



The new muon g-2 results from Fermilab: Physics and High Precision implications

Muon g-2 exp.'s incredible journey and the latest results from Fermilab

Yannis K. Semertzidis, IBS/CAPP & KAIST

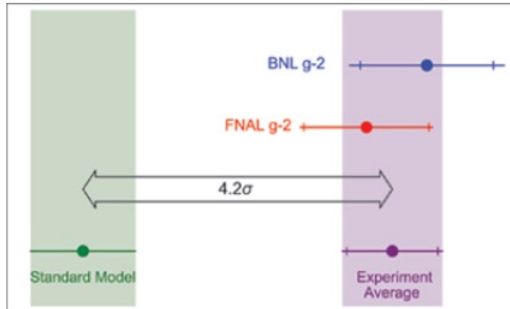
INP Demokritos colloquium (online), Athens, June 8, 2021

- The muon g-2 result was announced at Fermilab on April 7, 2021.
- The results are consistent with the BNL result 20-years earlier.
- There is a serious tension with the SM.

Muon g-2 on the cover of Phys. Rev. Letters

PHYSICAL REVIEW LETTERS

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ON THE COVER

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm

April 7, 2021

New muon magnetic moment data from a Fermilab experiment (red) combined with previous Brookhaven National Lab data (blue) is in 4.2σ tension with the value calculated by the Muon g-2 Theory Initiative (green). Selected for a [Viewpoint](#) in *Physics* and an Editors' Suggestion.

B. Abi *et al.* (Muon $g - 2$ Collaboration)

[Phys. Rev. Lett. **126**, 141801 \(2021\)](#)

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Yannis K. Semertzidis, IBS-CAPP and KAIST

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2. PRD 103 (2021) 7, 072002
3. PRA 103 (2021) 4, 042208
4. PRAB 24 (2021) 4, 044002
5. 2104.07805 (2021)

Subjects

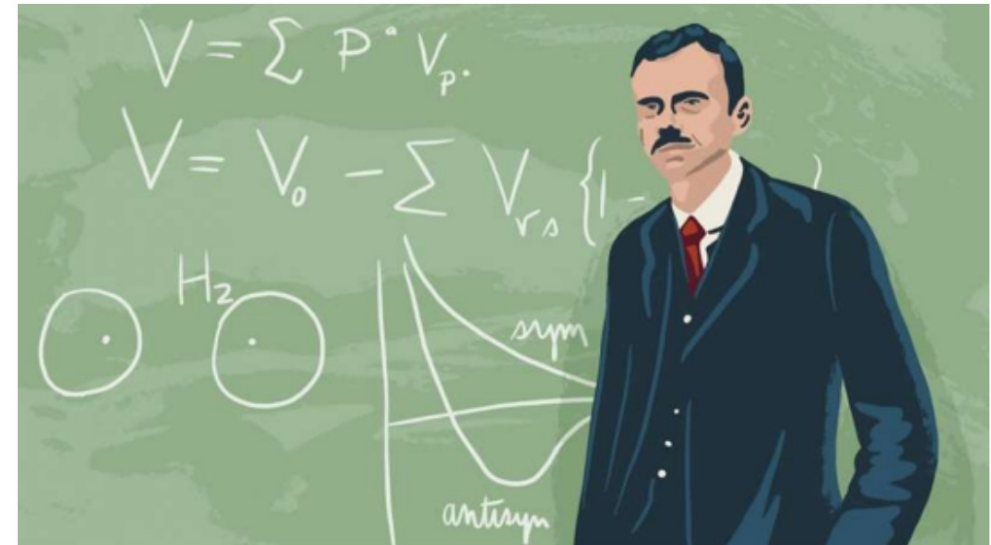
- Historical overview, how muon $g-2$ is done
- What matters in a real muon $g-2$ experiment
- Ideal vs. real experiment
- The results and what to expect in the future
- Outlook

Way back at the beginning, discovering spin

From Phys. Org., 2021 on muon g-2

Spectroscopy experiments in the 1920s (before the discovery of muons in 1936) revealed that the electron has an intrinsic spin and a magnetic moment. The value of that magnetic moment, g , was found experimentally to be 2. As for why that was the value—that mystery was soon solved using the new but fast-growing field of quantum mechanics.

In 1928, physicist Paul Dirac—building upon the work of Llewelyn Thomas and others—produced a now-famous equation that combined quantum mechanics and special relativity to accurately describe the motion and electromagnetic interactions of electrons and all other particles with the same spin quantum number. The Dirac equation, which incorporated spin as a fundamental part of the theory, predicted that g should be equal to 2, exactly what scientists had measured at the time.



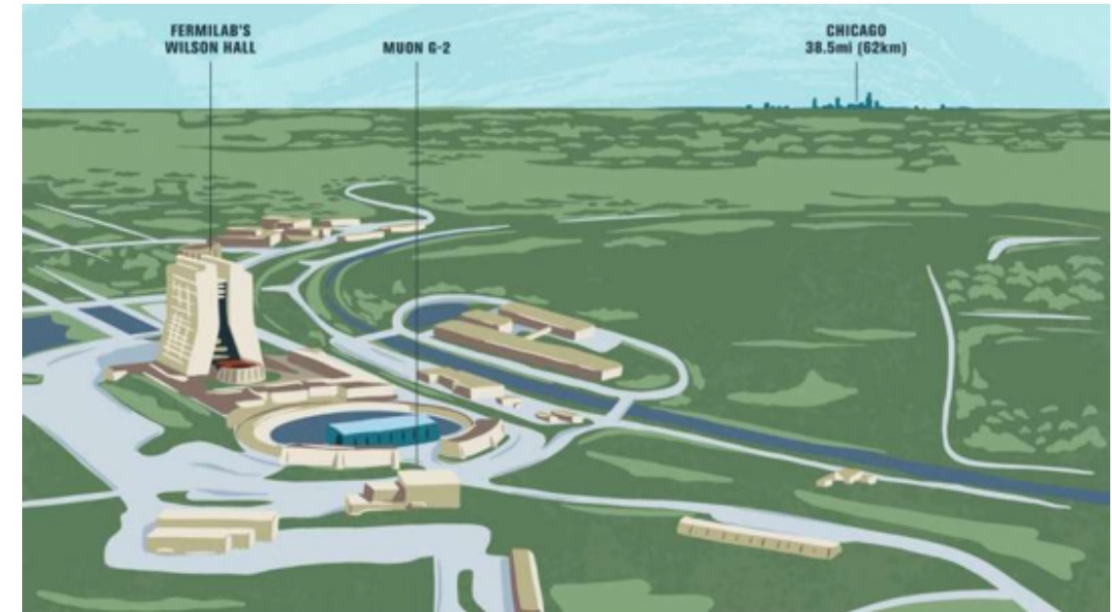
CERN, BNL & FNAL

From Phys. Org., 2021 on muon g-2

Early measurements of the muon's anomalous magnetic moment at Columbia University in the 1950s and at the European physics laboratory CERN in the 1960s and 1970s agreed well with theoretical predictions. The measurement's uncertainty shrank from 2% in 1961 to 0.0007% in 1979. It looked as if the same conspiracy of particles that affected the electron's g-2 were responsible for the magnetic moment of the muon as well.

But then, in 2001, the Brookhaven Muon g-2 experiment turned up something strange. The experiment was designed to increase the precision from the CERN measurements and look at the weak force's contribution to the anomaly. It succeeded in shrinking the error bars to half a part per million. But it also showed a tiny discrepancy—less than 3 parts per million—between the new measurement and the theoretical value. This time, theorists couldn't come up with a way to recalculate their models to explain it. Nothing in the Standard Model could account for the difference.

The theoretical models that describe these virtual interactions have been quite successful in describing the magnetism of electrons. For the electron's g-2, theoretical calculations are now in such close agreement with the experimental value that it's like measuring the circumference of the Earth with an accuracy smaller than the width of a single human hair.



Credit: Sandbox Studio, Steve Shanabrich

All of the evidence points to quantum mischief perpetrated by known particles causing any magnetic anomalies. Case closed, right?

Not quite. It's now time to hear the muon's side of the story.

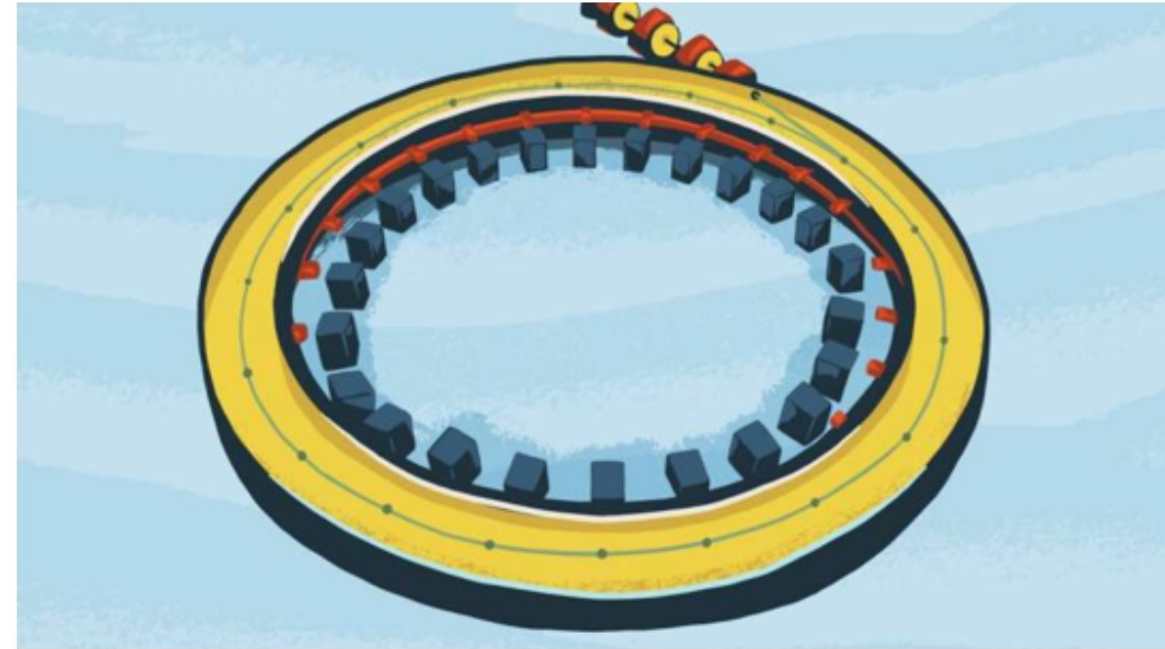
CERN, BNL & FNAL

From Phys. Org., 2021 on muon g-2

In the new Muon g-2 experiment, a beam of muons—their spins all pointing the same direction—are shot into a type of accelerator called a storage ring. The ring's strong magnetic field keeps the muons on a well-defined circular path. If g were exactly 2, then the muons' spins would follow their momentum exactly. But, because of the anomalous magnetic moment, the muons have a slight additional wobble in the rotation of their spins.

When a muon decays into an electron and two neutrinos, the electron tends to shoot off in the direction that the muon's spin was pointing. Detectors on the inside of the ring pick up a portion of the electrons flung by muons experiencing the wobble. Recording the numbers and energies of electrons they detect over time will tell researchers how much the muon spin has rotated.

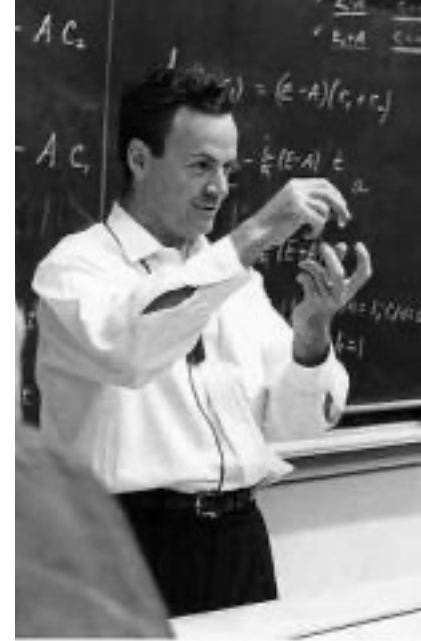
Physicists are now re-examining this "hair" at Fermilab, with support from the DOE Office of Science, the National Science Foundation and several international agencies in Italy, the UK, the EU, China, Korea and Germany.



Credit: Sandbox Studio, Steve Shanabruch

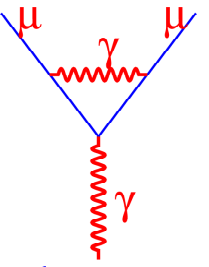
Using the same magnet from the Brookhaven experiment with significantly better instrumentation, plus a more intense beam of muons produced by Fermilab's accelerator complex, researchers are collecting 21 times more data to achieve four times greater precision.

Magnetic Dipole Moments probe Quantum Field Fluctuations



- A “soup” of virtual particles is coming in and out of existence affecting the **MDM** interaction of particles with B-fields.
- The interaction is estimated using Feynman diagrams.
- It is expressed with the so-called g-2 factor: $a = (g-2)/2$, the anomaly.

g-factors:



- Proton ($g_p=+5.586$) and the neutron ($g_n=-3.826$) are composite particles.
- The ratio $g_p/g_n=-1.46$ close to the predicted $-3/2$ was the first success of the constituent quark model.
- The g_e-2 (of the electron) is non-zero mainly due to quantum field fluctuations involving QED. A “soup” of virtual particles coming in and out of existence...
- The anomaly of the magnetic moment of leptons can be estimated with high accuracy, it's the same to first order for all leptons: $a_\mu = \frac{\alpha}{2\pi} \approx \frac{1}{800}$

Electron Magnetic Dipole Moment

D. Hanneke, S. Fogwell, and G. Gabrielse, PRL **100**, 120801 (2008)

$$\vec{\mu} = -g \left(\frac{e}{2m} \right) \vec{s}$$

$$\frac{d\vec{s}}{dt} = \vec{\mu} \times \vec{B}$$

$$g/2 = 1.001\,159\,652\,180\,73\,(28)\,[0.28\,\text{ppt}]$$

$$\begin{aligned} \frac{g}{2} = & 1 + C_2 \left(\frac{\alpha}{\pi} \right) + C_4 \left(\frac{\alpha}{\pi} \right)^2 + C_6 \left(\frac{\alpha}{\pi} \right)^3 + C_8 \left(\frac{\alpha}{\pi} \right)^4 \\ & + C_{10} \left(\frac{\alpha}{\pi} \right)^5 + \dots + a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}}, \quad (4) \end{aligned}$$

It's a triumph of QED!

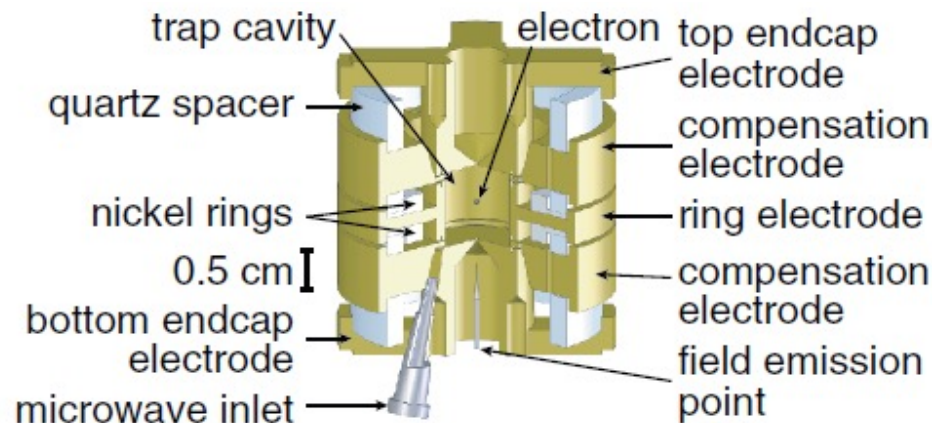


FIG. 2 (color). Cylindrical Penning trap cavity used to confine a single electron and inhibit spontaneous emission.

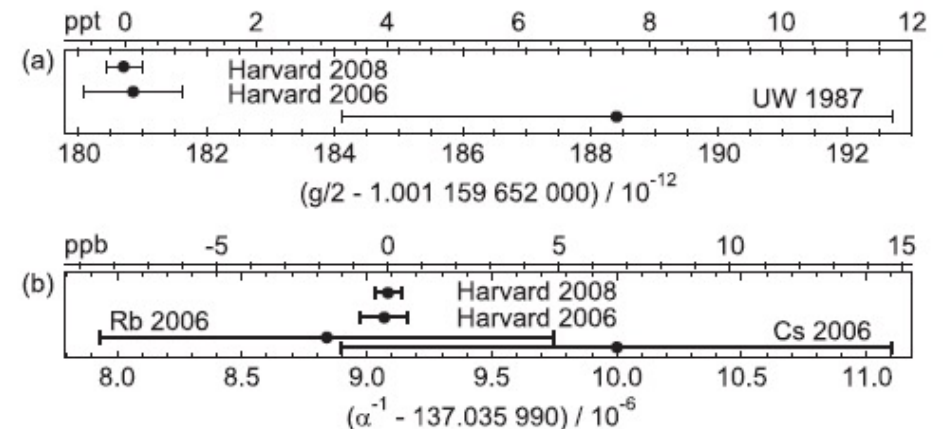


FIG. 1. Most accurate measurements of the electron $g/2$ (a), and most accurate determinations of α (b).

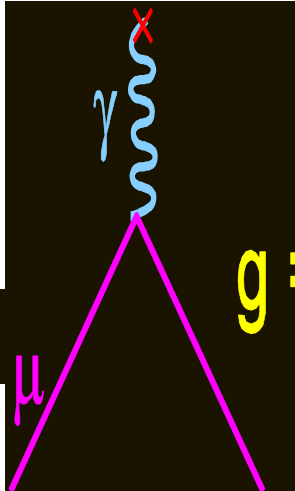
g-factors: Muon case

- The $g_\mu - 2$ is more sensitive to a class of particles than the $g_e - 2$ by $(m_\mu/m_e)^2 \sim 40,000$. A thicker “soup” of virtual particles coming in and out of existence...
- Muons are sensitive to W, Z, and new physics, e.g. SUSY: neutralino

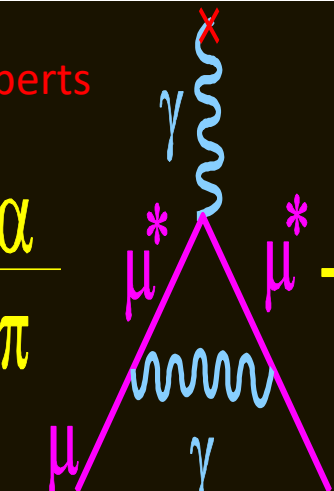
Radiative corrections change g from its Dirac value of 2. We symbolically express these corrections as Feynman diagrams

Lee B. Roberts

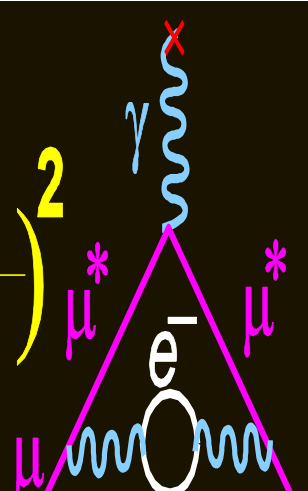
$$g = 2 + \frac{\alpha}{\pi} + c_2 \left(\frac{\alpha}{\pi} \right)^2 + \dots$$



Dirac
Stern-Gerlach



Schwinger
Kusch-Foley



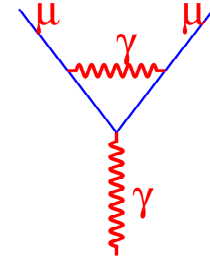
We have a perturbation expansion:

$$a(\text{QED}) = \frac{1}{2} \frac{\alpha}{\pi} + C_2 \left(\frac{\alpha}{\pi} \right)^2 + C_3 \left(\frac{\alpha}{\pi} \right)^3 + C_4 \left(\frac{\alpha}{\pi} \right)^4 + C_5 \left(\frac{\alpha}{\pi} \right)^5 + \dots$$

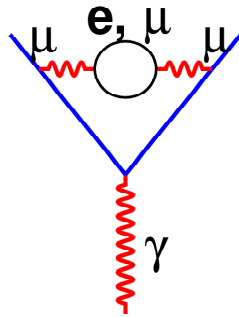


$g - 2$ for the muon, SM contributions

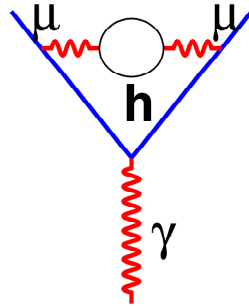
Largest contribution : $a_\mu = \frac{\alpha}{2\pi} \approx \frac{1}{800}$



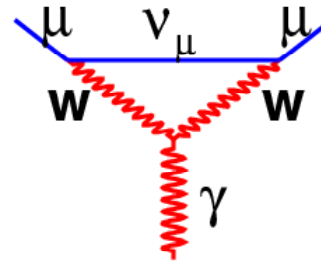
Other standard model contributions :



QED



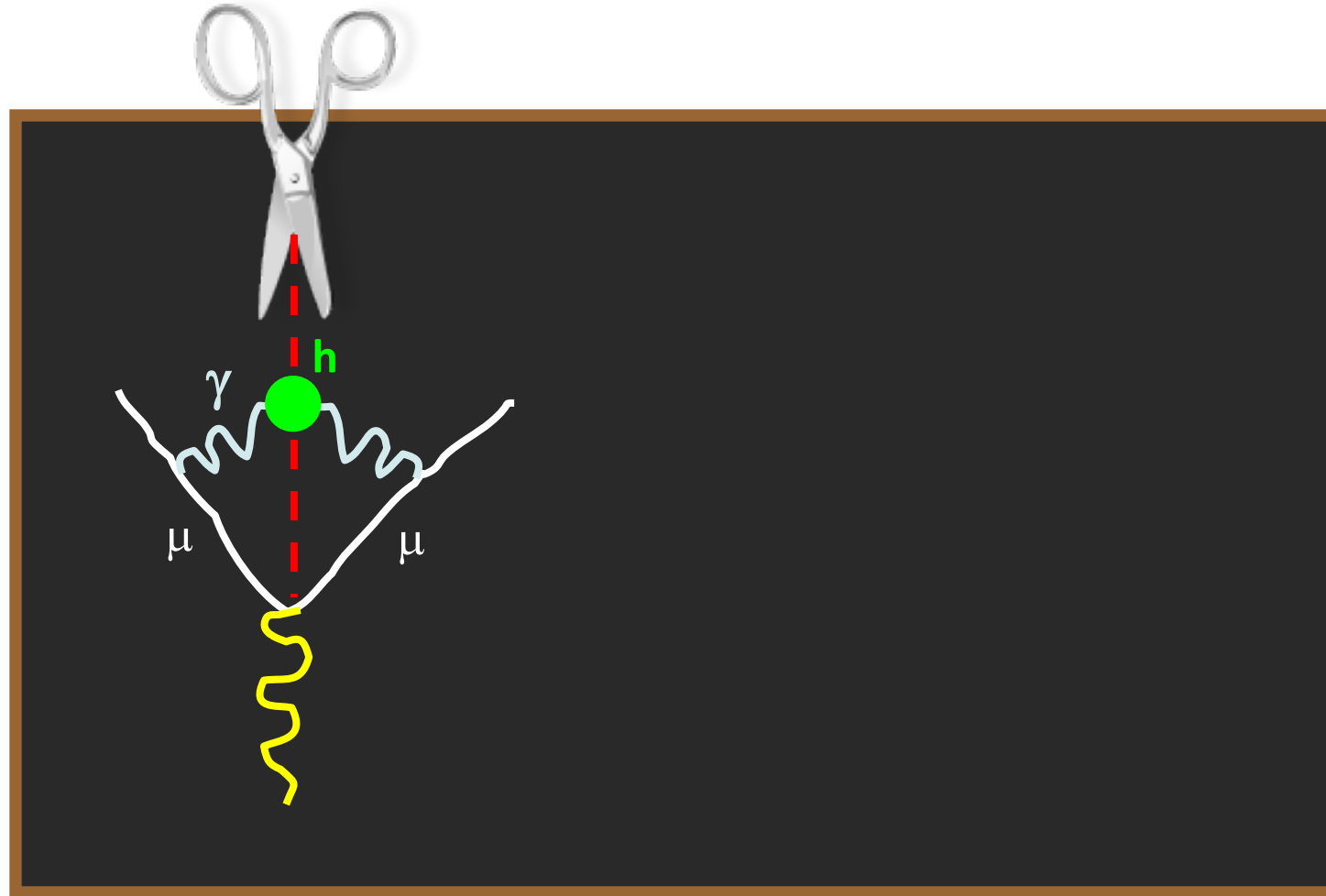
hadronic



weak

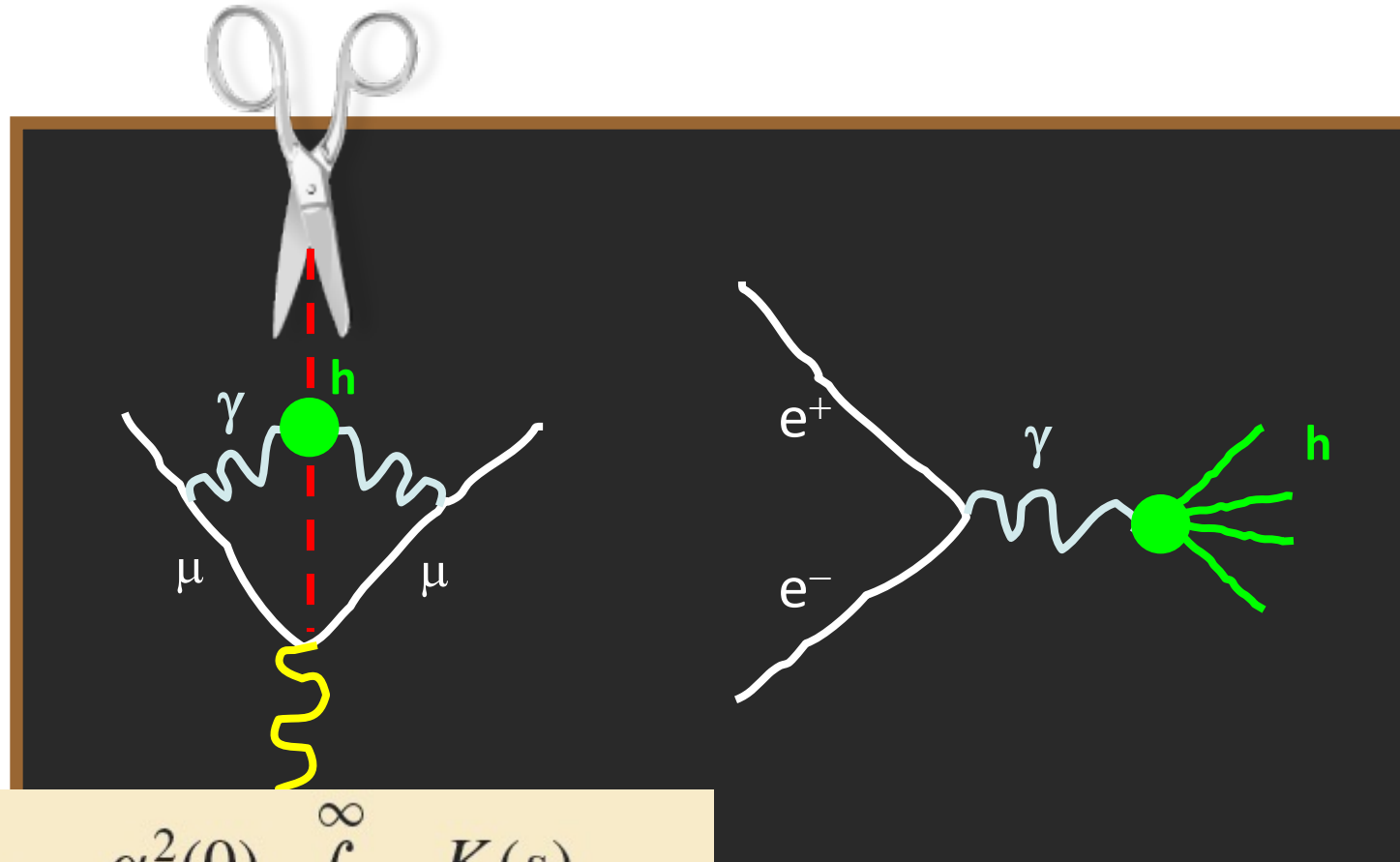
Hadronic vacuum polarization

Challenging but can link to experimental data!



Hadronic vacuum polarization

Challenging but can link to experimental data!

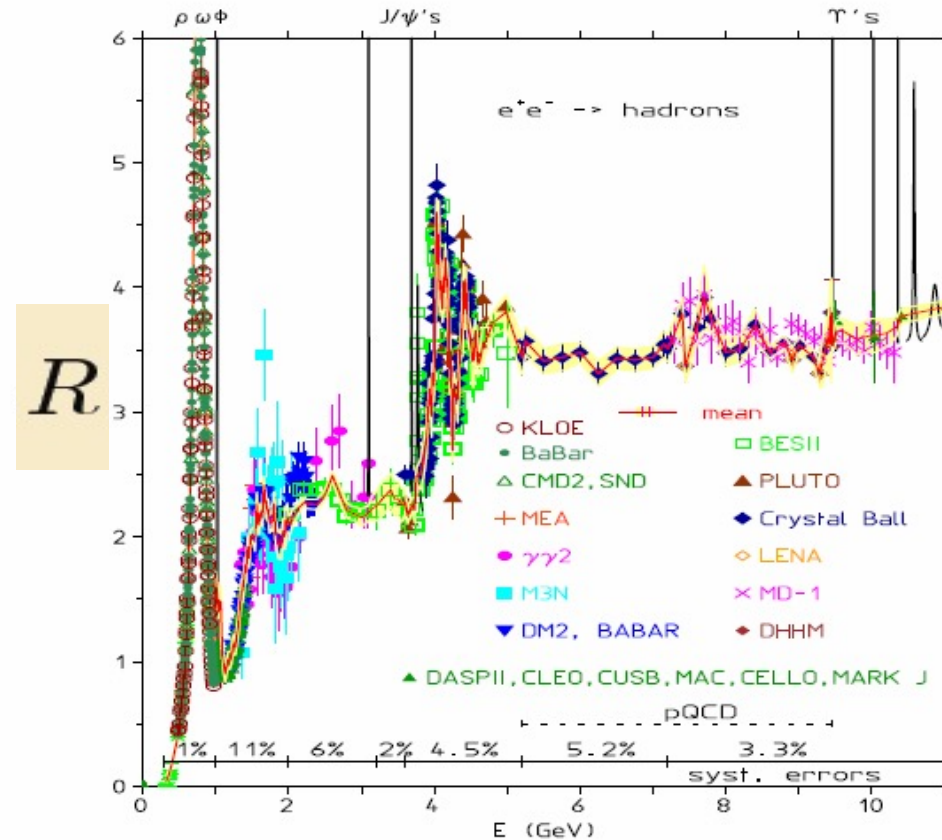


$$a_{\mu}^{\text{had,LO}} = \frac{\alpha^2(0)}{3\pi^2} \int_{4m_{\pi}^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \text{muons})}$$

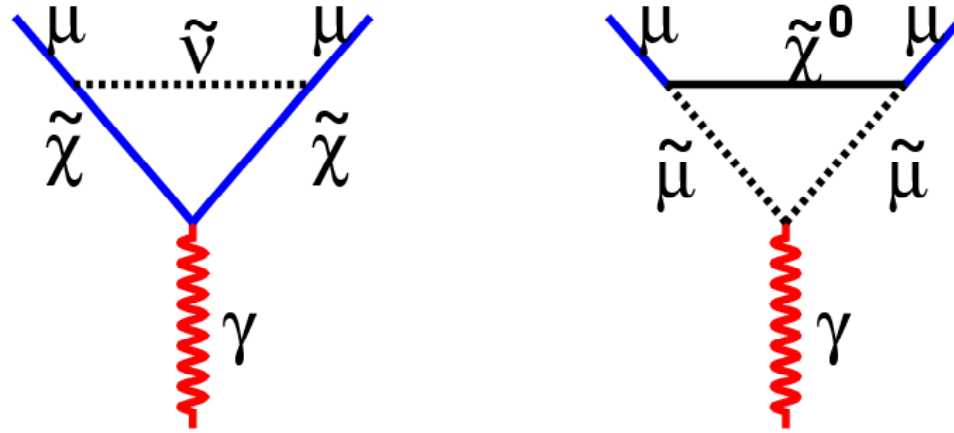
Hadronic vacuum polarization

$$a_{\mu}^{\text{had,LO}} = \frac{\alpha^2(0)}{3\pi^2} \int_{4m_{\pi}^2}^{\infty} ds \frac{K(s)}{s} R(s)$$



- A lot of precision data already available from many experiments
- Future improvements from VEP-2000, KLOE, BaBar, Belle, BES-III, ...

