

# The Particle World

CERN Summer School Lectures 2021

Lectures 1 and 2: The Standard Model

David Tong

# Further Reading

## Particle Physics

CERN Lectures

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**David Tong**

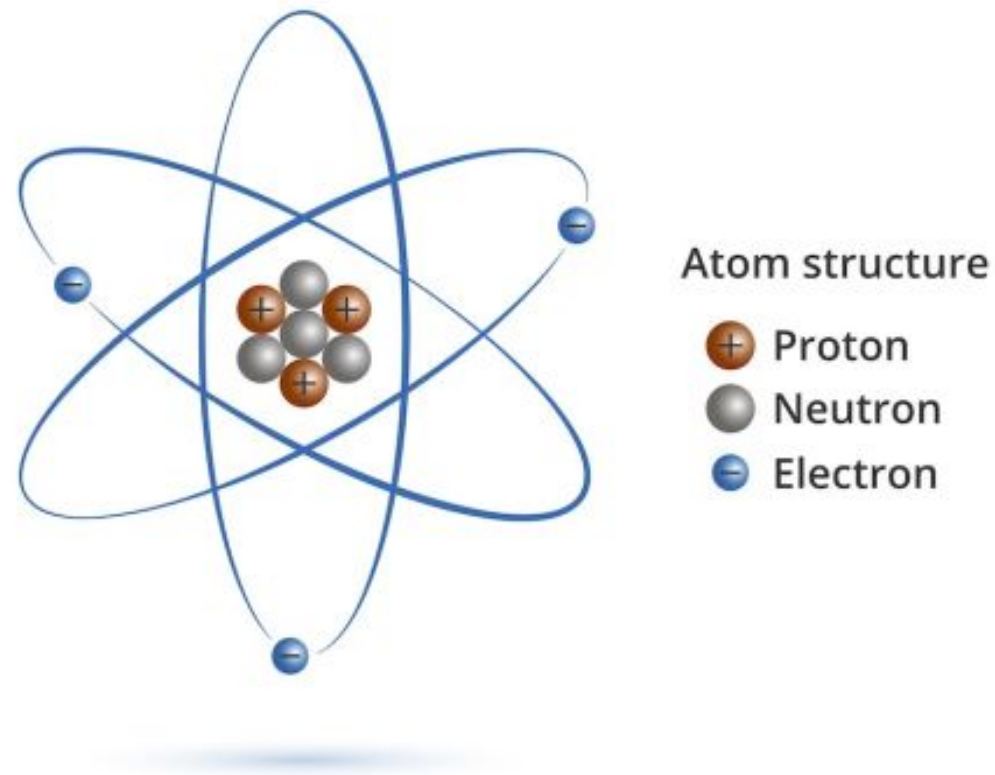
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around 240 pages! (Sorry)

# What are we made of?



"If we consider protons and neutrons as elementary particles, we would have three kinds of elementary particles [p,n,e].... This number may seem large but, from that point of view, two is already a large number."

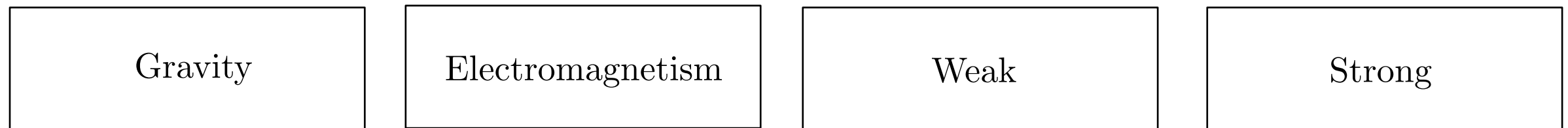
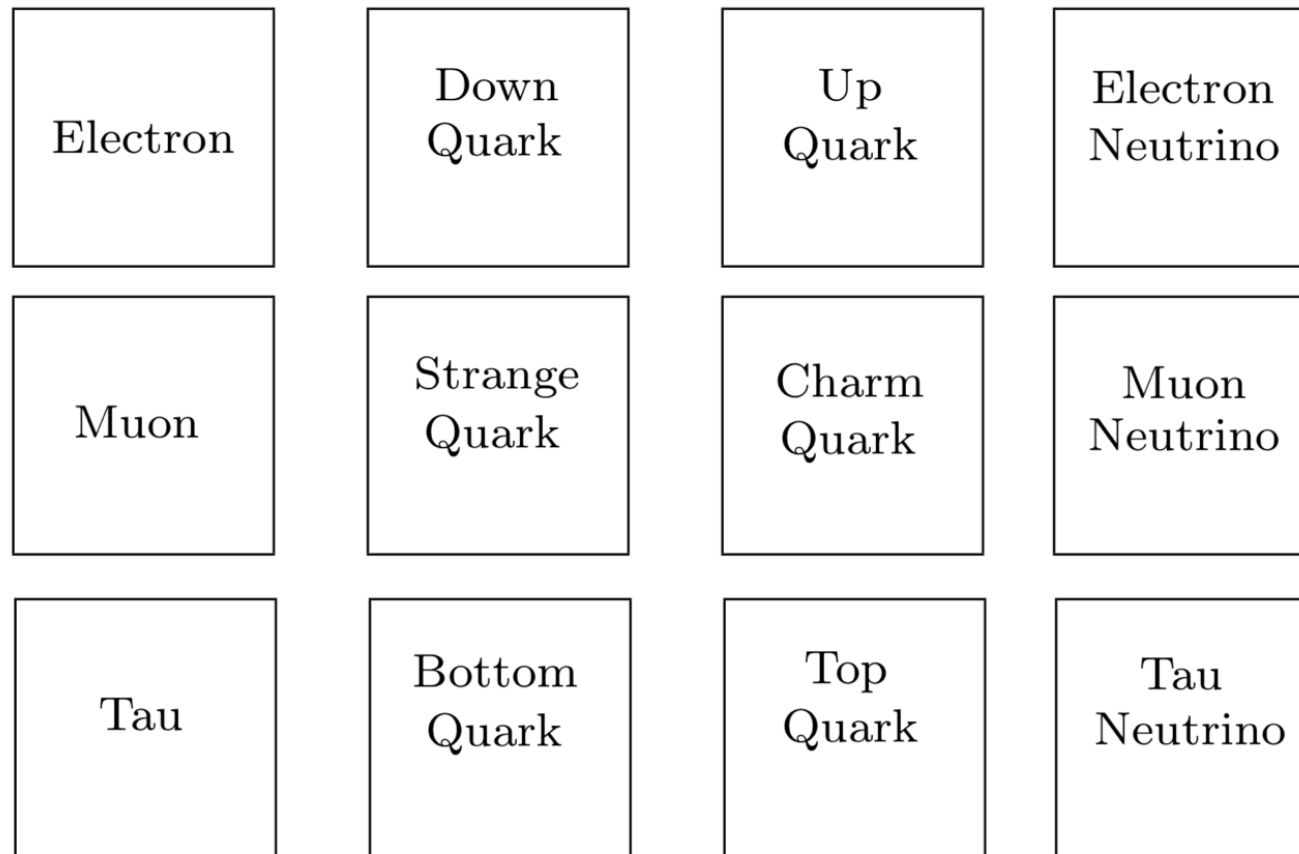
Paul Dirac 1933 Solvay Conference

