

# Gravitational waves from bubble collisions in first order phase transitions

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Gravitational Wave Probes of Physics Beyond Standard Model  
13 VII 2021

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UNIVERSITY  
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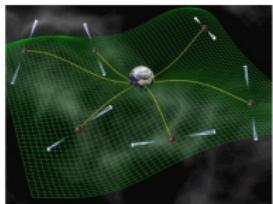
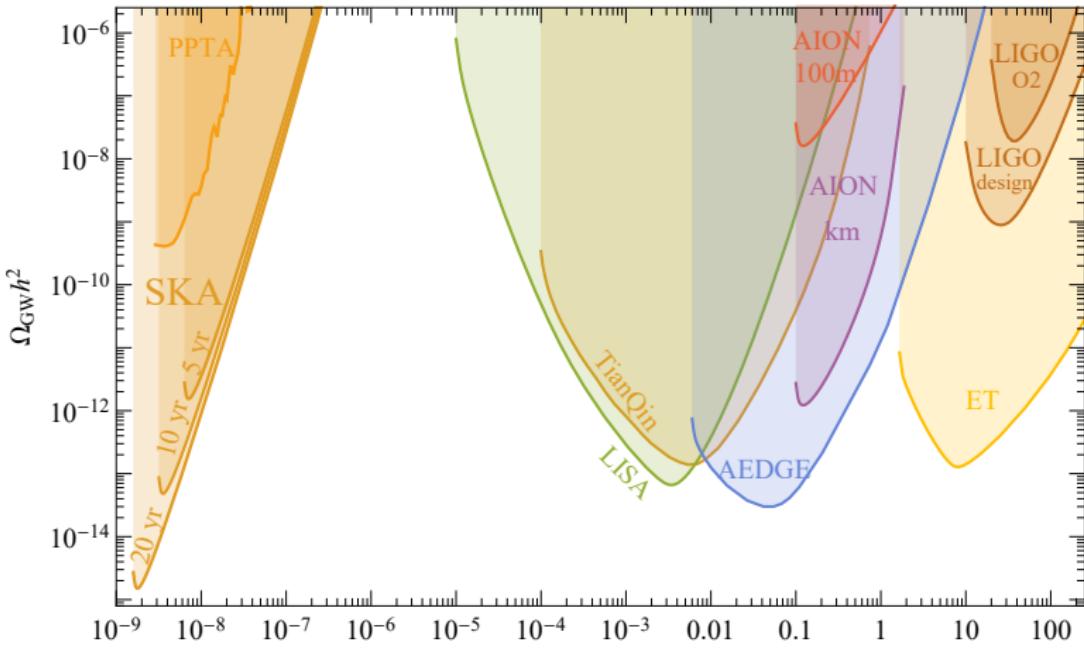


Based on:

J. Ellis, M.L., J. M. No arXiv:1809.08242, 2003.07360

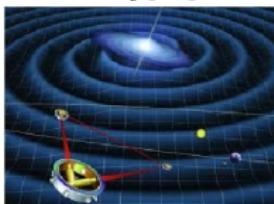
J. Ellis, M.L., J. M. No, V. Vaskonen arXiv:1903.09642

M.L., V. Vaskonen arXiv: 1912.00997, 2007.04967, 2012.07826



Pulsar Timing

[David Champion/NASA/JPL]



LISA

[wiki/Laser\\_Interferometer\\_Space\\_Antenna](https://en.wikipedia.org/wiki/Laser_Interferometer_Space_Antenna)



Einstein Telescope

[www.et-gw.eu](http://www.et-gw.eu)

# Gravitational waves from a PT

- Strength of the transition

$$\alpha \approx \left. \frac{\Delta V}{\rho_R} \right|_{T=T_*}, \quad \Delta V = V_f - V_t$$

