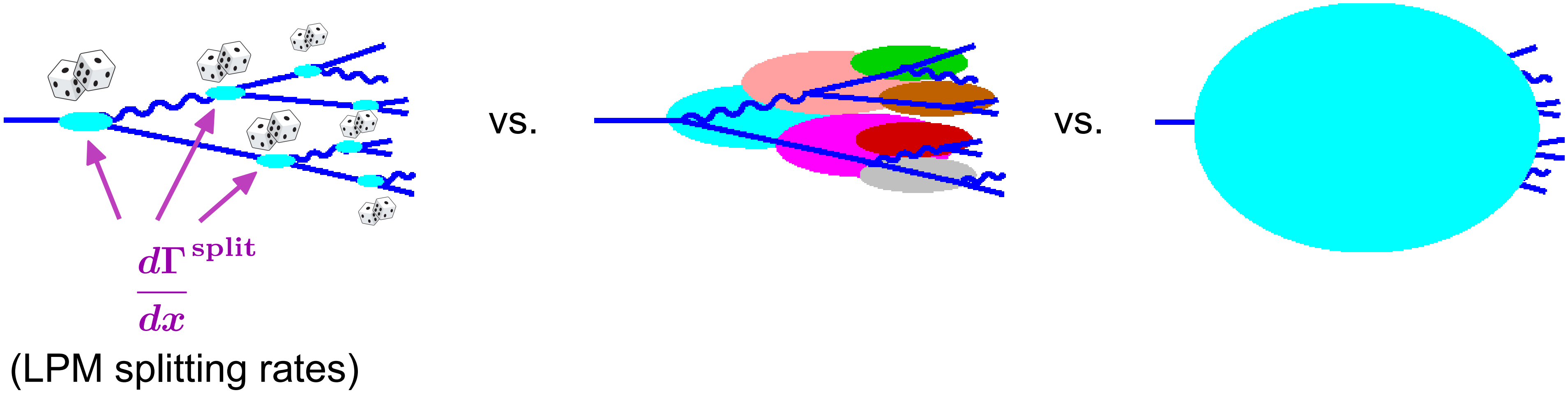


$$\frac{d\Gamma^{\rm split}}{dx}$$

(LPM splitting rates)

?

Or can splittings overlap?



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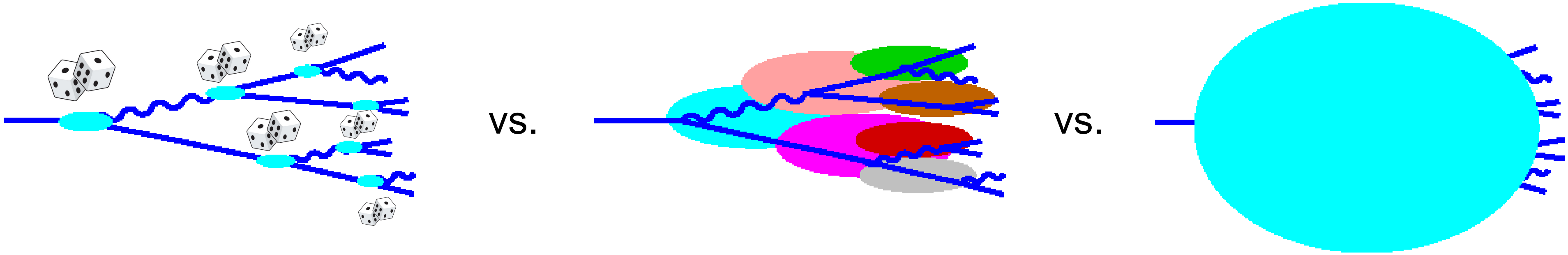


Prob. of brem ~ $\alpha(\mu)$ per formation time

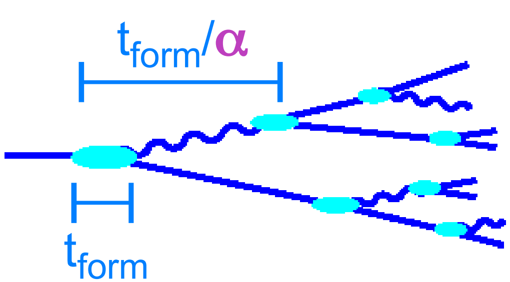
⇨ Prob. two consecutive splittings overlap ~ $\alpha(\mu)$

↑ a scale depending on energy

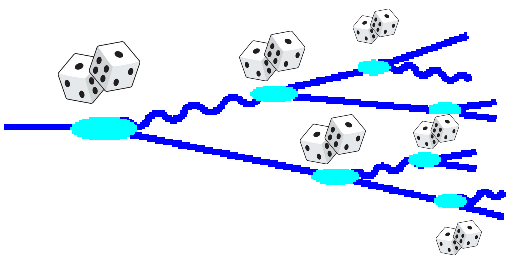
Not the $\alpha(T)$ of the QGP

All depends on how big $\alpha(\mu)$ is!