Beyond standard model, e.g. SUSY

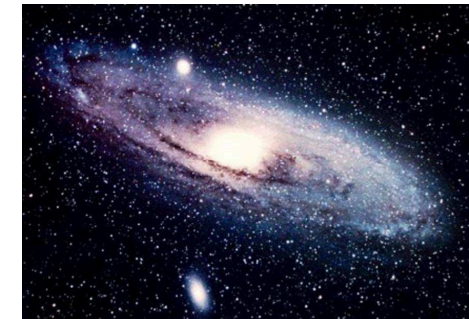
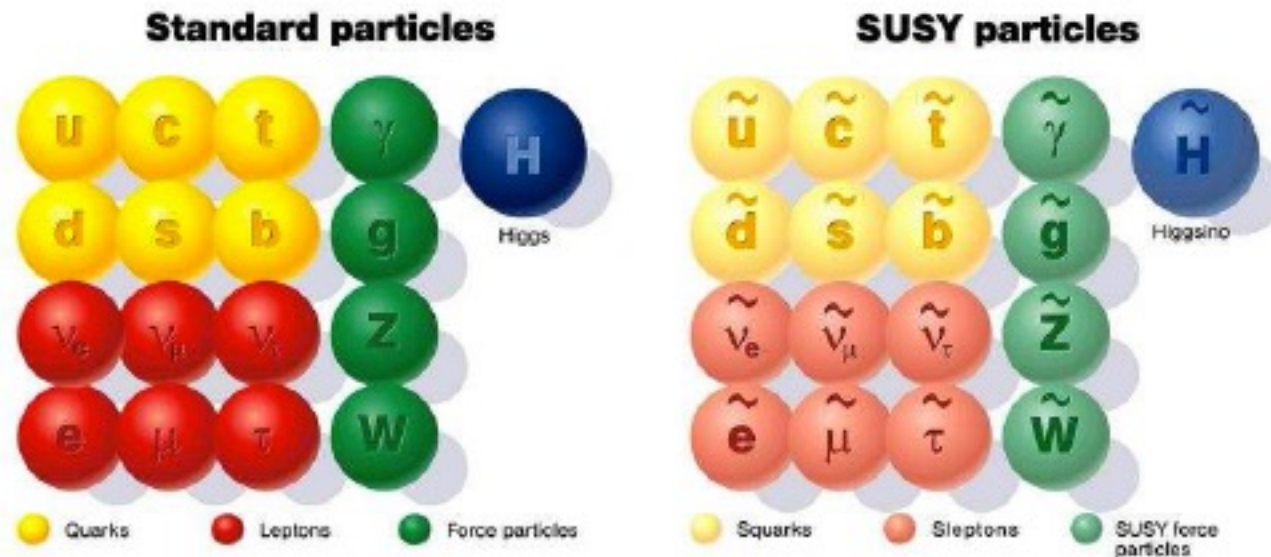


$$a_{\mu}^{\text{susy}} \cong \text{sgn}(\mu) \times 13 \times 10^{-10} \left(\frac{100 \text{ GeV}}{m_{\text{susy}}} \right)^2 \tan \beta$$

Muon g-2 sensitivity to the “image world” of SUSY

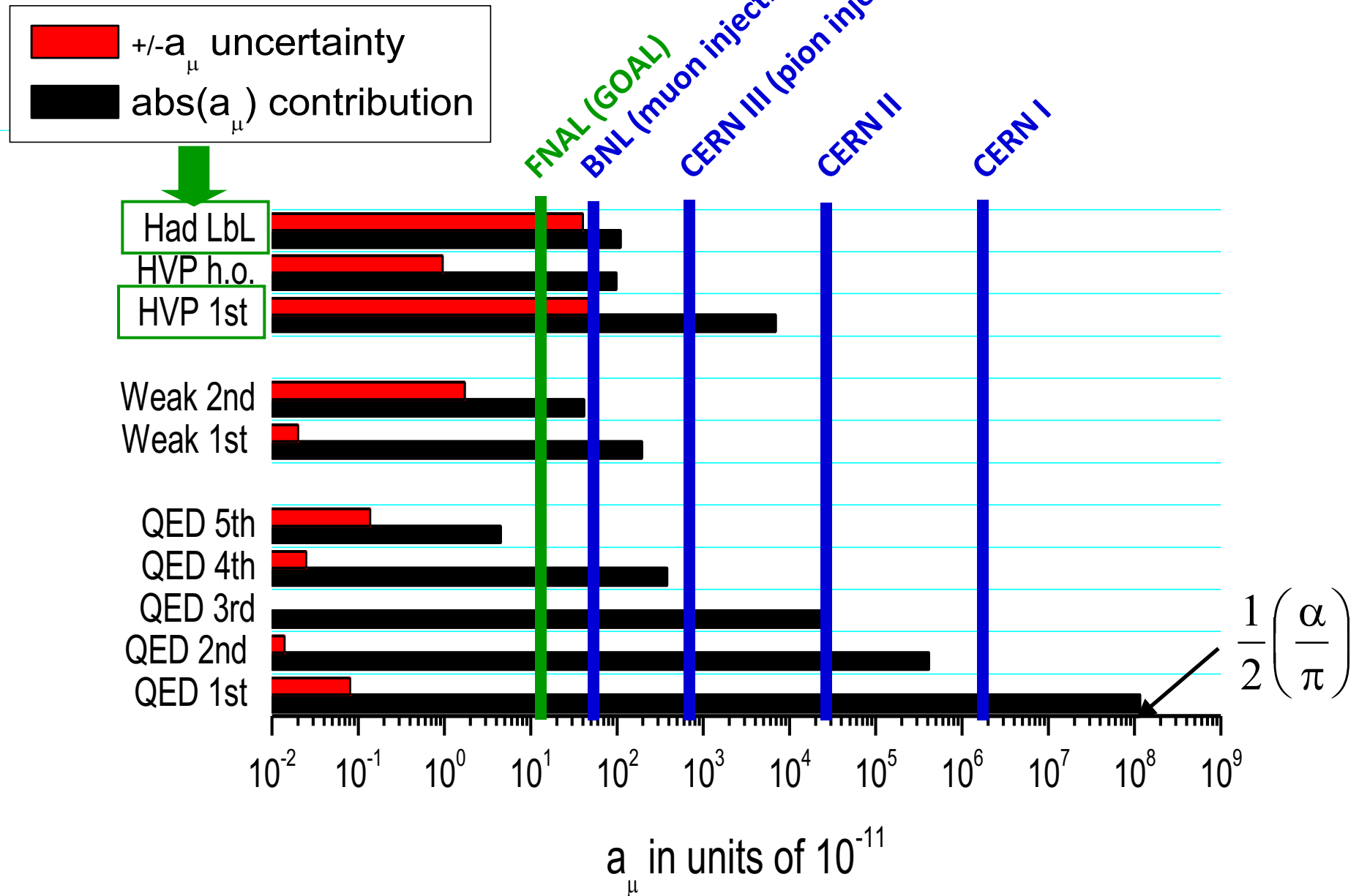
$$a_{\mu}^{\text{SUSY}} \approx 13 \times 10^{-10} \tan \beta \operatorname{sign}(\mu) \left(\frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2$$

Mass of Neutralino!



Historical overview of the muon $g-2$ experiment

Brief history



P. Winter

A brief history of the CERN muon $g-2$ experiments

The 47 years of muon $g - 2$

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Received 30 October 2003

F.J.M. Farley, Y.K. Semertzidis / Progress in Particle and Nuclear Physics 52 (2004) 1–83

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Fig. 10. The first experimental magnet in which muons were stored at CERN for up to 30 turns. Left to right: Georges Charpak, Francis Farley, Bruno Nicolai, Hans Sens, Antonio Zichichi, Carl York and Richard Garwin.

CERN I, 1958-1962

- Top view of first magnet, 1.6T dipole

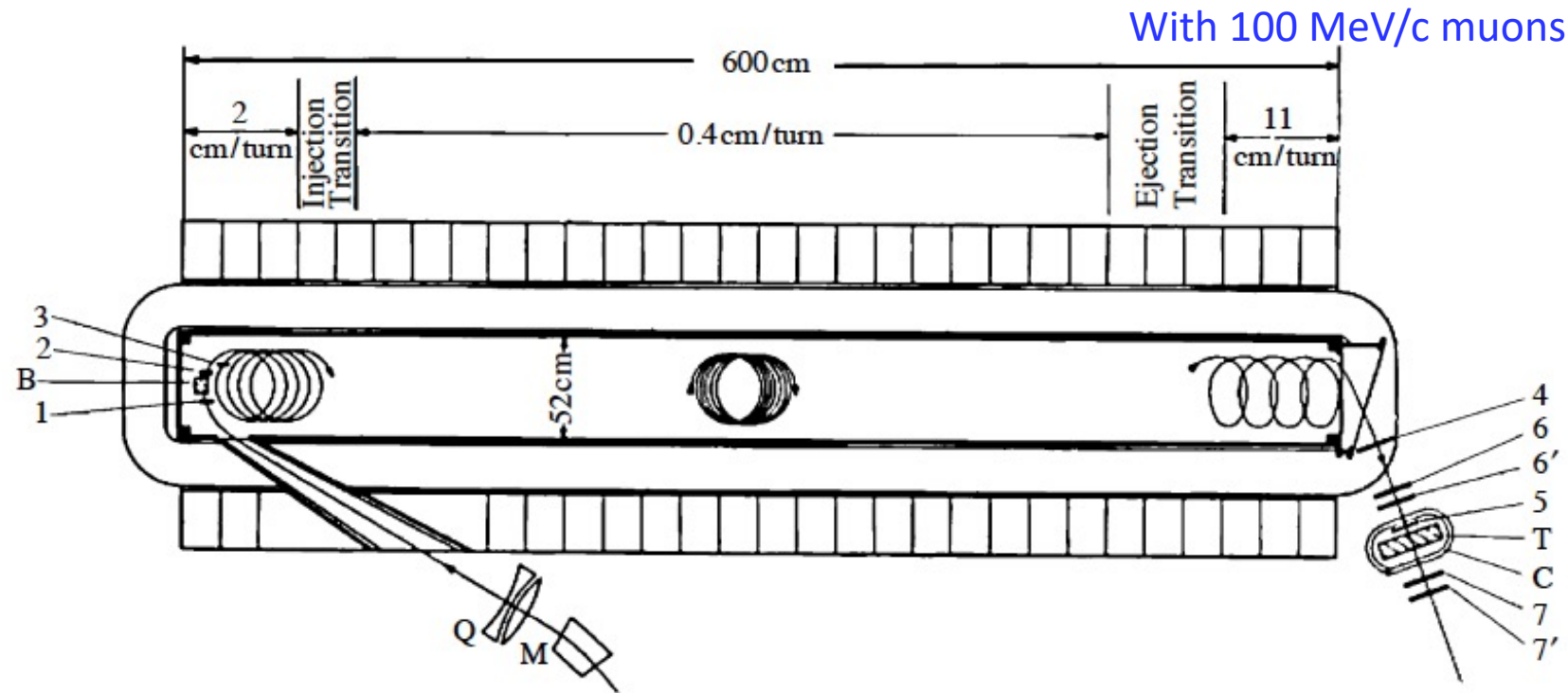


Fig. 12. The 6 m bending magnet used for storing of muons for up to 2000 turns. A transverse field gradient makes the orbit walk to the right. At the end a very large gradient is used to eject the muons which stop in the polarization analyzer. Coincidences 123 and $466'5\bar{7}$ signal an injected and ejected muon respectively. The coordinates used in the text are x (the long axis of the magnet), y (the transverse axis in the plane of the paper) and z (the axis perpendicular to the paper).

CERN I, 1958-1962

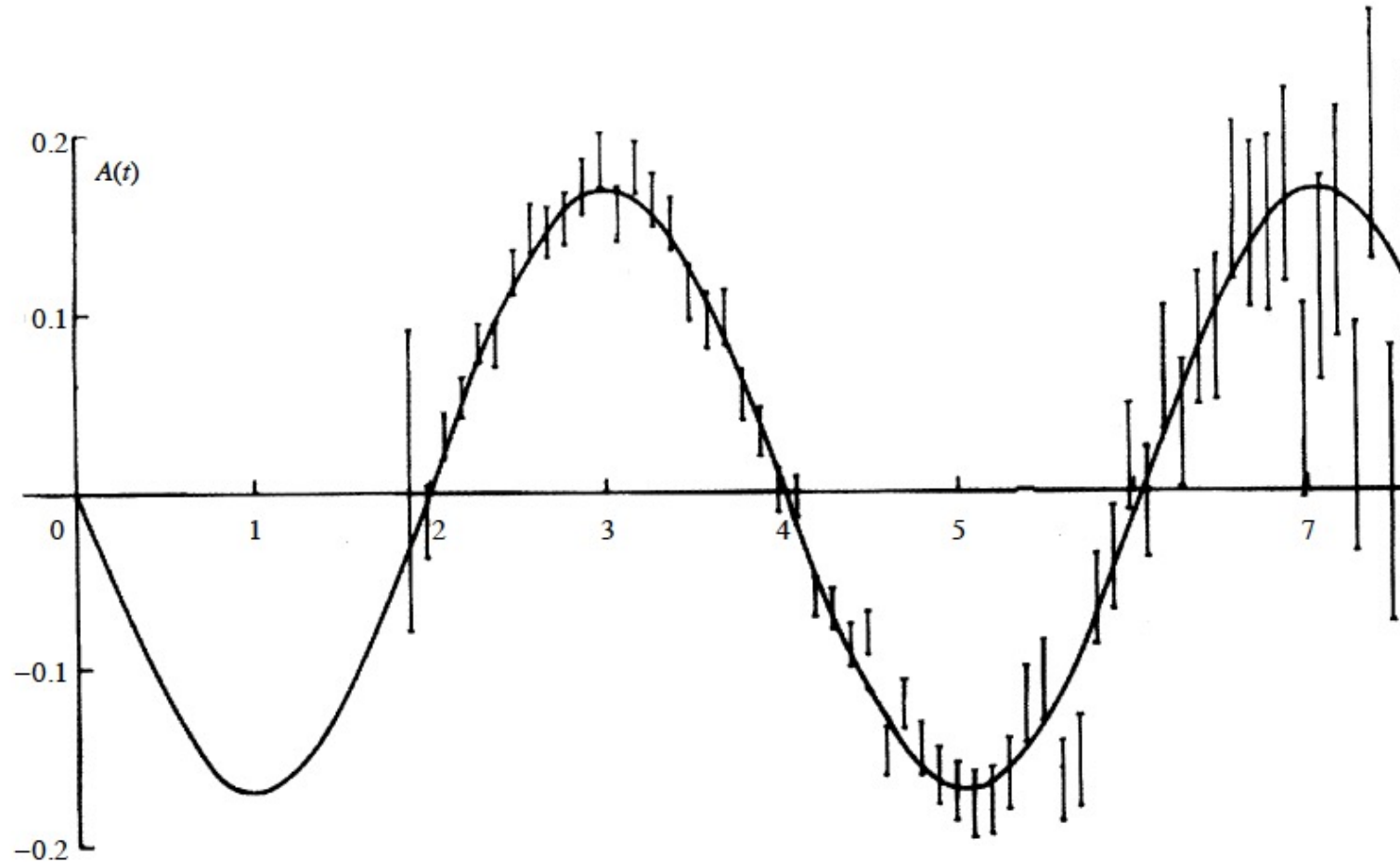


Fig. 14. Asymmetry A of observed decay electron counts as a function of the storage time t . The time t spent in the magnet depended on the transverse position of the orbit on the parabolic magnetic field (45). The muons that were stored for $7.5 \mu\text{s}$ made 1600 turns in the magnet and then emerged spontaneously at the far end. The sinusoidal variation results from the $(g - 2)$ precession; the frequency is measured to $\pm 0.4\%$.

CERN II, 1962-1968

- Top view of the second magnet 1.7T, proton injection.

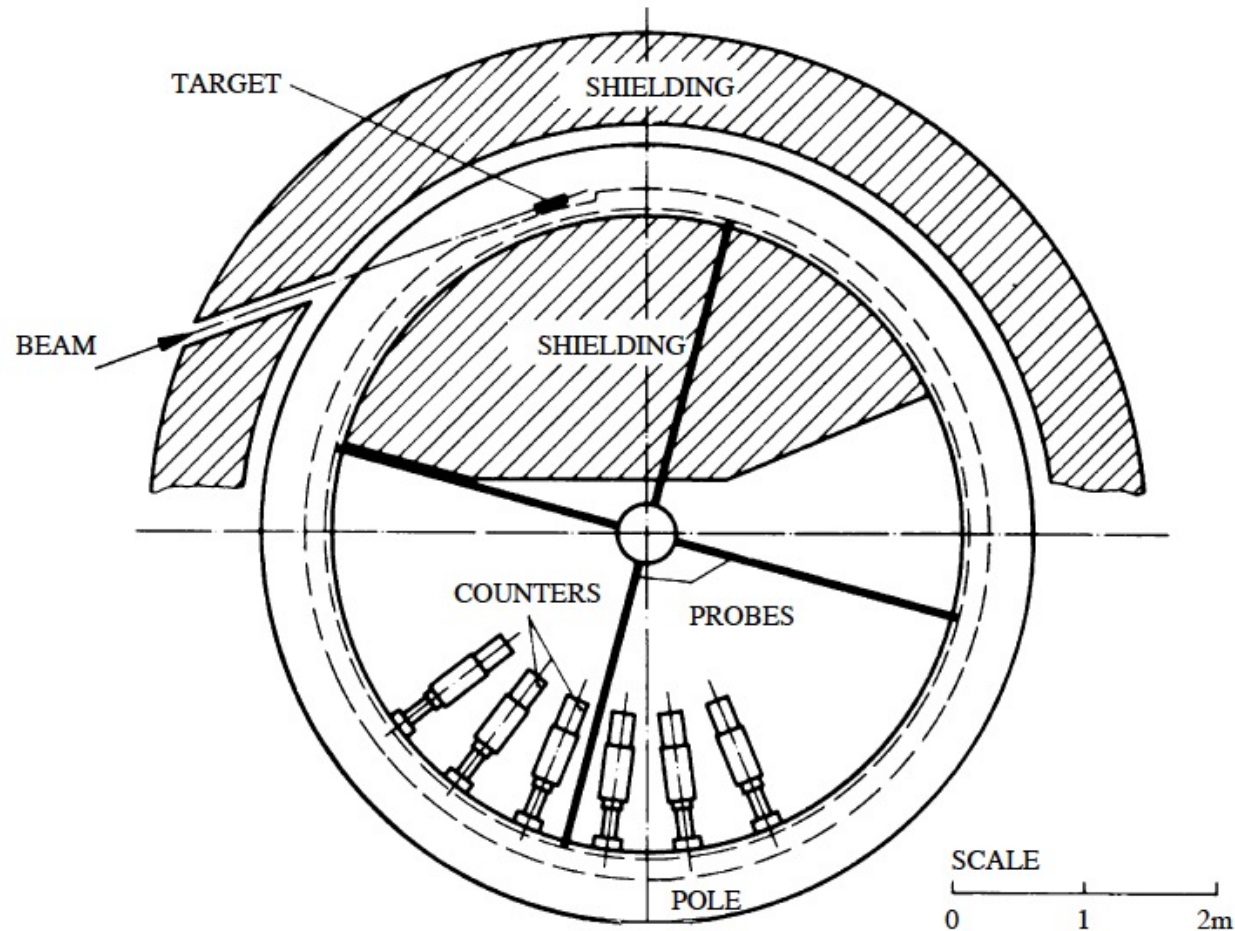


Fig. 17. The first muon storage ring: diameter 5 m, muon momentum 1.3 GeV/c, time dilation factor 12. The injected pulse of 10.5 GeV protons produces pions at the target, which decay in flight to give muons.

CERN II,
1962-1968.
270ppm in a .

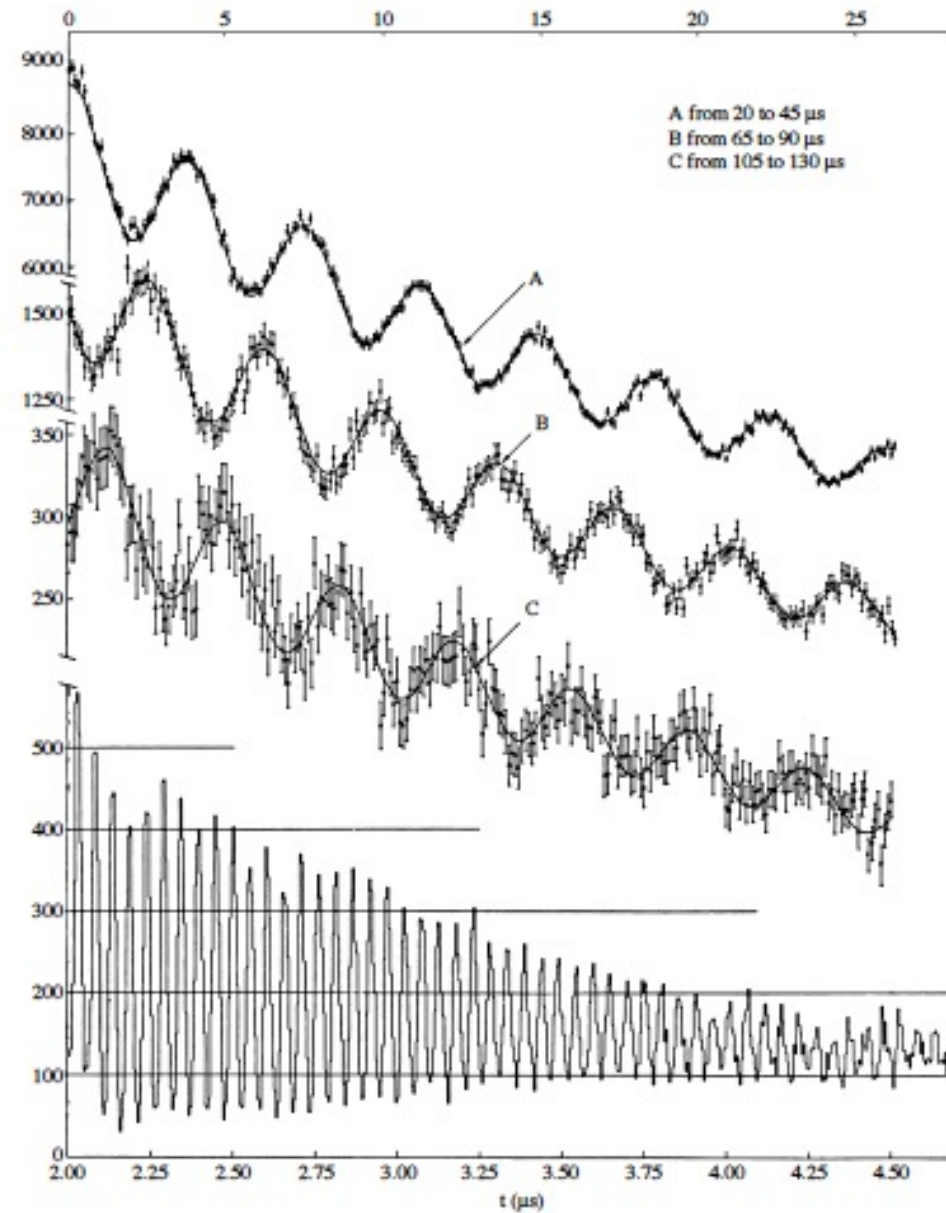


Fig. 19. The first muon storage ring: decay electron counts as a function of time after the injected pulse. The lower curve 1.5–4.5 μs (lower timescale) shows the 19 MHz modulation due to the rotation of the bunch of muons around the ring. As it spreads out the modulation dies away. This is used to determine the radial distribution of muon orbits. Curves A, B and C are defined by the legend (upper time scale); they show various sections of the experimental decay (lifetime 27 μs) modulated by the $(g - 2)$ precession. The frequency is determined to 215 ppm, B to 160 ppm leading to 270 ppm in a .

CERN III, 1969-1976

- The third magnet, second storage ring 1.47T.
- Pion injection, E-field focusing, Magic momentum

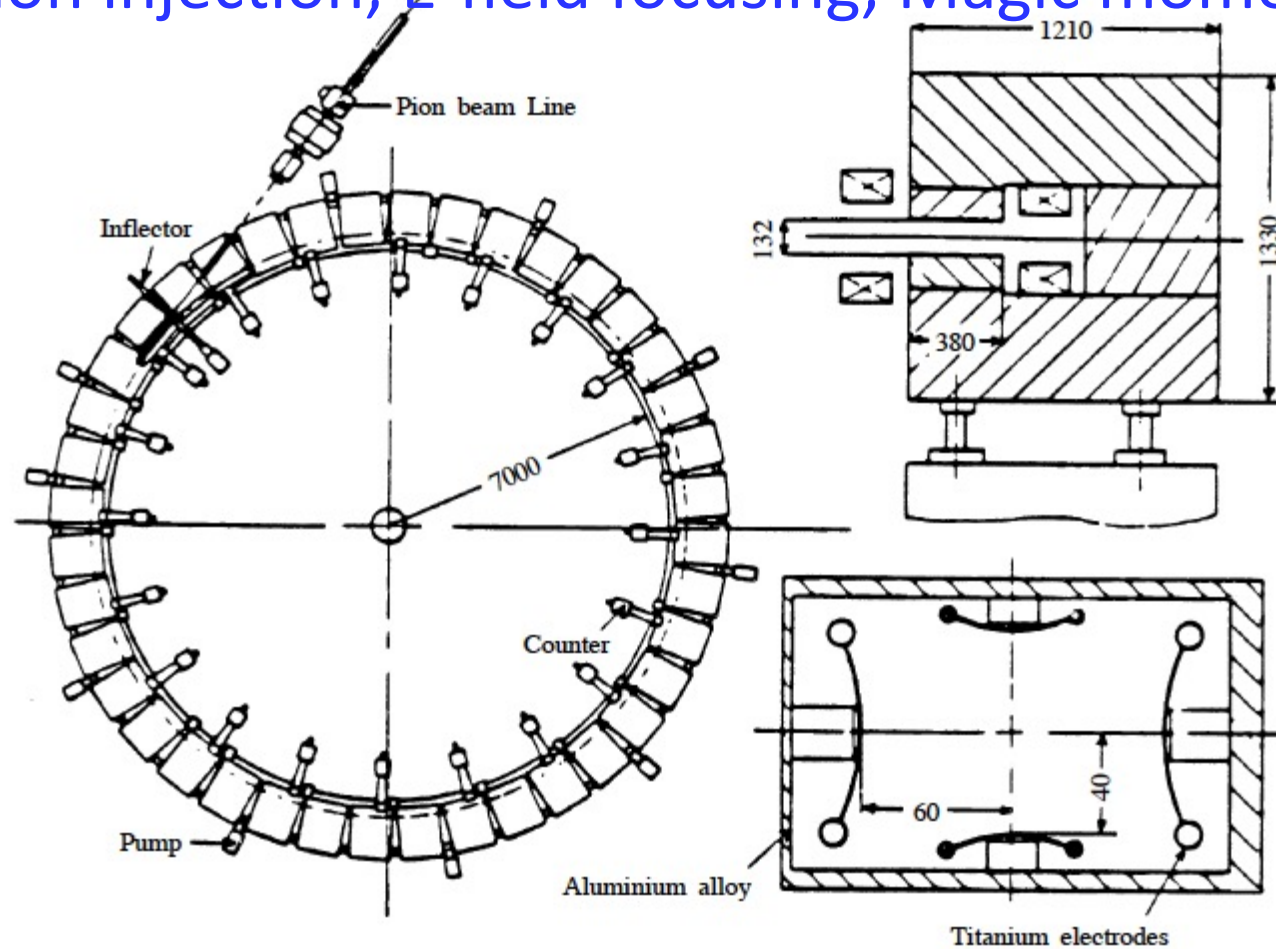


Fig. 21. The second muon storage ring, which consisted of 40 contiguous magnet blocks. The open side of the C-shaped yoke (upper right) faces the centre of the ring. The cross-section of the vacuum chamber and electric quadrupole is shown bottom right. The decay electrons are detected by 20 counters. Dimensions are in mm.

CERN III, 1969-1976. 7.3ppm in α .