- Characteristic scale

$$HR_* = (8\pi)^{\frac{1}{3}} \left( \frac{\beta}{H} \right)^{-1}$$

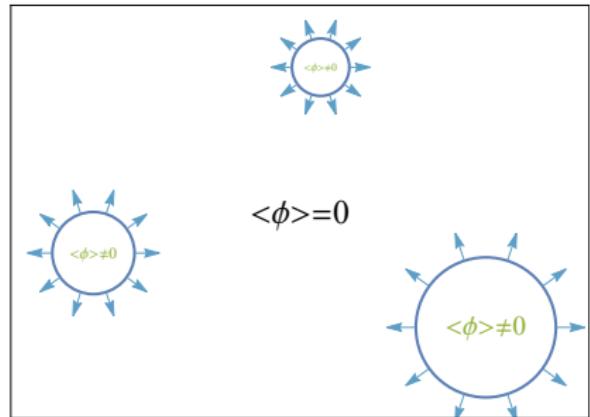
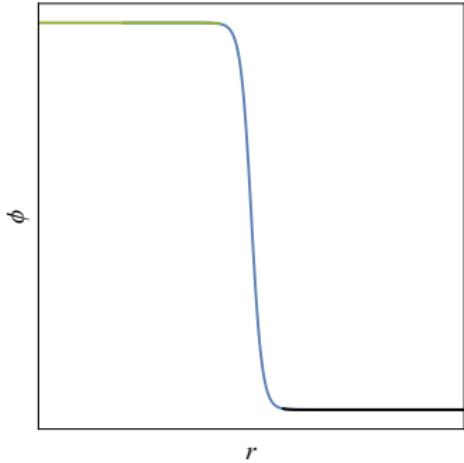
- Signals are produced by three main mechanisms:

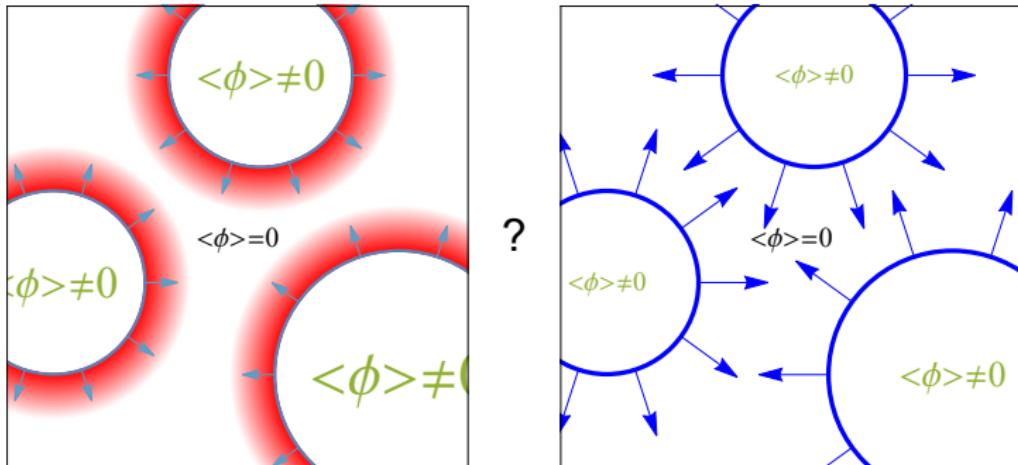
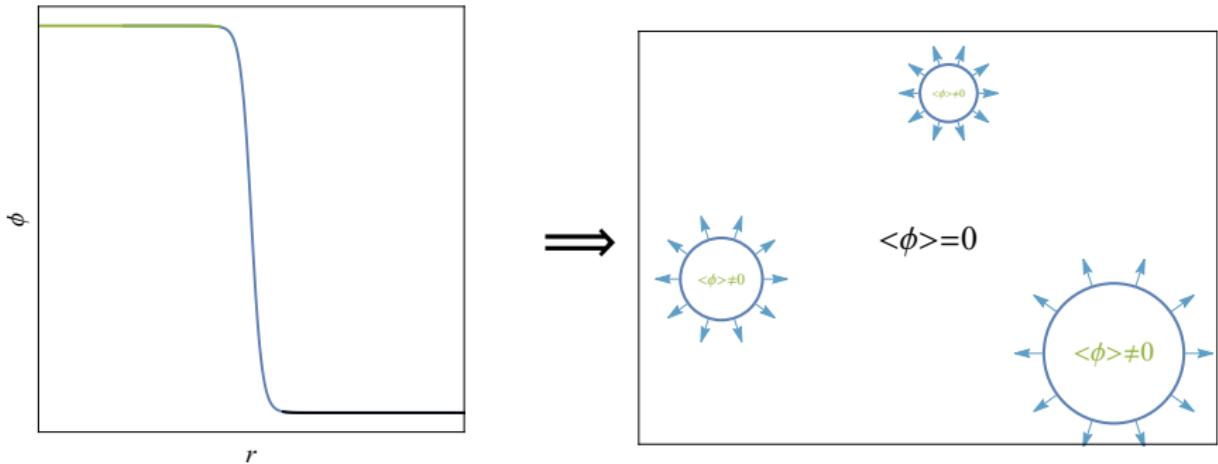
- collisions of bubble walls:  $\Omega_{\text{col}} \propto \left( \kappa_{\text{col}} \frac{\alpha}{\alpha+1} \right)^2 (HR_*)^2$   
Kamionkowski '93, Huber '08, Hindmarsh '18 '20 Lewicki '19 '20,