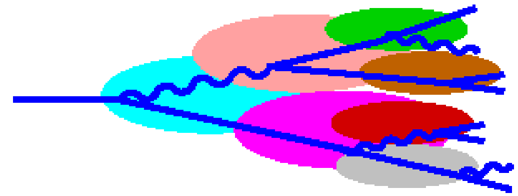
For <u>small</u> α, there is a hierarchy of scales that (typically) separates the splittings:



Summary so far



vs.



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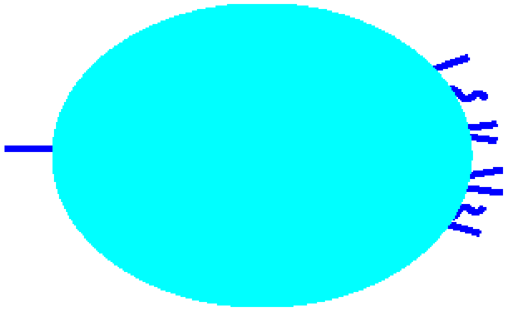


$\alpha_s(\mu)$ **small**

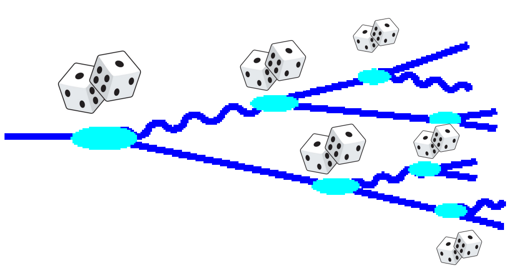
a “standard” picture
of a shower

$\alpha_s(\mu)$ **big**

HELP!

Turn to AdS/CFT for
qualitative insight

How do we tell if



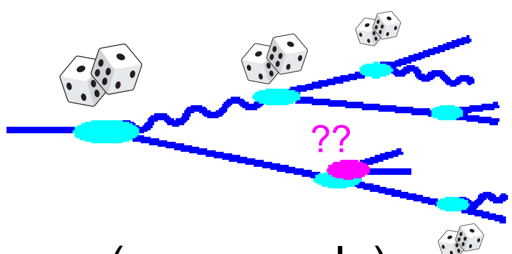
is a good or bad picture for reasonable values of $\alpha_s(\mu)$?

Two approaches

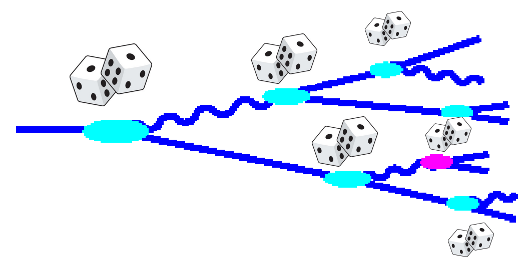
(1) EXTERNAL VALIDATION: Confront w/ experiment.
But…. many confounding factors.

(2) INTERNAL CONSISTENCY: Test with theory!

Question:
Are the first corrections



to



(an example)

small for reasonable values of $\alpha_s(\mu)$?

Perks for theorists:

- May avoid confounding factors by testing in simplified situations.
- Can test on simple shower characteristics not accessible to experiment.

So…

A theorist thought experiment

Simplifying assumptions

- Treat elastic scattering w/ medium in the $\hat{q}$ approximation:

$$\langle (\text{change in } p_\perp)^2 \rangle = \hat{q} \cdot (\text{distance traveled})$$

- A static, homogeneous, "infinite"-size QGP

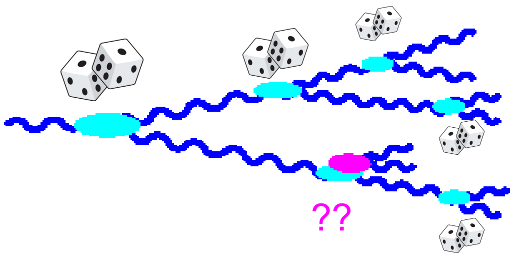
I can now reveal that scale for $\alpha_s(\mu)$ is $\mu \sim (\hat{q}E)^{1/4}$

and formation times are $t_{\text{form}} \sim \sqrt{E/\hat{q}}$

- Start with a parton that is (approx.) on-shell.