# The Standard Model

12 particles + 4 forces + Higgs boson



# The Standard Model



# The Standard Model

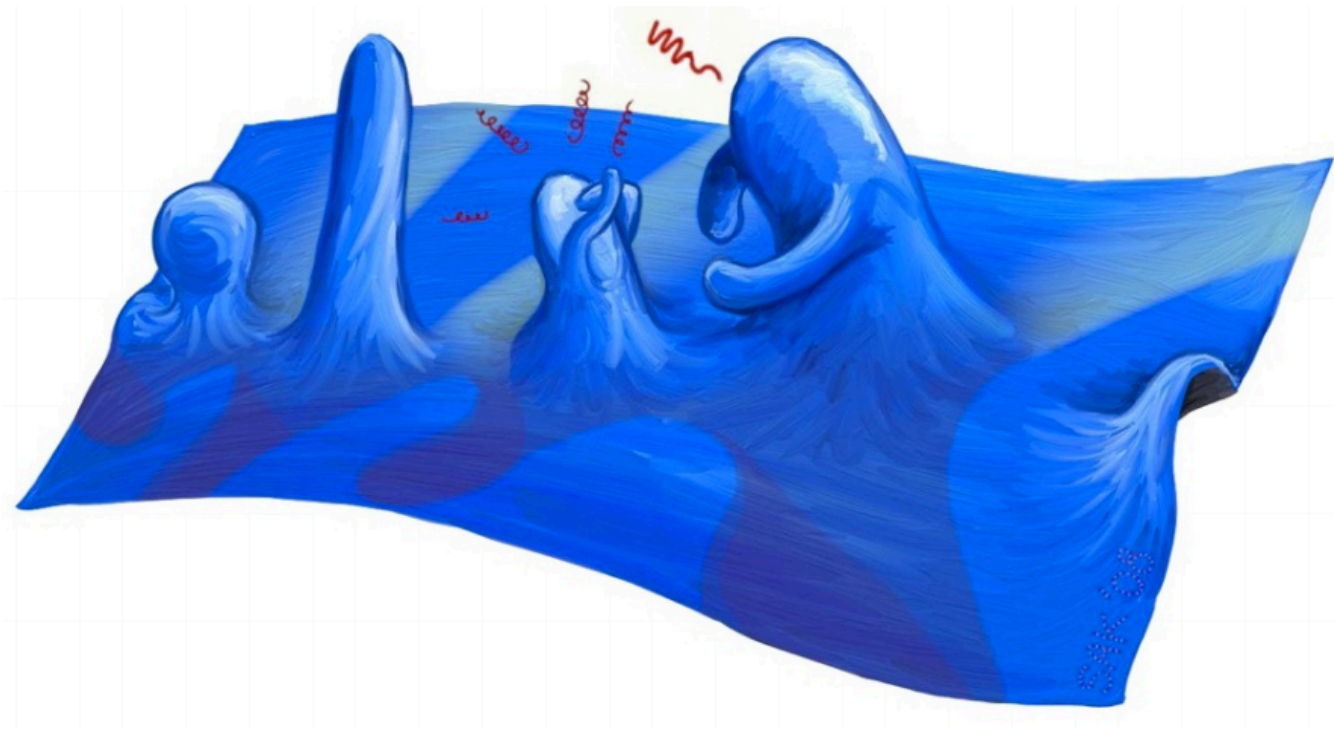
Electron	Down Quark	Up Quark	Electron Neutrino
Muon	Strange Quark	Charm Quark	Muon Neutrino
Tau	Bottom Quark	Top Quark	Tau Neutrino

Hypercharge

Gravity	<del>Electromagnetism</del>	Weak	Strong
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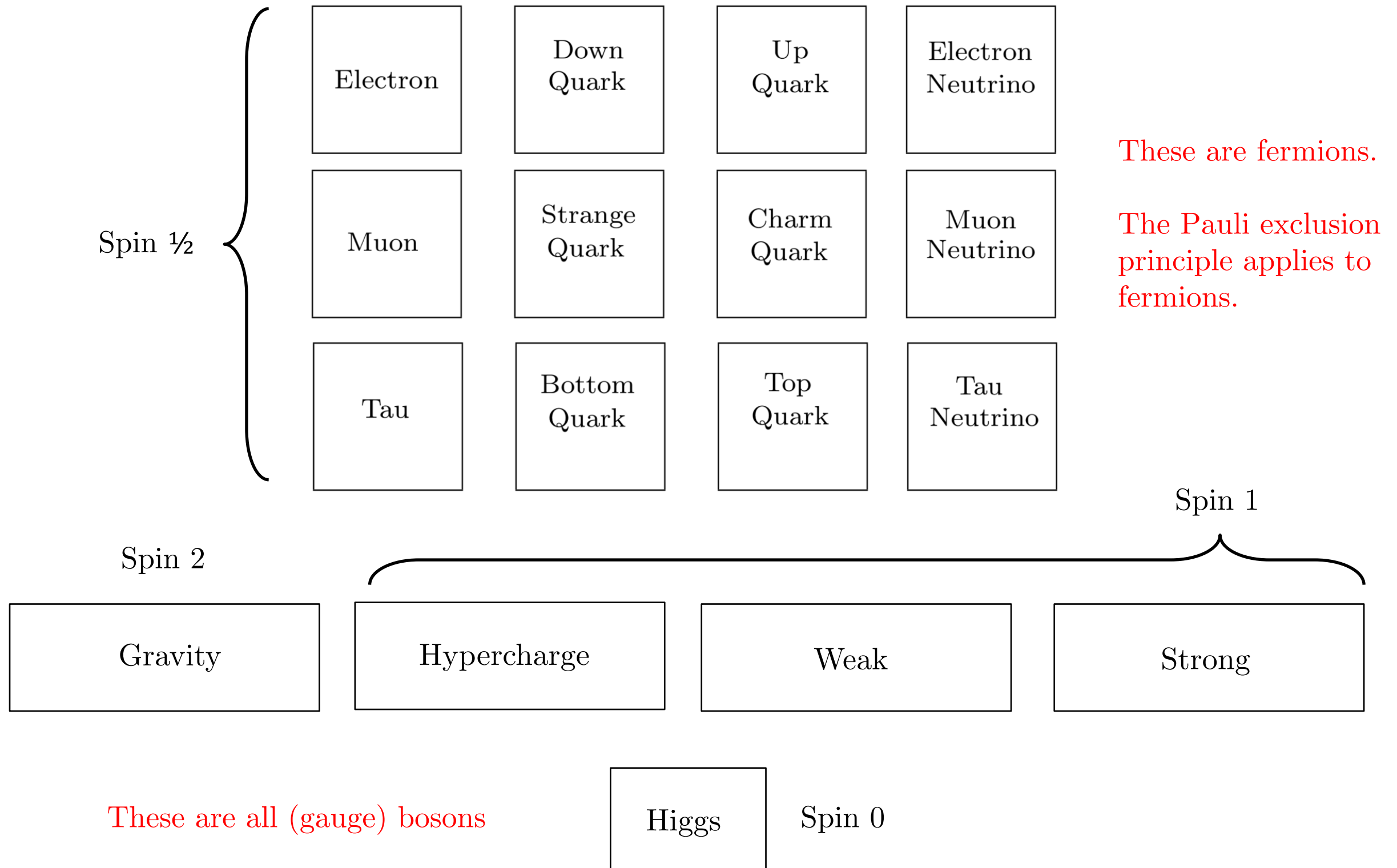
Higgs

# Our First Unification: Fields



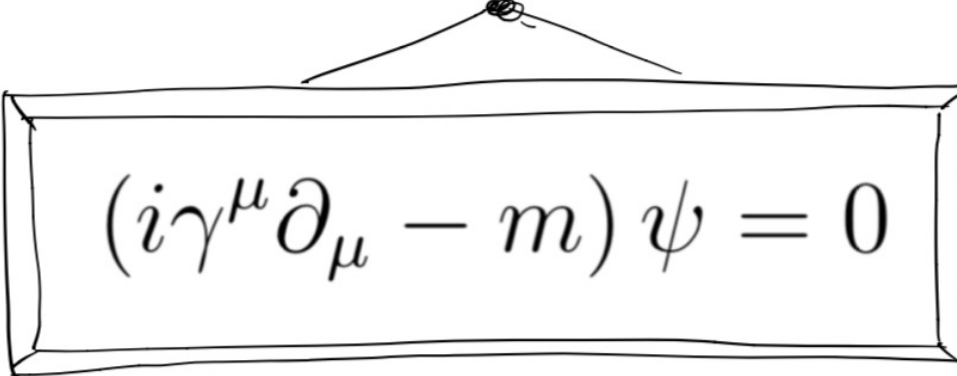
- Particles are excitations of underlying quantum fields
- Forces are also due to fields and have associated particles
  - Electromagnetism = photon
  - Strong = gluon
  - Weak = W and Z bosons
  - Gravity = graviton

# Intrinsic Angular Momentum = Spin



# A Remarkable Fact

All spin  $\frac{1}{2}$  particles are described by the same equation, discovered by Dirac


$$(i\gamma^\mu \partial_\mu - m) \psi = 0$$

Here  $m$  is the mass.