- sound waves:  $\Omega_{\text{sw}} \propto \left( \kappa_{\text{sw}} \frac{\alpha}{\alpha+1} \right)^2 (HR_*) (H\tau_{\text{sw}})$   
Hindmarsh '13 '15 '17 '19 '21, Ellis '18 '19 '20, Jinno '20

- turbulence  $\Omega_{\text{turb}} \propto \left( \kappa_{\text{sw}} \frac{\alpha}{\alpha+1} \right)^{\frac{3}{2}} (HR_*) (1 - H\tau_{\text{sw}})$   
Caprini '06 '09 '20, Brandenburg '10 '12 '17, Roper-Pol '17 '19 '21, Ellis '19 '20

- Sound wave period lasts  $H\tau_{\text{sw}} \equiv \min \left[ 1, \frac{HR_*}{U_f} \right]$





- Energy of the bubble

$$\mathcal{E} = 4\pi \textcolor{blue}{R}^2 \sigma \textcolor{green}{\gamma} - \frac{4\pi}{3} \textcolor{blue}{R}^3 p, \quad \textcolor{green}{\gamma} = \frac{1}{\sqrt{1 - \dot{\textcolor{blue}{R}}^2}}$$

- Vacuum pressure on the wall

Coleman '73

$$p_0 = \Delta V$$

- Energy of the bubble

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- Leading order plasma contribution

Bodeker '09 Caprini '09

$$p_1 = \Delta V - \Delta P_{\text{LO}} \approx \Delta V - \frac{\Delta m^2 \textcolor{red}{T}^2}{24},$$

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- Next-To-Leading order plasma contribution

Bodeker '17

$$p = \Delta V - \Delta P_{\text{LO}} - \textcolor{green}{\gamma} \Delta P_{\text{NLO}} \approx \Delta V - \frac{\Delta m^2 \textcolor{red}{T}^2}{24} - \textcolor{green}{\gamma} g^2 \Delta m_V \textcolor{red}{T}^3.$$

- Next-To-Leading order plasma contribution with resummation

Hoche '20

$$P = \Delta V - P_{1 \rightarrow 1} - \textcolor{green}{\gamma}^2 P_{1 \rightarrow N} \approx \Delta V - 0.04 \Delta m^2 \textcolor{red}{T}^2 - 0.005 g^2 \textcolor{green}{\gamma}^2 \textcolor{red}{T}^4.$$

- terminal velocity  $\gamma$  factor and the value in absence of friction

$$\gamma_{\text{eq}} = \sqrt{\frac{\Delta V - P_{1 \rightarrow 1}}{P_{1 \rightarrow N}}} \simeq \sqrt{\frac{\Delta V - 0.04\Delta m^2 \textcolor{red}{T}^2}{0.005g^2 \textcolor{red}{T}^4}}, \quad \gamma_* \equiv \frac{2}{3} \frac{\textcolor{blue}{R}_*}{\textcolor{blue}{R}_0},$$

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- Amount of energy stored in the wall

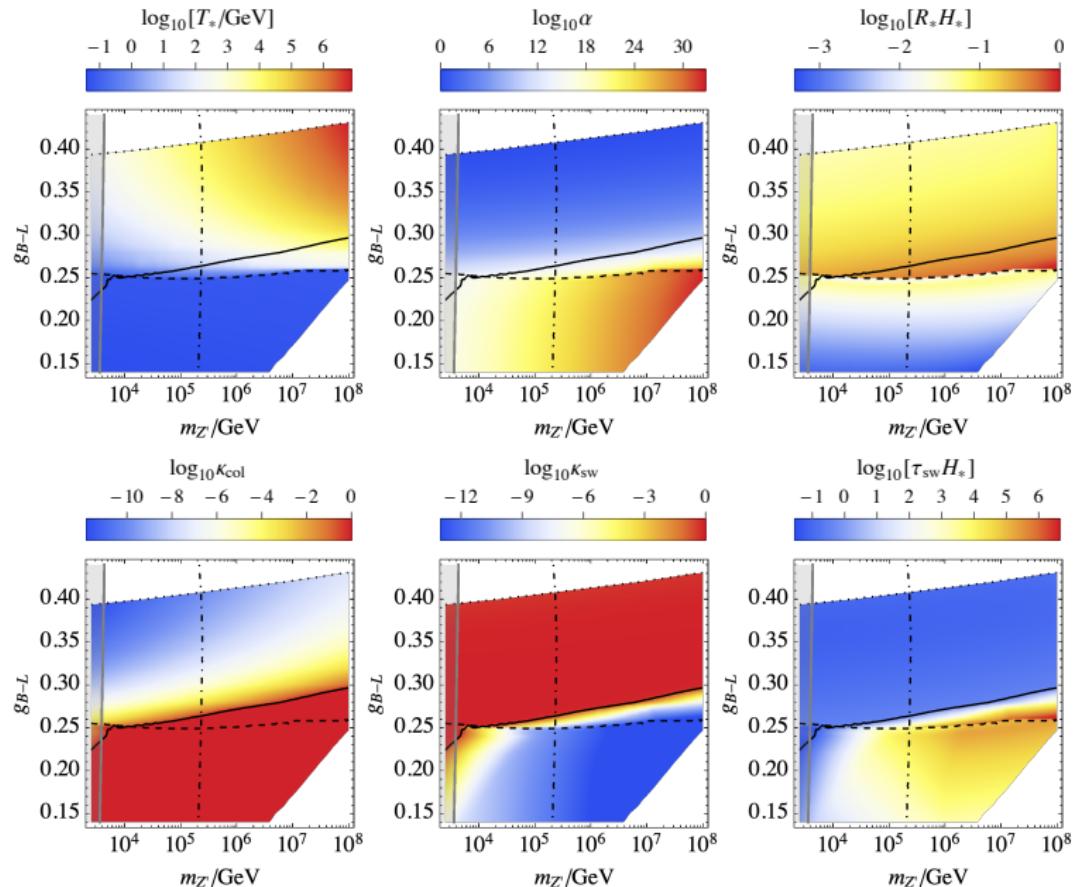
$$E_{\text{wall}} = \frac{4\pi R^2}{3} \int_0^R dR' [\Delta V - P_{1 \rightarrow 1} - \gamma^2(R') P_{1 \rightarrow N}],$$