- Study gluon-initiated showers in large-N_c limit (w/ N_f fixed)

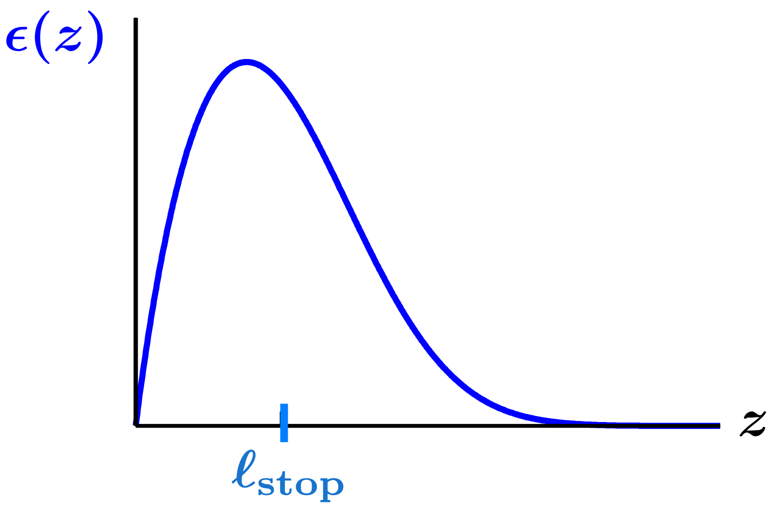
⇒ Only g→gg splittings consider (so far!)



A theorist thought experiment

Something theorists could "observe":

(statistically averaged) distribution of energy deposited by shower as a function of distance *z*



$\ell_{\rm stop} \equiv \langle z \rangle$ (1st moment of energy deposition distribution)

$$\ell_{\rm stop} \sim \frac{t_{\rm form}}{\alpha} \sim \frac{1}{\alpha}\sqrt{\frac{E}{\hat{q}}}$$

Note: $\ell_{\rm stop}$ depends on $\hat{q}$

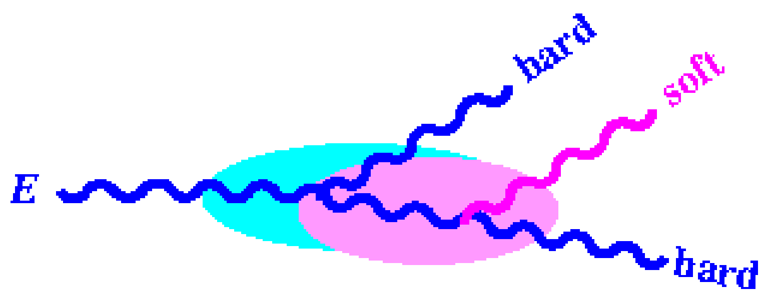
How big are the overlap corrections to ε(z)?

Answer: **BIG !** … which has been know since Iancu (2014), Blaizot and Mehtar-Tani (2014), Wu (2014)

[building on radiative corrections to $\hat{q}$ found by Liou, Mueller, Wu (2013)]

(1) BIG because there is a double-log enhancement coming from SOFT radiation:

Prob. of overlap



suppressed by (in my application)

$$\alpha_s(\mu)\ln^2\left(\frac{E}{T}\right)$$ ⇒ BIG result for large *E*

(2) But these BIG soft-radiation effects can be absorbed into an effective value of $\hat{q}$:

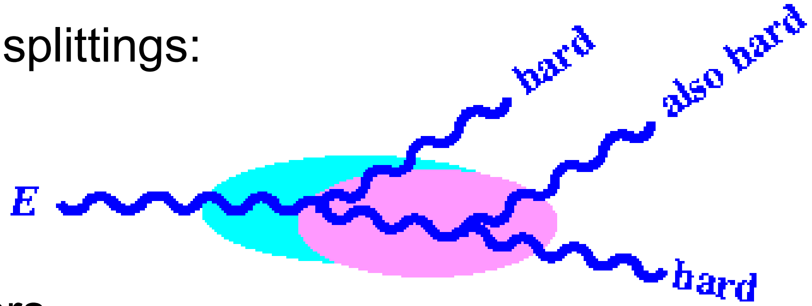
$$\hat{q} \longrightarrow \hat{q}_{\rm eff}(E) = \hat{q}\left[1 + \#\alpha_s \ln^2\left(\frac{E}{T}\right)\right]$$

Can even be re-summed at leading log to all orders in α_s

A REFINED QUESTION

How big are overlap effects that cannot be absorbed in $\hat{q}$?

(1) Need to calculate overlap of two hard splittings:



Extremely difficult calculation.

After lots of QFT and many (!!) years ...

Completed (for gluons) in 2022 with S. Iqbal and

Tyler Gorda

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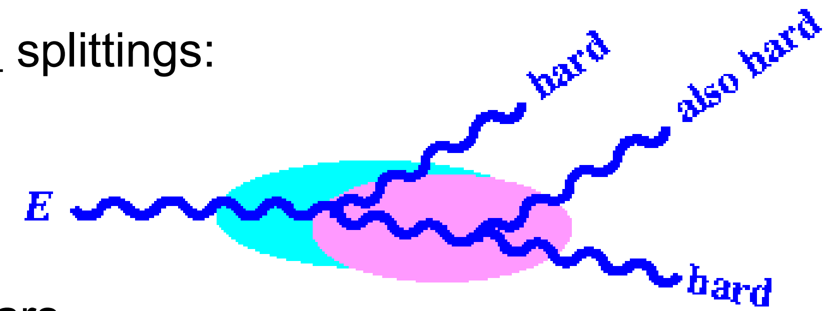

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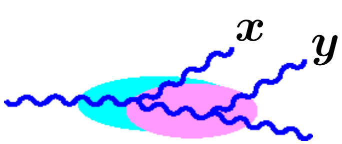
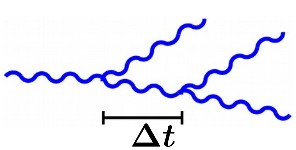
Technical note

The drawing above is short-hand for what we call



$\Delta \frac{d\Gamma}{dx\,dy} \equiv$ the overlap correction to two independent splittings

$$= \Bigg[\text{full calculation of double splitting rate} \quad \left\langle \left| \int_0^\infty d(\Delta t) \right. \right.$$  $$\left. \left. + \cdots \right|^2 \right\rangle_{\text{medium avg}} \Bigg] - \Bigg[\text{pretending the two splittings are independent dice roles} \quad \frac{d\Gamma^{\text{split}}}{dx} \text{ and } \frac{d\Gamma^{\text{split}}}{dy} \Bigg]$$

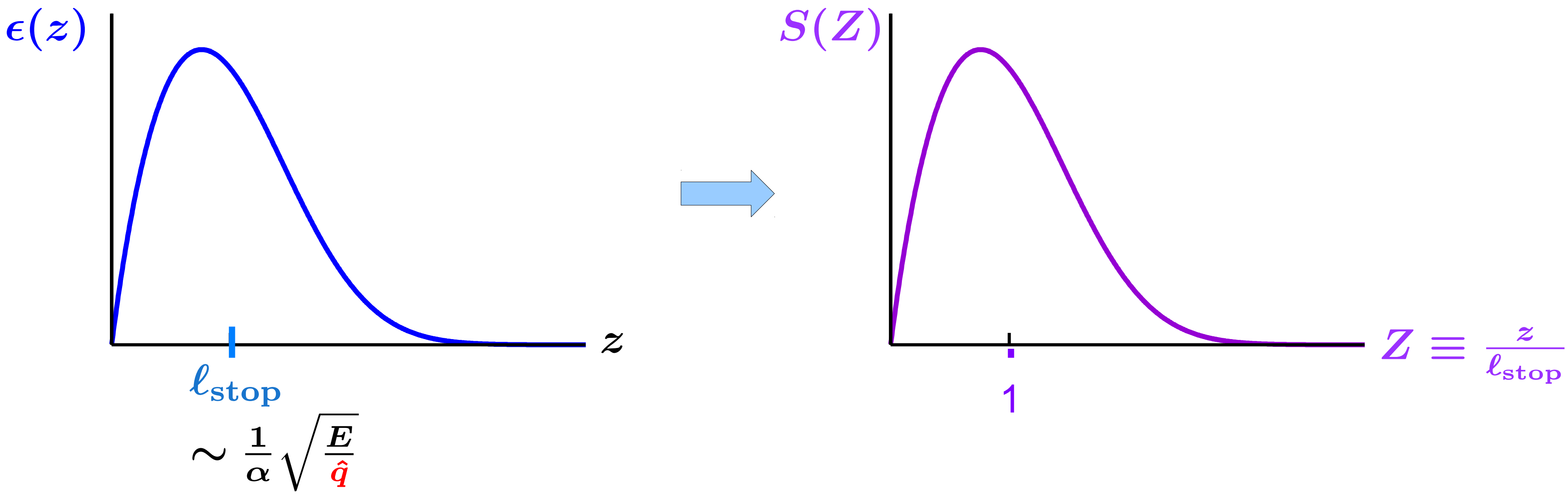
which cancels except for contributions from splittings separated by $\Delta t \lesssim t_{\text{form}}$