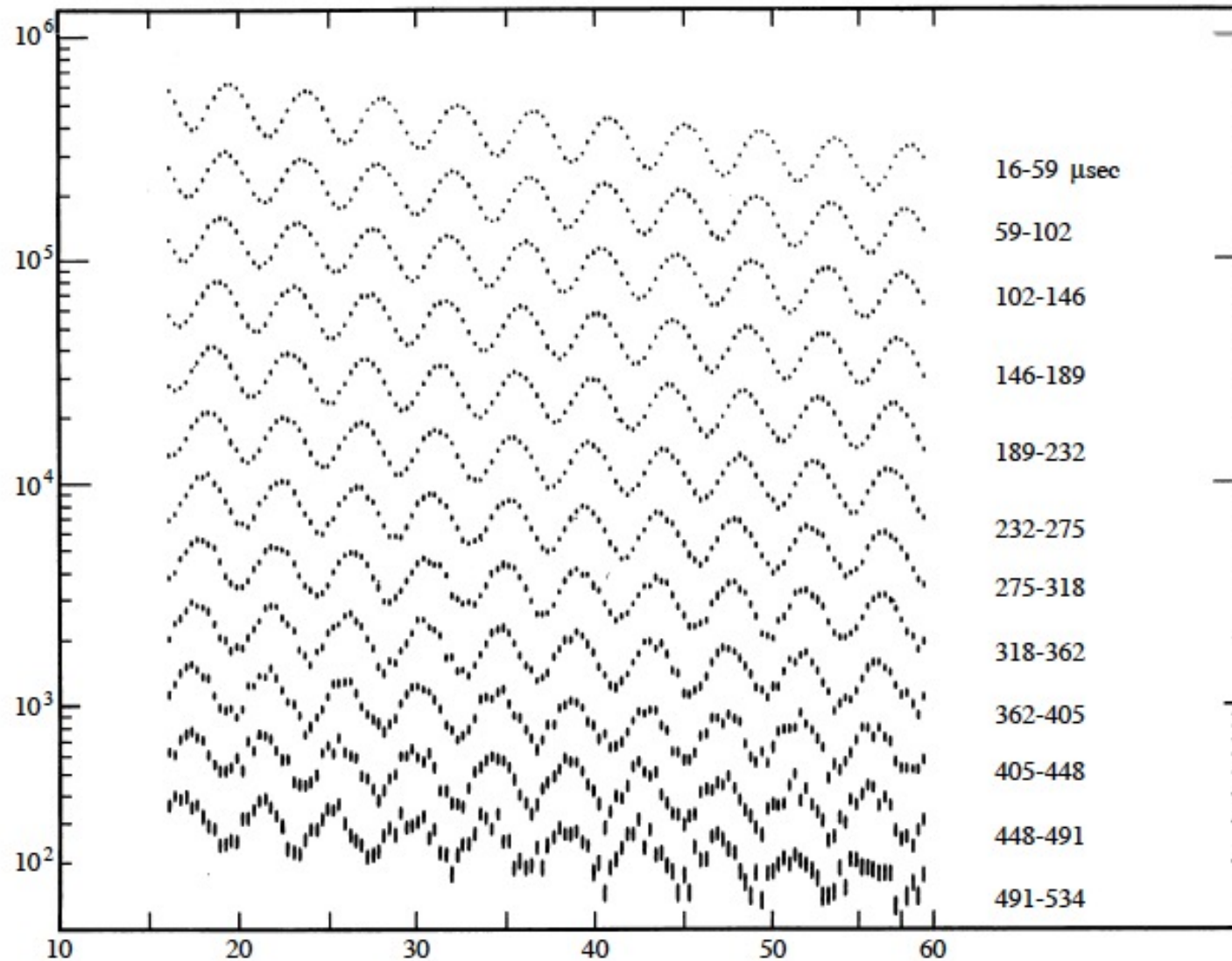


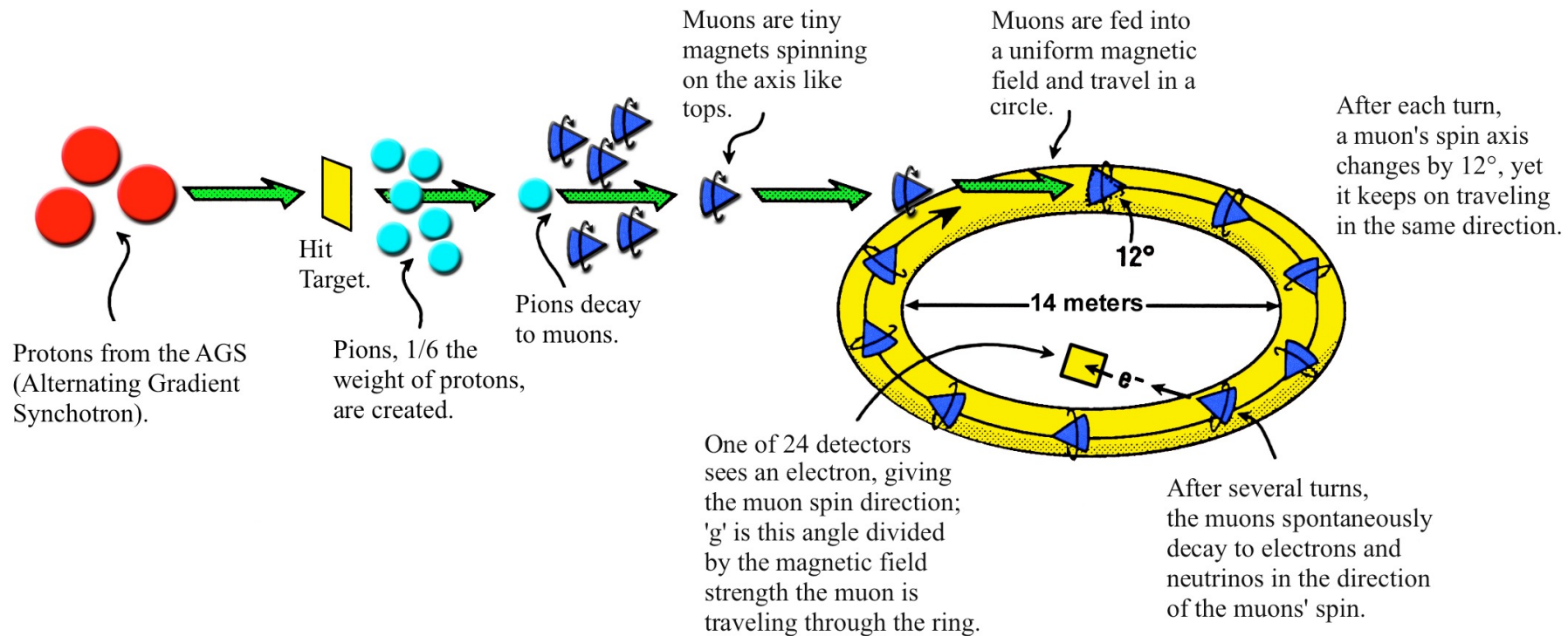
Fig. 25. The second muon storage ring: decay electron counts versus time (in microseconds) after injection. The range of time for each line is shown on the right (in microseconds).

Storage Ring Muon g-2: Rigorous Test of the Standard Model

Muon g-2 experiment: major challenge to the Standard Model

- E821 at BNL: 1997-2004
- E989 at FNAL: first data in 2017

LIFE OF A MUON: THE g-2 EXPERIMENT



Spin Precession Rate at Rest

$$\frac{d\vec{s}}{dt} = \vec{\mu} \times \vec{B} + \vec{d} \times \vec{E}$$

There is a large asymmetry in this equation: μ is relatively large, d is compatible with zero

The Principle of g-2

At rest : $\frac{d\vec{s}}{dt} = \vec{\mu} \times \vec{B}$

Spin vector Moving: Thomas precession!

Momentum vector



Momentum vector



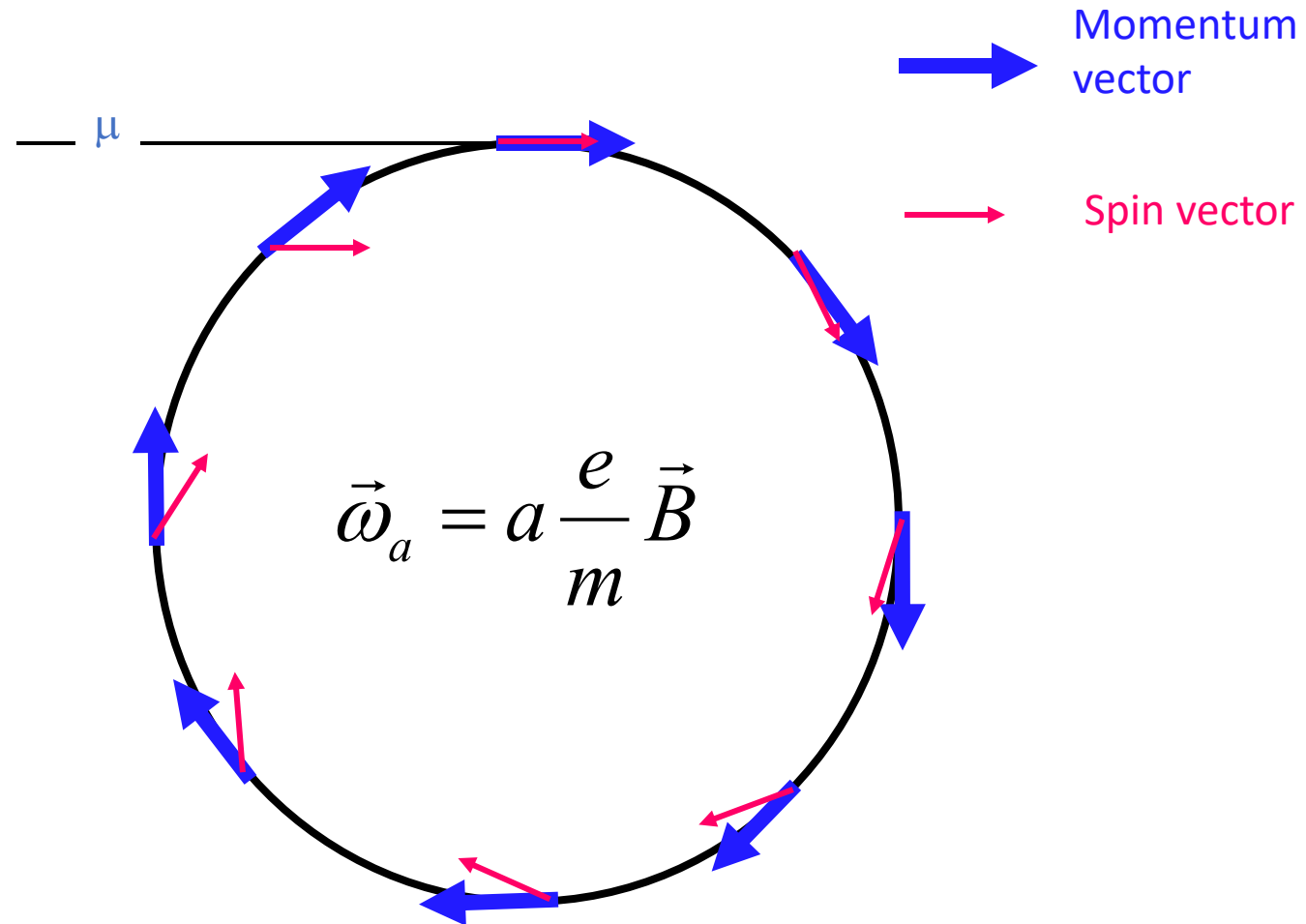
$$\omega_c = \frac{eB}{m\gamma}$$

$$\omega_s = \frac{g}{2} \frac{eB}{m} + (1 - \gamma) \frac{eB}{m\gamma}$$

$$\omega_a = \omega_s - \omega_c = \left(\frac{g-2}{2} \right) \frac{eB}{m} \Rightarrow \omega_a = a \frac{eB}{m}$$

Independent of velocity!

Spin Precession in g-2 Ring (Top View)



Reality check: need vertical focusing!

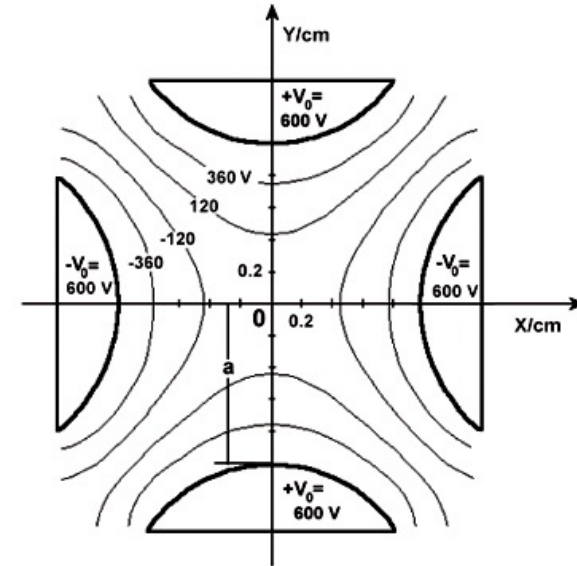
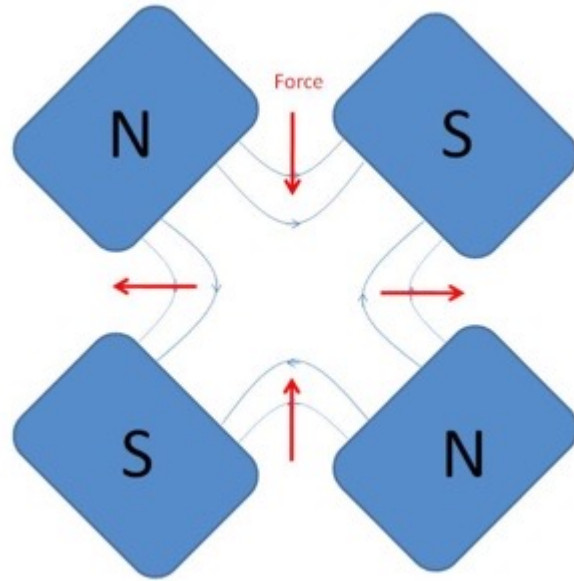
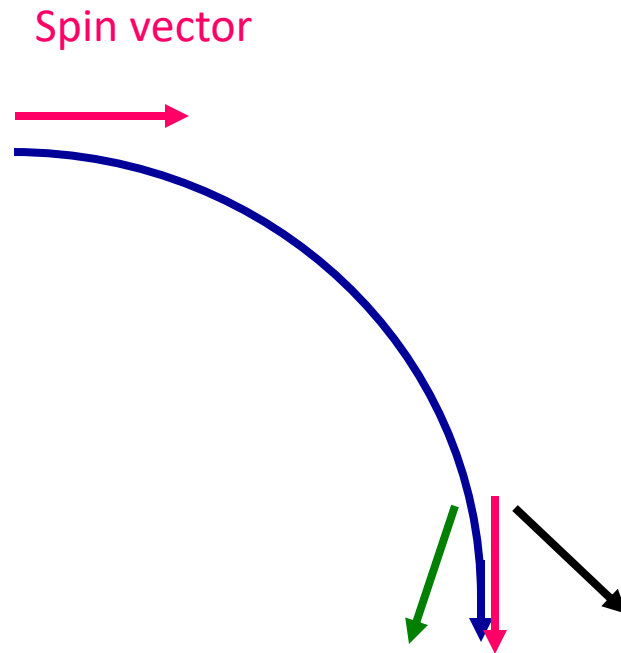


Figure 1. Electrodes and equipotential lines in an electrostatic quadrupole.

Focusing implies field gradients! Magnetic focusing would require knowledge of the muon distribution with micron resolution. Not a starter...

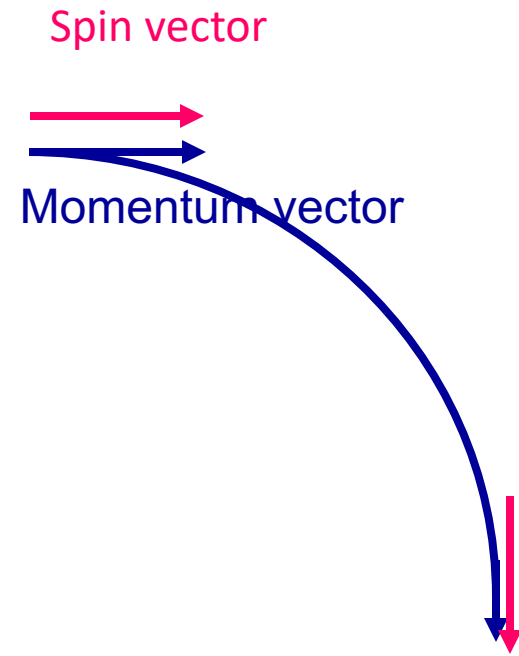
Electric fields look like magnetic fields in the muon rest frame, similar problem, unless...

Effect of Radial Electric Field



- Low energy particle
- ...just right
- High energy particle

Effect of Radial Electric Field



- ...just right, $\gamma \approx 29.3$
for muons
($\sim 3\text{GeV}/c$)

Breakthrough concept: Freezing the horizontal spin precession due to E-field

$$\vec{\omega}_a = -\frac{q}{m} \left\{ a\vec{B} - \left[a - \left(\frac{mc}{p} \right)^2 \right] \frac{\vec{\beta} \times \vec{E}}{c} \right\}$$

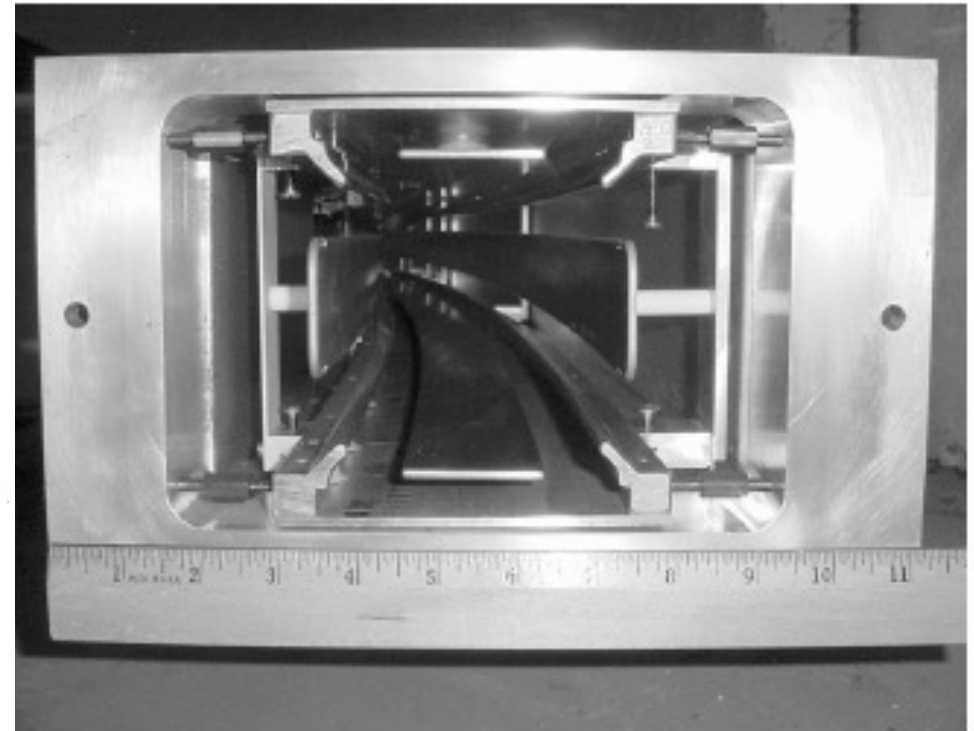
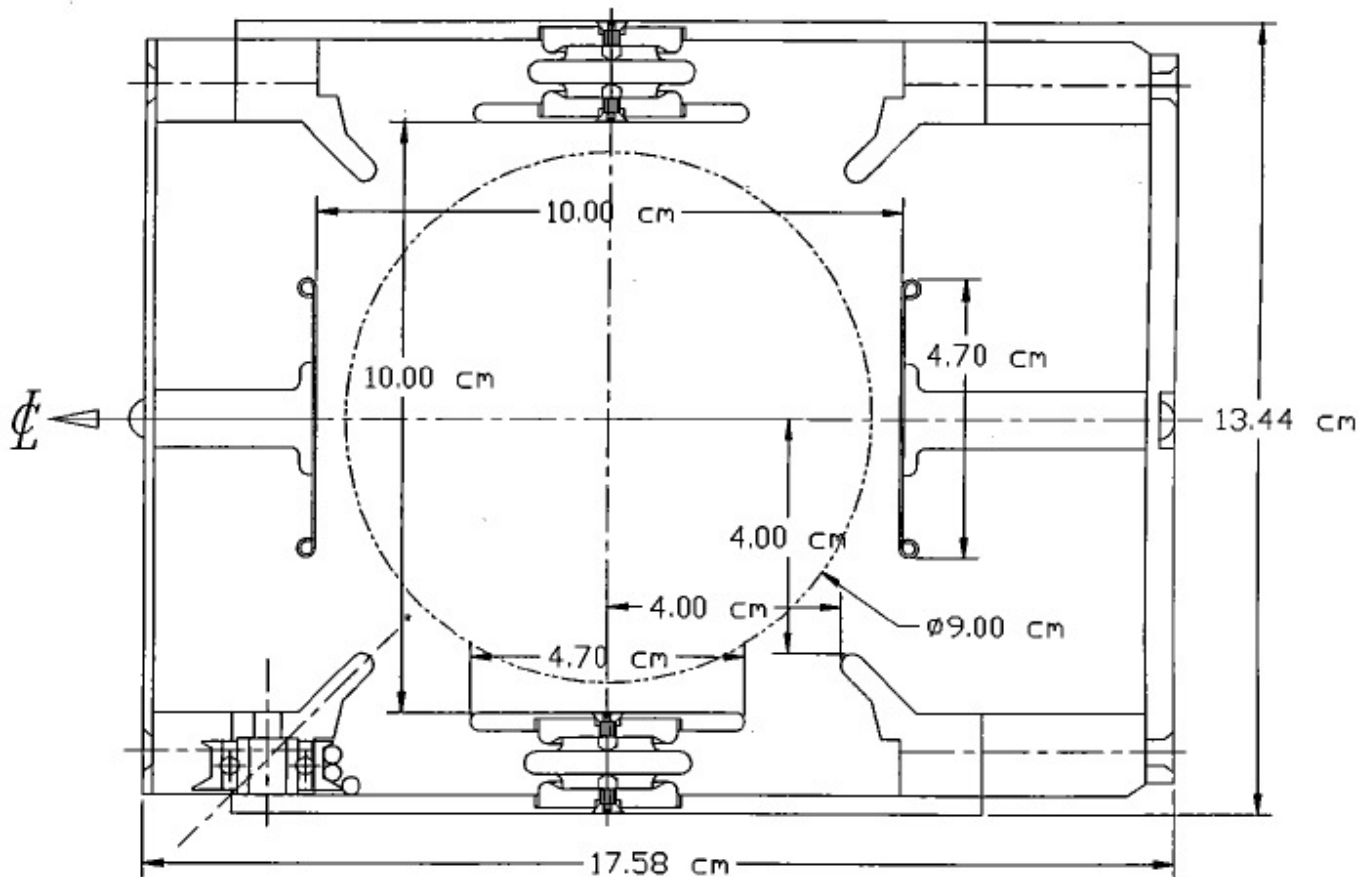
Muon g-2 focusing is electric: The spin precession due to E-field is zero at “magic” momentum (3.1GeV/c for muons, 0.7 GeV/c for protons,...)

$$p = \frac{mc}{\sqrt{a}}, \text{ with } G = a = \frac{g-2}{2}$$

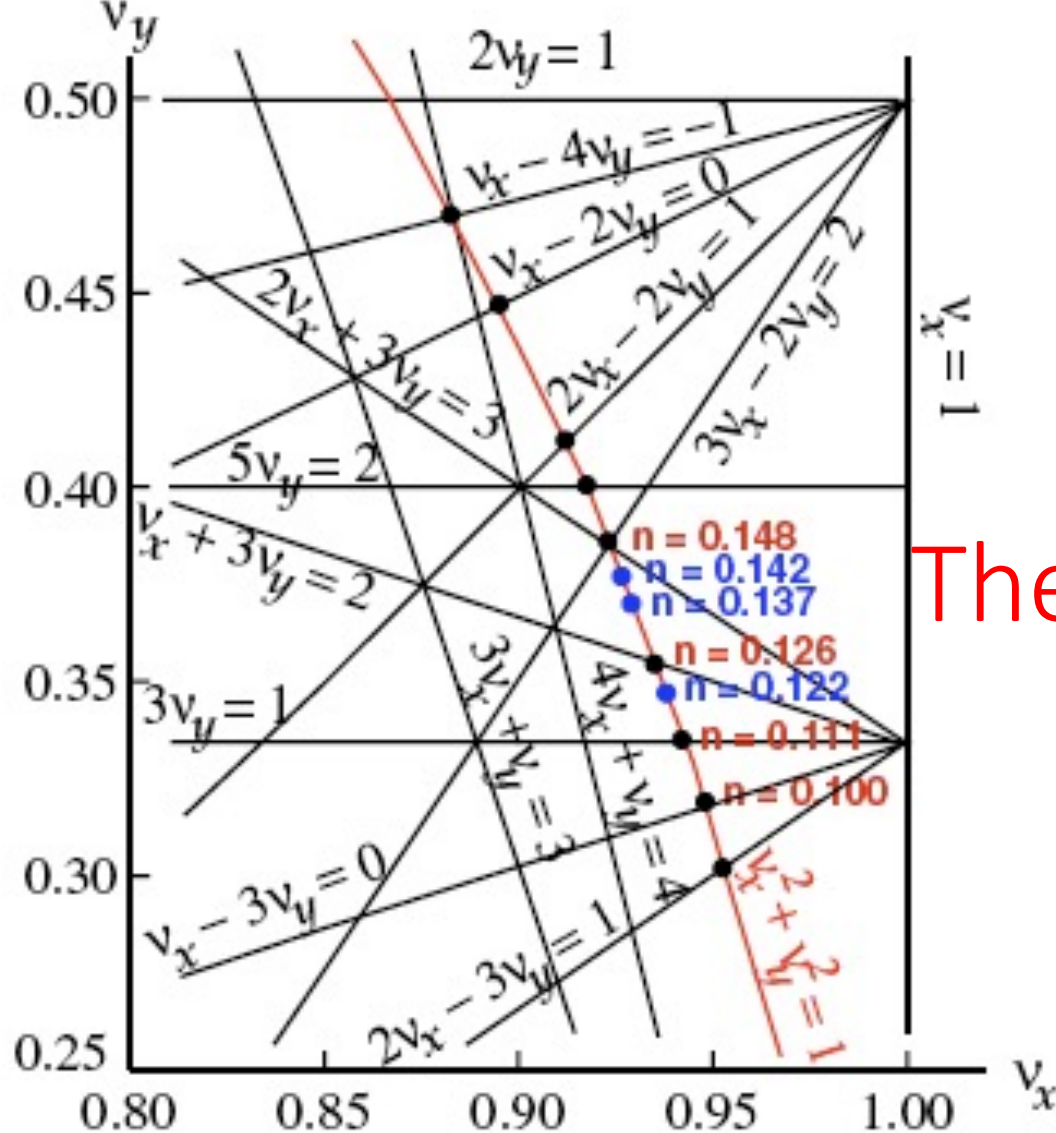
The “magic” momentum concept was used in the muon g-2 experiments at CERN, BNL, and now at FNAL.

Flat, Aluminum plate Quadrupoles

- Pulsed (for $\sim 1\text{ms}$) electrostatic quadrupoles focus the muon beam vertically



h taken from the end of a vacuum chamber housing the quadrupole plates; the ring center is on the left. The dрупole plates at equal potential is 10 cm. The bottom left and the top right rails are where the cable NMR trolley ; the magnetic field. The other two rails were used to keep the symmetry in the quadrupole region. The ruler units



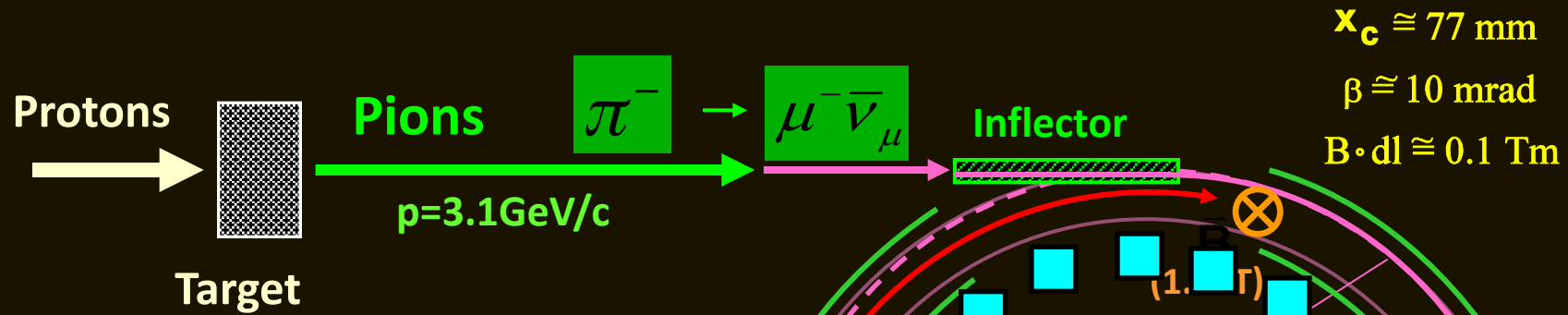
The tune plane

$$\nu_x = \sqrt{1 - n}$$

$$\nu_y = \sqrt{n}$$

FIG. 18 (color online). The tune plane showing resonance lines. Three of the n values used to run the experiment, 0.122, 0.137, 0.142, are indicated on the arc of the circle defined as $\nu_x^2 + \nu_y^2 = 1$. They do not intersect any of the resonance lines, contrary to nearby tunes, which are also shown on the arc.