- Finally the efficiency factors read

$$\kappa_{\text{col}} = \frac{E_{\text{wall}}}{E_V} = \begin{cases} \left[ 1 - \frac{1}{3} \left( \frac{\gamma_*}{\gamma_{\text{eq}}} \right)^2 \right] \left[ 1 - \frac{P_{1 \rightarrow 1}}{\Delta V} \right], & \gamma_* < \gamma_{\text{eq}}, \\ \frac{2}{3} \frac{\gamma_{\text{eq}}}{\gamma_*} \left[ 1 - \frac{P_{1 \rightarrow 1}}{\Delta V} \right], & \gamma_* > \gamma_{\text{eq}}, \end{cases}$$

$$\kappa_{\text{sw}} = \frac{\alpha_{\text{eff}}}{\alpha} \frac{\alpha_{\text{eff}}}{0.73 + 0.083\sqrt{\alpha_{\text{eff}}} + \alpha_{\text{eff}}} \quad , \quad \text{with } \alpha_{\text{eff}} = \alpha(1 - \kappa_{\text{col}}).$$

# $U(1)_{B-L}$ Example



# Purely thermal transition: no $T = 0$ potential barrier

- Simple polynomial potential

$$V(\phi) = m^2 \phi^2 - a \phi^3 + \lambda \phi^4$$

- Has a semi-analytical solution

$$\frac{S_3}{T} = \frac{a}{T\lambda^{3/2}} \frac{8\pi\sqrt{\delta} (\beta_1\delta + \beta_2\delta^2 + \beta_3\delta^3)}{81(2-\delta)^2}, \text{ where } \delta \equiv \frac{8\lambda m^2}{a^2}$$

Adams '93

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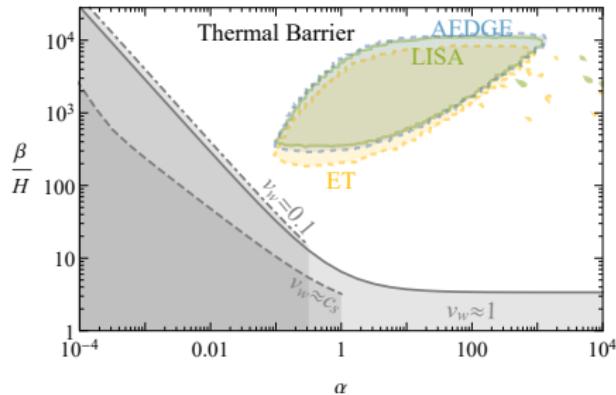
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Adams '93