A REFINED QUESTION

How big are overlap effects that <u>cannot</u> be absorbed in $\hat{q}$?

(2) Choose a theorist observable that is insensitive to $\hat{q}$:

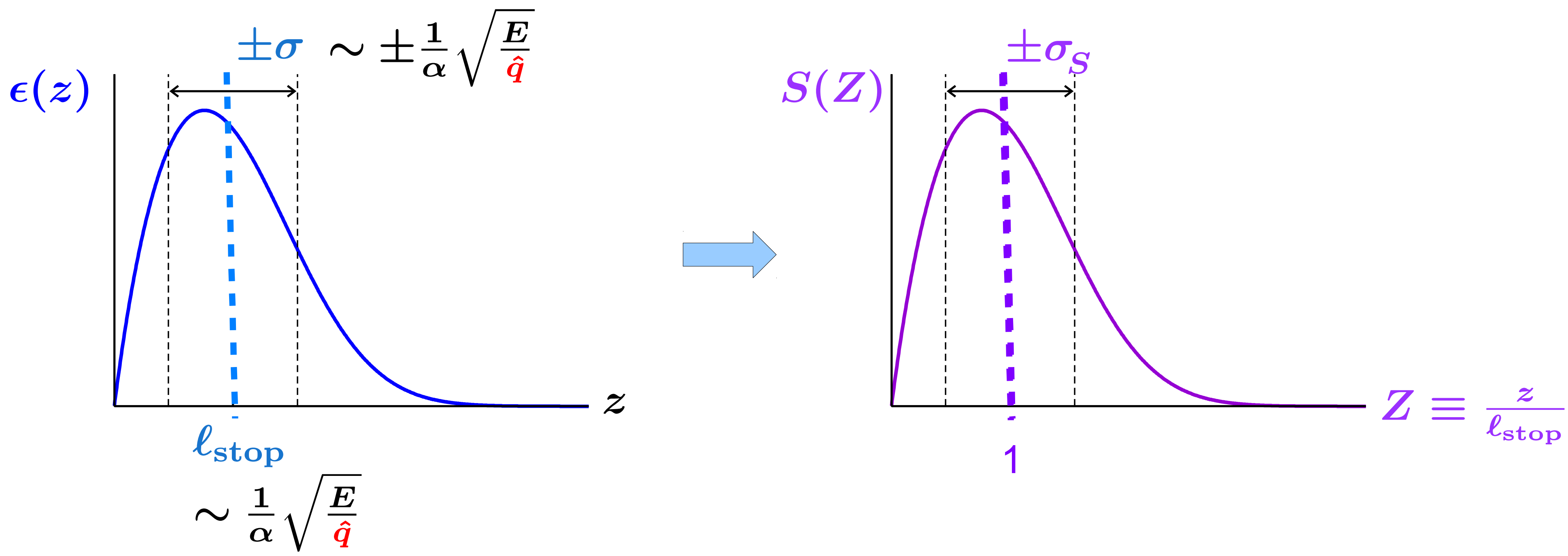
consider the <u>shape</u> $S(Z)$ of the energy deposition distribution:



A REFINED QUESTION

How big are overlap effects that <u>cannot</u> be absorbed in $\hat{q}$?