Consequence: all matter particles come with anti-particles

# Mass

Electron 1	Down Quark 9	Up Quark 4	Electron Neutrino $\sim 10^{-6}$
Muon 207	Strange Quark 186	Charm Quark 2495	Muon Neutrino $\sim 10^{-6}$
Tau 3483	Bottom Quark 8180	Top Quark 340,000	Tau Neutrino $\sim 10^{-6}$

An aside: In the Standard Model, the masses of all particles are determined by the strength of interaction with the Higgs field.

# Units for Mass

We measure mass in terms of energy, using  $E=mc^2$ . The unit of choice is the electronvolt

$$1 \text{ eV} \approx 1.6 \times 10^{-19} \text{ J}$$

Or  $\text{MeV} = 10^6 \text{ eV}$  or  $\text{GeV} = 10^9 \text{ eV}$  or  $\text{TeV} = 10^{12} \text{ eV}$ .

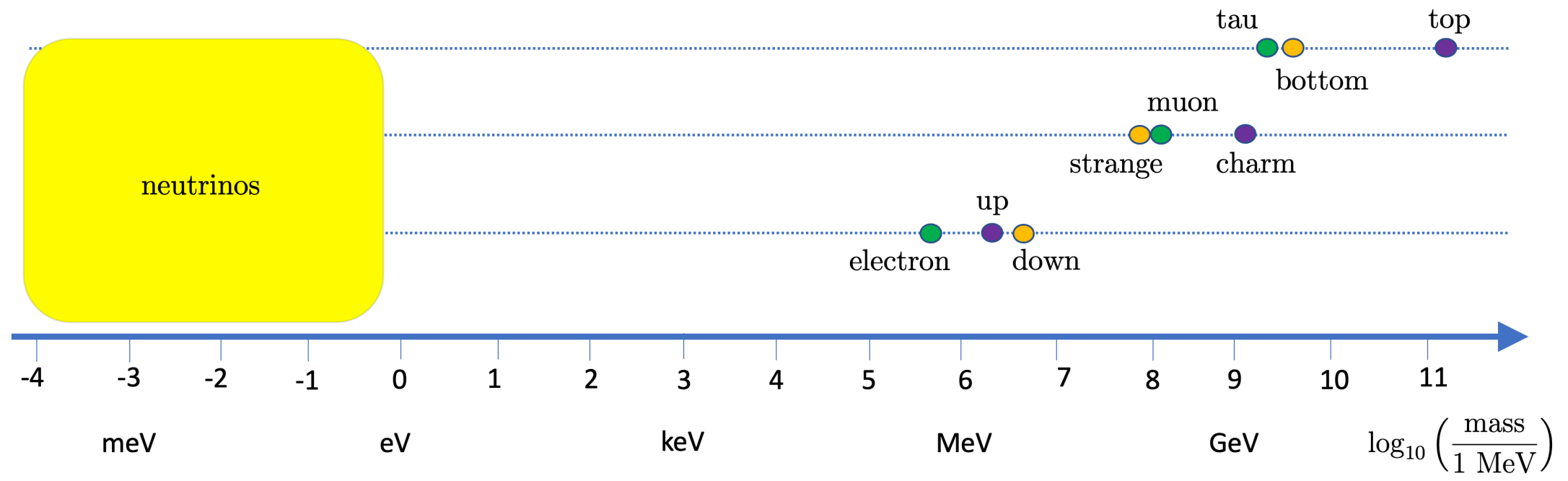
More confusingly, we also measure distance in terms of inverse energy, using

$$\lambda = \frac{\hbar c}{E}$$

For a particle of mass  $E$ , this is the “Compton wavelength”, or the size of the particle.

Note: heavier particles are smaller!