- Useful for high temperature expansion

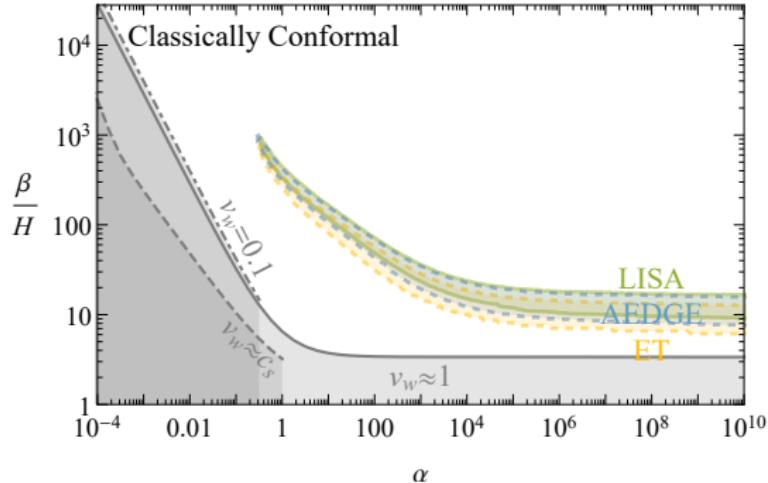
$$V(\phi, T) = \frac{g_m^2}{24} (T^2 - T_0^2) \phi^2 - \frac{g_m}{12\pi} T \phi^3 + \lambda \phi^4, \quad T_0^2 > 0$$



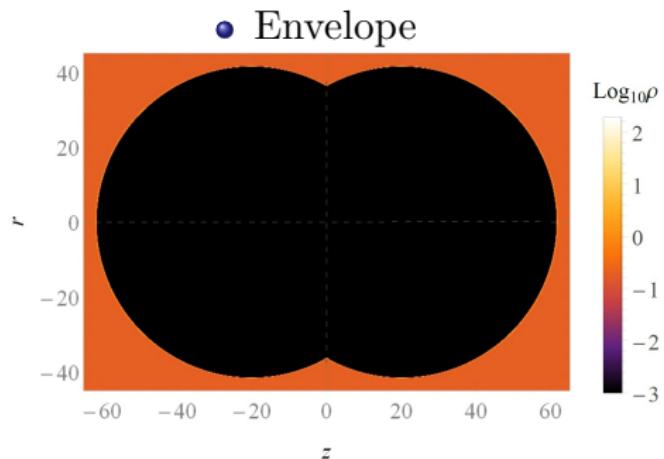
# Classically scale-invariant CW-like potential

- Generic classically scale-invariant potential

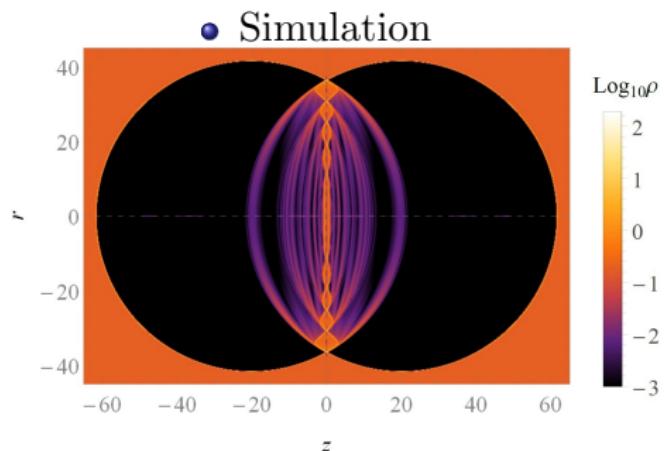
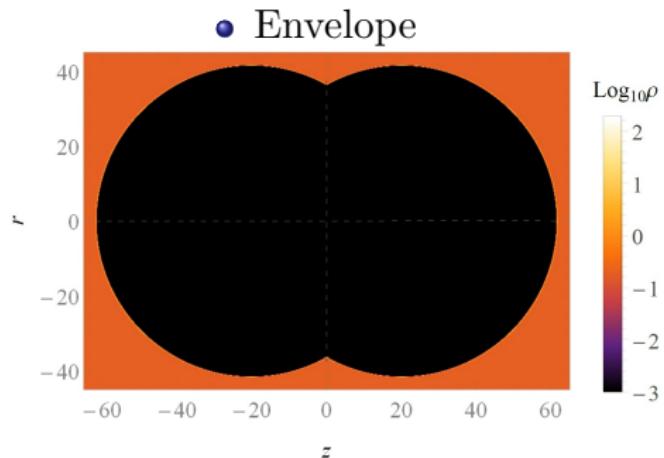
$$V(\phi, T) = g^2 T^2 \phi^2 + \frac{3g^4}{4\pi^2} \phi^4 \left( \log\left(\frac{\phi^2}{v^2}\right) - \frac{1}{2} - \frac{g^2 T^2}{2v^2} \right)$$