<u>Example</u>

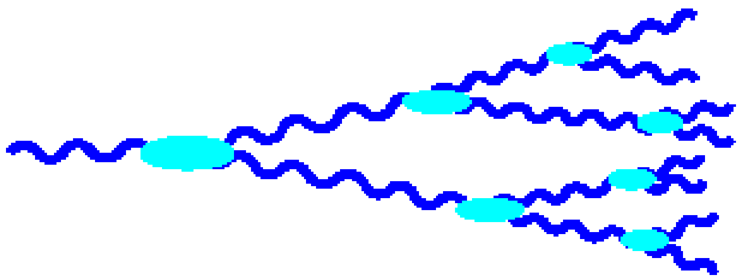


$$\sigma_S = \frac{\sigma}{\ell_{\text{stop}}} \text{ is independent of } \hat{q}$$ *

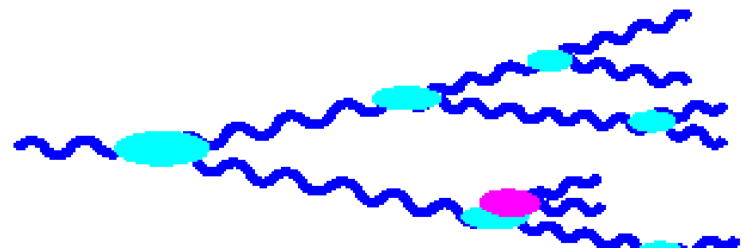
* Important, interesting, and resolvable caveats that I may not have time to explain.

How to account for overlaps in showers

Think of

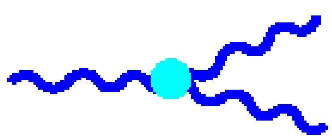


+



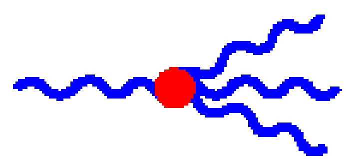
(for example)

as “standard” shower development with independent splittings but two types of localized, independent vertices:



1→2 (normal LPM)

$$\frac{d\Gamma^{\rm split}}{dx}$$



1→3 (overlap correction)

$$\Delta \frac{d\Gamma}{dx\,dy}$$

Then treat these “splitting” probabilities as purely classical.

RESULTS

To start: the width of the shape *S*(*Z*) of energy deposition

Large-N_f QED [2018 w/ S. Iqbal]:

"LO" means "ignoring overlaps"

charge deposition

$$\sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 - 0.87\, N_{\rm f}\,\alpha(\mu)\right]$$

Large-N_c QCD (gluons only) [2022 w/ S. Iqbal and O. Elgedawy]:

energy deposition

$$\sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 + \text{???}\, N_{\rm c}\,\alpha_{\rm s}(\mu)\right]$$

DRUM ROLL
PLEASE

RESULTS

To start: the width of the shape S(Z) of energy deposition

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energy deposition

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RESULTS

To start: the width of the shape S(Z) of energy deposition

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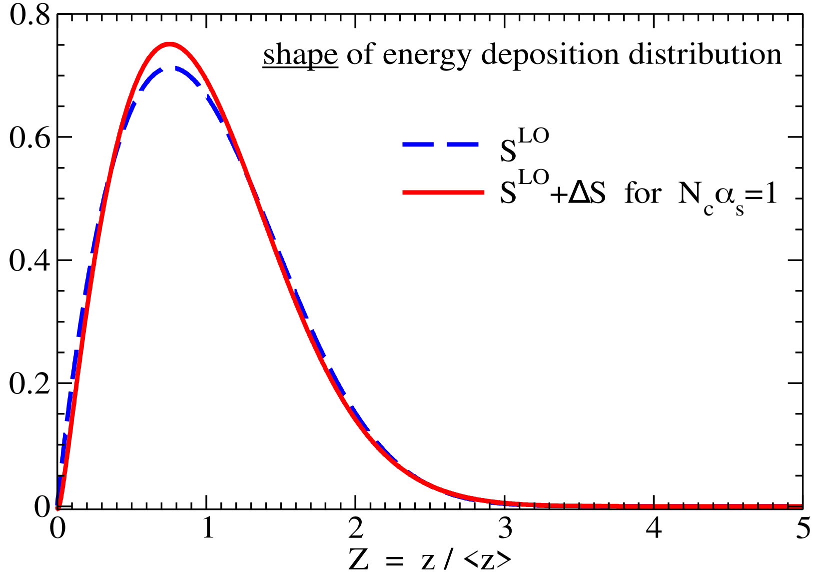