# The Masses of Particles

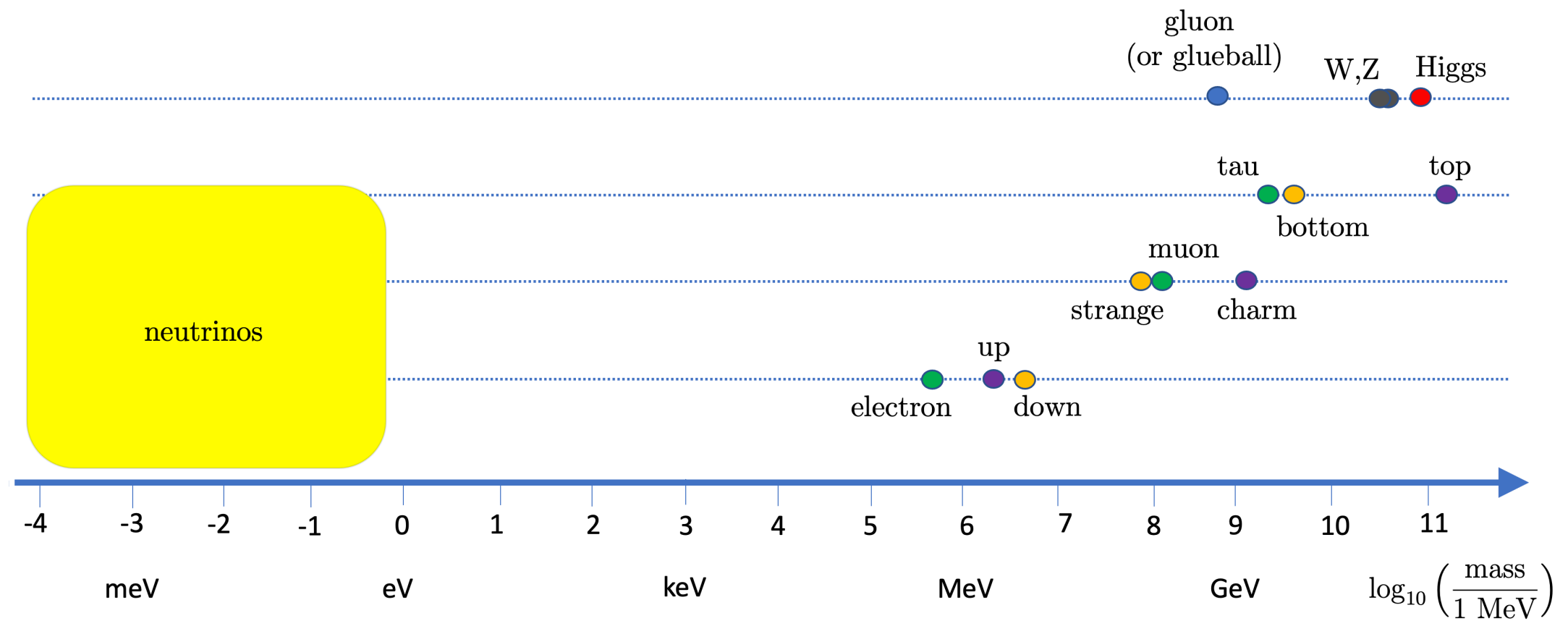


↑

Compton wavelength of electron  $\lambda_e \approx 2 \times 10^{-12} \text{ m}$



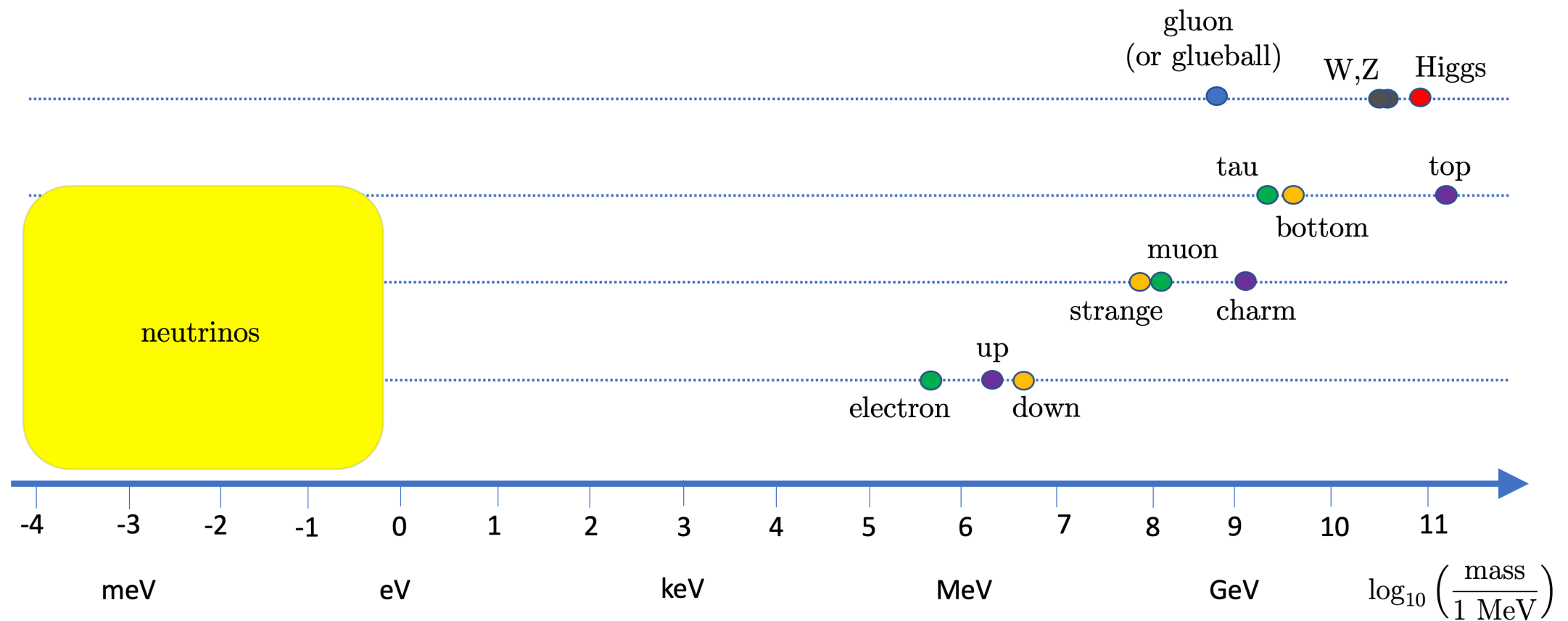
# The Masses of Particles



Note: photon and graviton both massless.

(The gluon is a little subtle...see later.)

# The Masses of Particles



← The biggest

$$H \approx 2 \times 10^{-33} \text{ eV}$$

$$L_{\text{universe}} \approx 9 \times 10^{26} \text{ m}$$

The smallest →

$$M_{\text{pl}} = \sqrt{\frac{\hbar c}{8\pi G}} \approx 2 \times 10^{18} \text{ GeV}$$

$$L_{\text{pl}} = \sqrt{\frac{8\pi \hbar G}{c^3}} \approx 8 \times 10^{-35} \text{ m}$$

# Electric Charge

Charge =

	-1	-1/3	+2/3	0
	Electron 1	Down Quark 9	Up Quark 4	Electron Neutrino $\sim 10^{-6}$
	Muon 207	Strange Quark 186	Charm Quark 2495	Muon Neutrino $\sim 10^{-6}$
	Tau 3483	Bottom Quark 8180	Top Quark 340,000	Tau Neutrino $\sim 10^{-6}$

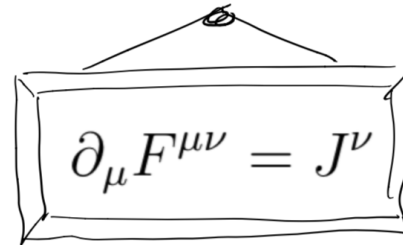
The electric charge characterizes the (relative) strength of the electromagnetic interaction

# Electromagnetism (or QED)

## The Maxwell Equations

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} & , & & \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0 & , & & \nabla \times \mathbf{B} &= \mu_0 \left( \mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)\end{aligned}$$

or


$$\partial_\mu F^{\mu\nu} = J^\nu$$

This implies the Coulomb force which, in natural units, reads

$$F = \alpha \frac{Q_1 Q_2}{r^2}$$

with the fine structure constant

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$$

# Feynman Diagrams

An important fact: quantum field theory is hard!

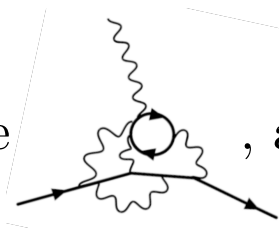
We are saved in QED because  $\alpha \approx \frac{1}{137} \ll 1$ . This allows us to write down an approximate solution

e.g. what is the probability for a photon to scatter off an electron in some direction?

$$\text{Probability} = \left| \begin{array}{c} \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots \end{array} \right|^2$$

$\mathcal{O}(\alpha^2)$ 
 $\mathcal{O}(\alpha^4)$

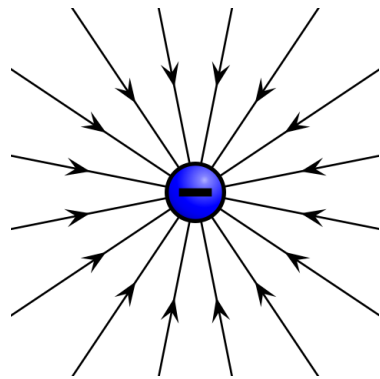
More complicated diagrams, like



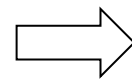
, are less and less important.

# Renormalisation

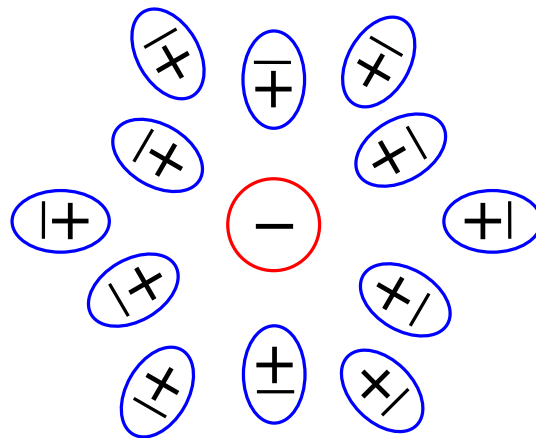
Look close at the electron.