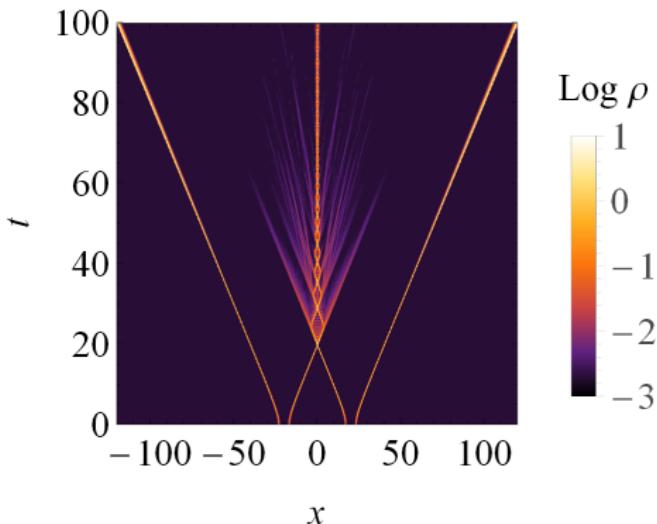
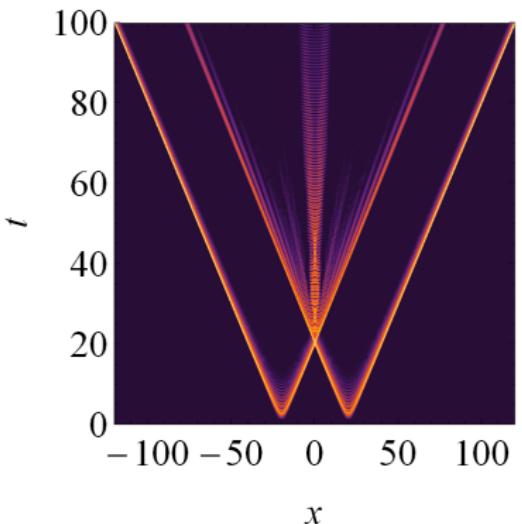
# Bubble Collisions



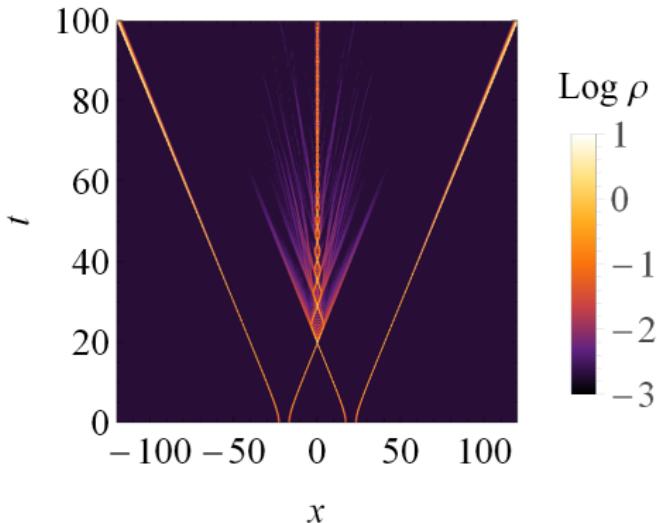
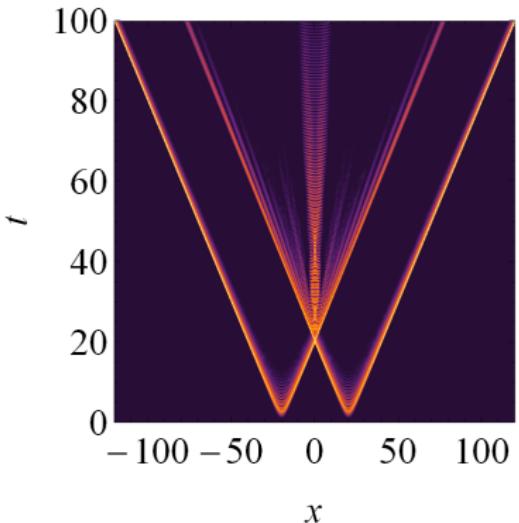
# Bubble Collisions