Conclusion for this test

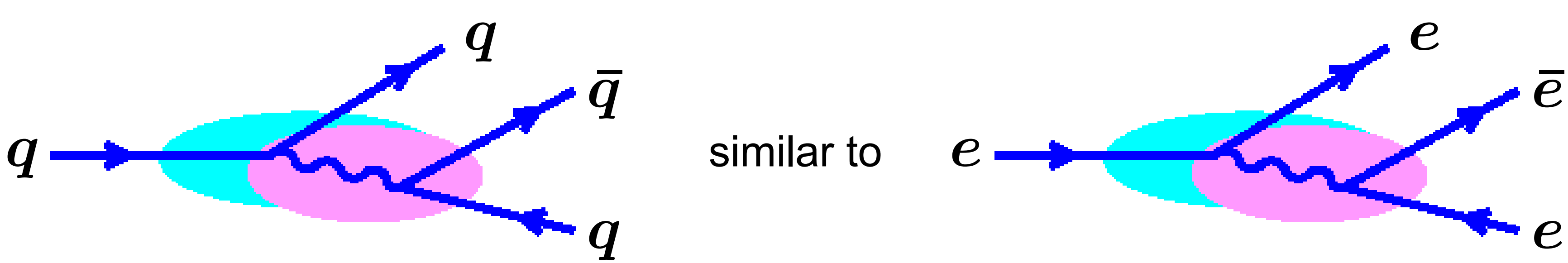
Overlap corrections that cannot be absorbed into $\hat{q}$ are negligible.

The QED and gluon results are very different: Discuss!

Large-N_f QED $\quad \sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 - 0.87\, N_{\rm f}\alpha(\mu)\right]$

Large-N_c gluons $\quad \sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 - 0.02\, N_{\rm c}\alpha_{\rm s}(\mu)\right]$

***A concern*:** QCD with quarks has some overlap diagrams that look similar to QED



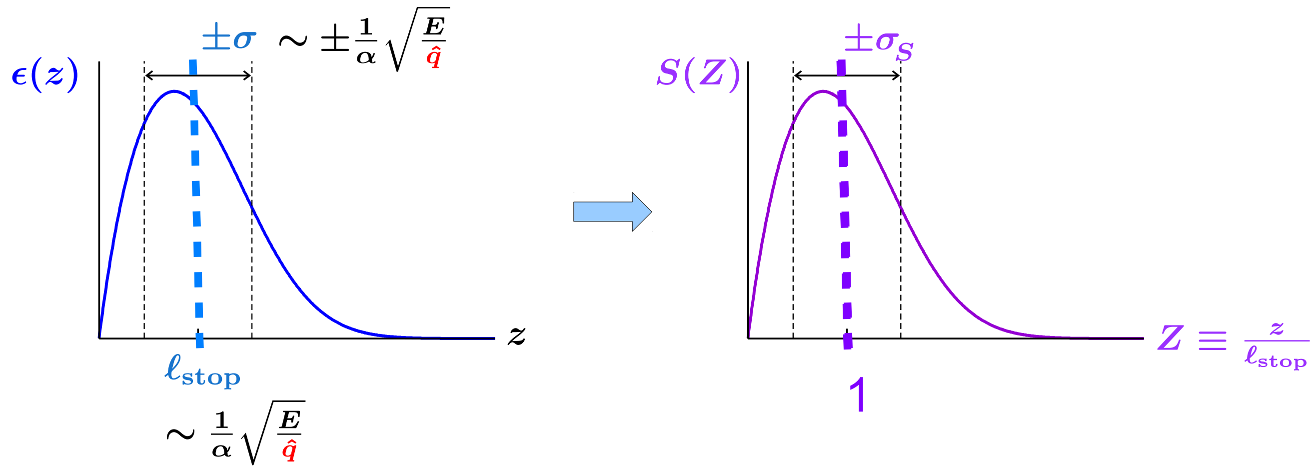
Will adding quarks to the analysis qualitatively change the conclusion for QCD?

Answer: Work in progress.

Shrouded from view in this presentation ...