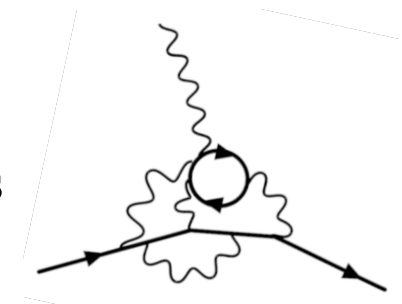
$$\mathbf{E} = \frac{e}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}}$$



Large energy density near electron, This allows for the creation of particle-anti-particle pairs

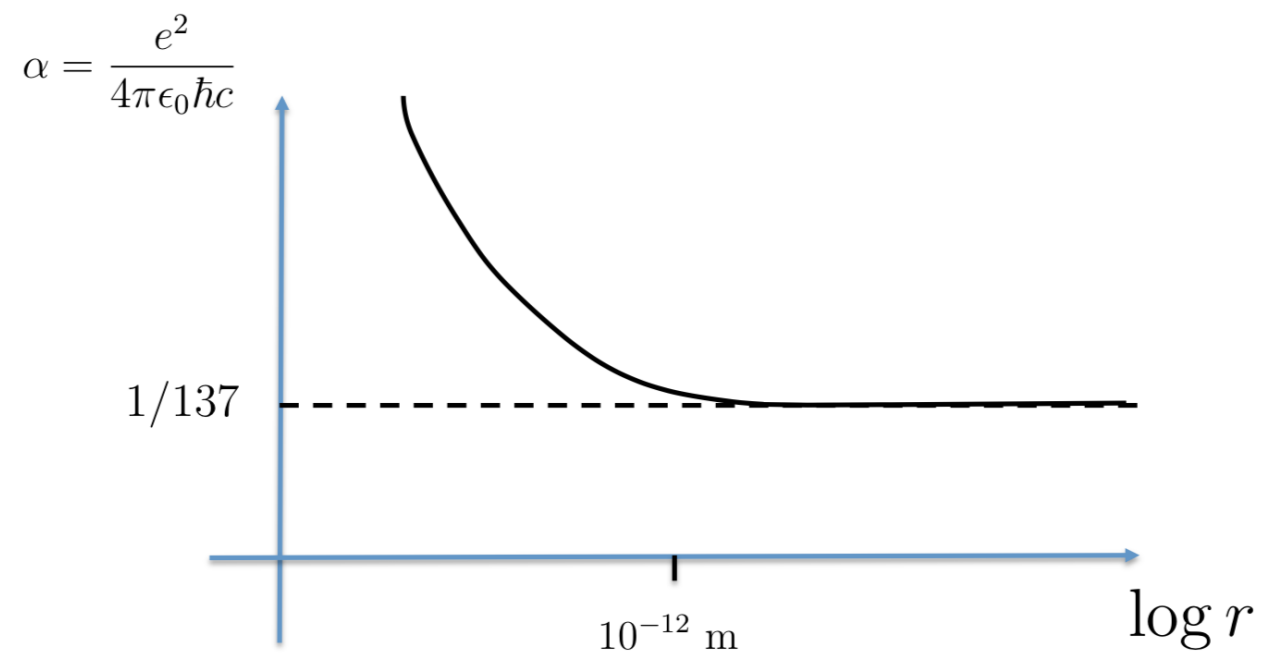


This is the physics behind the increasingly complicated diagrams like this



# Renormalisation

As you look more closely, the charge of an electron gets bigger!



Constants of nature are not constant!

# The Strong and Weak Force

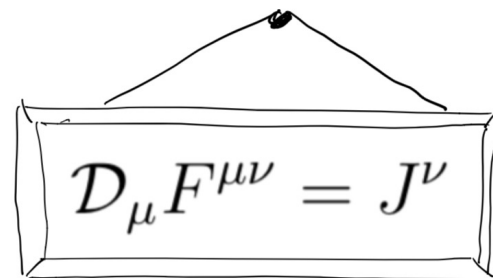
Both nuclear forces have associated “electric” and “magnetic” fields

$$\mathbf{E} = (E_x, E_y, E_z) \quad \mathbf{B} = (B_x, B_y, B_z)$$

But each component is now itself a matrix.

- 1 x 1 matrix  $\implies$  Electromagnetism
  - 2 x 2 matrix  $\implies$  Weak force
  - 3 x 3 matrix  $\implies$  Strong force
- or  $U(1) \times SU(2) \times SU(3)$

These fields are governed by the Yang-Mills equations


$$\mathcal{D}_\mu F^{\mu\nu} = J^\nu$$



# The Strong Force (or QCD)

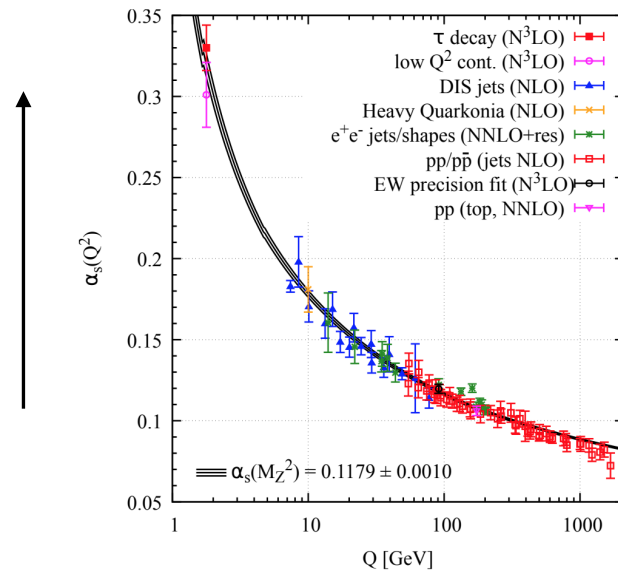
No	Yes	Yes	No
Electron 1	Down Quark 9	Up Quark 4	Electron Neutrino $\sim 10^{-6}$
Muon 207	Strange Quark 186	Charm Quark 2495	Muon Neutrino $\sim 10^{-6}$
Tau 3483	Bottom Quark 8180	Top Quark 340,000	Tau Neutrino $\sim 10^{-6}$

Each quark comes in three *colours*, which we take to be red, green and blue.

(Note: a better counting is that each generation contains  $1+3+3+1=8$  particles.)

# Why is the Strong Force Strong?

strong coupling constant



energy = 1/distance

At high energy, say  $E=100$  GeV, we have  $\alpha_s \approx 0.1$ . But the strong force gets stronger as we go to larger distances. (Asymptotic freedom.)

Taken naively,  $\alpha_s \rightarrow \infty$  at the energy scale:

$$\Lambda_{\text{QCD}} \approx 200 \text{ MeV}$$

This corresponds to a distance scale  $R_{\text{QCD}} = \frac{1}{\Lambda_{\text{QCD}}} \approx 5 \times 10^{-15} \text{ m}$

# Confinement

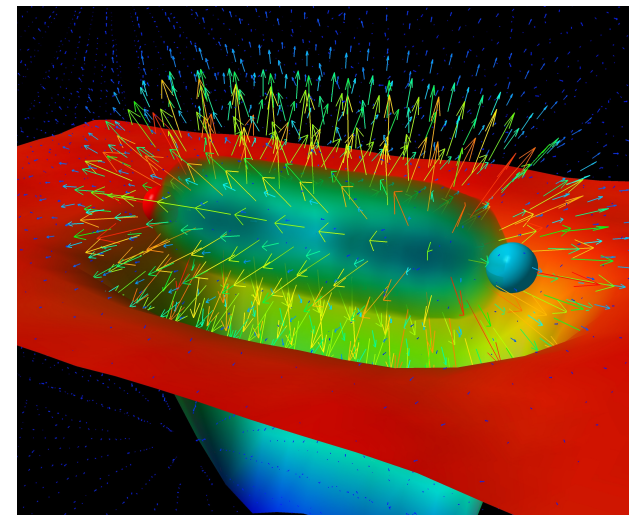
At short distances,  $F(r) \sim \frac{\alpha_s}{r^2}$  but at long distances  $F(r)$  becomes constant.