Experimental Technique

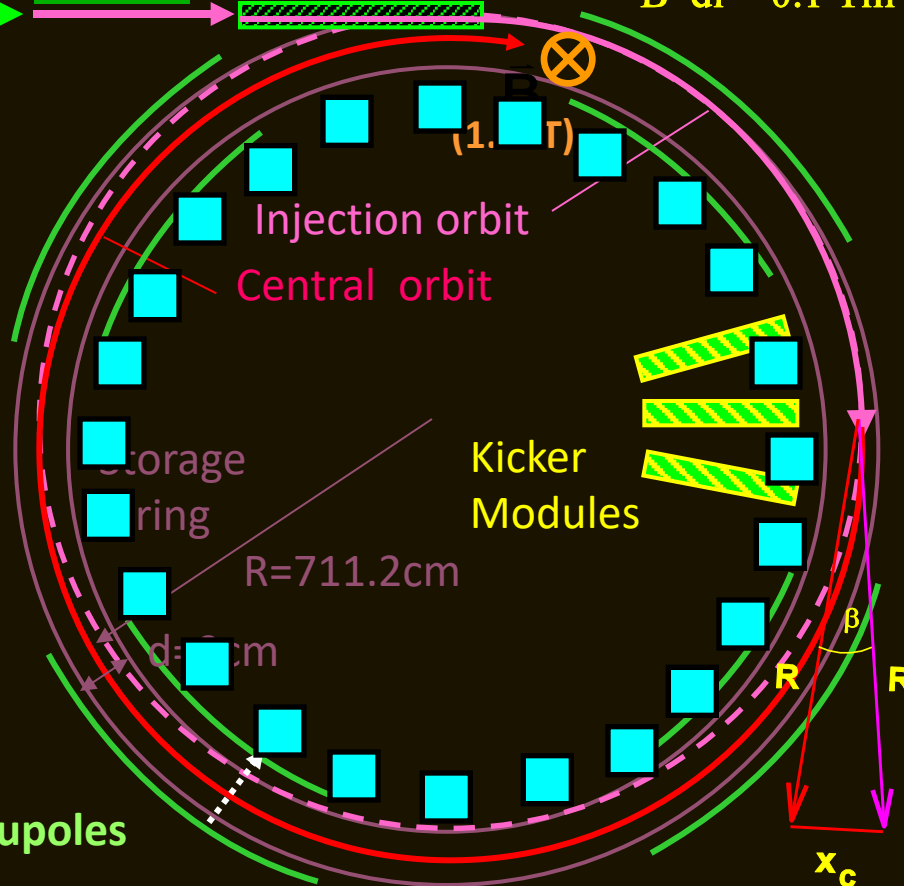


slide by B. Lee Roberts

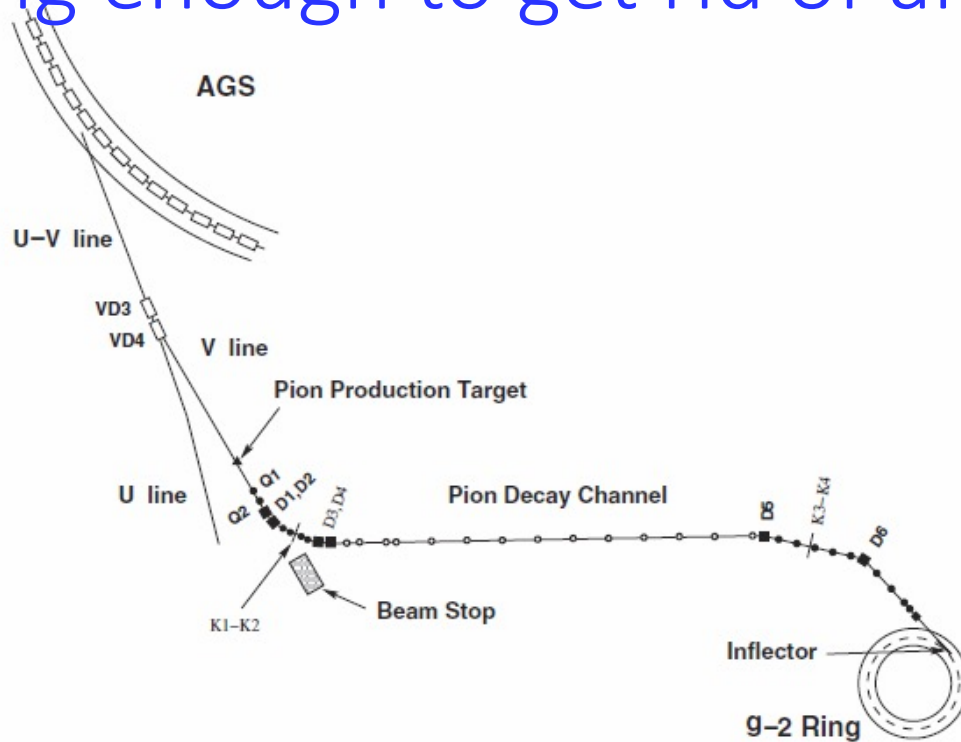
- Muon polarization
- Muon storage ring
- injection & kicking
- focus by Electric Quadrupoles
- 24 electron calorimeters

$$\vec{\omega}_a = - \frac{e}{m} a_\mu \vec{B}$$

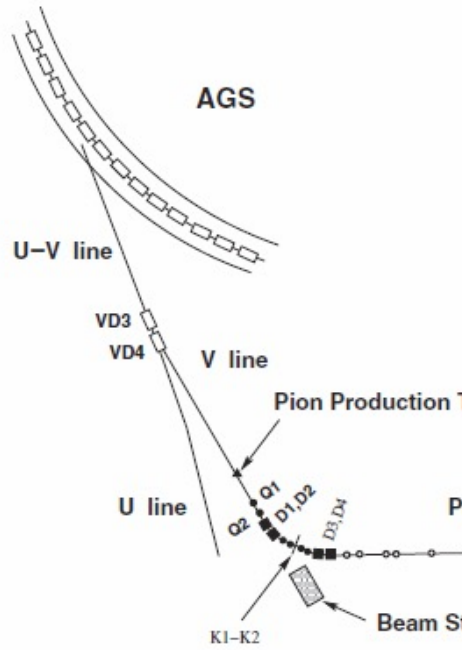
Electric Quadrupoles



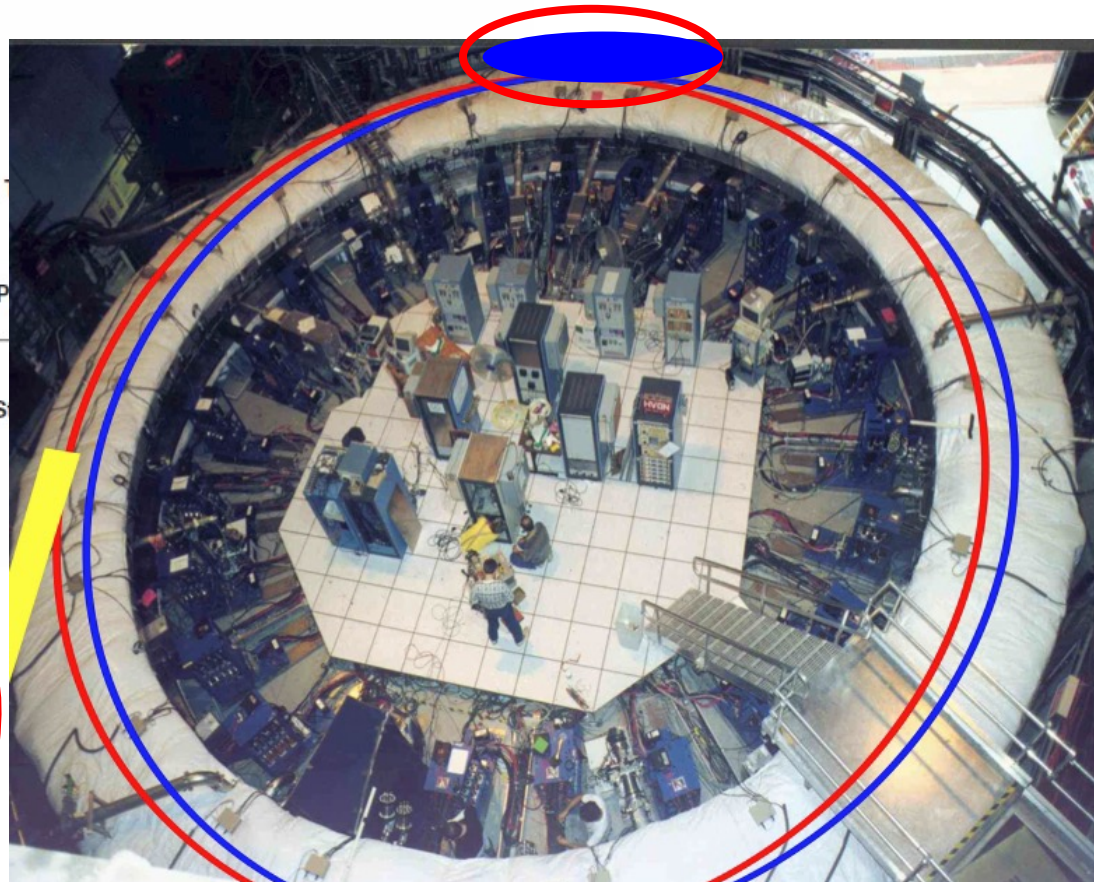
BNL E821: Muon injection, beamline not quite long enough to get rid of all pions at injection



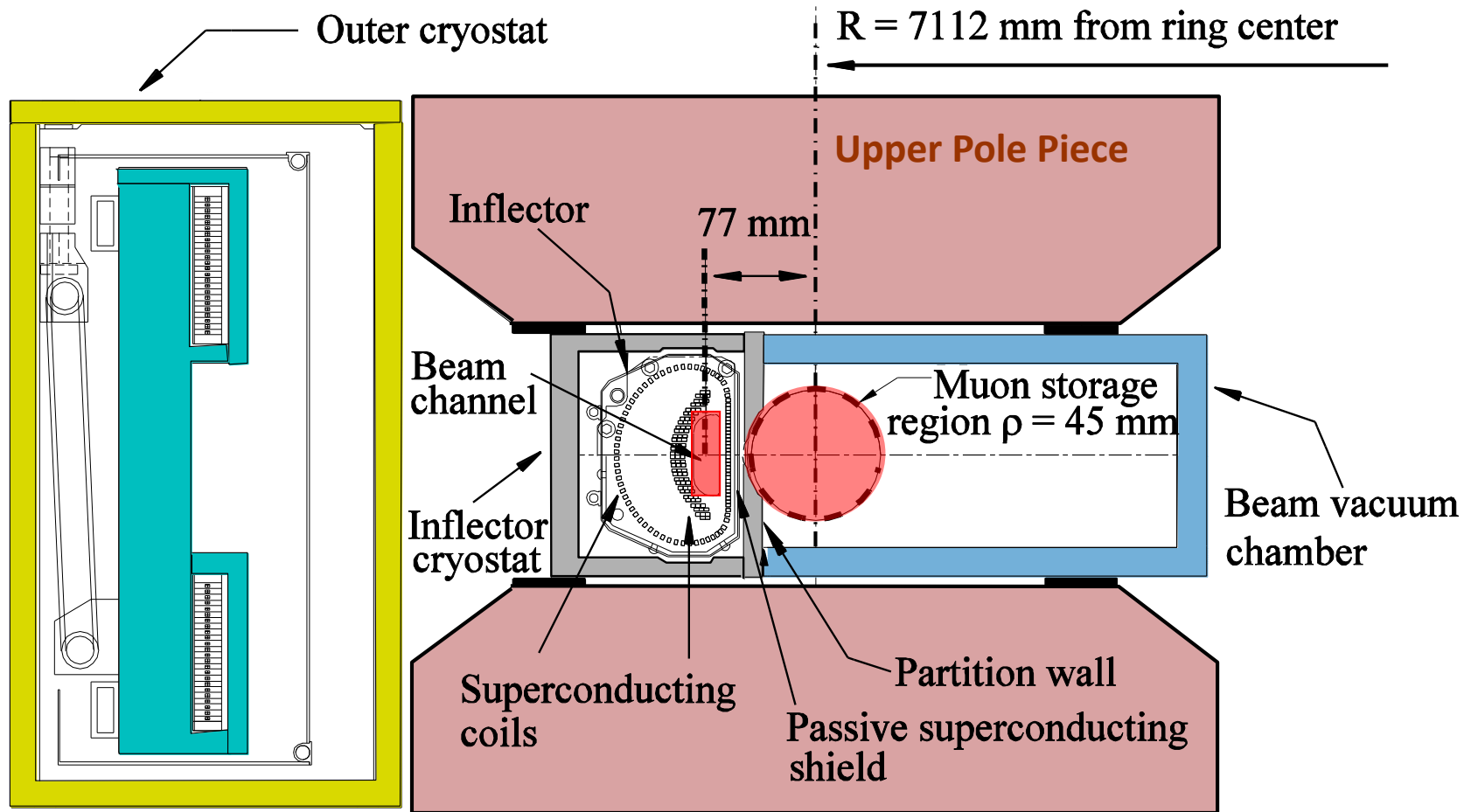
BNL E821: Muon injection and kicker



Need to kick the muons onto stable orbit

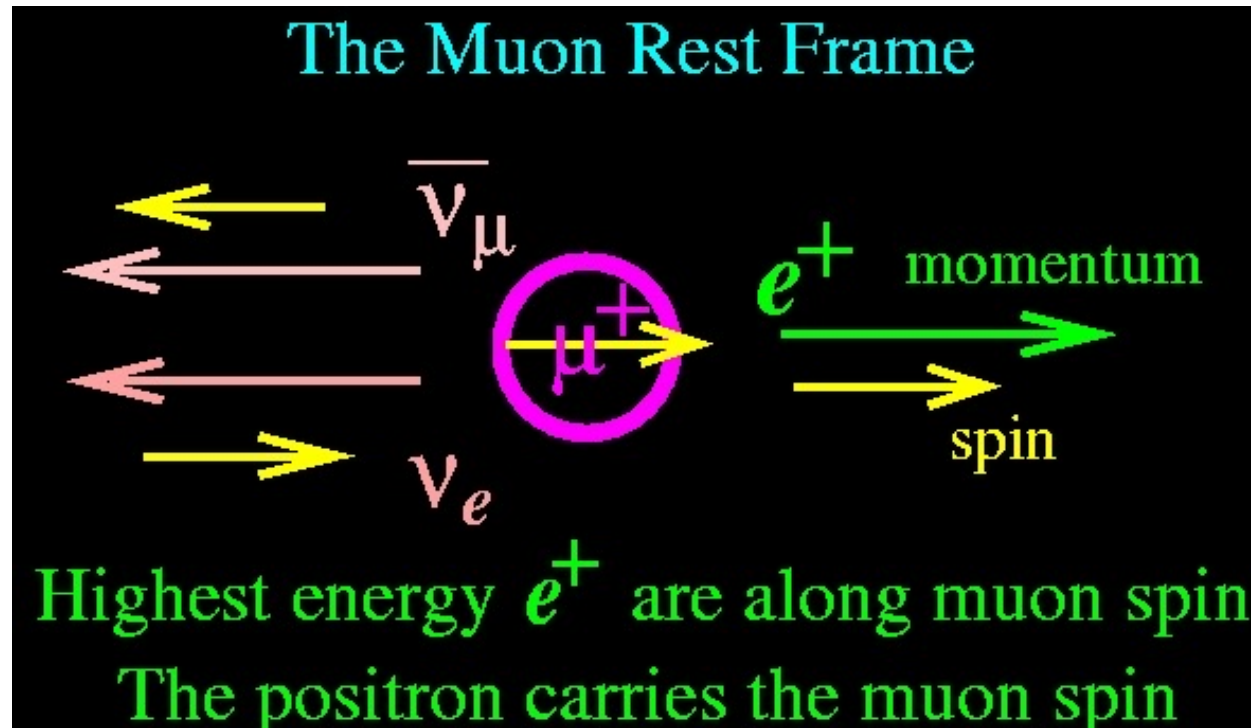


Space limitations prevent matching the inflector exit to the storage aperture



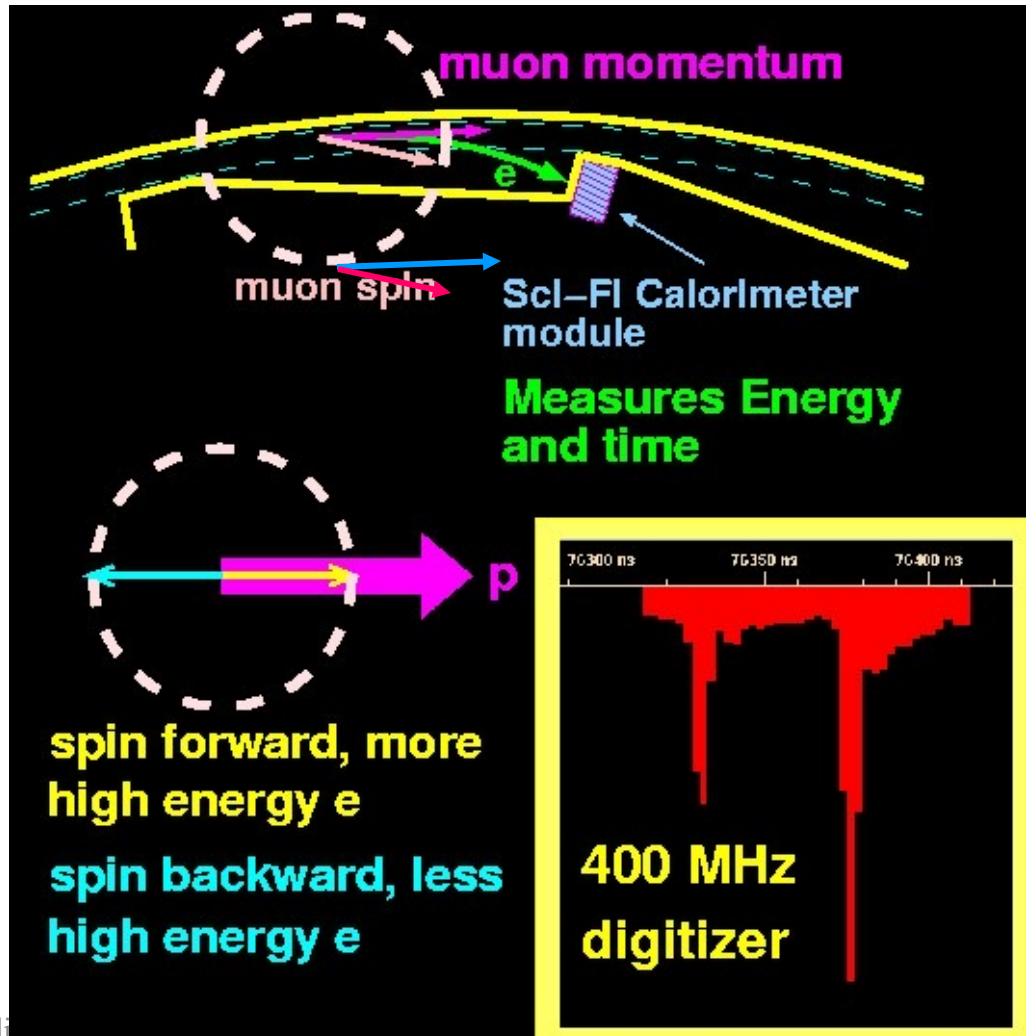
Muon decay

- Decay is self analyzing



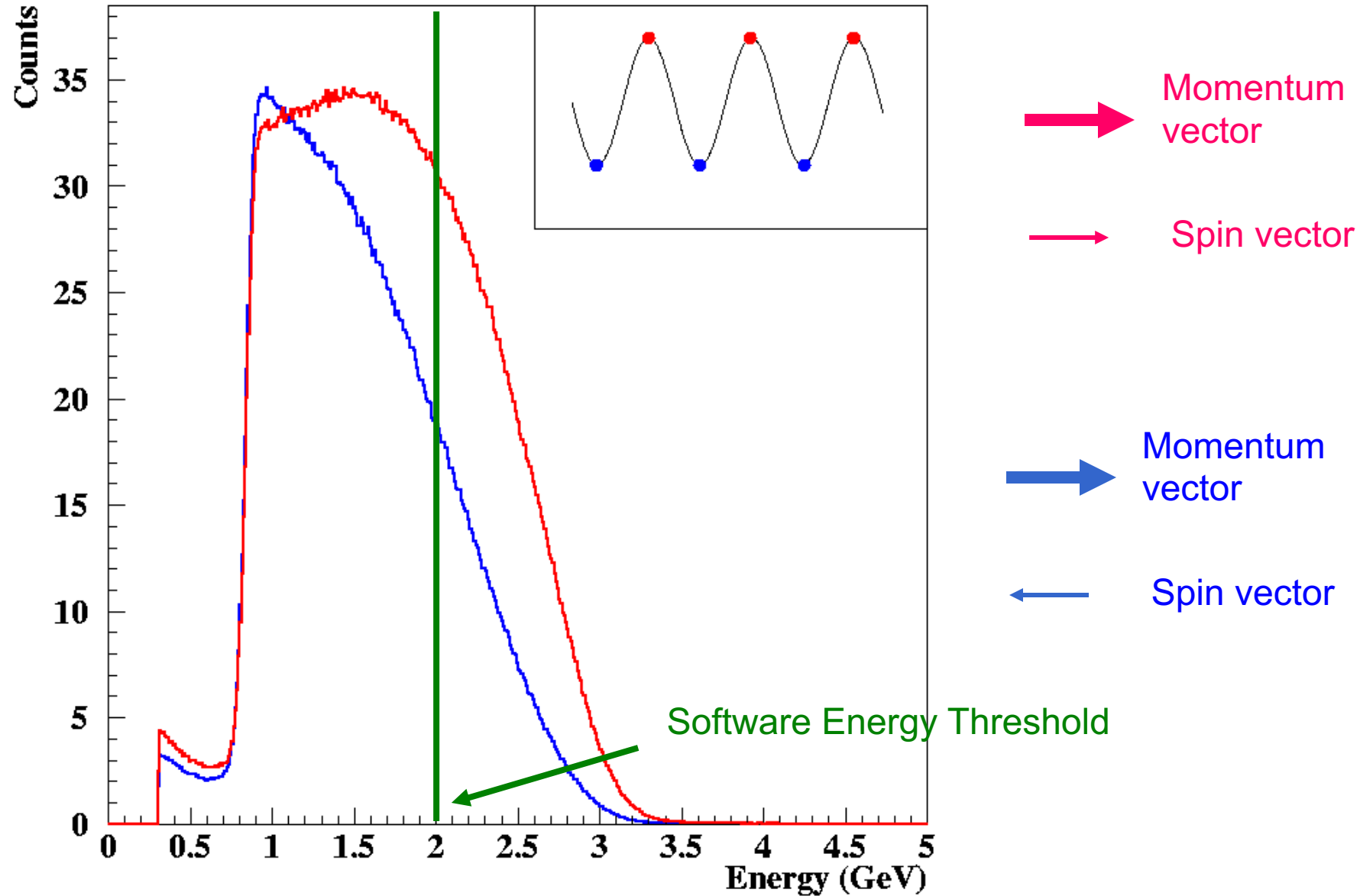
- The highest energy e^\pm from μ^\pm decay carry information on the muon spin direction.

Detectors and vacuum chamber



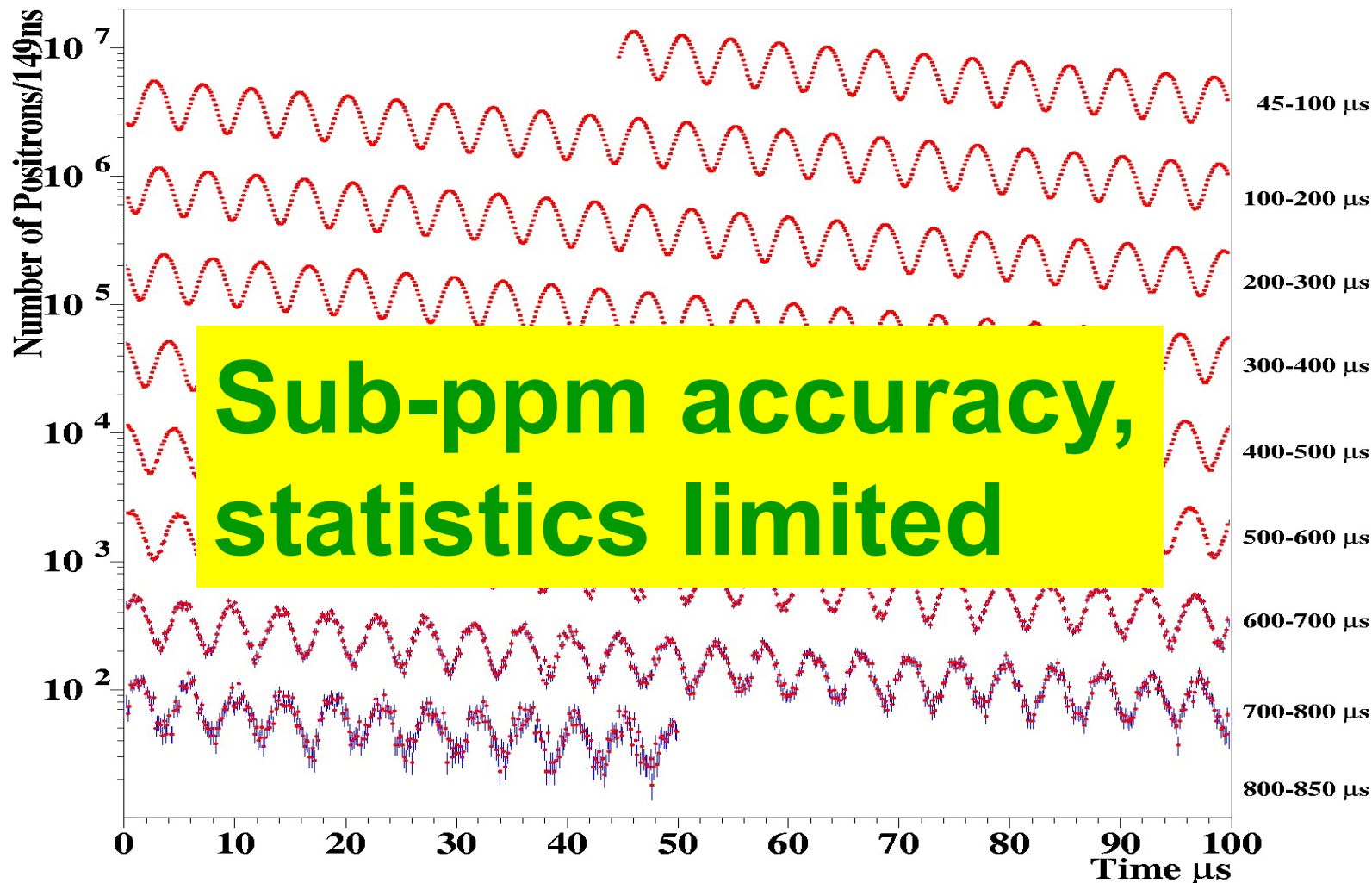
Lee B. Roberts

Energy Spectrum of Detected Positrons depends on spin direction

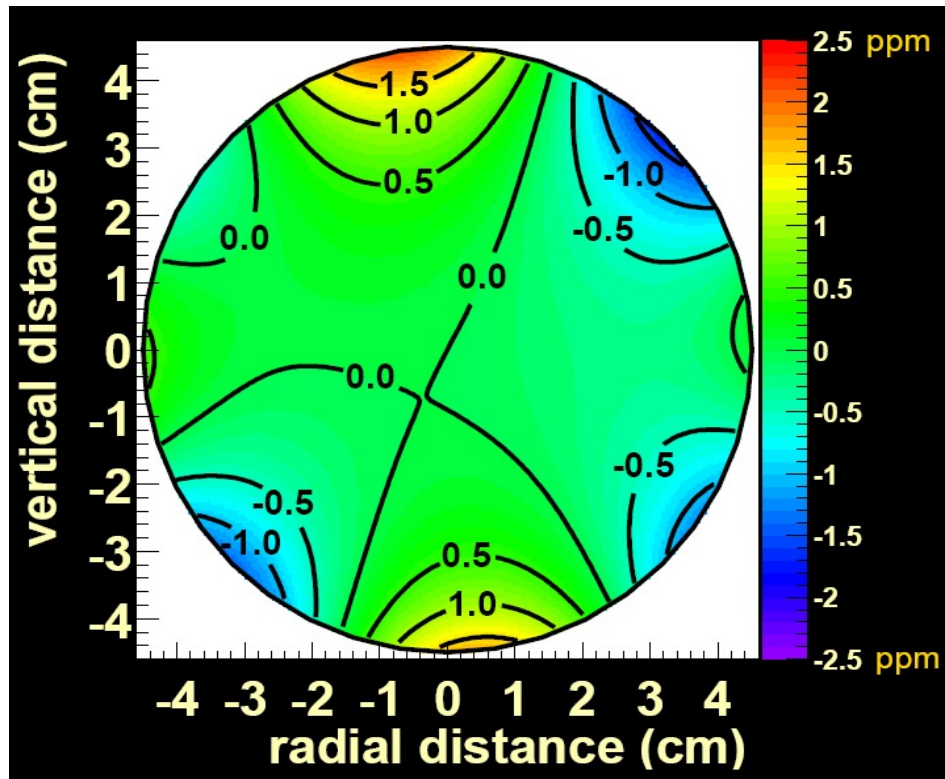


Muon g-2: 4 Billion e^+ with $E > 2\text{GeV}$

$$dN / dt = N_0 e^{-\frac{t}{\tau}} \left[1 + A \cos(\omega_a t + \phi_a) \right]$$



The ± 1 ppm uniformity in the average field is obtained with special shimming tools.

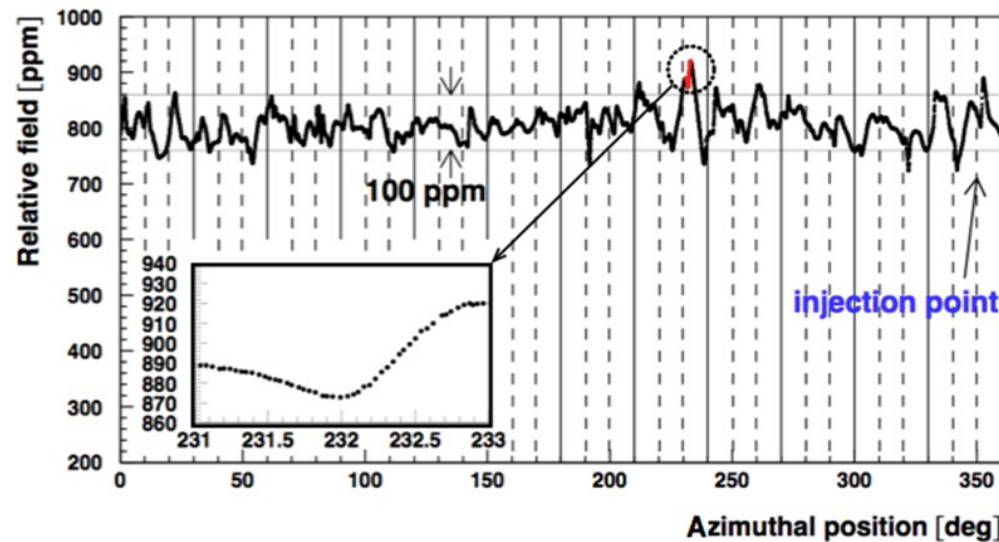


$\langle B \rangle_{\text{azimuth}}$

$$\sigma_{\text{sys}} \text{ on } \langle B \rangle_{\mu\text{-dist}} = \pm 0.03 \text{ ppm}$$

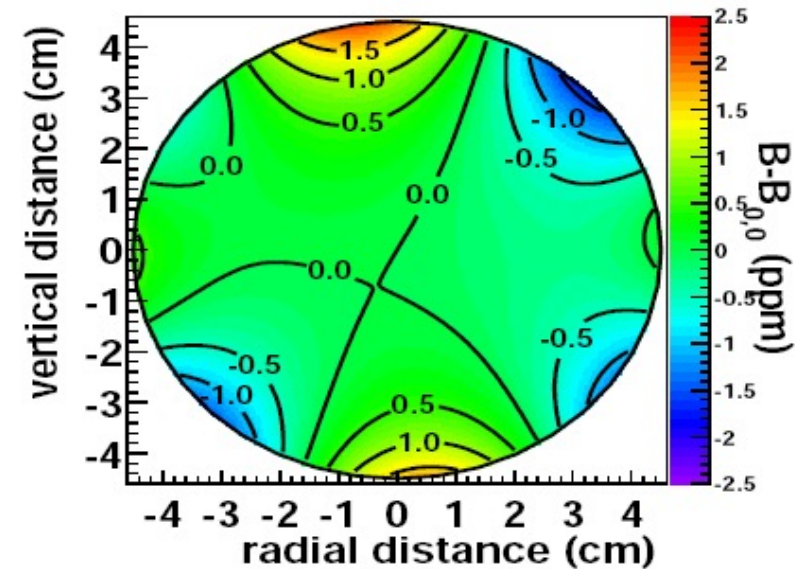
Goal for the shimming of the storage ring magnet

Field vs. Azimuth



Goal: ± 25 ppm

Azimuthally Averaged
Field vs r,z



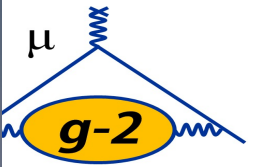
Bennett et al. 10.1103/PhysRevD.73.072003

Goal: ± 0.5 ppm

• Muon g-2: Precision physics in a Storage Ring

• Statistics limited... to improve sensitivity by a factor of 4 at Fermilab





Courtesy of Themis Bowcock/Liverpool

The pitch effect

Volume 42B, number 1

PHYSICS LETTERS

13 November 1972

PITCH CORRECTION IN $(g-2)$ EXPERIMENTS

F.J.N. FARLEY[‡]

CERN, Geneva, Switzerland

Received 14 September 1972

The pitch correction to the $(g-2)$ precession frequency of the electron or muon, arising from the axial oscillations of the particle rotating in an almost uniform magnetic field, is computed in a new way and extended to the case of axial focusing by electric fields. The main results are confirmed by a direct physical argument.

$$\omega_z = \omega_0 \left\{ 1 - \frac{\gamma-1}{\gamma} \psi^2 \right\} \quad (4)$$

$$\omega_x = -\omega_0 \frac{\gamma-1}{\gamma} \psi, \quad (5)$$

For the axial focusing forces we remark that when the momentum vector is deflected through angle ψ in the xz -plane the spin will rotate about the y -axis through angle $f\psi$ where [6]

$$\text{and } \left. \begin{aligned} f &= (1 + \gamma a) && \text{for magnetic focusing,} \\ f &= (1 + \beta^2 \gamma a - \gamma^{-1}) && \text{for electric focusing} \end{aligned} \right\}. \quad (6)$$

Precision spin/beam dynamics simulations

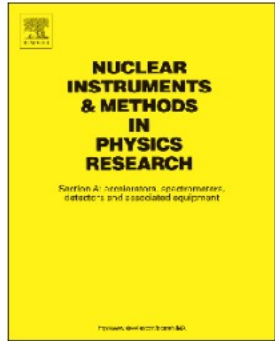
Nuclear Instruments and Methods in Physics Research A 797 (2015) 311–318



Contents lists available at [ScienceDirect](#)

Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima



Analytical benchmarks for precision particle tracking in electric and magnetic rings

E.M. Metodiev^{a,c,d,f}, I.M. D'Silva^a, M. Fandaros^a, M. Gaisser^{d,f}, S. Hacıömeroğlu^{a,d,e,f},
D. Huang^a, K.L. Huang^{a,c}, A. Patil^a, R. Prodromou^a, O.A. Semertzidis^a, D. Sharma^a,
A.N. Stamatakis^a, Y.F. Orlov^b, Y.K. Semertzidis^{a,d,f,*}



Precision beam/spin dynamics simulations often good to sub-part-per-billion

Precision tracking based on Predictor-Corrector/Runge-Kutta integration; Benchmarking

Most common tracking programs may ignore second order effects!

