ALPACA
AMY Lorentz Invariant Parton Cascade

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MOTIVATION

- Recent observations of signs of collective behaviour in small systems: need for *new models* to confront experimental data.
- *QCD effective kinetic theory (AMY)*: good candidate.
- Detailed comparisons between theory and data needed: *Monte Carlo Event Generator*.
- ALPACA: Lorentz Invariant parton cascade which evolves discrete parton ensembles according to AMY collision kernels.

POINCARÉ INVARIANCE

- Relativistic mechanics in 8N-dimensional phase space [1] [2] [3].
- Simplified Hamiltonian, assuming particles to behave as *free between binary scatterings*, time ordered by frame independent τ.
- Equations of motion from above, minimize the *Poincaré invariant distance*

$$d_{ij}^2(\tau) = -\left(\Delta x_\mu - \frac{\Delta x_\nu P^\nu}{P^2} P_\mu\right),$$

with $\Delta x^\mu(\tau) = x_i^\mu - x_j^\mu$ and $P^\mu(\tau) = p_i^\mu + p_j^\mu$, to find closest approach.

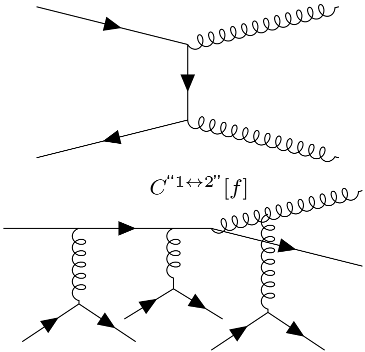
[1] G. Peter, C. Noack, and D. Behrens, Phys. Rev. C 49 (1994).
[2] D. Behrens, C. Noack, and G. Peter, Phys. Rev. C 49 (1994).
[3] V. Börchers, J. Meyer, St. Gieseke, G. Martens, C.C. Noack, Phys. Rev. C 62 (2000).

QCD EFFECTIVE KINETIC THEORY

- Boltzmann equation

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}})\, f(\mathbf{x}, \mathbf{p}, t) = -C[f(\mathbf{x}, \mathbf{p}, t)].$$

  Relevant LO scattering processes[4]:
  - $2 \leftrightarrow 2$ elastic scattering.
  - "$1 \leftrightarrow 2$" collinear radiation.
- Out-of-equilibrium QGP description.
- Bottom-up thermalization in large systems[5].



[4] P. Arnold, G.D. Moore, and L.G. Yaffe, Journal of High Energy Physics (2003).
[5] R. Baier, A.H. Mueller, D. Schiff, D.T. Son, Phys. Lett. B 502 (2001).

QCD EFFECTIVE KINETIC THEORY

ELASTIC SCATTERING

- Collision kernel controls local scattering rate

$$C_a^{2\leftrightarrow 2}[f] = \frac{1}{4|\mathbf{p}|}\sum_{bcd}\int_{\mathbf{k}\mathbf{p}'\mathbf{k}'} \nu_b |\mathcal{M}^{ab}_{cd}|^2 (2\pi)^4 \delta^{(4)}(P + K - P' - K') \times \left\{ f_a(\mathbf{p}) f_b(\mathbf{k})[1 \pm f_c(\mathbf{p}')][1 \pm f_d(\mathbf{k}')] - f_c(\mathbf{p}') f_d(\mathbf{k}')[1 \pm f_a(\mathbf{p})][1 \pm f_b(\mathbf{k})] \right\}.$$

- Effective screening mass regulates divergent matrix elements

$$m_g^2 = \sum_s 2\nu_s \frac{g^2 C_s}{d_A} \int_V \frac{d^3\mathbf{x}}{V} \int \frac{d^3\mathbf{p}}{2|\mathbf{p}|(2\pi)^3} f_s(\mathbf{p}, \mathbf{x}).$$