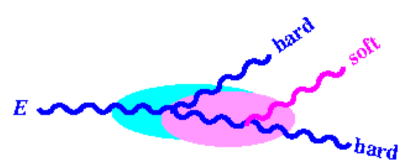
I half-lied about something

Remember



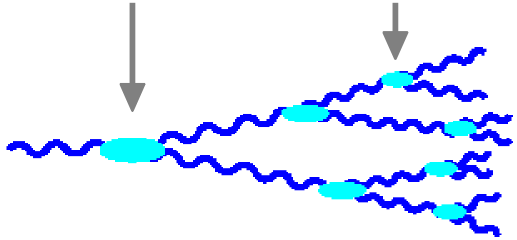
$\sigma_S = \dfrac{\sigma}{\ell_{\rm stop}}$ is independent of $\hat{q}$ *

and why we did that:



$$\hat{q} \longrightarrow \hat{q}_{\rm eff}(E) = \hat{q}\left[1 + \#\alpha_{\rm s} \ln^2\left(\tfrac{E}{T}\right)\right]$$

But then $\hat{q}_{\rm eff}(E)$ is different *here* and *there*.



Those difference don't quite cancel in $\sigma_S{=}\sigma/\ell_{\rm stop}$ and $S(Z)$.
They cancel at leading log but leave behind BIG single-log corrections to σ_S and $S(Z)$:

overlap corrections $\sim \alpha_{\rm s}(\mu)\ln\left(\tfrac{E}{T}\right)$

Factorization

Remember that soft radiation can be absorbed into $\hat{q}$.

When factorizing away some IR or UV physics in QFT, we must introduce a *factorization scale* to do NLO calculations.

Examples

UV divergences absorbed into couplings:	renormalization scale μ
Collinear divergences absorbed into PDFs:	factorization scale M_{fac}

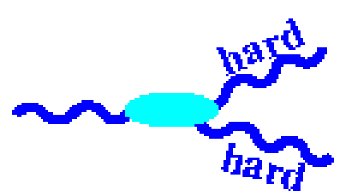
Such factorization scales appear explicitly inside logarithms in NLO results.

- Set them to the appropriate physics scale for the process.
- Check sensitivity to the precise choice of scale.

Our problem

To factorize *all* the soft radiation effects into $\hat{q}_{\mathbf{eff}}$,
we introduce an energy factorization scale

$$\Lambda_{\mathbf{fac}} = \# \left(\text{min energy of daughters of} \right)$$



where # =
any reasonable O(1) number.

The overlap result shown earlier was the result for # = 1.

Now showing dependence on the normalization # of the factorization scale:

$$\sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 - (0.02 + 0.001 \ln \#) N_c \alpha_s(\mu)\right]$$

Extremely weak dependence on factorization scale.

Return to Conclusions

Large-N_f QED $$\sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 - 0.87\, N_{\rm f}\alpha(\mu)\right]$$

Large-N_c gluons $$\sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 - 0.02\, N_{\rm c}\alpha_{\rm s}(\mu)\right]$$

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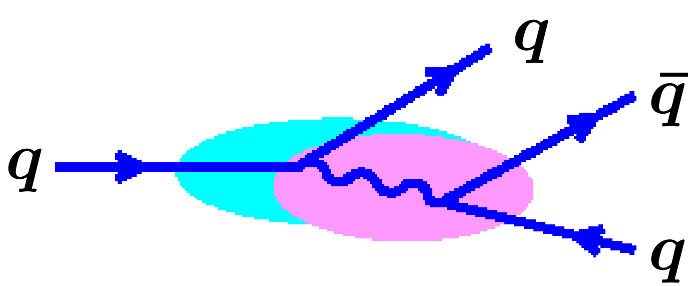
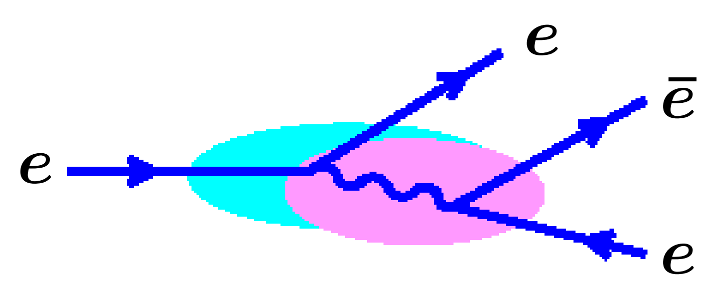


similar to



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