Predictor-Corrector/Runge-Kutta integration: very precise with 1-10ps time step.

Benchmarked our method to sub ppb

Estimate pitch correction; dp/p effect, E-field correction; distortion of closed orbit,...

High precision spin and beam dynamics simulation

E.M. Metodiev et al. / Nuclear Instruments and Methods in Physics Research A 797 (2015) 311–318

For this work we have used the differential equations from J. D. Jackson's equations [3] shown here in rationalized MKS units for both the particle motion and the spin precession:

$$\frac{d\vec{\beta}}{dt} = \frac{q}{\gamma m} \left[\frac{\vec{E}}{c} + \vec{\beta} \times \vec{B} - \frac{\vec{\beta}(\vec{\beta} \cdot \vec{E})}{c} \right] \quad (1.1)$$

$$\frac{d\vec{s}}{dt} = \frac{q}{m} \vec{s} \times \left[\left(\frac{g}{2} - 1 + \frac{1}{\gamma} \right) \vec{B} - \left(\frac{g}{2} - 1 \right) \frac{\gamma}{\gamma + 1} (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left(\frac{g}{2} - \frac{\gamma}{\gamma + 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right] \quad (1.2)$$

ω_a can be obtained by taking the scalar product of \vec{s} and $\vec{\beta}$.

Pitch effect: Uniform B-field plus E-field focusing

1. Farley's formula can be used directly
2. Vertical oscillations have a small effect we need to correct for
3. 0.5mrad $\rightarrow C = 0.25 \times \langle \theta_0^2 \rangle = 62.5$ ppb

$$\omega_a = a \frac{e}{m} B$$

Measured g-2 value

$$\omega_m = \omega_a (1 - C)$$

Correct g-2 value

Pitch effect: Uniform B-field plus E-field focusing

E.M. Metodiev et al. / Nuclear Instruments and Methods in Physics Research A 797 (2015) 311–318

315

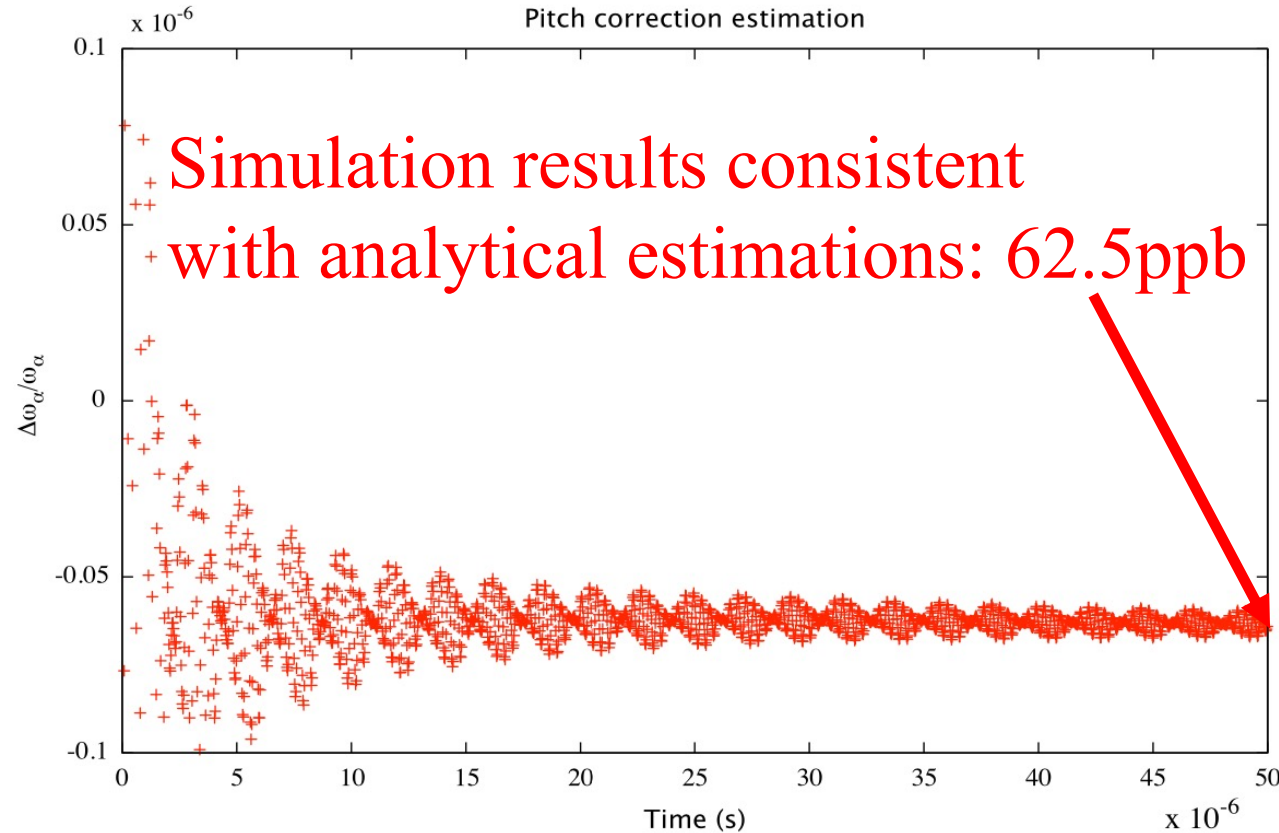


Fig. 6. The relative difference between the ideal ($g-2$) and the actual precession of a particle with a maximum pitch angle of $\theta_0 = 0.5$ mrad, estimated from tracking using electric focusing in a magnetic ring. To obtain the correct $g-2$ frequency, the correction $0.25 \times \theta_0^2$ needs to be added to the observed frequency. Again, the tracking results are consistent with the predictions to sub-ppb level.

Benchmarking, rigorous checking

Pitch correction, Electric field correction (folding in momentum dispersion):

1. Vertical oscillations in a uniform B-field, electric focusing
2. For off magic momentum muons we need electric field correction

E. Metodiev et al., NIMA **797**, 311-318 (2015)

Our high-precision spin and beam dynamics software estimates those to better than 1 ppb. Several independent programs are currently used.

Ideal experiment characteristics

- The ideal muon $g-2$ experiment has the beam-line and storage ring phase-space matched.
- All the stored muons have the same lifetime.
- All muons have same spin direction!
- No muon losses!

Muon g-2 experiment in practice

- Pitch effect: Particle velocity component along the B-field
- Pileup events due to high rates
- Coherent betatron oscillations (due to lack of phase-space matching between beamline and storage ring with muon injection)

Pion vs. muon injections

Injection method	Pion	Muon
Light flash on dets (neutrons, etc)	Large! Some dets did not gate on but after 140micro- sec!	Down by $\alpha \sim 1/137$, when eliminate pions
Statistics	Limited	>10 improvement
Phase-space	Uniform	Large CBO*
Kicker needed?	No, kinematics assisted	Pulsed magnet needed (300Gauss, 100ns)

* CBO: Coherent betatron oscillations

Recasting a_μ in fundamental constants

$$\omega_a = (e/m_\mu) \mathbf{a}_\mu \mathbf{B}$$

$$\mu_e = g_e e \hbar / 4 m_e \quad \downarrow \quad B = \frac{\hbar \omega_p}{2 \mu_p}$$

$$a_\mu = \frac{\omega_a}{\omega_p} \frac{\mu_p}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

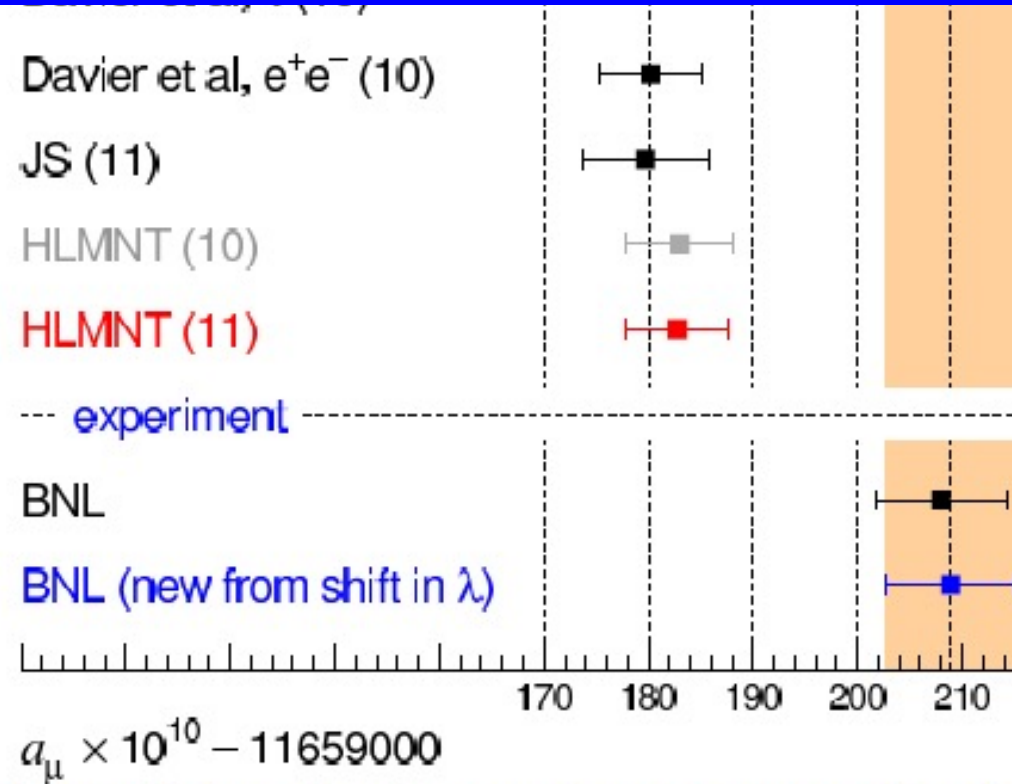
$\Delta \mu_e / \mu_p \sim 8 \text{ ppb}$

$\Delta m_\mu / m_e \sim 25 \text{ ppb}$

$\Delta g_e / 2 \sim 0.3 \text{ ppt}$

Comparison of Theory/Experiment

The result is 3.5 s.d. away from theory! What is it?



Yannis Semertzidis

Figure 1: Standard model predictions of a_μ by several groups compared to the measurement from BNL

Systematic errors for the muon g-2 exp. at BNL and at FNAL (projections)

Category	E821 [ppb]	E989 Improvement Plans	Goal [ppb]
Gain changes	120	Better laser calibration low-energy threshold	20
Pileup	80	Low-energy samples recorded calorimeter segmentation	40
Lost muons	90	Better collimation in ring	20
CBO	70	Higher n value (frequency)	
		Better match of beamline to ring	< 30
E and pitch	50	Improved tracker	
		Precise storage ring simulations	30
Total	180	Quadrature sum	70

The muon ring moved to Fermilab (22 June – 25 July 2013)





The muon g-2 coil moved in 2013 to Fermilab for more intense beam

Contacts: C. Polly – Project Manager (polly@fnal.gov)
K.W. Merritt – Deputy Project Manager (wyatt@fnal.gov)
D. Hertzog – Co-Spokesperson (hertzog@uw.edu)
B. L. Roberts – Co-Spokesperson (roberts@bu.edu)

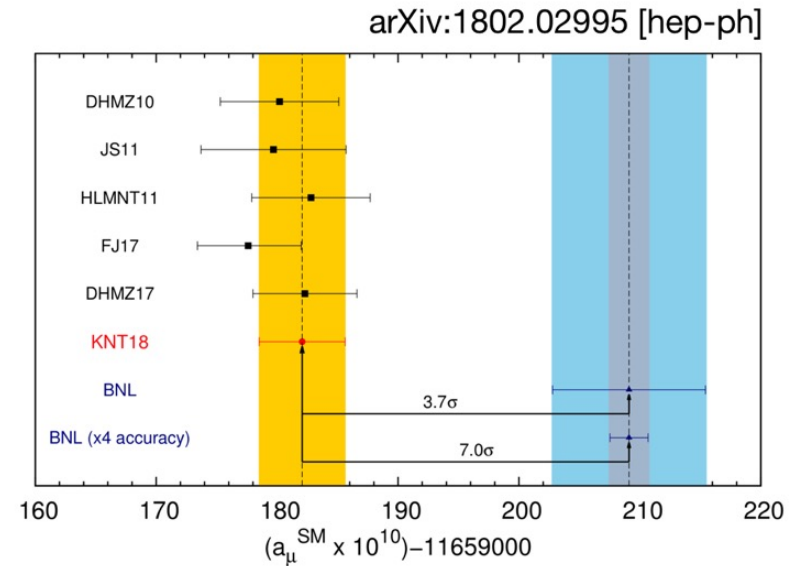
The muon ring arrived at Fermilab





Current status of a_μ in Standard Model

	Value ($\times 10^{-11}$)
QED	$116\,584\,718.951 \pm 0.009 \pm 0.019 \pm 0.007 \pm 0.077$
HVP (lo)	6949 ± 34
HVP (ho)	-98.4 ± 0.7
HLBL	105 ± 26
EQ	154 ± 1
Total SM	$116\,591\,818 \pm 43$



$$a_\mu^{\text{Expt.}} - a_\mu^{\text{SM}} = (271 \pm 73) \times 10^{-11} \quad (3.7 \sigma)$$

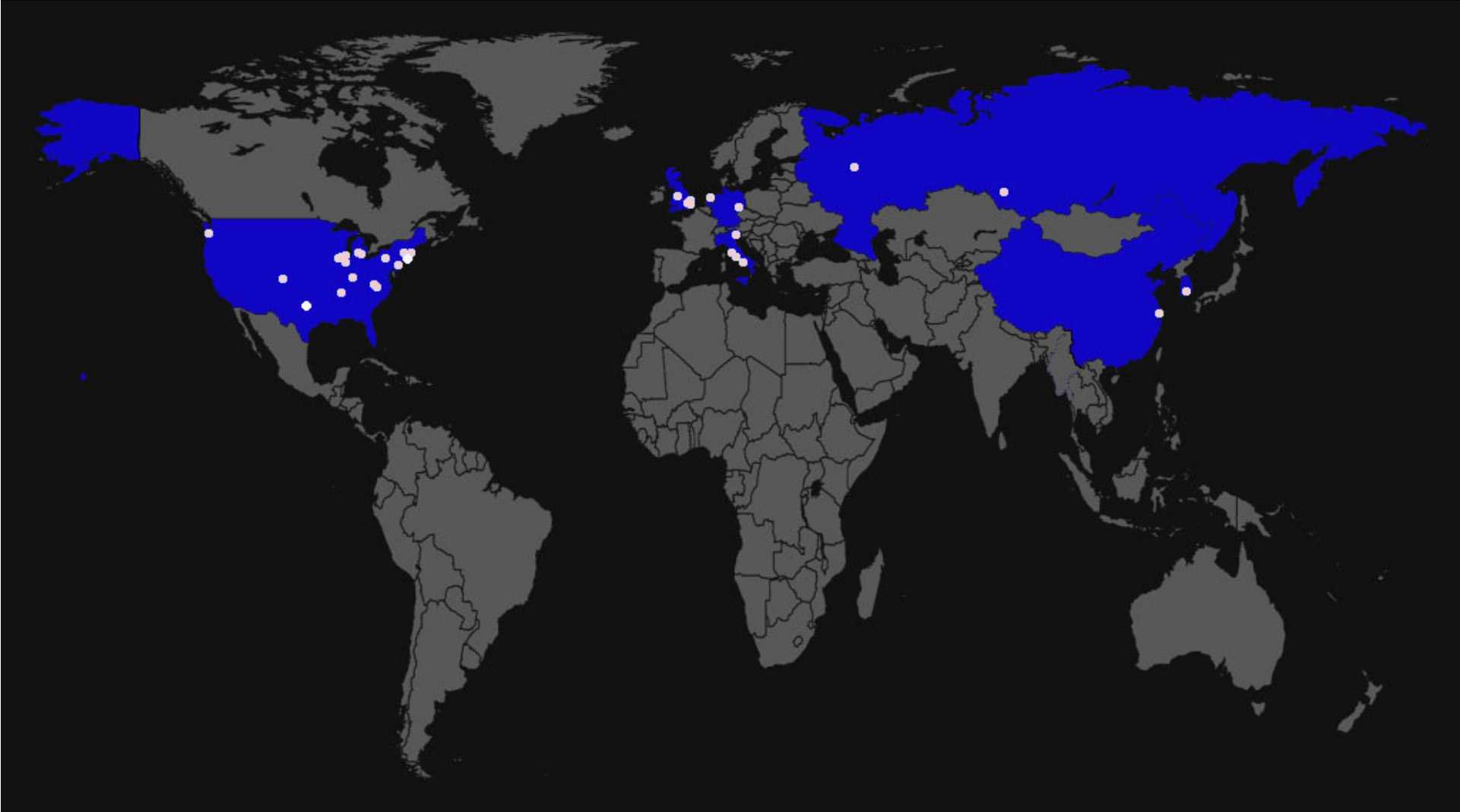
- New E989 experiment will reduce experimental uncertainty by a factor of 4 to 16×10^{-11} (140 ppb)
- If current discrepancy remains this would yield $>7\sigma$
- Together with theory improvements could give $>9\sigma$

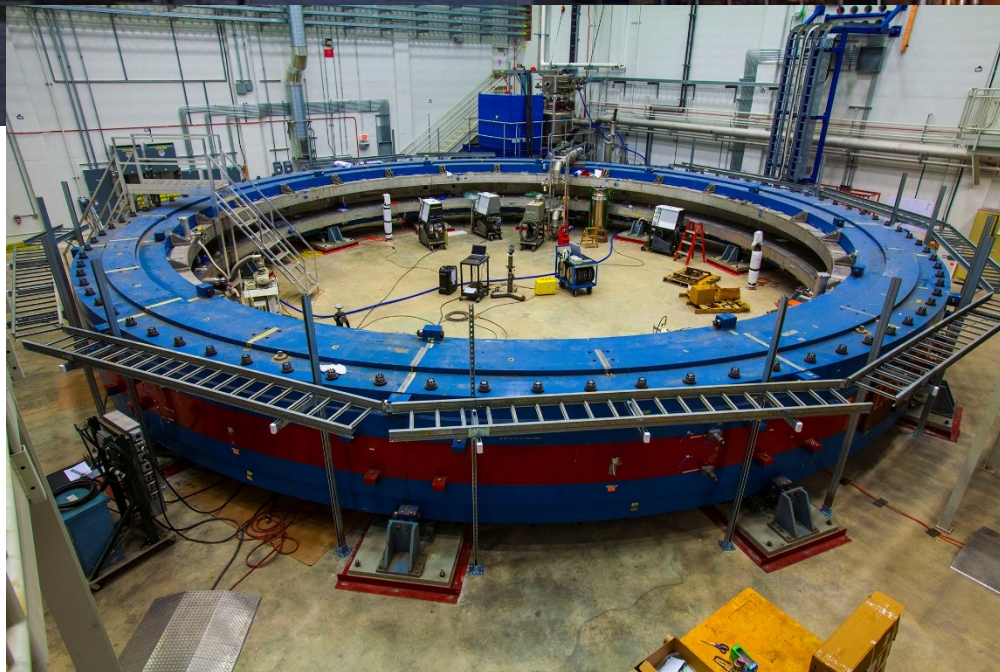
The muon ring at Fermilab, goal 140ppb, 20x the statistics

- ✓ More injections per unit time, run longer. Statistics is being collected.
- ✓ Longer beam-line to double the number of muons/proton. No pion or proton contamination, minimal impact on electronics (gain stability). Elaborate laser system to improve detector gain stability as a function of time.
- ✓ Three times better B-field uniformity around the ring. Independently calibrate the absolute B-field of the ring using ^3He magnetometry.
- ✓ Instrument the ring with effective tracker to monitor the position of the stored muon beam as a function of time.
- ✓ Instrument the ring for fast magnetic pickup (systematic error control)

E989 Muon g-2 Collaboration

8 Countries, 33 Institutions

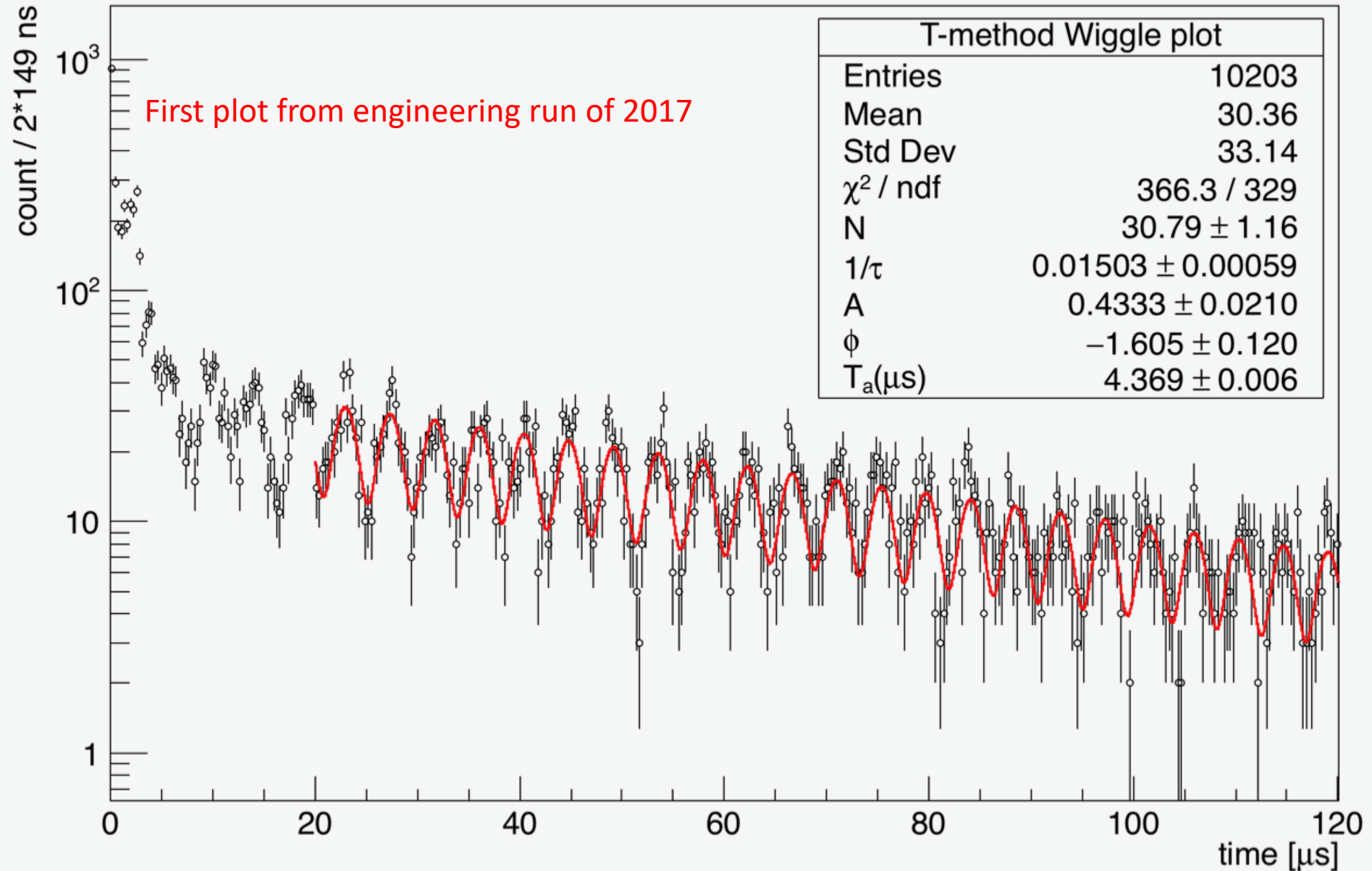




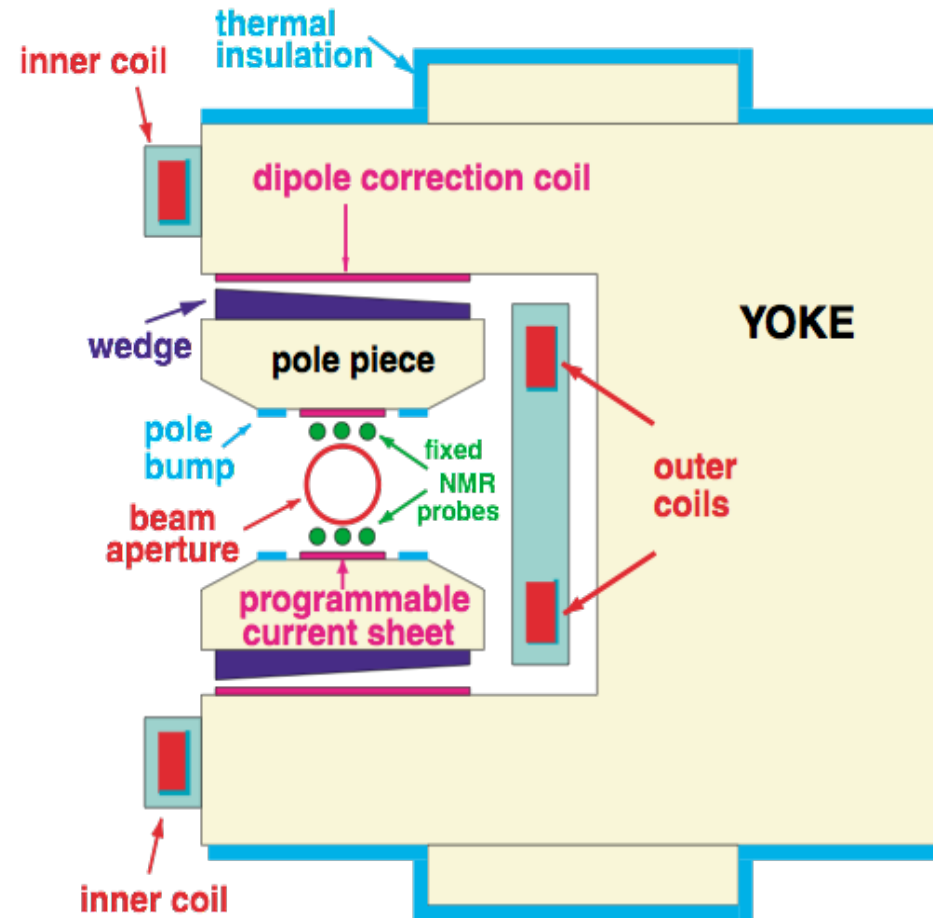


The ring has been reassembled and fully powered to 1.45T! First data taken: 2017

T-method Wiggle plot



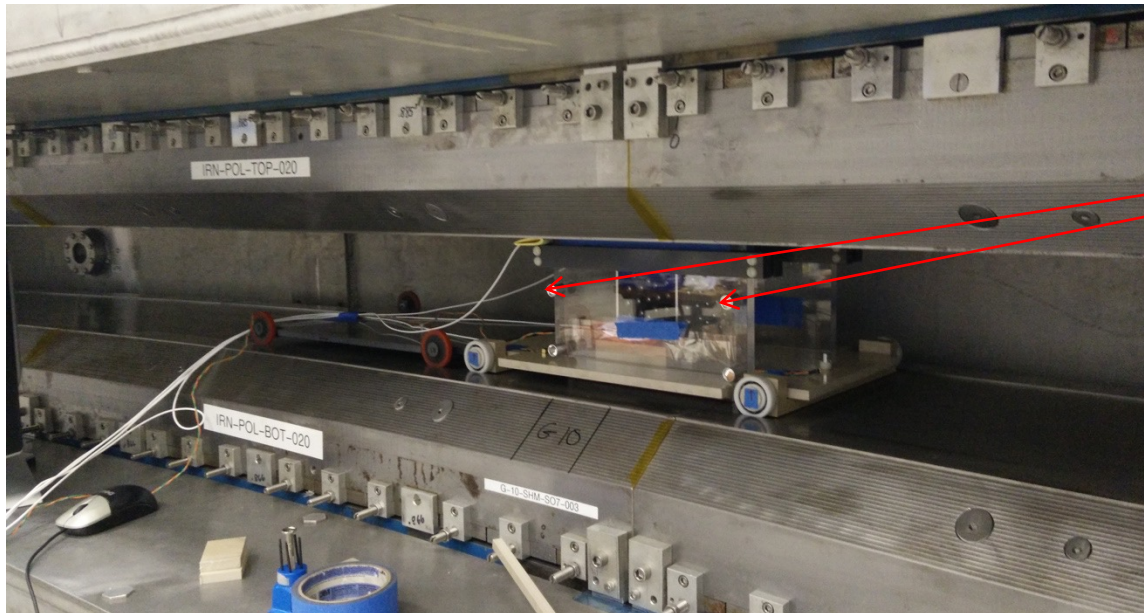
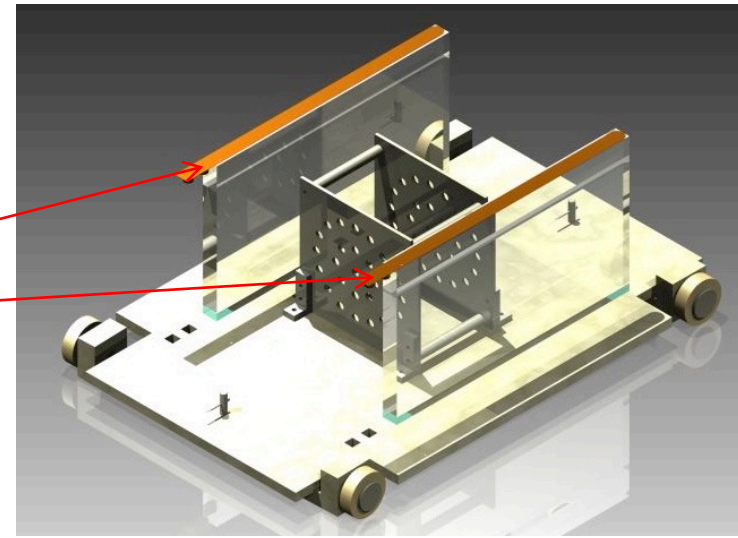
Cross section of the storage ring magnet



P. Winter

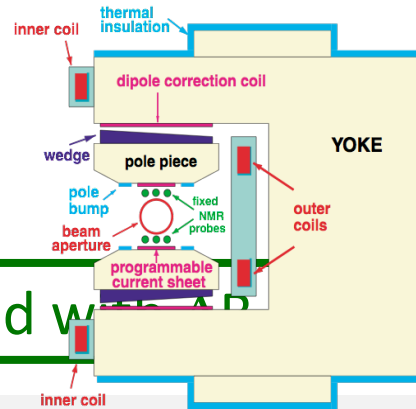
Shimming tool

- Multipurpose instrument
 - 25 **NMR Probes** for field
 - 4 **capacitive gap sensors**
 - Measure pole alignment
 - 70 nm resolution
 - Few micron reproducibility
- Laser tracked position with μm resolution

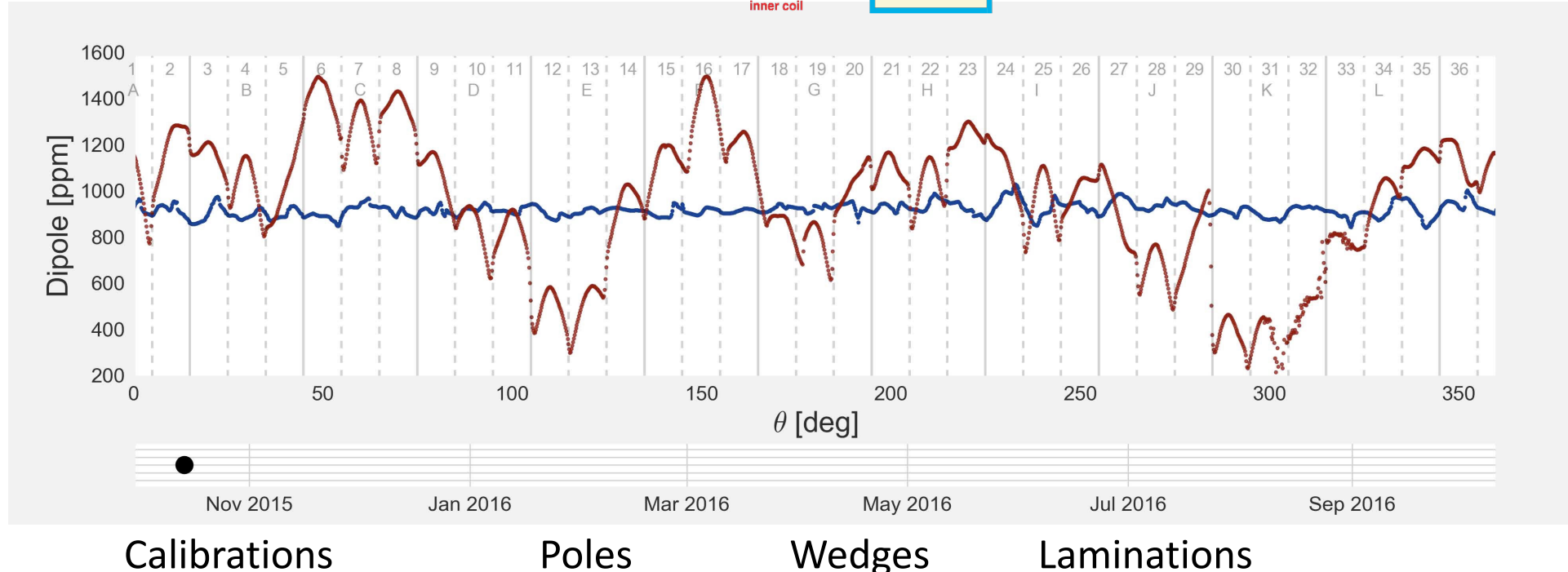


Shimming goal achieved as of September 2016:

- Many improvements on entire experiment to reach 140 ppb
- Precision alignment to reach dipole gradients of $\Delta B < \pm 25$ ppm
 - 72 poles
 - 840 wedge shims
 - 9000 thin iron foils



10 mT Shimming completed with 140 ppm ✓ minute...