QCD EFFECTIVE KINETIC THEORY

SPLITTING AND MERGING

- Local collinear splitting/merging rate from kernel

$$C_a^{“1\leftrightarrow 2”}[f] = \frac{(2\pi)^3}{2|\mathbf{p}|^2\nu_a}\sum_{b,c}\int_0^\infty d\mathbf{p}'\, d\mathbf{k}'\delta(p-p'-k')\gamma^a_{bc}(\mathbf{p};p'\hat{\mathbf{p}},k'\hat{\mathbf{p}})$$
$$\times\left\{f_a(\mathbf{p})[1\pm f_b(p'\hat{\mathbf{p}})][1\pm f_c(k'\hat{\mathbf{p}})] - f_b(p'\hat{\mathbf{p}})f_c(k'\hat{\mathbf{p}})[1\pm f_a(\mathbf{p})]\right\} + C^{“1\leftrightarrow 2”}_{\mathbf{p},\mathrm{merge}}.$$

- Depends on *effective temperature*

$$T_* = \frac{1}{m_g^2}\frac{1}{2}g^2\sum_s\frac{\nu_s C_s}{d_A}\frac{1}{V}\int d^3\mathbf{x}\int\frac{d^3\mathbf{p}}{(2\pi)^3}f_s(p)[1+f_s(p)].$$

- γ piecewise divergent for small energy transfer x.

ELASTIC SCATTERING

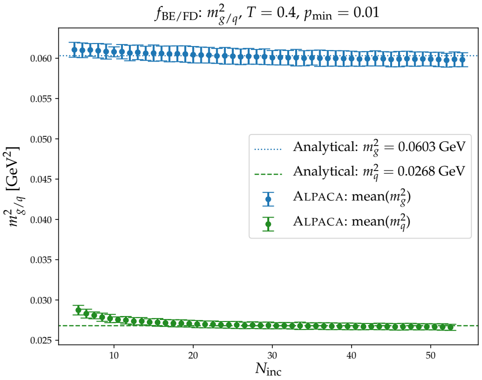
- Evolution in τ.
- Black disk: scattering if $\min(d_{ij}) < \sqrt{\sigma/\pi}$.
- Regulating matrix elements as

$$\frac{1}{t^2} \rightarrow \frac{1}{(t - \zeta_s m_s^2)^2}.$$

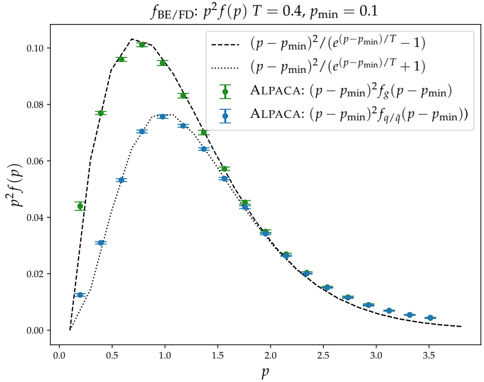
- Due to Bose/Pauli factors depending on outgoing kinematics Monte Carlo accept/reject sampling is used with an overestimate of the cross section.

ELASTIC SCATTERING

- $m^2_{g,q}$: sum N closest (pointlike) particles, $\int d^3\mathbf{x}d^3\mathbf{p} \to \sum_{\text{particles}}$.



- $f_{g,q}(p)$: Similar to $m^2_{g,q}$, except counting N in fixed phase space volume $d^3\mathbf{x}d^3\mathbf{p}$.



INELASTIC SPLITTING/MERGING

SPLITTING

- Splitting: *Veto algorithm* to get τ_{split} and x.
- Assign (close by) *recoil parton* to keep all momenta on shell, kinematics following SHERPA's Catani-Seymour shower[6].
- Small $k_\perp^2$ sampled from

$$\frac{dk_\perp^2}{k_\perp^2 + k_{\text{reg}}^2}.$$

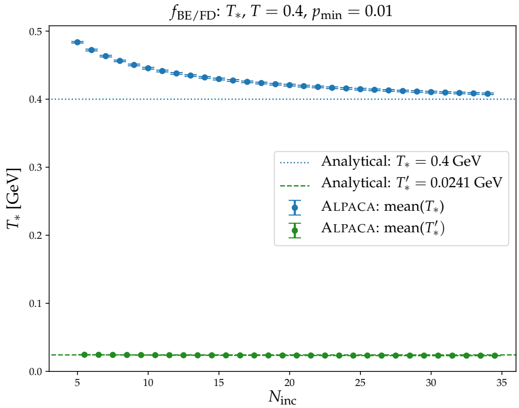
- Introduce global $p_{\min}$ due to divergent split/merge rate for small x (and $f_{\text{Bose-Einstein}}(p \to 0)$).