In terms of the potential energy,  $V(r) \sim -\frac{\alpha_s}{r}$  at short distances, but at long distances

$$V(r) \sim \Lambda_{\text{QCD}}^2 r$$

This is *confinement*. We don't see isolated quarks.

Also, the force carrying field is not massless. The gluons stick together to form glueballs, with mass around  $m_{\text{gluon}} \approx \Lambda_{\text{QCD}}$ . This is the “mass gap” problem.

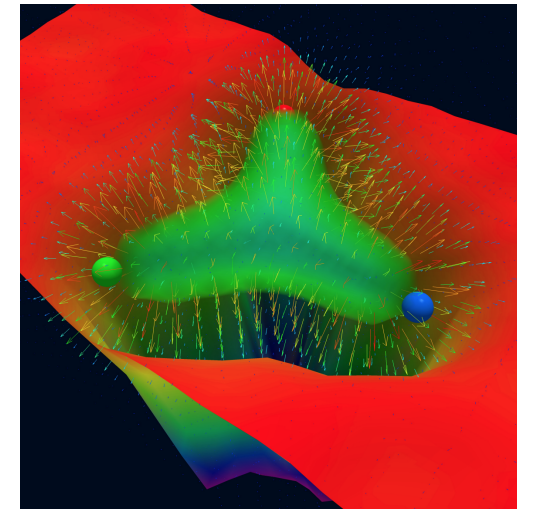


# Hadrons (Stuff Made of Quarks)

- Baryons: three quarks. For example

$$n (ddu) \quad m_n \approx 939.57 \text{ MeV}$$

$$p (uud) \quad m_p \approx 938.28 \text{ MeV}$$



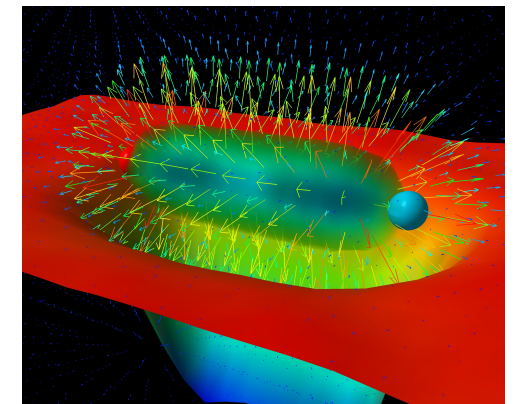
A puzzle:  $m_{\text{down}} = 5 \text{ meV}$  and  $m_{\text{up}} = 2 \text{ MeV}$ . Where does the mass come from?

- Mesons: quark-anti-quark pair. For example, pions

$$\pi^+ (\bar{d}u) \quad m \approx 139 \text{ MeV}$$

$$\pi^0 \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d) \quad m \approx 135 \text{ MeV}$$

$$\pi^- (\bar{u}d) \quad m \approx 139 \text{ MeV}$$



Note: Pions have spin 0 and so should be thought of as “force carrying” particles! So ...

# Decay

All hadrons other than the proton are unstable. They decay.

- Strong decay:  $\sim 10^{-22}$  to  $10^{-24}$  seconds.
- Electromagnetic decay:  $\sim 10^{-16}$  to  $10^{-21}$  seconds.
- Weak decay:  $\sim 10^{-7}$  to  $10^{-13}$  seconds.

$$\Delta^{++} (uuu) \rightarrow p + \pi^+$$

$$\Sigma^0 (dus) \rightarrow \Lambda^0 + \gamma, \text{ with } \Lambda^0 (dus)$$

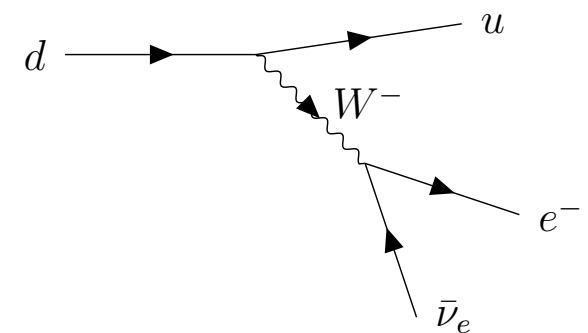
$$\pi^+ (ud) \rightarrow \mu^+ + \nu_\mu$$

The most famous weak decay is how we first discovered the weak force

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Or, if you look more closely,

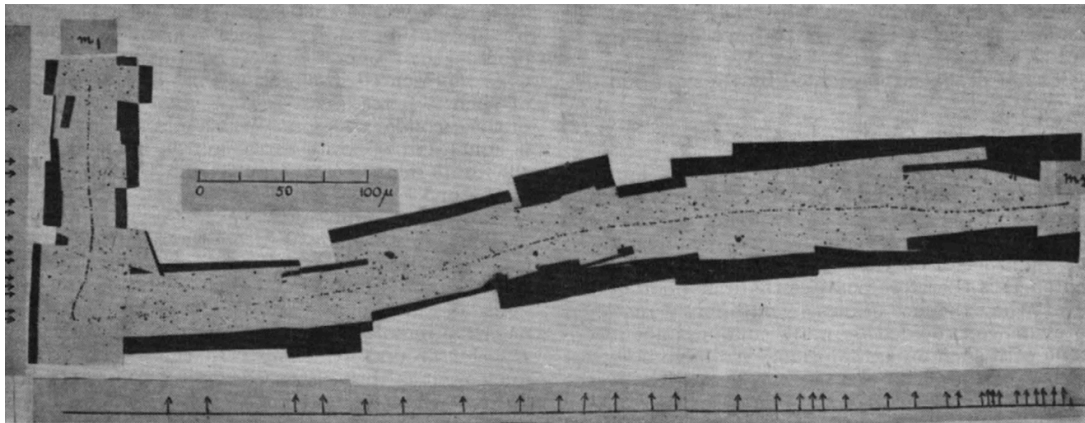
$$d \rightarrow u + e^- + \bar{\nu}_e$$



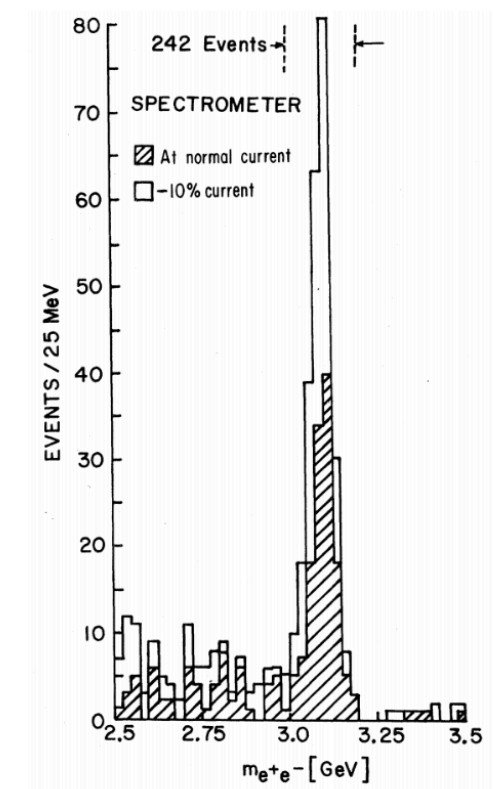
# Particles vs Resonances

- Strong decay:  $\sim 10^{-22}$  to  $10^{-24}$  seconds.
- Electromagnetic decay:  $\sim 10^{-16}$  to  $10^{-21}$  seconds.
- Weak decay:  $\sim 10^{-7}$  to  $10^{-13}$  seconds.

If a particle decays through the weak force, we can take a photograph of it!



If it decays through the strong force, or EM, then we see it more indirectly

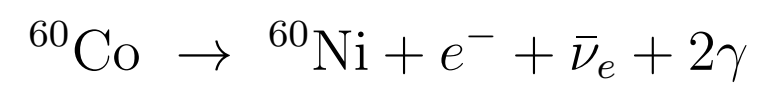


# The Weak Force

half of each particle!