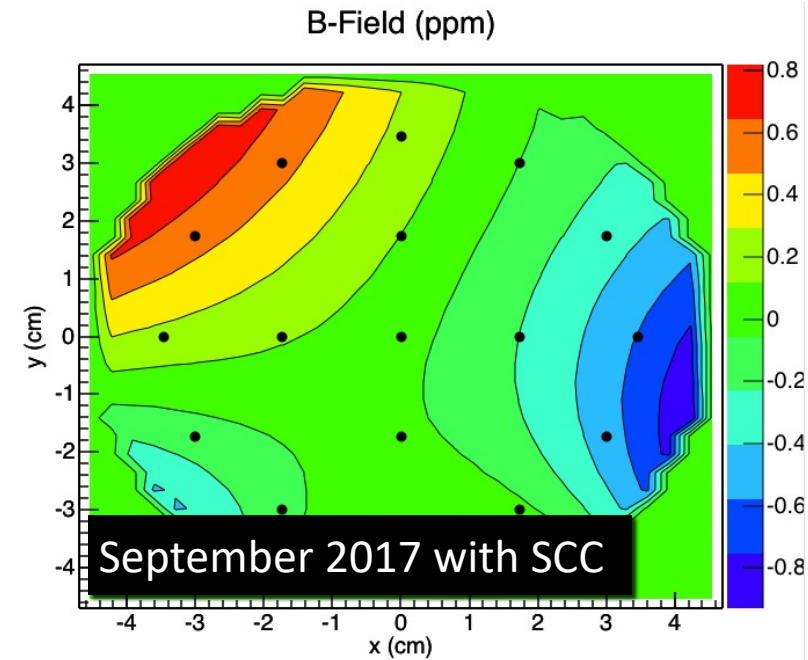


Surface coils and power supply feedback

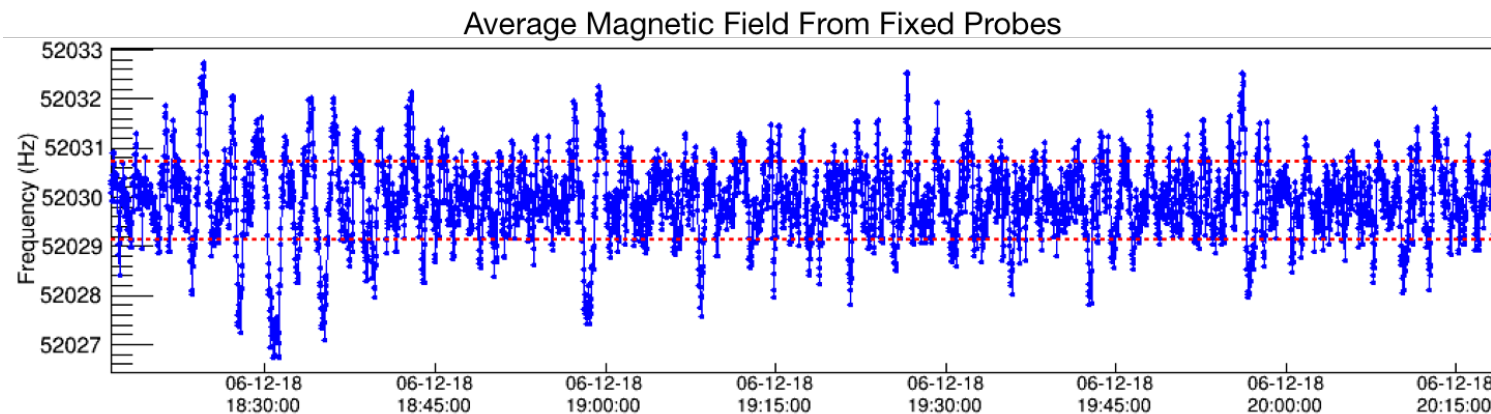
- 200 continuous current traces around the ring individual tunable between $\pm 2\text{A}$
- Used to cancel higher multipoles



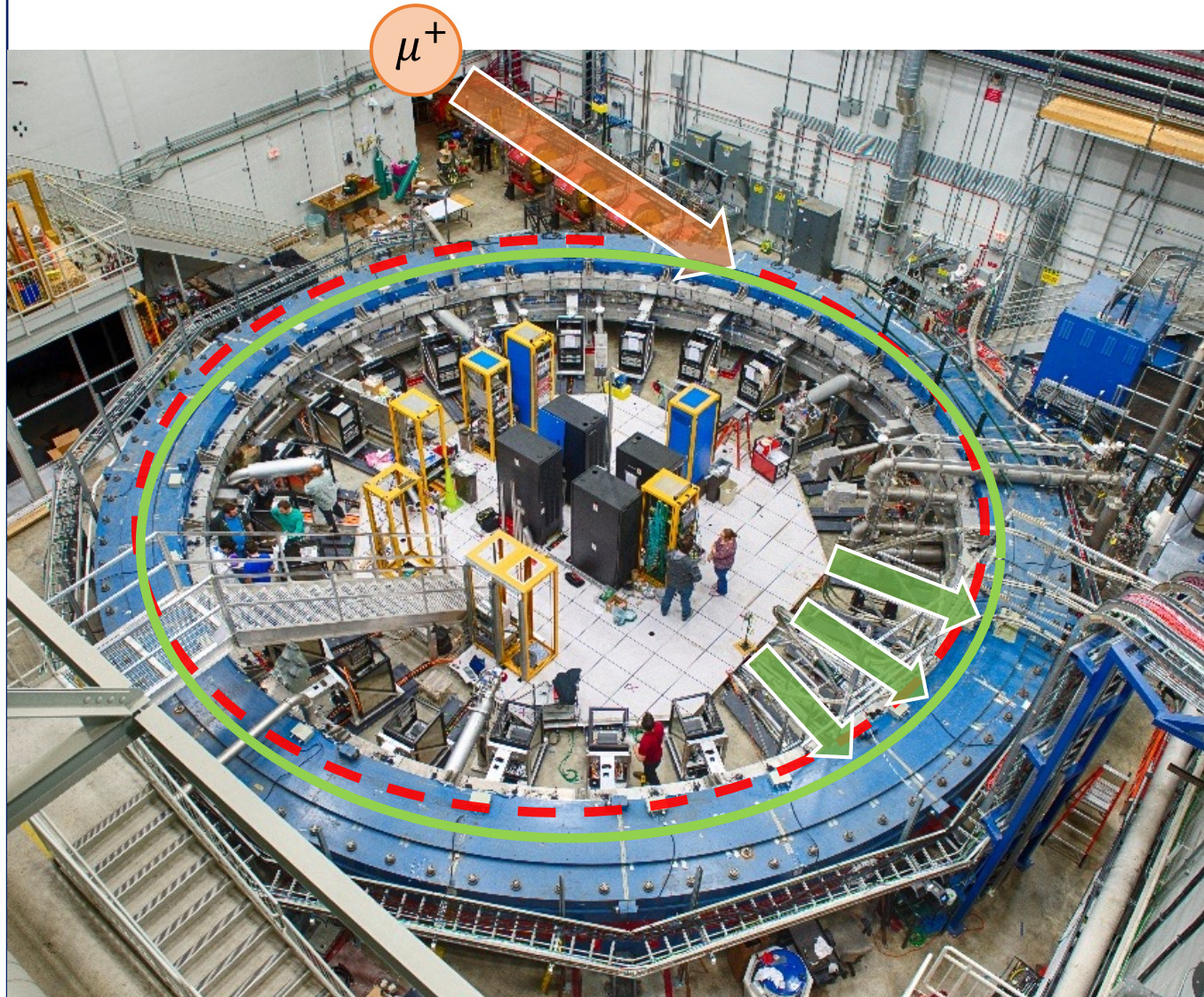
- Power supply feedback stabilizes main dipole field to $\pm 15\text{ ppb}$



	Normal	Skew
Quad	-0.46	0.22
Sext	-0.32	-0.29
Octu	-0.13	0.28
Decu	0.05	0.07
Dipole	-0.0	

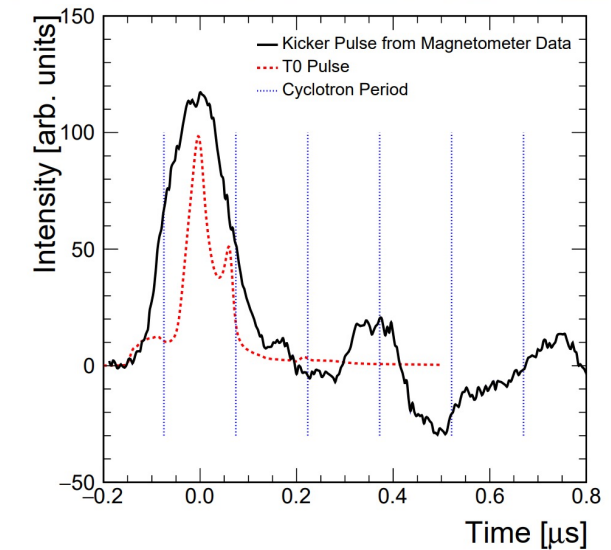
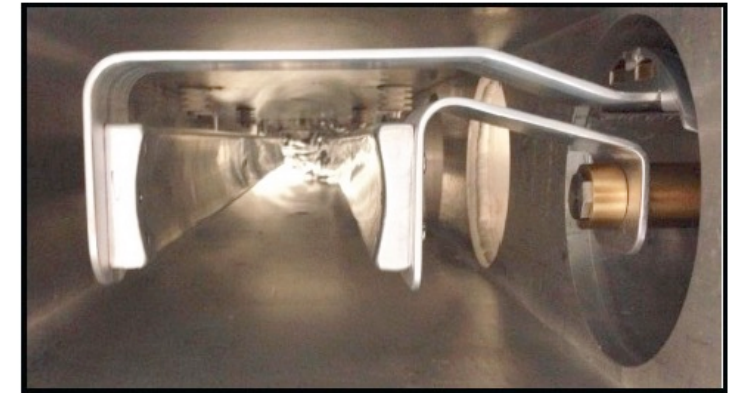


P. Winter

Overview of Muon $g - 2$ Experiment at Fermilab (E989)

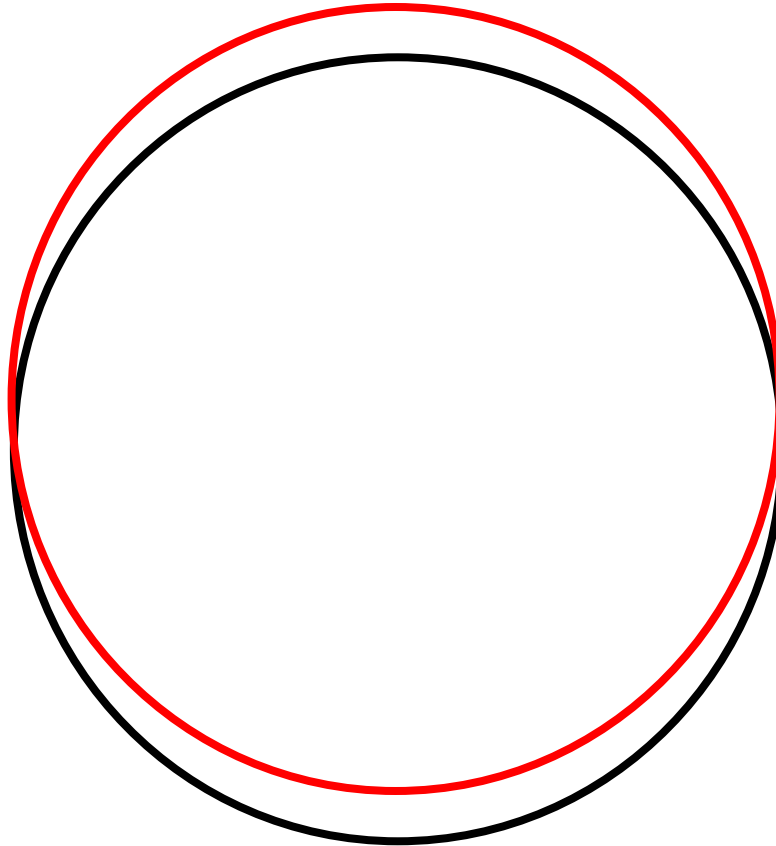
Kick

- Muons are kicked onto the design orbit by the fast non-ferric **kicker magnet** system.



Coherent betatron oscillations influence the g-2 phase

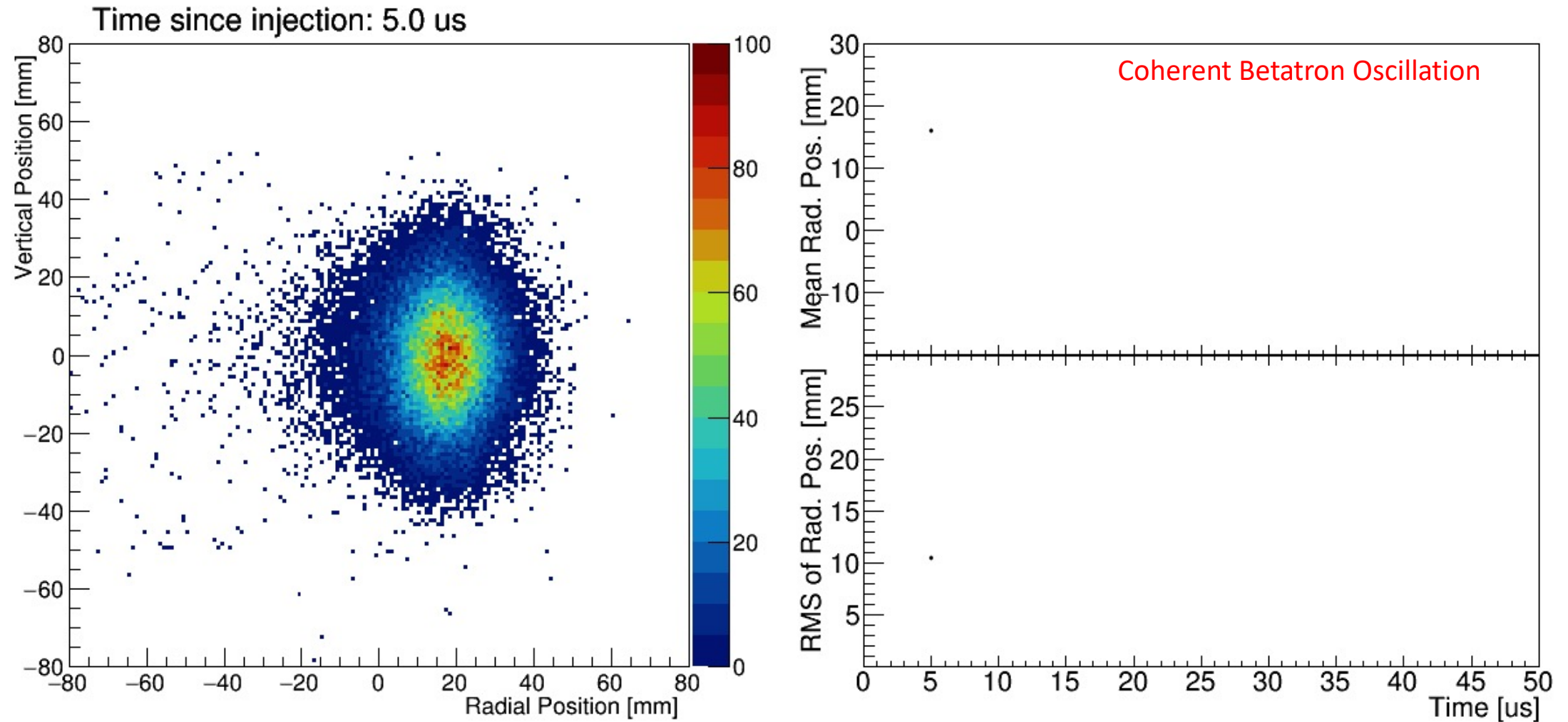
- CBO frequency $f_{cbo} = f_c (1 - \sqrt{1 - n})$. Radial oscillations, through aliasing, became a problem
- A very high-frequency, cascaded through various effects down to g-2 frequency



Straw trackers

► Straw trackers

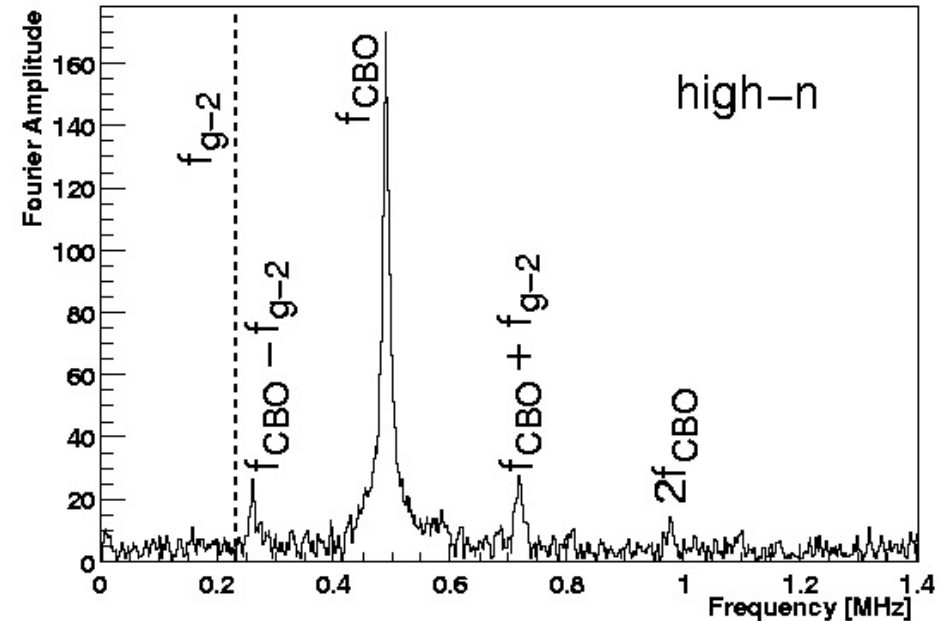
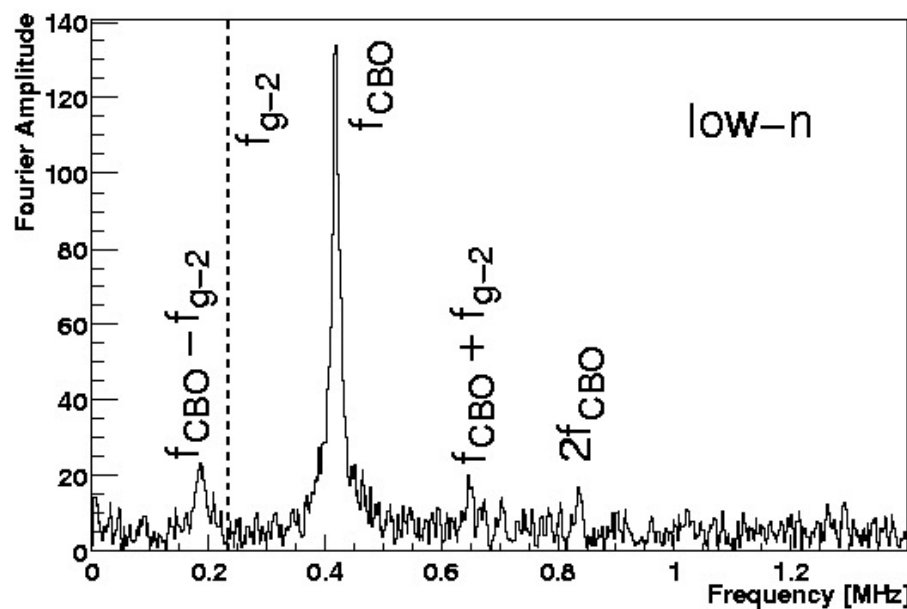
- Measures trajectories of the decay positrons and extrapolates to find the muon distribution.



CBO in the 2001 Data Set

$$f(t) = N_0 e^{-\lambda t} [1 + A \cos(\omega_a t + \phi)]$$

Residuals from fitting the 5-parameter function



CBO in the Data Set

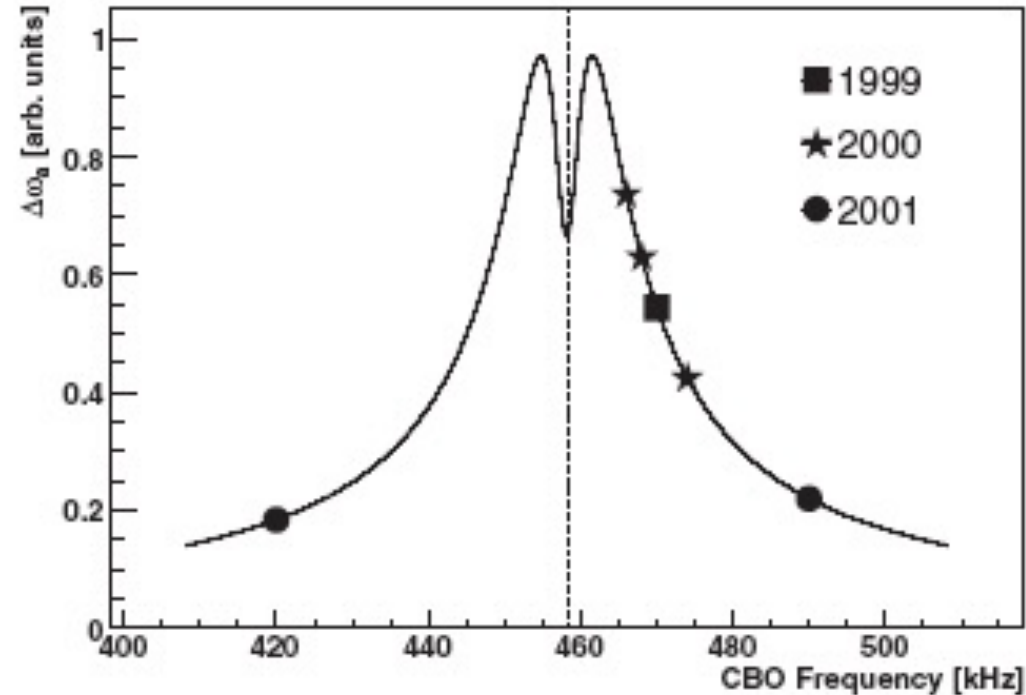


FIG. 36. The relative pull ($\Delta\omega$) versus the CBO modulation frequency *if not* addressed by the fitting function. A typical full vertical scale is several ppm; the actual scale depends on the specifics of the fit and the data set used. The R00 data were acquired under run conditions in which ω_a was very sensitive to CBO. This sensitivity was minimized in the R01 period where low- and high- n subperiods, each having CBO frequencies well below or above twice the $(g - 2)$ frequency, were employed.

Yannis K. Semertzidis, IBS-CAPP and KAIST

The effect depends on the CBO frequency

Coherent betatron oscillations

- Coherent betatron oscillations affect the acceptance (N_0), the decay asymmetry (A), and the g-2 phase (ϕ).

$$N(t) = \frac{N_0}{\gamma\tau_\mu} e^{-t/\gamma\tau_\mu} \cdot \Lambda(t) \cdot V(t) \cdot B(t) \cdot C(t) \cdot [1 - A(t) \cos(\omega_a t + \phi(t))]$$

$$\Lambda(t) = 1 - A_{\text{loss}} \int_0^t L(t') e^{-t'/\gamma\tau_\mu} dt'$$

$$V(t) = 1 - e^{-t/\tau_{\text{vw}}} A_{\text{vw}} \cos(\omega_{\text{vw}} t + \phi_{\text{vw}})$$

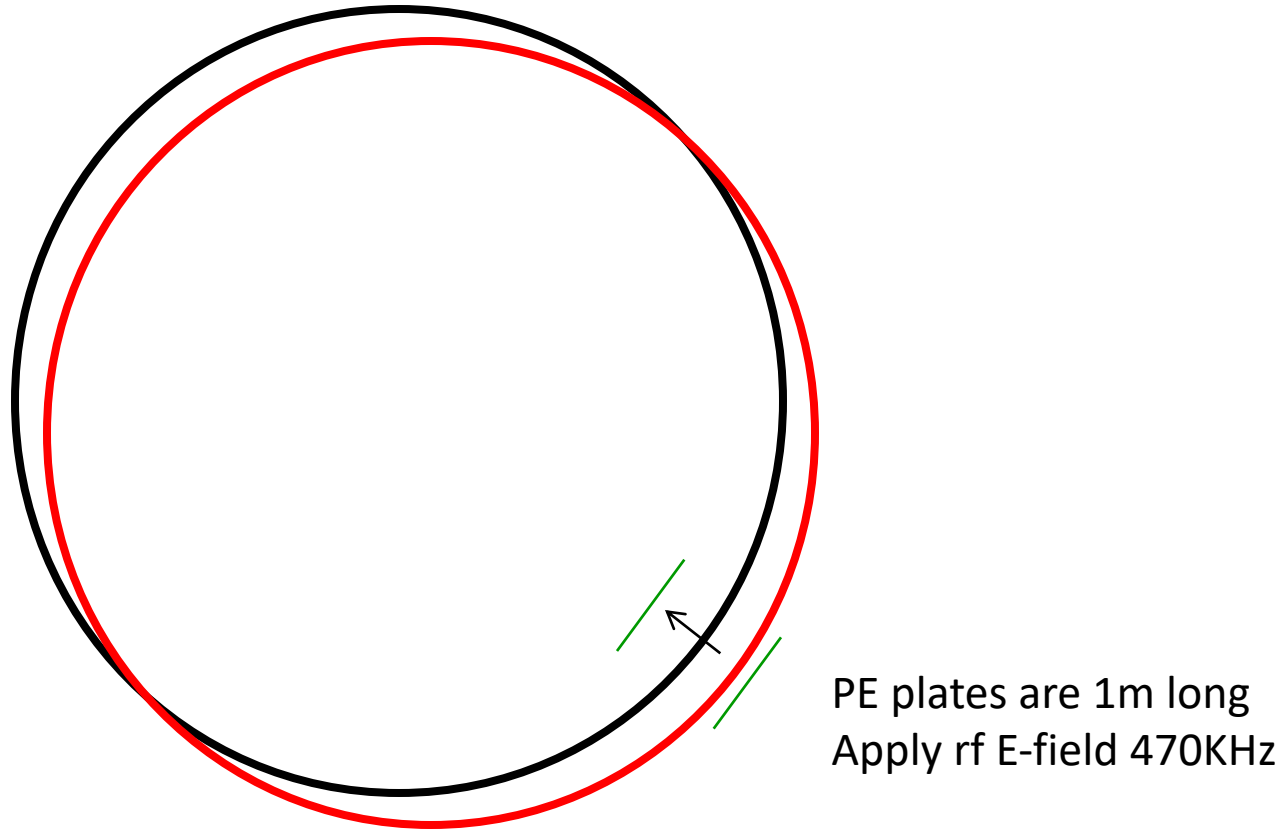
$$C(t) = 1 - e^{-t/\tau_{\text{cbo}}} A_1 \cos(\omega_{\text{cbo}} t + \phi_1)$$

$$A(t) = A(1 - e^{-t/\tau_{\text{cbo}}} A_2 \cos(\omega_{\text{cbo}} t + \phi_2))$$

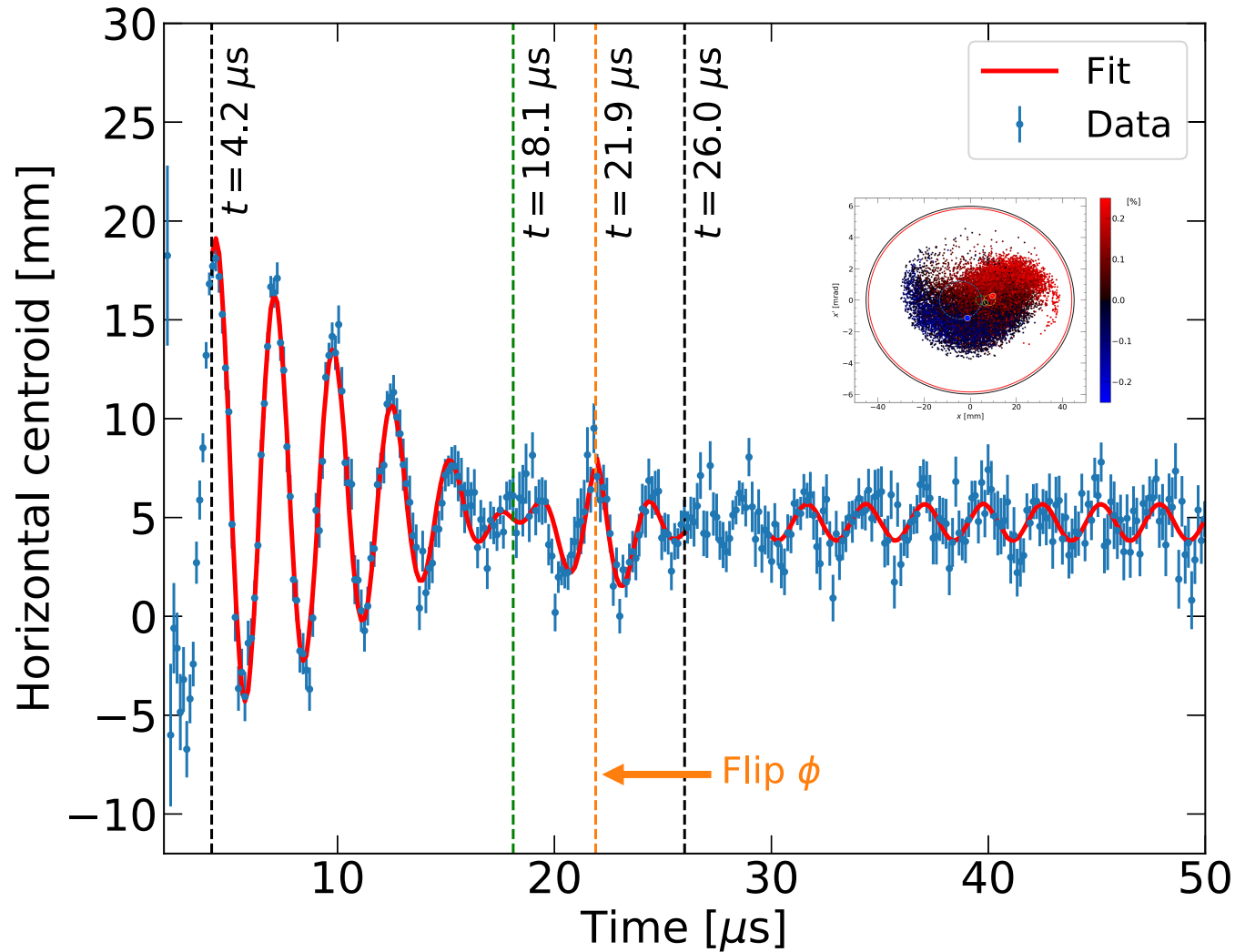
$$\phi(t) = \phi_0 + e^{-t/\tau_{\text{cbo}}} A_3 \cos(\omega_{\text{cbo}} t + \phi_3).$$

$$B(t) = 1 - A_{br} e^{-t/\tau_{br}}$$

Yuri Orlov suggested to fix it by using a pair of plates (PE) as mini-kicker: We tried his method at Fermilab; it worked.