[6] S. Schumann and F. Krauss, JHEP 03 (2008).

INELASTIC SPLITTING/MERGING

MERGING, T_*

- τ_{merge}: same as elastic scattering.
- Merging kinematics: same as splitting.
- T_* extraction: similar to m_g^2.
  [Gaussian approximation of particles instead of Dirac deltas to include cross terms from $f(p)^2$.]



THERMAL VALIDATION

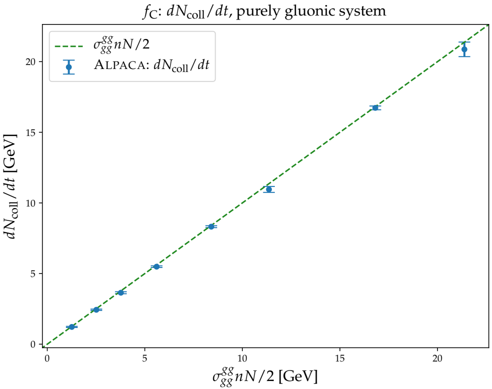
- Validate implementation in thermal equilibrium,

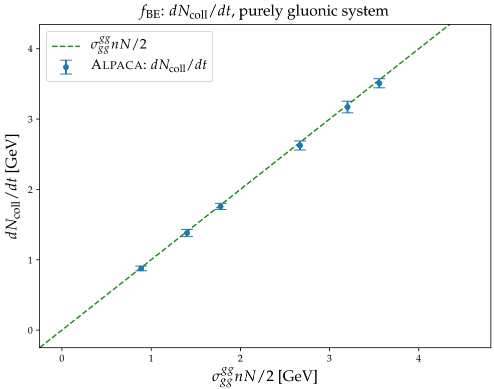
$$f_{\mathrm{C}}(p) = e^{-p/T}, \qquad f_{\mathrm{BE}}(p) = \frac{1}{e^{p/T} - 1}, \qquad f_{\mathrm{FD}}(p) = \frac{1}{e^{p/T} + 1}.$$

- Initialize in box with periodic boundary conditions.