# Vacuum Trapping



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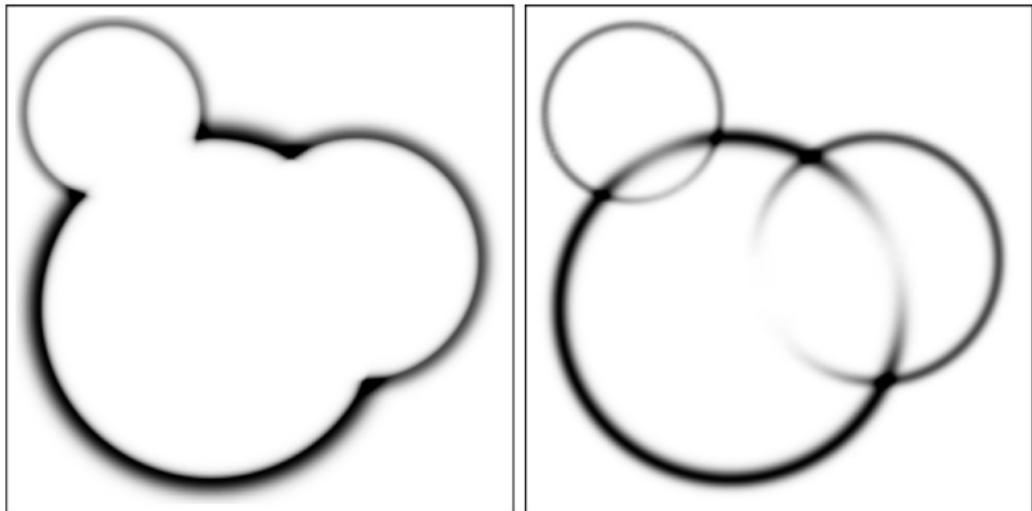


- scale-invariant vs. polynomial
- can also be verified analytically:  
R. Jinno, T. Konstandin and M. Takimoto: 1906.02588