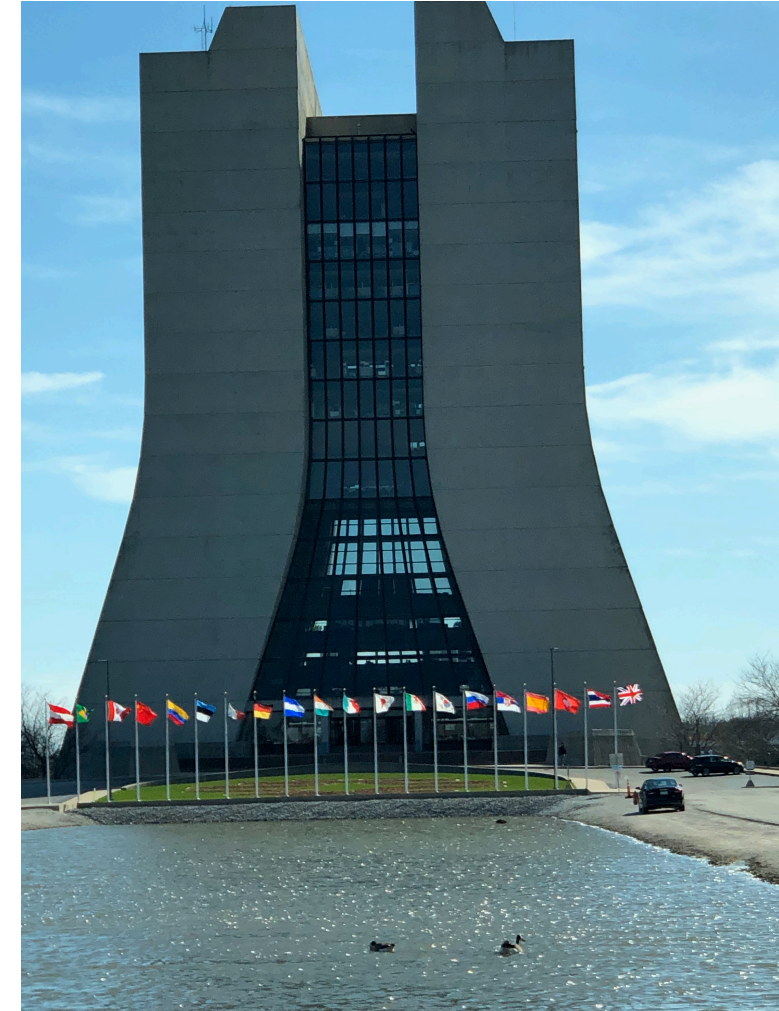


RF CBO amplitude reduction (data from muon g-2 experiment)



On Kim *et al*, New J. Phys. **22** (2020) 063002

Yannis K. Semertzidis, IBS-CAPP and KAIST



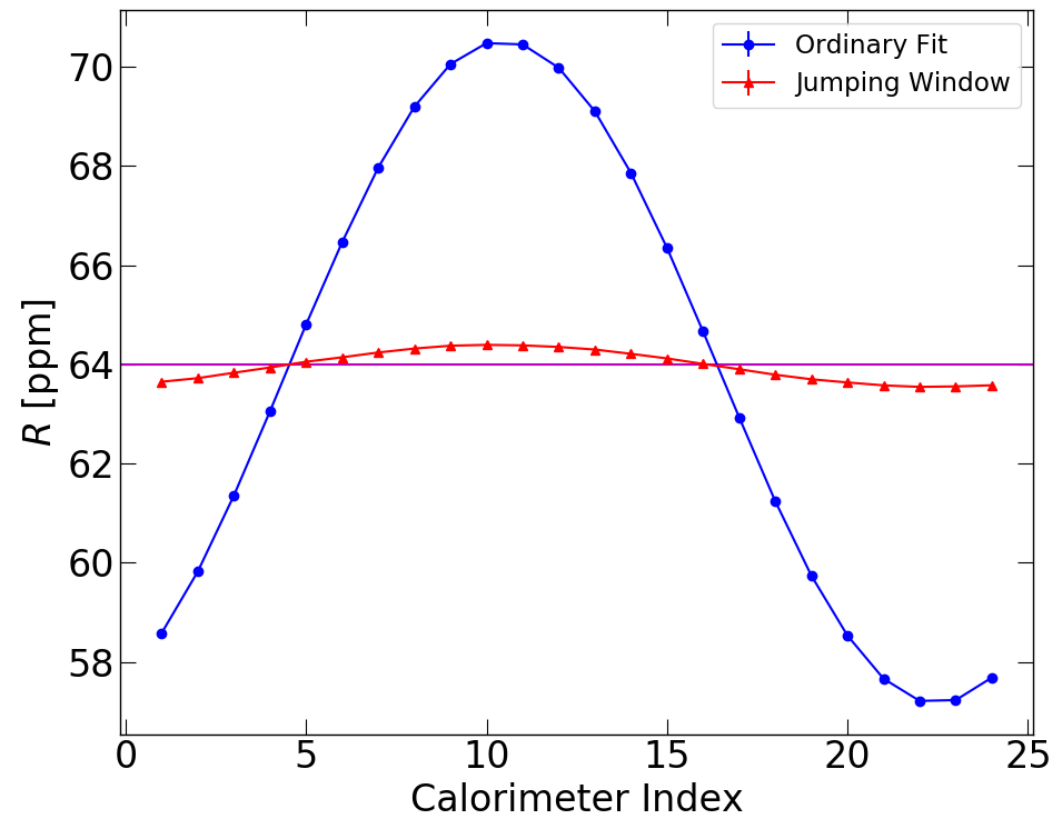
Stroboscopic analysis method by Yuri Orlov

- When we realized the extent of the problem we had already taken the data. Actually, as it turns out, we were stuck with the CBO.
- What do you do with the data?
- Yuri Orlov: this is not a real resonance, but an observational one.
- Look at it at its own frequency... hence stroboscopic method, without needing to know the CBO functional form.

MC simulation (Constant f_{CBO})

- Ordinary fit vs. Stroboscopic fit (with five-parameter function)
 - Systematic biases are reduced by an order of magnitude!

- When our results are checked out using this method, we know our CBO models are correct

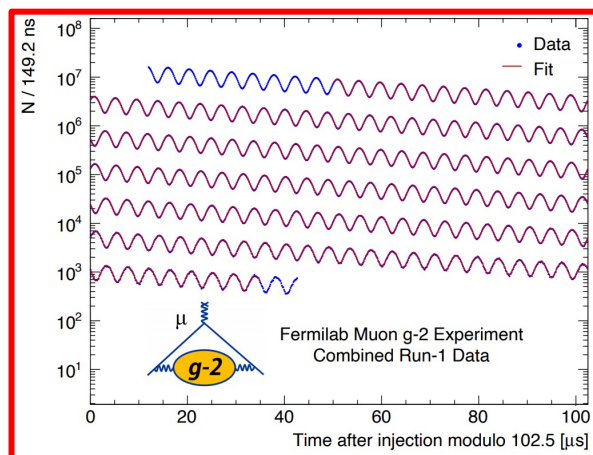


Measuring a_μ (detailed)▶ Revisited a_μ expression

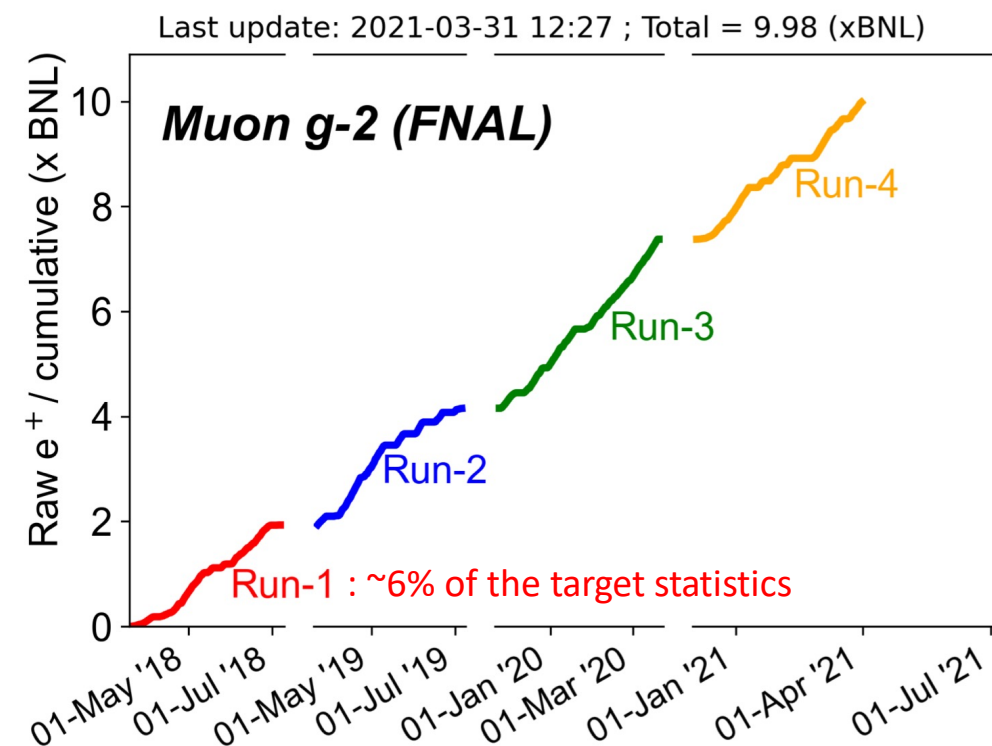
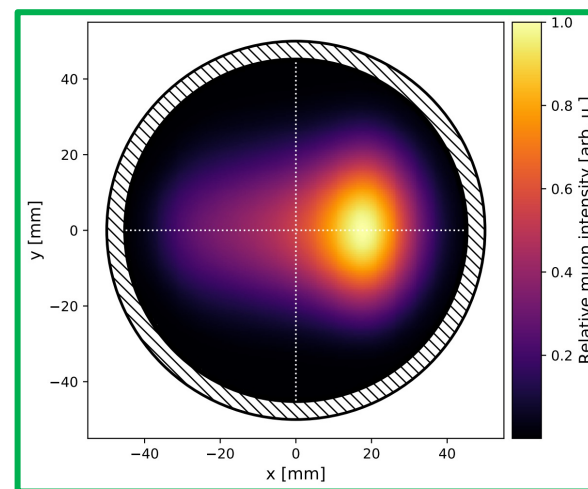
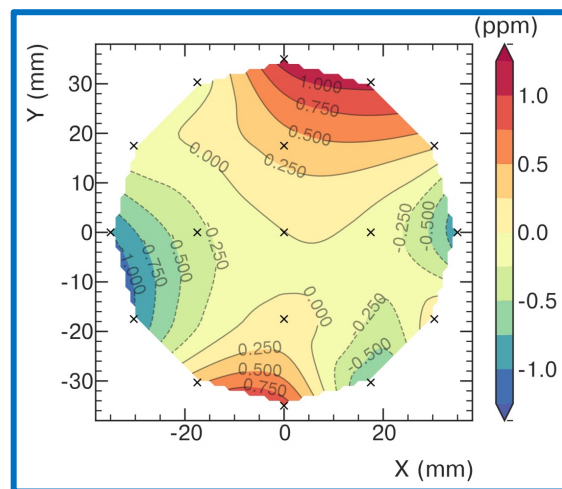
$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \underbrace{\frac{\mu'_p(T_r)}{\mu_e(H)}}_{11 \text{ ppb}} \underbrace{\frac{\mu_e(H)}{\mu_e}}_{0 \text{ ppb}} \underbrace{\frac{m_\mu}{m_e}}_{22 \text{ ppb}} \underbrace{\frac{g_e}{2}}_{0.3 \text{ ppt}}$$

$$\mathcal{R}'_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} = \frac{f_{\text{clock}} \omega_a^m (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{\text{calib}} \underbrace{\langle \omega_p(x, y, \phi) \times M(x, y, \phi) \rangle}_{\text{Magnetic field weighted over the muon distribution and azimuthally averaged}} (1 + \underbrace{B_k + B_q}_{\text{Corrections from the transient magnetic field}})$$

Unblinding conversion factor $\rightarrow \frac{\omega_a}{\tilde{\omega}'_p(T_r)}$
 Measured $g-2$ frequency $\rightarrow f_{\text{clock}} \omega_a^m$
 Corrections from the beam dynamics systematic effects $\rightarrow (1 + C_e + C_p + C_{ml} + C_{pa})$
 NMR probe calibration factor $\rightarrow f_{\text{calib}}$
 Corrections from the transient magnetic field $\rightarrow (1 + B_k + B_q)$

Measuring a_μ (detailed)

$$\mathcal{R}'_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} = \frac{f_{\text{clock}} \omega_a^m (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{\text{calib}} \langle \omega_p(x, y, \phi) \times M(x, y, \phi) \rangle (1 + B_k + B_q)}$$

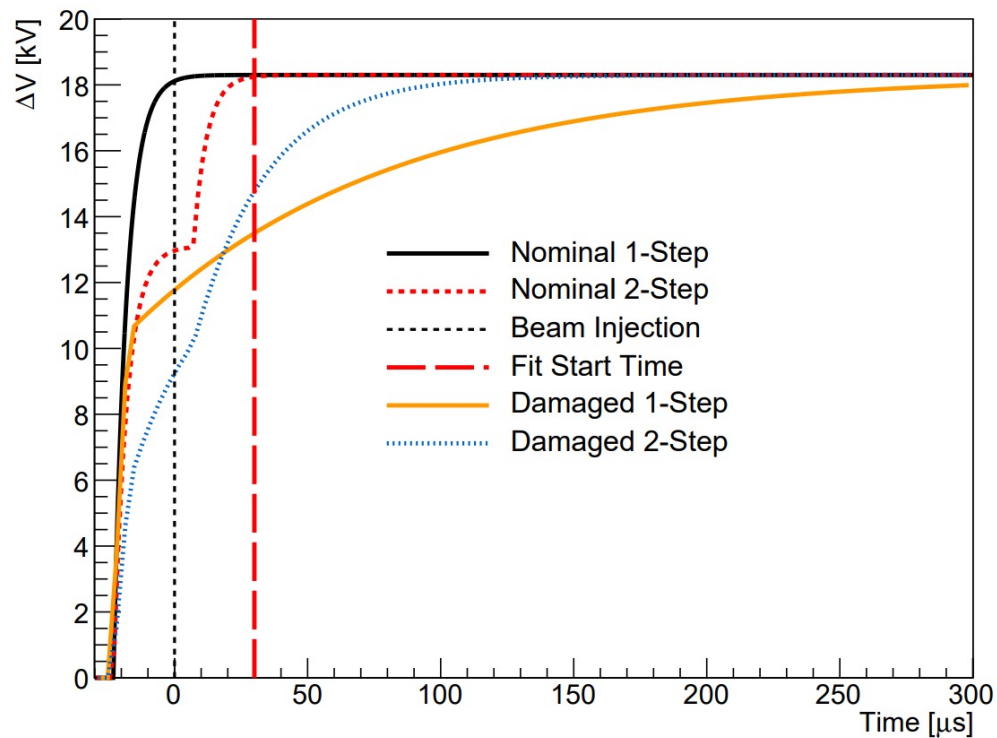


Data Set	$\delta\omega_a^m$ (stat) ppb	ESQ kV	Effective Field Index	Kicker kV
Run-1a	1206	18.3	0.108	130
Run-1b	1024	20.4	0.120	137
Run-1c	825	20.4	0.120	130
Run-1d	676 [†]	18.3	0.107	125

Challenge in Run-1: Damaged ESQ resistors

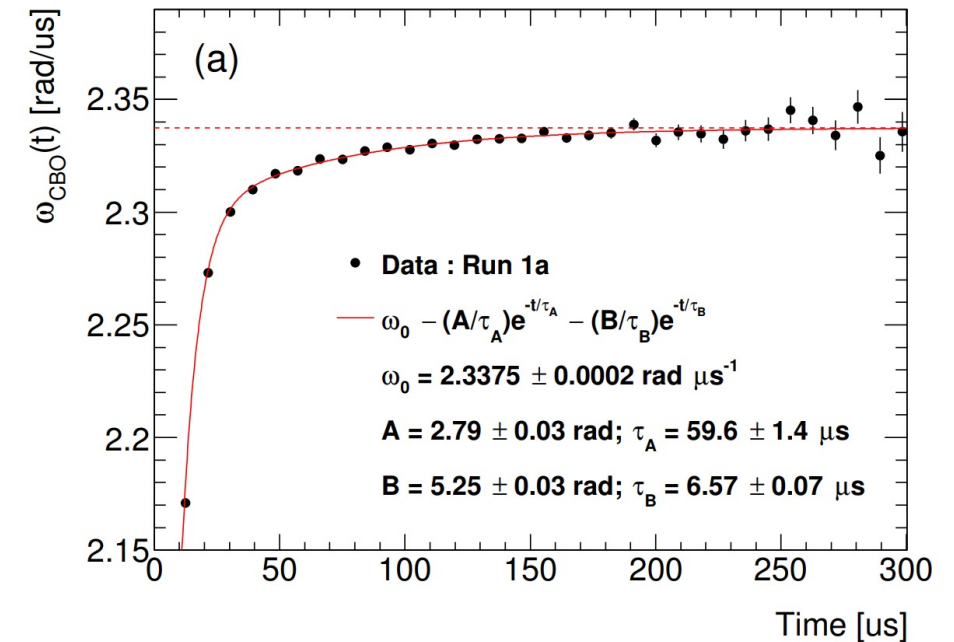
▶ Damaged ESQ HV resistors

- The HV resistance together with the Quad plate capacitance determines the **RC time constant** of the Quad charging (designed to be $\sim 5 \mu\text{s}$).
- 2 resistors were discovered to be damaged and have far larger resistance than the desired value \rightarrow induced **slow changes to beam dynamics**.



▶ Estimated effects

- Most noticeably, the **CBO frequency drifted in time** (designed to be constant during the ω_a fit time window).



- Vertical width changed slowly in time.
- Amplified the **phase-acceptance systematic effect**.

Experimental systematic errors seem to be under control

- Experiment:

TABLE II. Values and uncertainties of the \mathcal{R}'_μ correction terms in Eq. (4), and uncertainties due to the constants in Eq. (2) for a_μ . Positive C_i increase a_μ and positive B_i decrease a_μ .

Quantity	Correction terms (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	...	434
ω_a^m (systematic)	...	56
C_e	489	53
C_p	180	13
C_{ml}	-11	5
C_{pa}	-158	75
$f_{\text{calib}} \langle \omega_p(x, y, \phi) \times M(x, y, \phi) \rangle$...	56
B_k	-27	37
B_q	-17	92
$\mu'_p(34.7^\circ)/\mu_e$...	10
m_μ/m_e	...	22
$g_e/2$...	0
Total systematic	...	157
Total fundamental factors	...	25
Totals	544	462

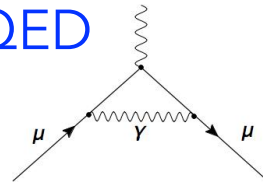
Chris Polly: The two highest systematic errors are associated with the QUADs. They will be reduced by large factors with the next data analysis. Essentially, the experiment has reached its systematic error goals.

Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{Weak}) + a_\mu(\text{Hadronic})$$

- Theory :

QED

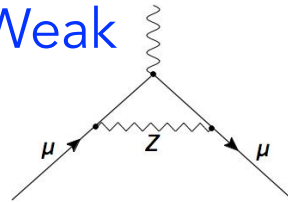


+ ...

$$116\,584\,718.9(1) \times 10^{-11}$$

0.001 ppm

Weak



+ ...

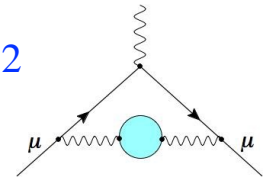
$$153.6(1.0) \times 10^{-11}$$

0.01 ppm

Hadronic...

...Vacuum Polarization (HVP)

α^2



+ ...

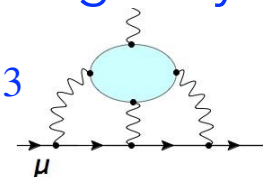
$$6845(40) \times 10^{-11}$$

[0.6%]

0.37 ppm

...Light-by-Light (HLbL)

α^3



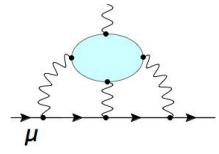
+ ...

$$92(18) \times 10^{-11}$$

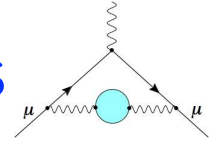
[20%]

0.15 ppm

Muon g-2 announcement, theory vs. theory



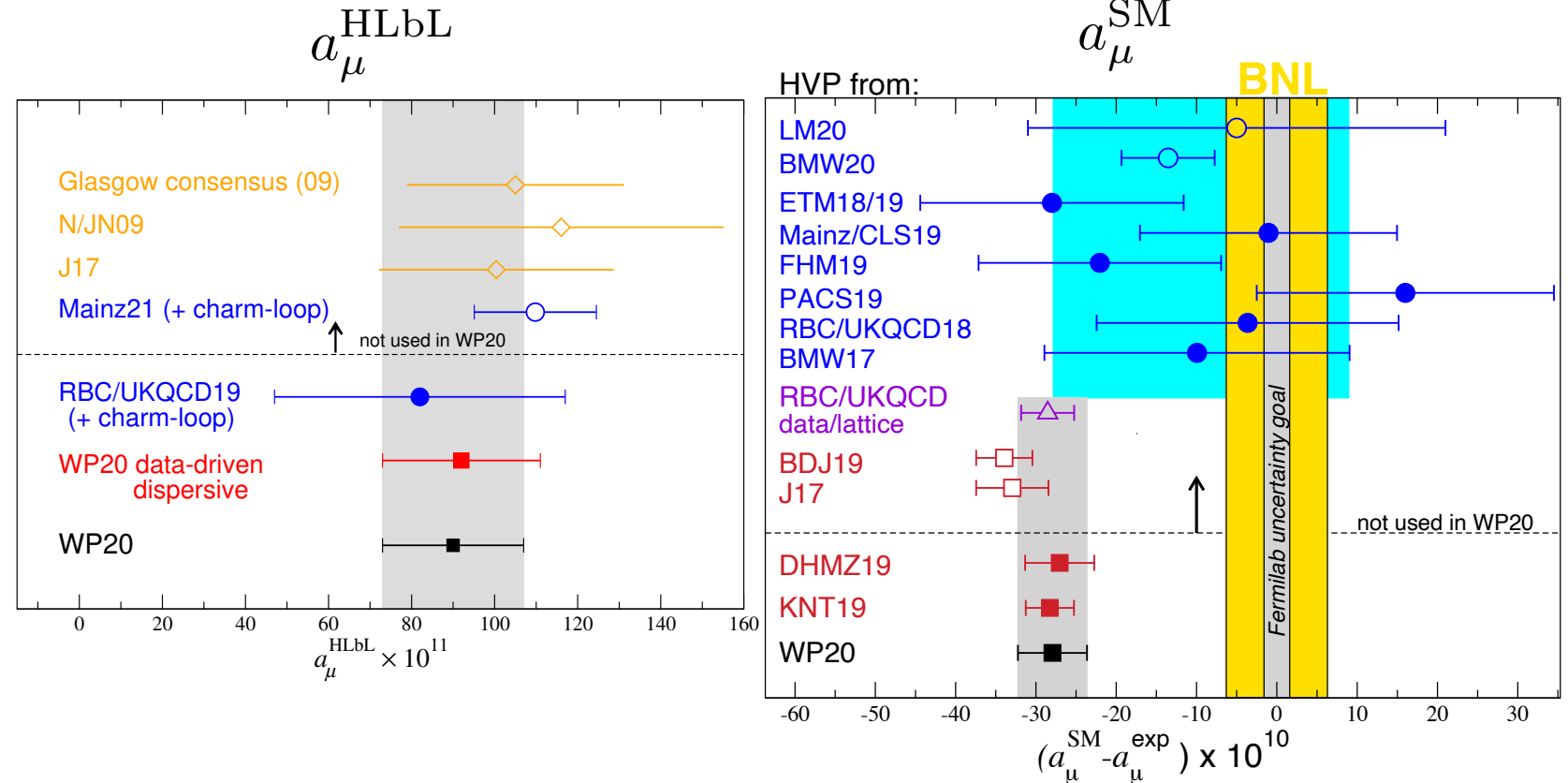
Hadronic Corrections: Comparisons



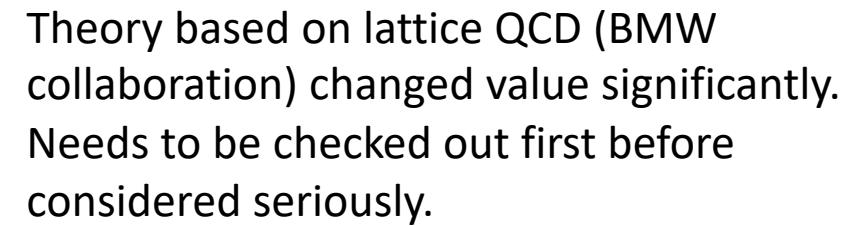
- Theory :

$$a_{\mu}^{\text{HVP}} + [a_{\mu}^{\text{QED}} + a_{\mu}^{\text{Weak}} + a_{\mu}^{\text{HLbL}}]$$

$$a_{\mu}^{\text{SM}}$$



- Theory:



Experiment and theory 4.2 sigma (theory based on e^+e^- data)

- Experiment:

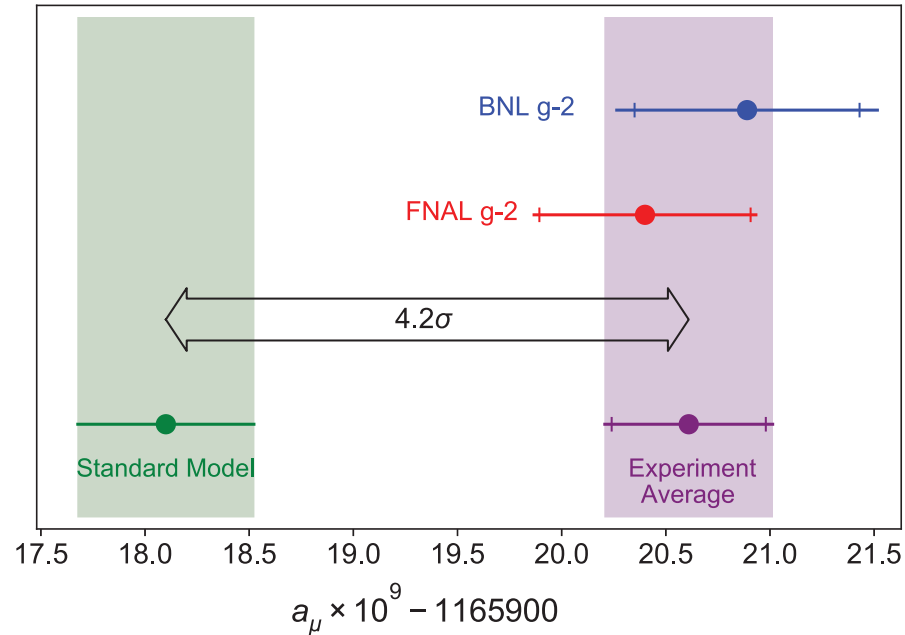


FIG. 4. From top to bottom: experimental values of a_μ from BNL E821, this measurement, and the combined average. The inner tick marks indicate the statistical contribution to the total uncertainties. The Muon $g - 2$ Theory Initiative recommended value [13] for the standard model is also shown.

Muon g-2 announcement

- Physics: >8500 participants dialed in on “zoom” and Youtube channel during the announcement on April 7. It was estimated that the muon g-2 news reached ~2.7B people. Chris Polly gave a great presentation of the experiment.
- Blind analysis, meaning the frequency has a constant offset, so you don’t know the result when analyzing. The offset was set by Fermilab people outside the collaboration.
- The result is right on with the BNL value. An experimental triumph!
- The theory on hadronic contribution based on e^+e^- and lattice are at odds. The lattice work needs to be cross-checked and confirmed. Until then, we use e^+e^- .

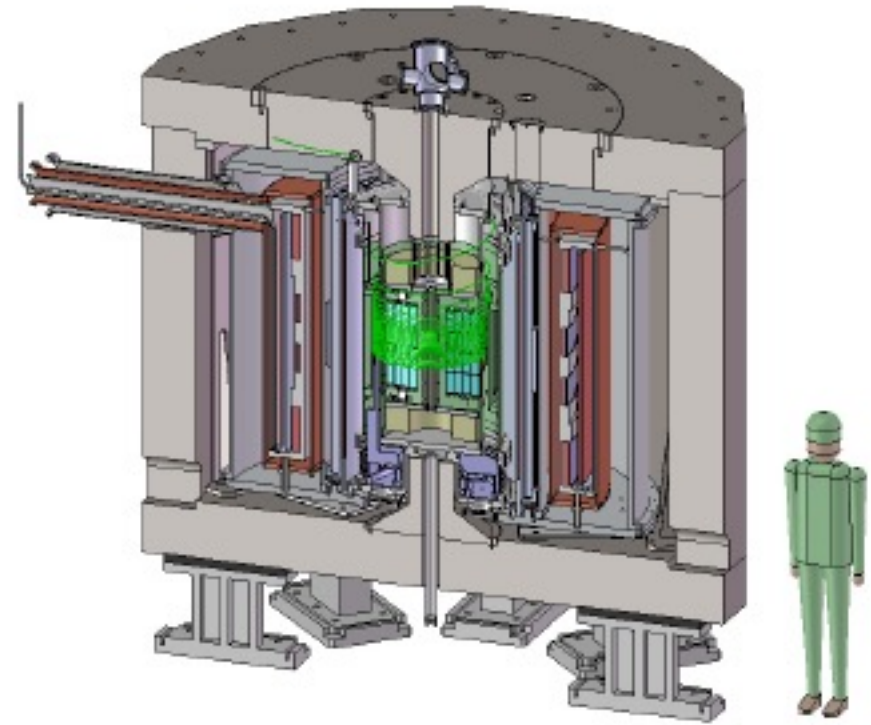
Significance of the muon g-2 new results

- The field is mature, the storage ring method can be considered mainstream
- The resources on this experiment were larger than at BNL having the critical mass and time to go further in-depth. The systematic errors have been understood at much higher level, analyzed independently by several groups.
- The experiment has proven that we can reliably estimate things to $<10\text{ppb}$, which is the level required, e.g., for better sensitivity muon g-2 experiments, for the storage ring proton EDM (SR pEDM), etc., see <https://indico.fnal.gov/event/48469/timetable/?view=standard>
- SR pEDM is ready for prime time, goal 10^{-29}e-cm , physics reach $\sim 10^3\text{TeV}$.
- The JPARC muon g-2 experiment uses lower energy muons in a 3T field, cold muon source (muonium) and very weak magnetic focusing.