THERMAL VALIDATION

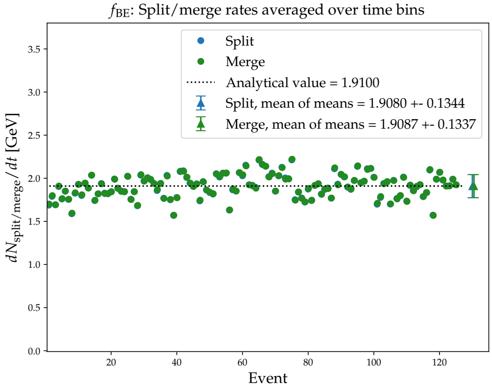
ELASTIC SCATTERING

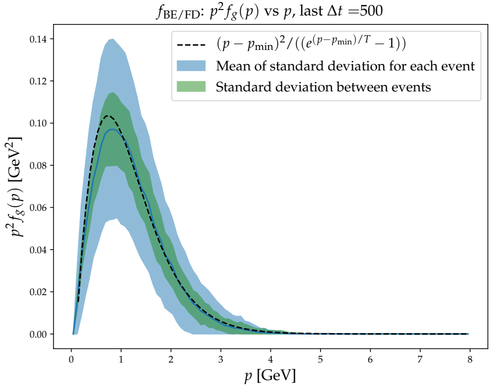




THERMAL VALIDATION

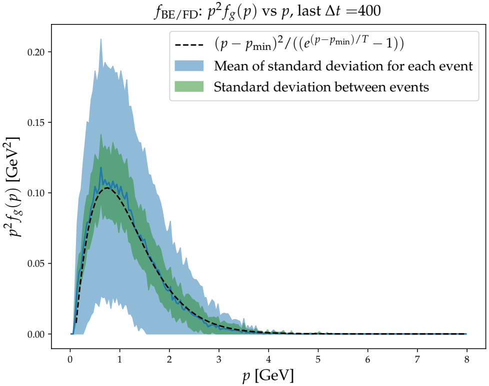
INELASTIC SPLITTING/MERGING

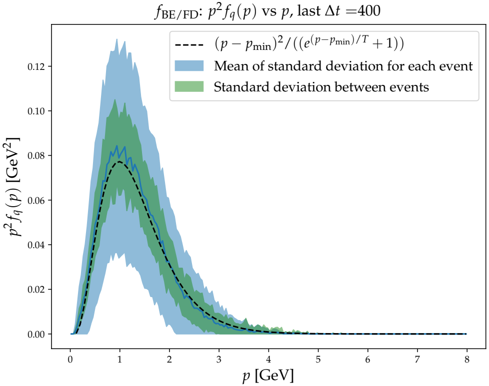




THERMAL VALIDATION

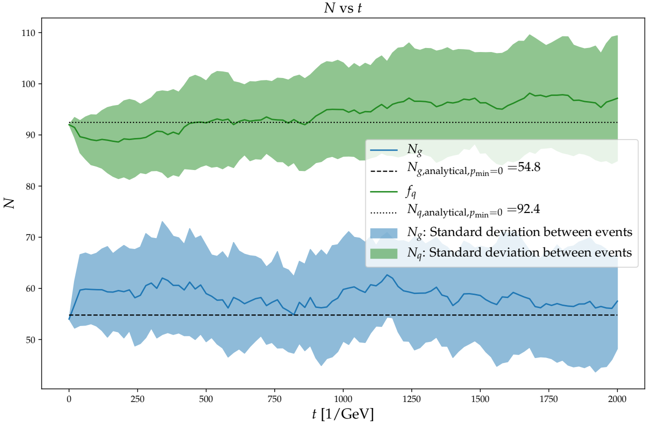
GLUONS AND QUARKS, DYNAMIC σ, γ



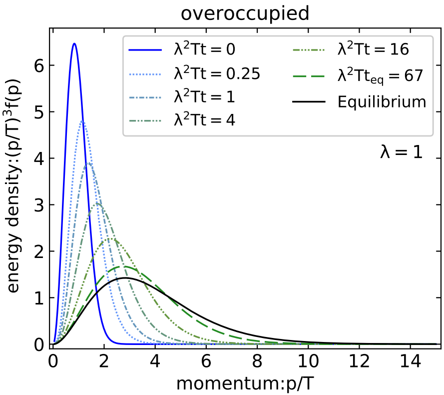


THERMAL VALIDATION

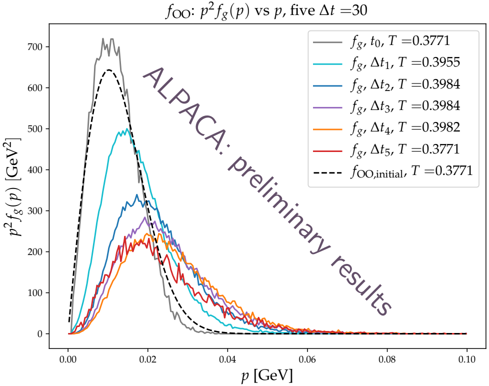
GLUONS AND QUARKS, DYNAMIC σ, γ



PRELIMINARY: THERMALIZATION



Y. Fu, J. Ghiglieri, S. Iqbal, A. Kurkela, Phys.Rev.D 105 (2022).



SUMMARY AND OUTLOOK

Summary:

- Monte Carlo Event Generator implementing full AMY collision kernels.
- Validated in thermal equilibrium.
- Started looking at thermalization.

Outlook

- Full event generator.

Thank you for listening!