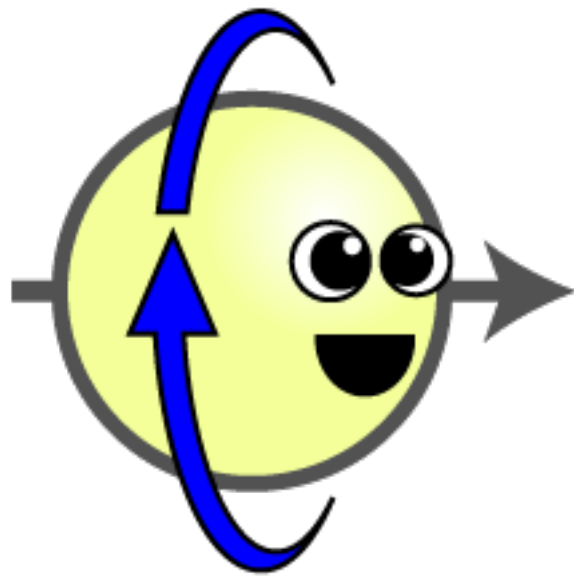
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# Parity Violation

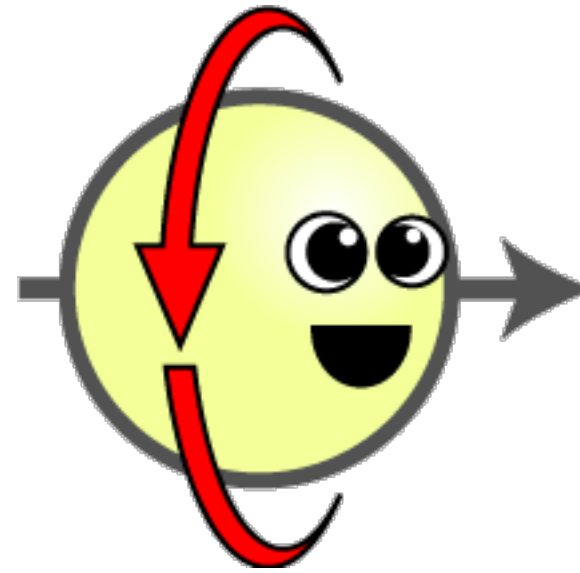




# Chiral Fermions



left-handed fermion

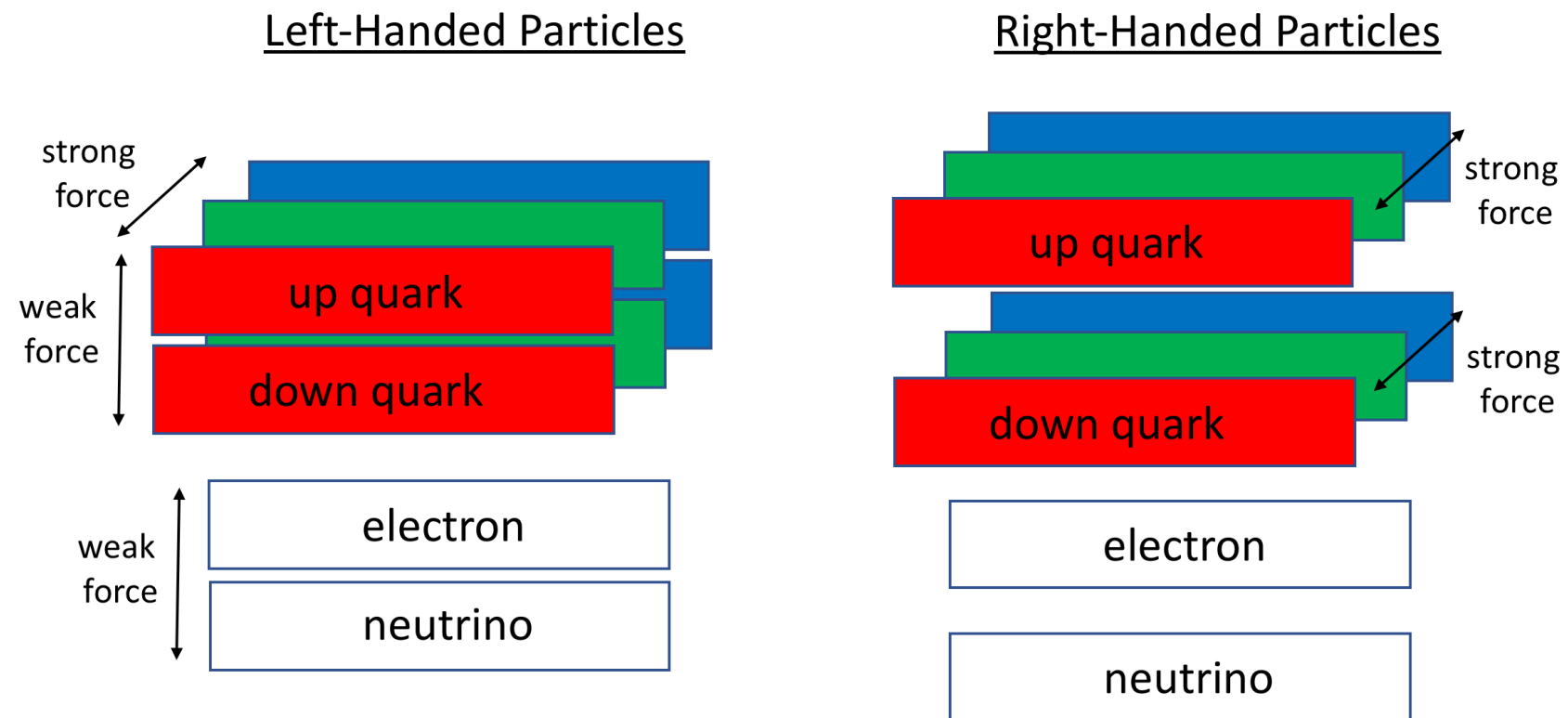


right-handed fermion

Left-handed particles experience the weak force, right-handed do not.

# The Forces of the Standard Model

Each generation splits into 8x2 sets of particles



Note: We don't yet know if the right-handed neutrino exists.

# The Structure of the Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

strong
weak
hypercharge

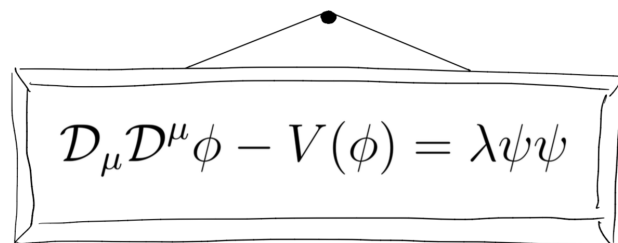
Particles		Strong	Weak	Hypercharge
Left-handed	quarks	yes	yes	+1/6
	leptons	no	yes	-1/2
Right-handed	up quark	yes	no	+2/3
	down quark	yes	no	-1/3
	electron	no	no	-1
	neutrino	no	no	0

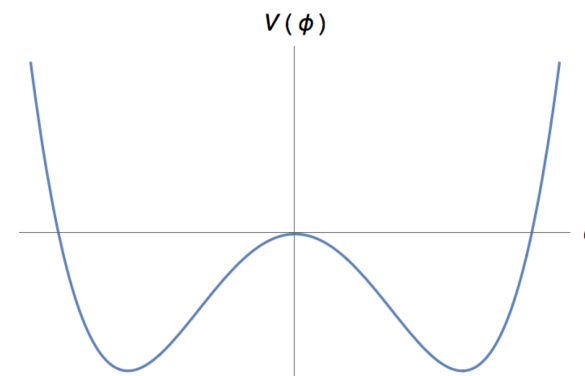
A perfect jigsaw: Anomaly cancellation means that it could hardly be any other way!