J-PARC Muon g-2
experiment with very
weak magnetic focusing.

B-field: 3T

$\gamma = 3$



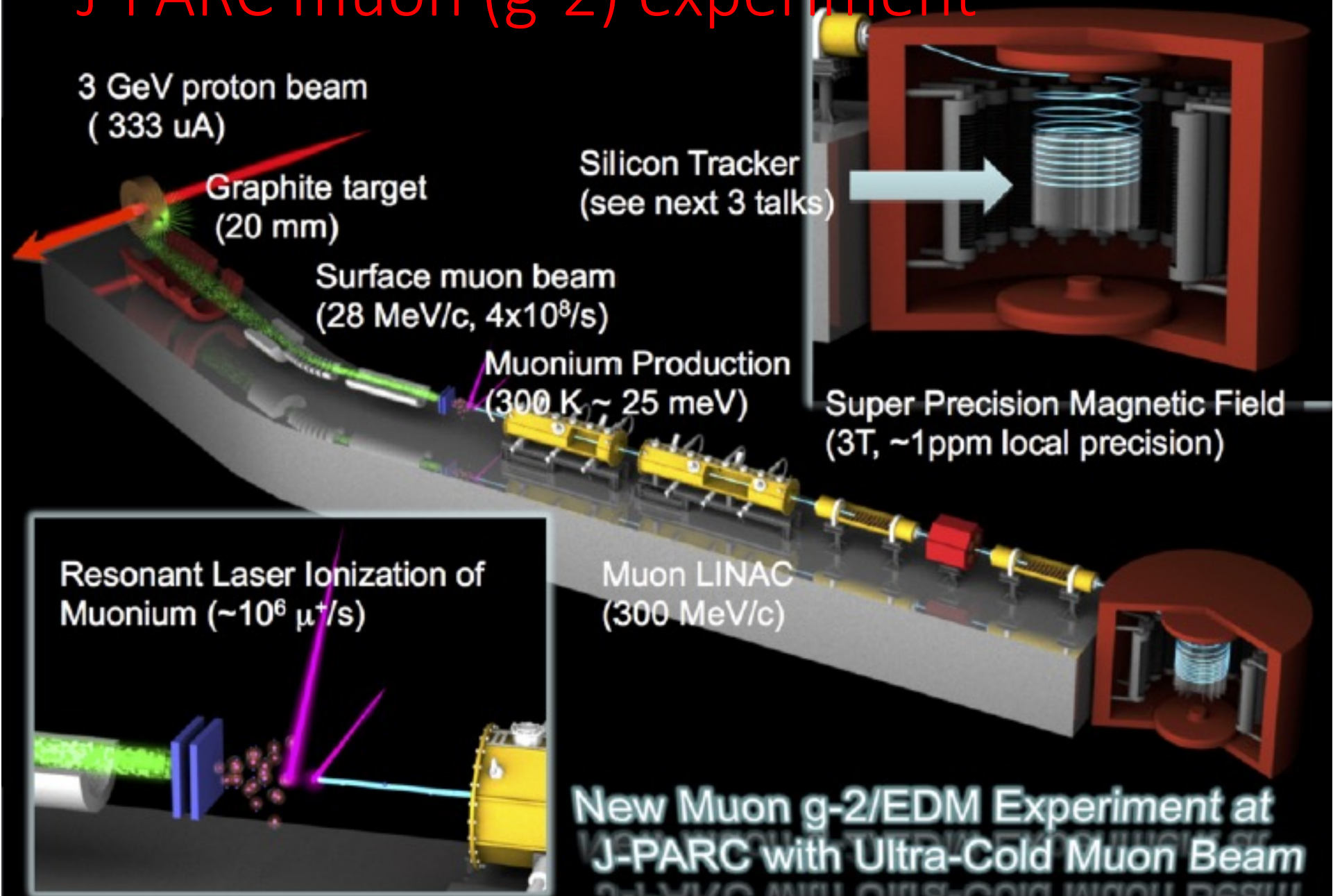
- Totally independent experiment, storing cold muons, lower energy, more muons
- Very different systematic errors
- Much more uniform B-field
- Accepting all muon decays
- Planned start data taking in 2025

Summary

- Measuring the energy and time of the decay positrons defines the “muon $g-2$ ” frequency.
- At Fermilab, the systematic errors have been understood in depth. Different approaches have been used to understand the effects of potential systematic error sources. There is a serious tension with the SM.
- Much more data to be analyzed soon.
- Next level experiments, e.g., SR pEDM are being considered

Extra slides

J-PARC muon (g-2) experiment



With RF in the g-2 ring

- Francis Farley new g-2 ring idea works, without RF it doesn't
- Next generation muon g-2 experiment: larger energy muons, more g-2 cycles, use protons to map the magnetic field.

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F.J.M. Farley, Y.K. Semertzidis / Progress in Particle and Nuclear Physics 52 (2004) 1–83

- Problem: muon and proton average radius to sub-micron level. It can be done with RF!

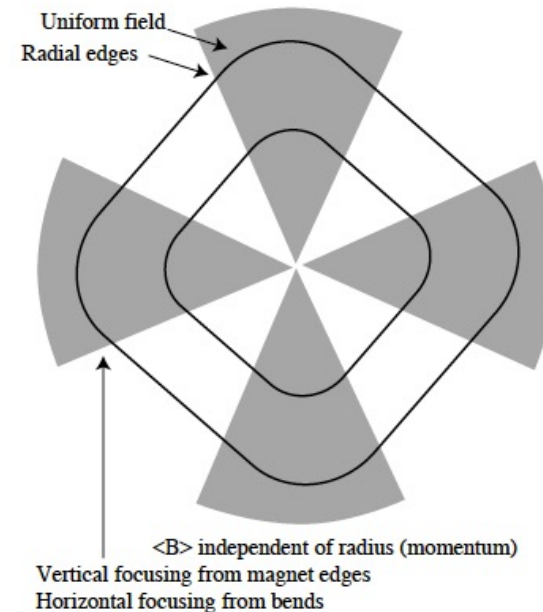


Fig. 47. A storage ring with edge focusing. The field in the magnets is uniform and the edges are radial to the centre of the ring. The mean field is independent of the orbit radius.

More precisely (Y.O.), and electric field correction for quads covering 43% of the ring

$$C = 2F(n)n(1-n) \left[\left\langle \left(\frac{f - f_m}{f_m} \right)^2 \right\rangle + \left[\left\langle \frac{f - f_m}{f_m} \right\rangle \frac{f_m - f_0}{f_m} \right] \right]$$

- f , the muon revolution freq., f_0 , the revolution frequency at $E=0$, f_m , the rev. freq. of the magic mom. muons.

Checking the approximations

- Beam dimensions stable at the 4% level around the ring
- $F(n)$ is 1 within $\sim 1\%$.

$\alpha(n)$ and $F(n)$ (see text) resulting from the discrete nature of the quadrupole structure in the $(g - 2)$ ring

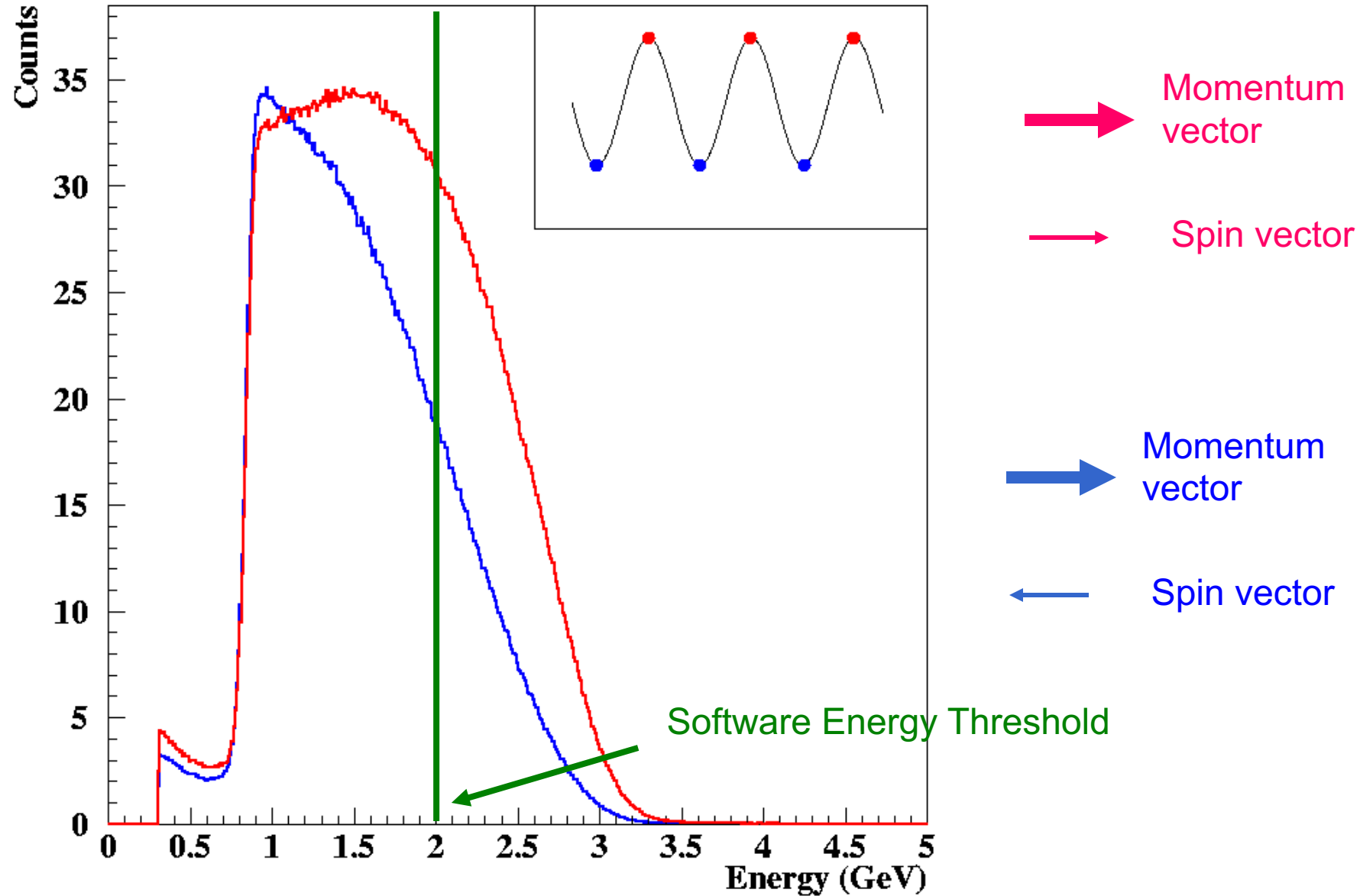
n	0.121	0.123	0.125	0.127	0.129	0.131	0.133	0.135	0.137	0.139
$\alpha(n)$	1.136	1.139	1.142	1.144	1.147	1.149	1.152	1.155	1.157	1.160
$F(n)$	0.994	0.993	0.992	0.992	0.991	0.992	0.992	0.992	0.992	0.991

Beta functions for two, four and eight-fold lattice symmetry

N	2	4	8
$\beta_x(\text{max})$	24.4m	7.9m	7.7m
$\beta_x(\text{min})$	2.30m	7.35m	7.6m
$\sqrt{\text{max/min}}$	3.26	1.04	1.01
$\beta_y(\text{max})$	21.8m	19.9m	19.5m
$\beta_y(\text{min})$	16.8m	18.8m	19.2
$\sqrt{\text{max/min}}$	1.15	1.03	1.01

Pileup effect in $g-2$ experiment

Energy Spectrum of Detected Positrons depends on spin direction



Pileup: View from top

Pileup and Gain Systematics

Pete Alonzi – University of Washington
g-2 collaboration meeting – Dec 14, 2012

Pileup < 0.04 ppm

Background

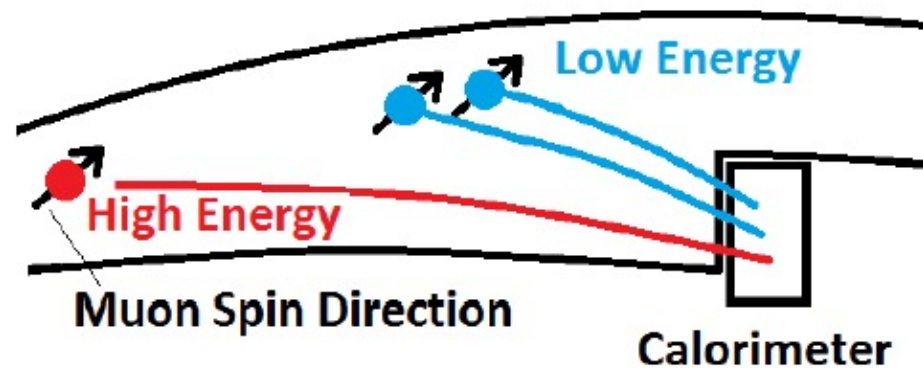
Answers

Progress

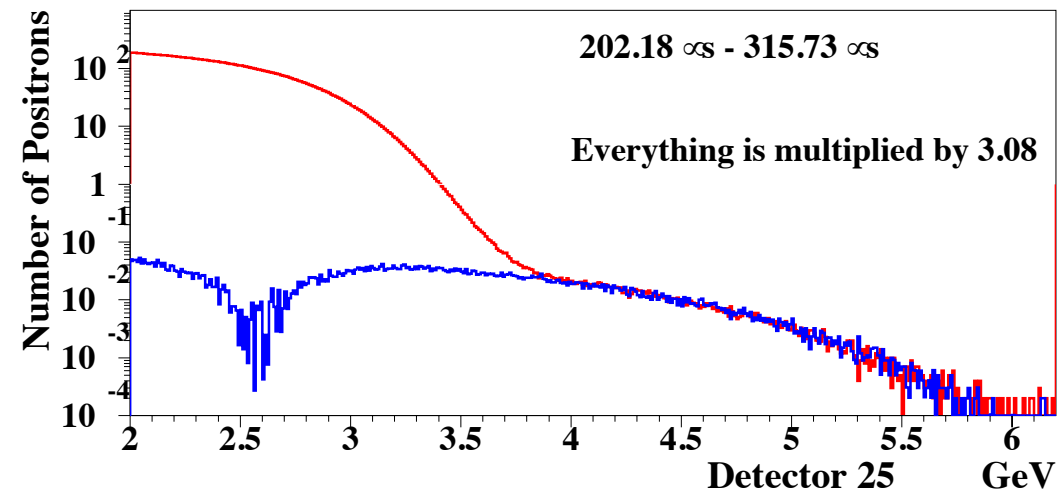
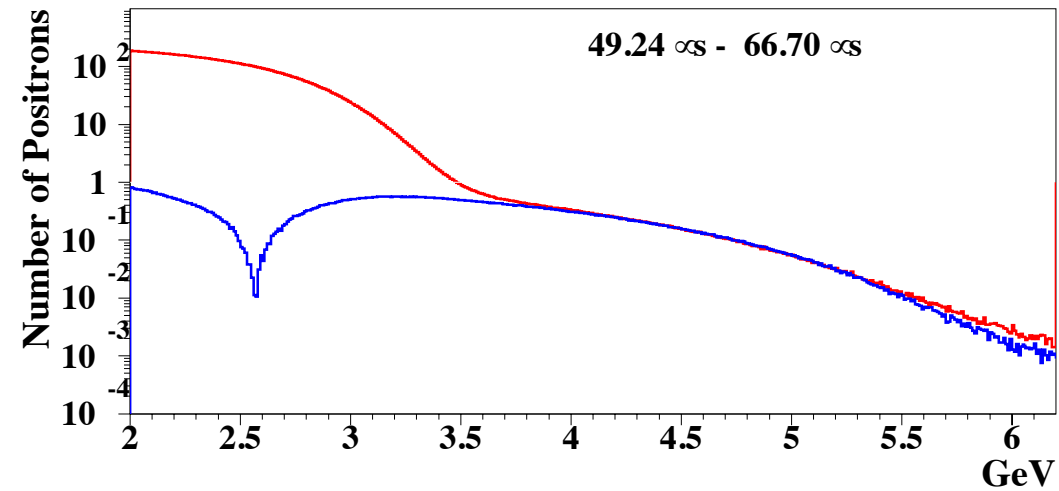
Gain < 0.02 ppm

Background

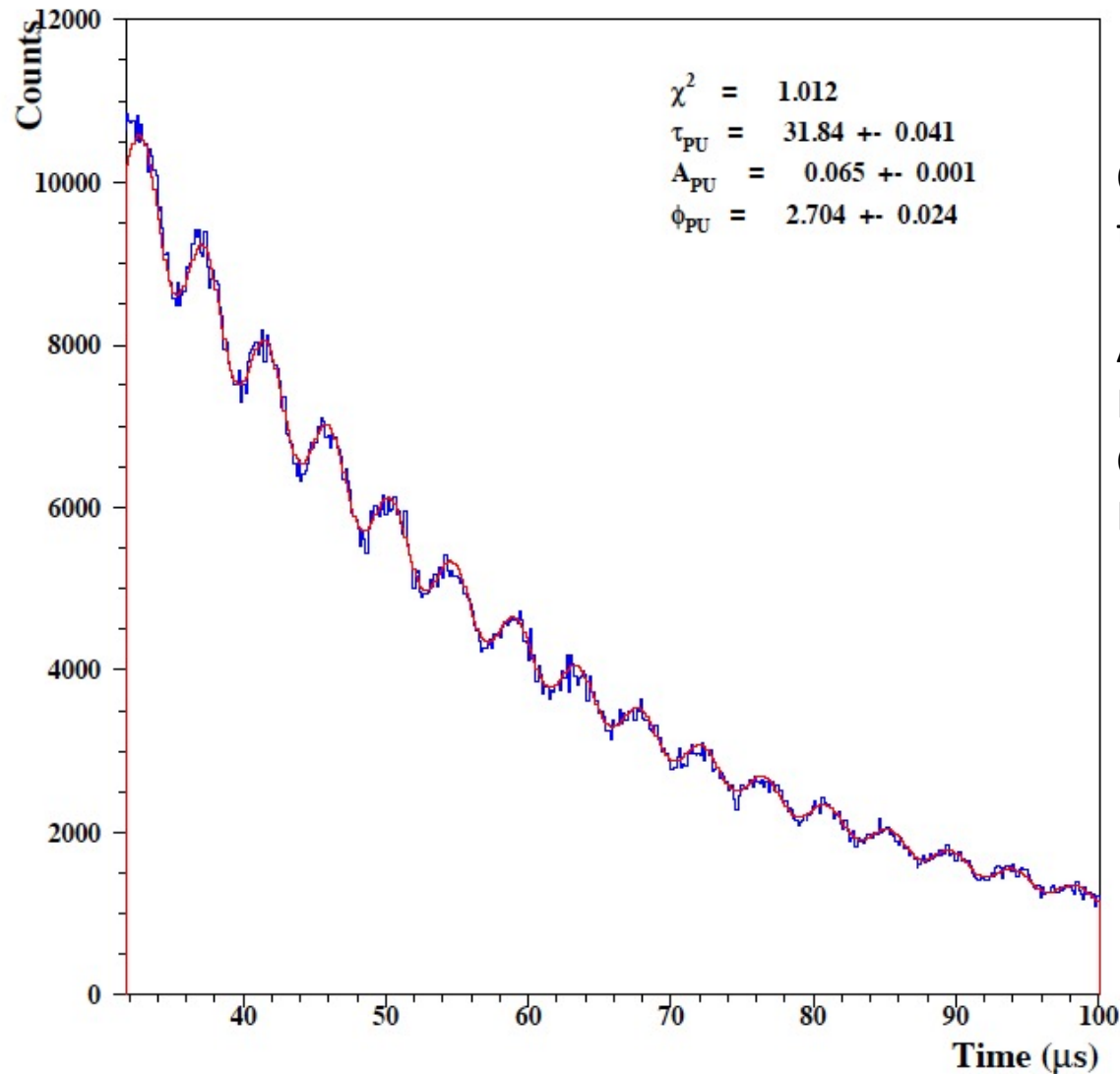
Progress



Decay positron energy spectrum



Decay positron time spectrum



Question: what is the frequency of the pileup spectrum?

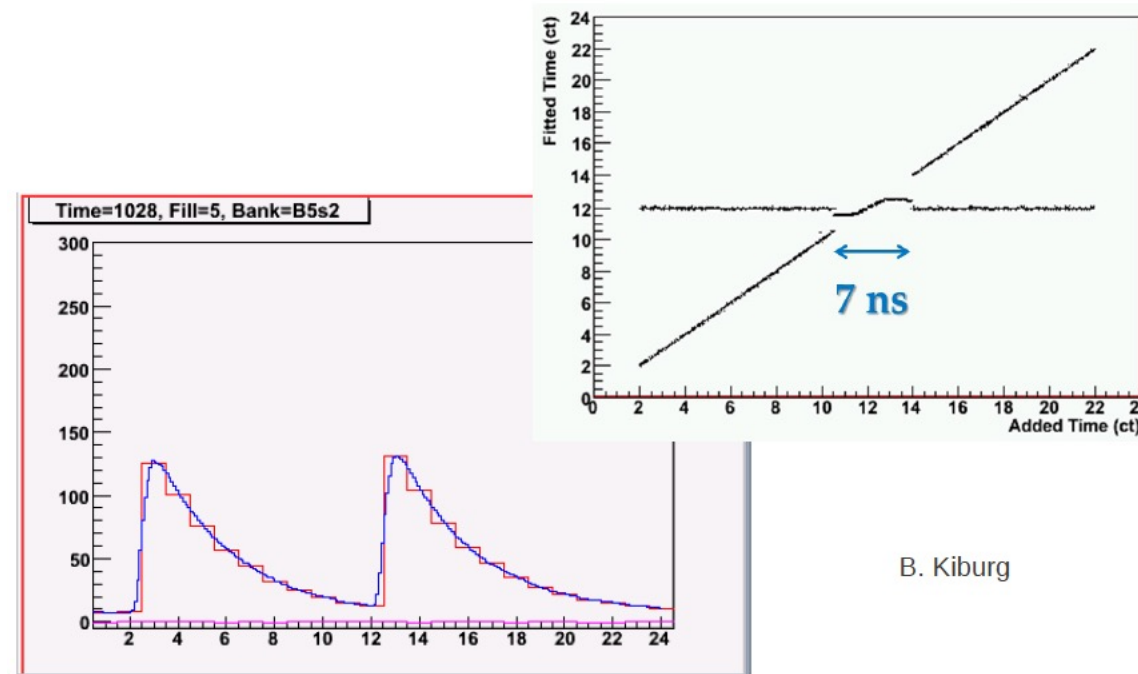
- A. $0.5 \times \Omega_a$
- B. $1.0 \times \Omega_a$
- C. $2.0 \times \Omega_a$
- D. $3.0 \times \Omega_a$

Can we fit for it in the data?
Answer: yes, with a loss in statistical sensitivity of 2. In practice we don't!

Figure 1: Fit results to pseudo-pileup time histogram (D-S1-S2) when the correct error definitions per bin content are used.

Two pulses can be too close!

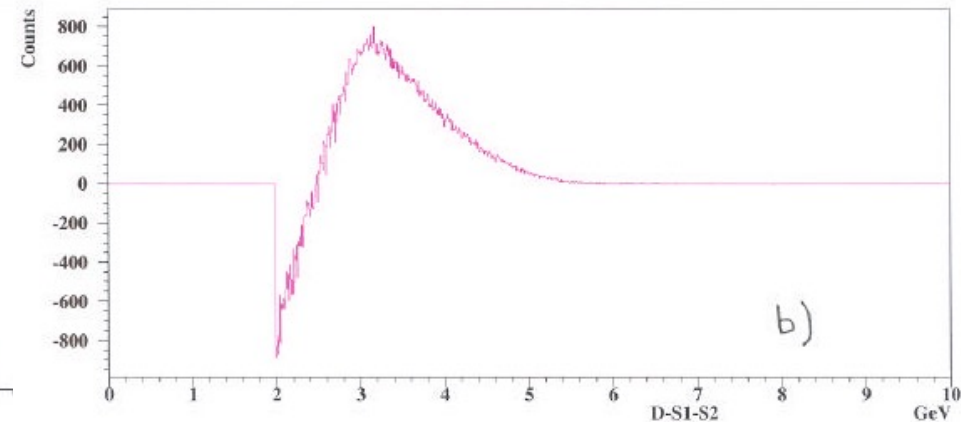
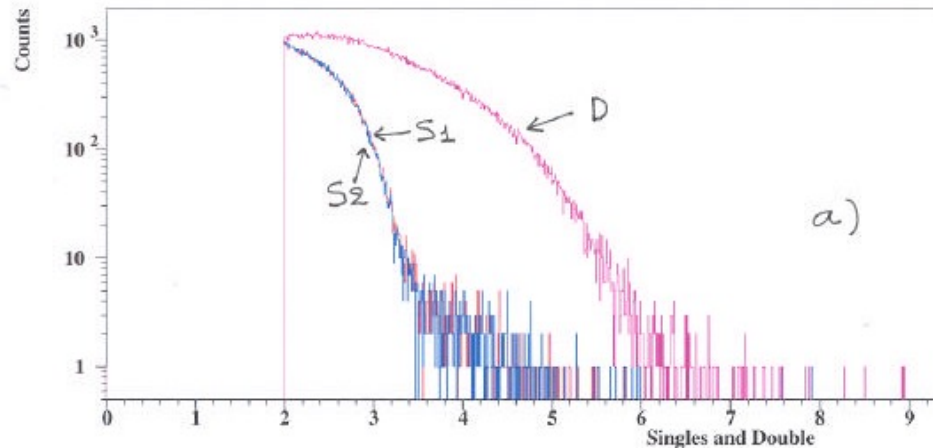
MuLan Style Pulse Fitting



B. Kiburg

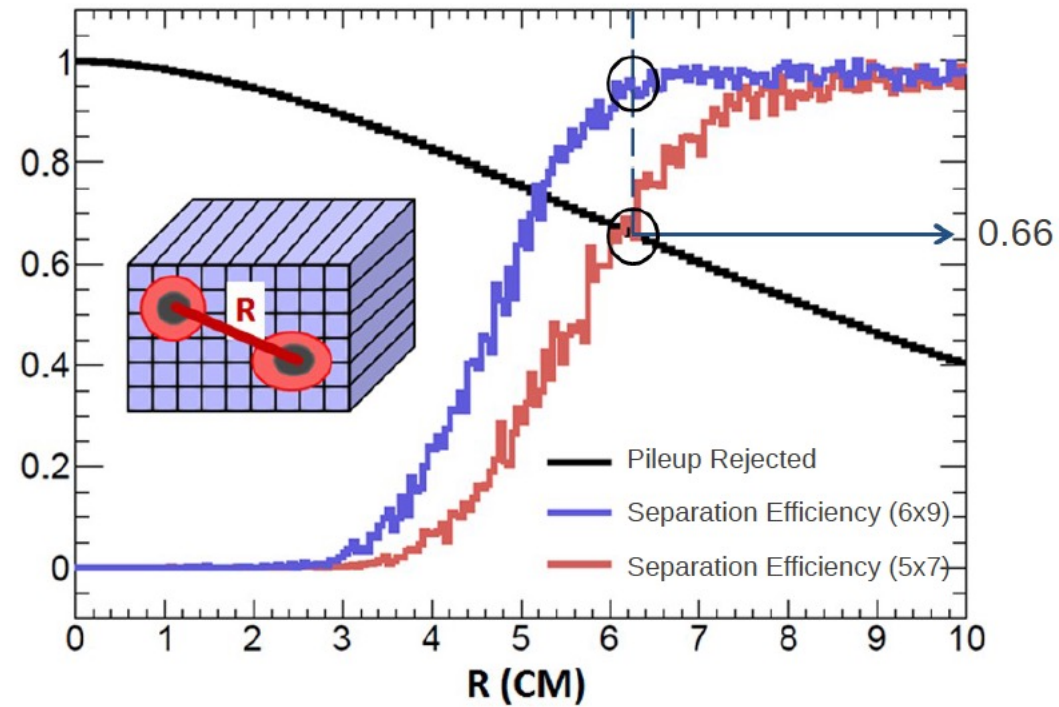
Pileup subtraction procedure

1. Set a threshold on E_1 e.g. $E_1 > 1\text{GeV}$ (the hardware energy threshold; in order to ensure early to late stability)
2. If $E_D = 0.96 \times (E_1 + E_2) > 2\text{GeV}$ then we have a double (D).
3. If we have a double then ask if $E_1 > 2$ (if yes there is a single pileup (S1)), and if $E_2 > 2\text{GeV}$ (again if yes, there is another single pileup (S2)).
4. If we have a double and $E_2 < 1\text{GeV}$ then count all pileup pulses twice with the appropriate timing.



Plan at Fermilab

Segmentation Effectiveness – Lower Bound



Blue option is chosen

Definition of g-Factor

$$g \equiv \frac{\frac{\text{magnetic moment}}{e\hbar/2m}}{\frac{\text{angular momentum}}{\hbar}}$$

g-2 measures the difference between the charge and mass distribution. g-2=0 when they are the same all the time...

From Dirac equation g-2=0 for point-like, spin ½ particles, e.g leptons.

Magnetic Dipole Moments: μ

- Nuclear Magnetic Resonance: a new direct method of detecting NMR, I. Rabi *et al.*, 1938

$$\frac{d\vec{s}}{dt} = \vec{\mu} \times \vec{B}$$

- Used in Magnetic Resonance Imaging

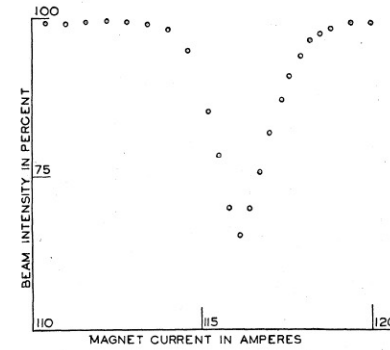
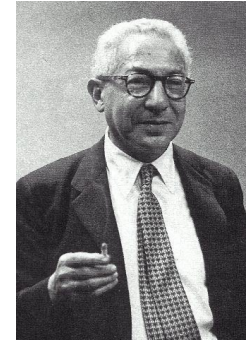


FIG. 1. Curve showing refocused beam intensity at various values of the homogeneous field. One ampere corresponds to about 18.4 gauss. The frequency of the oscillating field was held constant at 3.518×10^6 cycles per second.



Electric field correction (FF) for continuous quads

The change of the g-2 freq. for off-momentum muons:

$$\vec{\omega}_a = \frac{e}{m} \left\{ a \vec{B} - \left[a - \left(\frac{mc}{p} \right)^2 \right] \frac{\vec{\beta} \times \vec{E}}{c} \right\}$$

$$C = 2n(1-n) \left(\frac{x_e^2}{R^2} \right)$$