# Computation of the GW spectrum

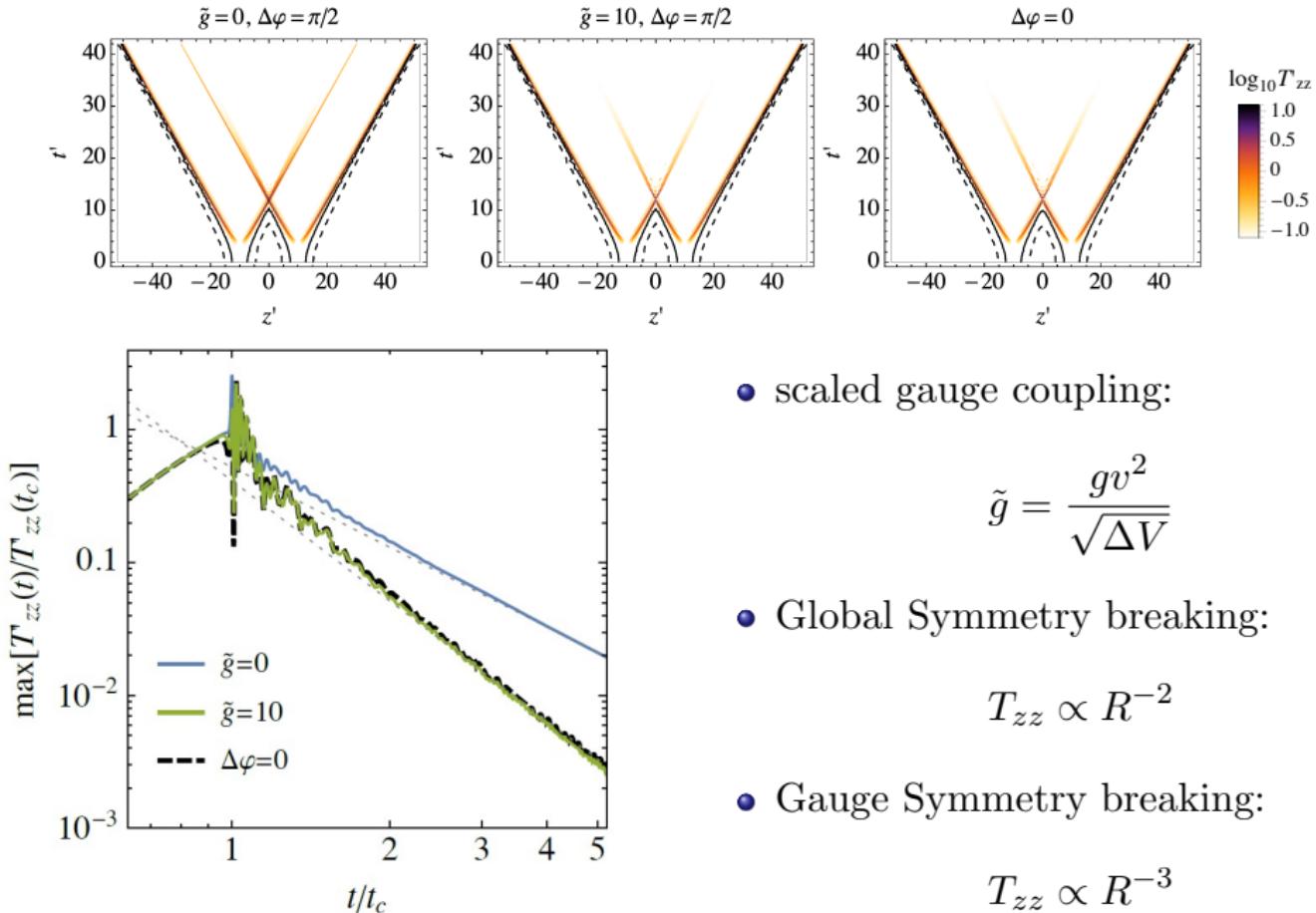
- Bulk Flow model:  $E \propto R^{-2}$

R. Jinno and M. Takimoto 1707.03111, T. Konstandin 1712.06869

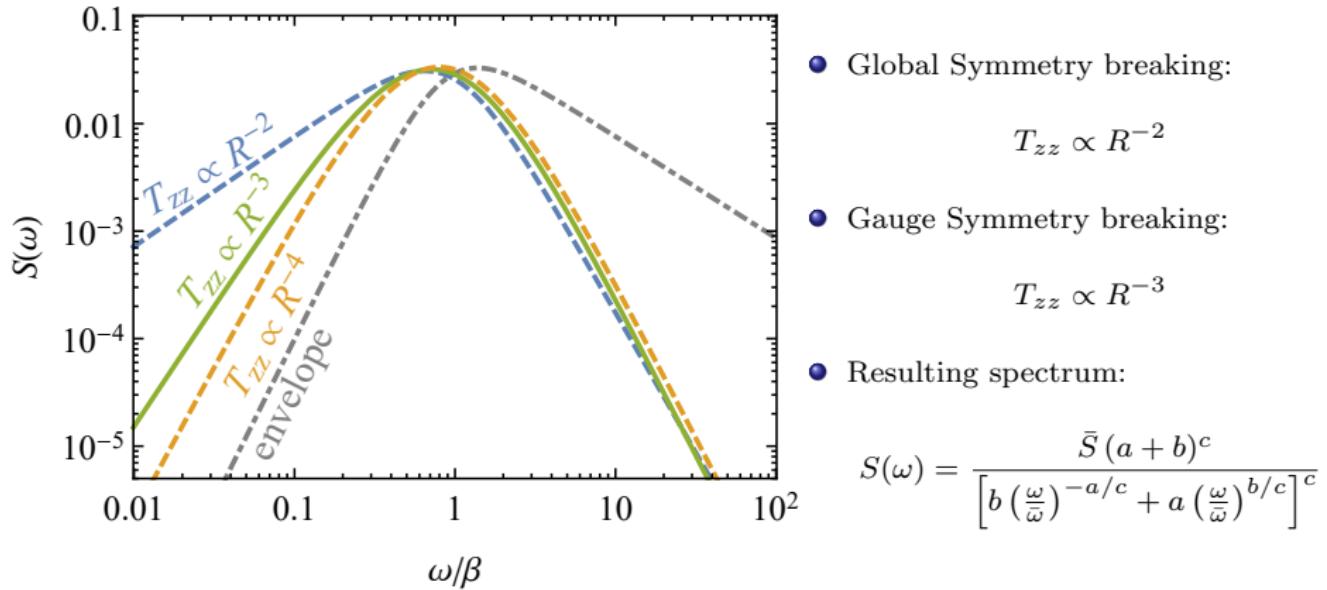


- We follow a similar approach using scaling from lattice simulations

# Abelian Higgs Model: Energy Scaling



# Bubble Collision Spectrum



	$100\bar{S}$	$\bar{\omega}/\beta$	$a$	$b$	$c$
$T_{zz} \propto R^{-2}$	$3.1 \pm 0.1$	$0.64 \pm 0.01$	$1.00 \pm 0.01$	$2.61 \pm 0.06$	$1.5 \pm 0.1$
$T_{zz} \propto R^{-3}$	$3.2 \pm 0.1$	$0.71 \pm 0.01$	$2.25 \pm 0.02$	$2.94 \pm 0.02$	$3.5 \pm 0.1$
$T_{zz} \propto R^{-4}$	$3.3 \pm 0.1$	$0.80 \pm 0.01$	$2.78 \pm 0.02$	$2.91 \pm 0.02$	$3.9 \pm 0.1$
env.	$3.3 \pm 0.1$	$1.38 \pm 0.03$	$3.01 \pm 0.01$	$0.94 \pm 0.03$	$1.5 \pm 0.1$

# Conclusions

- Observable bubble collision GW signal requires very significant supercooling  $\alpha > 10^{10}$ .
  - Observing a bubble collision signal would indicate a scale invariant potential for the field undergoing the transition.
- Shape of the spectrum encodes details of the underlying particle physics models
  - Less steep spectrum at low frequencies  $\Omega \propto f$  would point to global symmetry breaking while more steep spectra  $\Omega \propto f^{2.2}$  would indicate breaking of a gauge symmetry.