# The Higgs Field

This is both the simplest and most complicated field in the Standard Model!

Particle	Strong	Weak	Hypercharge
Higgs	no	yes	+1/2


$$\mathcal{D}_\mu \mathcal{D}^\mu \phi - V(\phi) = \lambda \psi \psi$$



Two relevant scales:

- Mass  $m_H \approx 125$  GeV
- Condensate  $\langle \phi \rangle \approx 246$  GeV

It is the condensate that gives the Higgs its Midas touch: everything that it touches gets a mass

# How Particles Get a Mass

In the Standard Model, all fermions and gauge bosons are obliged to be fundamentally massless

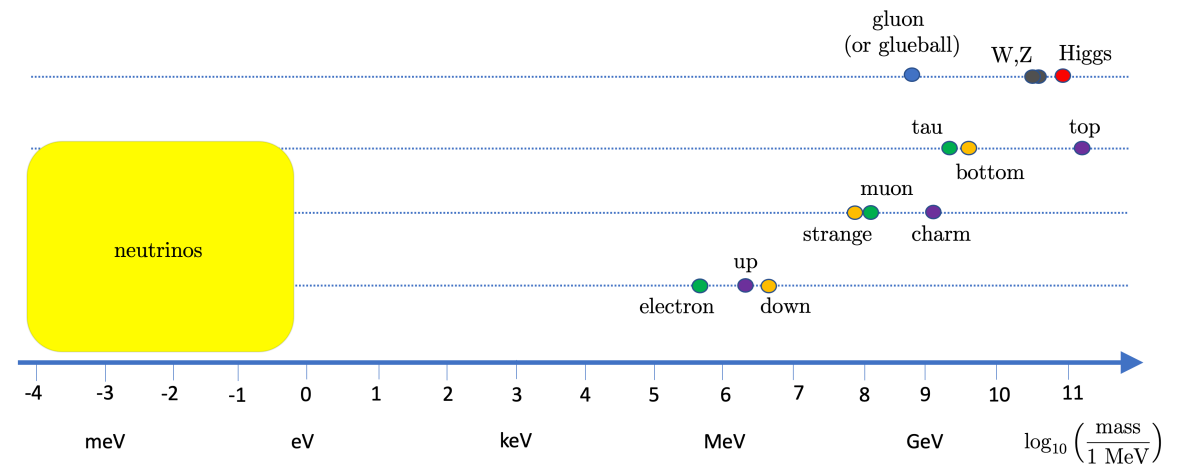
They get a mass by interaction with the Higgs.

$$\text{Mass} = g \times \langle \phi \rangle$$



some dimensionless coupling

$$\langle \phi \rangle \approx 246 \text{ GeV}$$



- The Higgs gives mass to the W-boson and Z-boson and all fermions.
- The photon remains massless: it is the one that got away!
- Recall: the mass of the proton and neutron do not come from the Higgs!

# One Last Thing: Quark Mixing

There is a misalignment between the interactions with the Higgs and the interaction with the weak force.

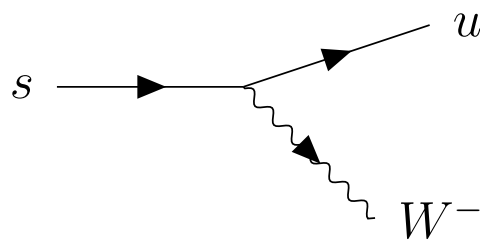
It turns out that you can choose to have the up-sector aligned. But then the down sector is not. The result is a superposition of particles.

These particles interact with weak force

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

These particles interact with Higgs, and so definite mass.

This is how, for example, mesons with strange quarks decay



# One Last Thing: and Lepton Mixing

There is a similar statement for neutrinos

These particles interact with weak force and are produced in, say, beta decay

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

These particles have definite mass. These are energy eigenstates that travel unchanged through space.

This gives rise to neutrino oscillations

# The Mixing Matrices

For quarks, we have the CKM matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.97 & 0.22 & 0.004 \\ 0.22 & 0.97 & 0.04 \\ 0.009 & 0.04 & 0.999 \end{pmatrix}$$

For neutrinos, we have the PMNS matrix

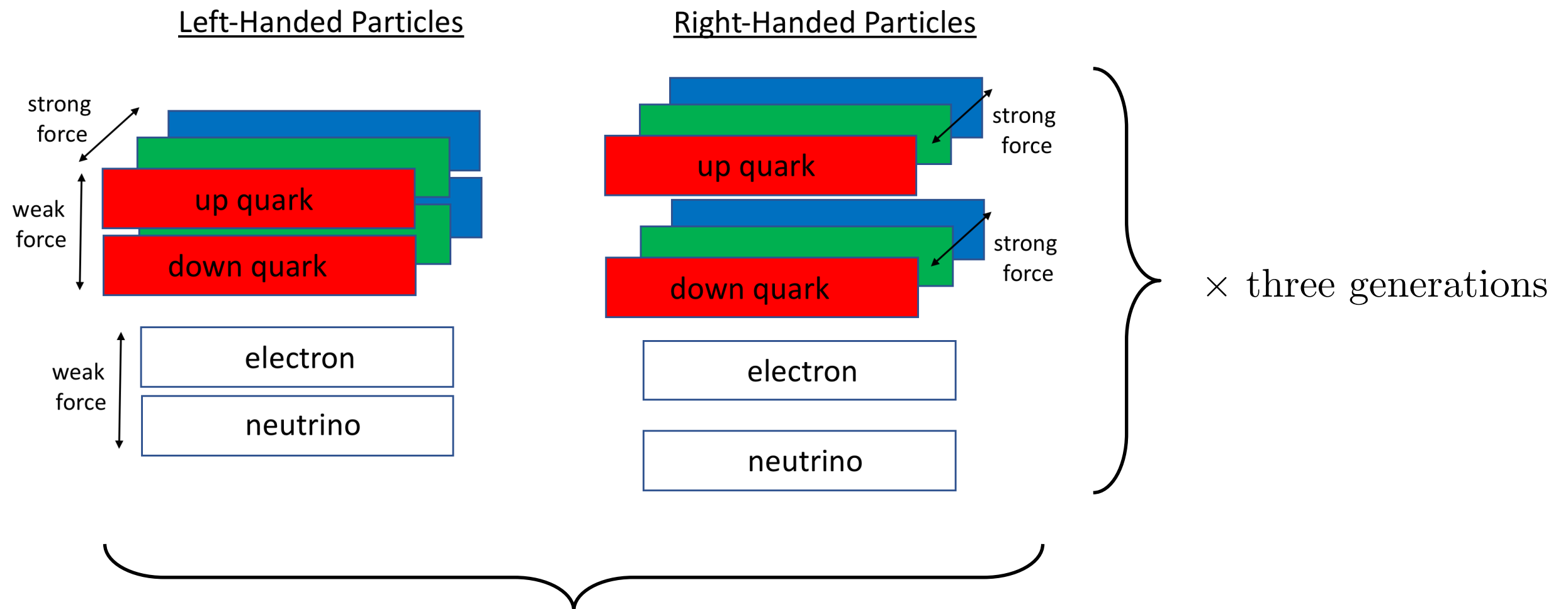
$$\begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix} \approx \begin{pmatrix} 0.8 & 0.5 & 0.1 \\ 0.3 & 0.5 & 0.7 \\ 0.4 & 0.6 & 0.6 \end{pmatrix}$$

We only know these parameters by experimental measurement. Why do they take these values?  
Why are the matrices so different?



# Summary: The Greatest Theory of All Time

$$G = SU(3) \times SU(2) \times U(1)$$



with all complications coming from interactions with Higgs!