Cosmological bubble friction in local equilibrium

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in collaboration with...

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The aim:

Provide **new understanding** and **explicit calculations** for **bubble friction in local equilibrium**

The novelty:

We **relate previous results** in the literature and provide a new **understanding** in terms of **entropy conservation**

We confirm directly the friction effect by studying time-dependent solutions, and **relate local friction** to the **field-dependence of enthalpy**

We also illustrate the effect for **detonations** in the wall frame

The plan:

Usual understanding of bubble friction

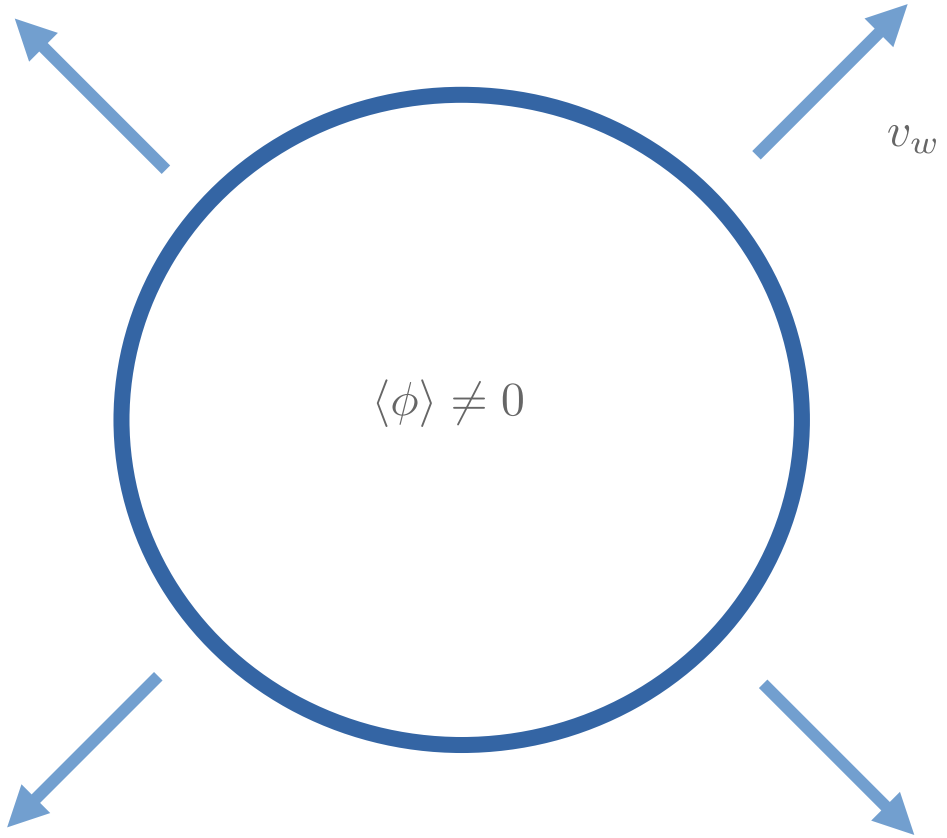
Friction in local equilibrium: previous literature

Friction in local equilibrium from local stress-energy conservation

Numerical studies

Usual understanding of bubble friction

Bubble basics

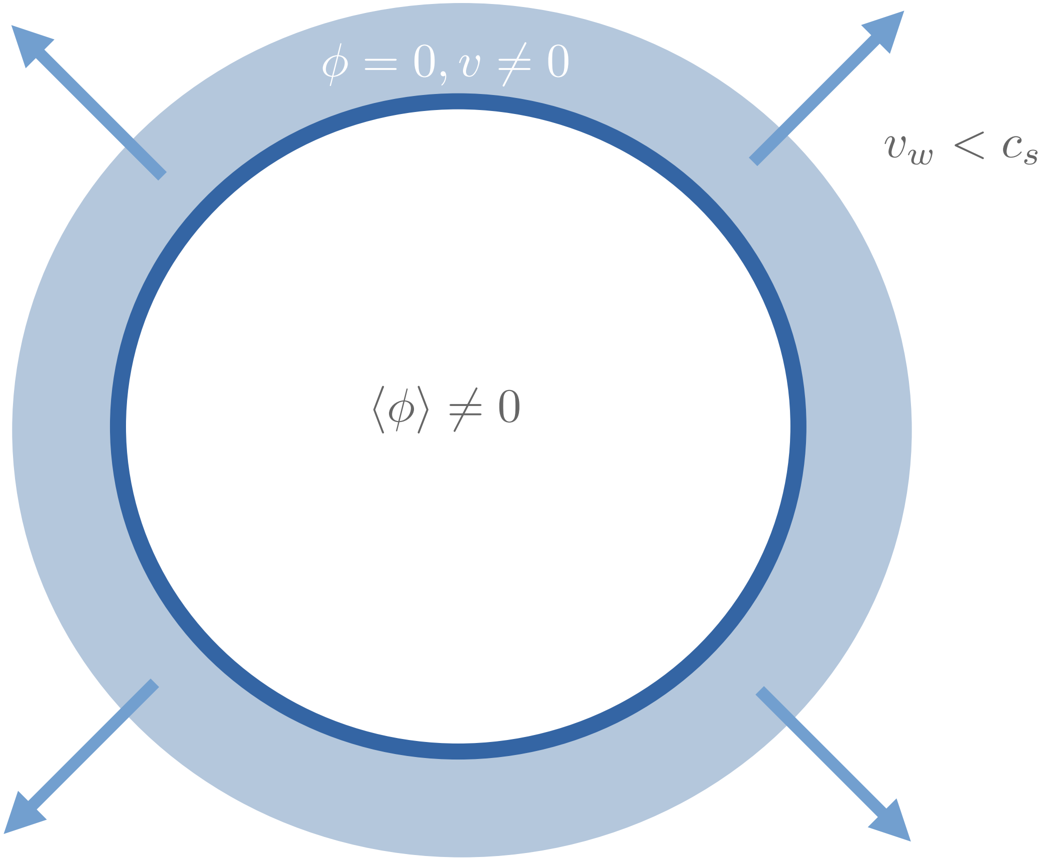


v_w

$\langle\phi\rangle \neq 0$

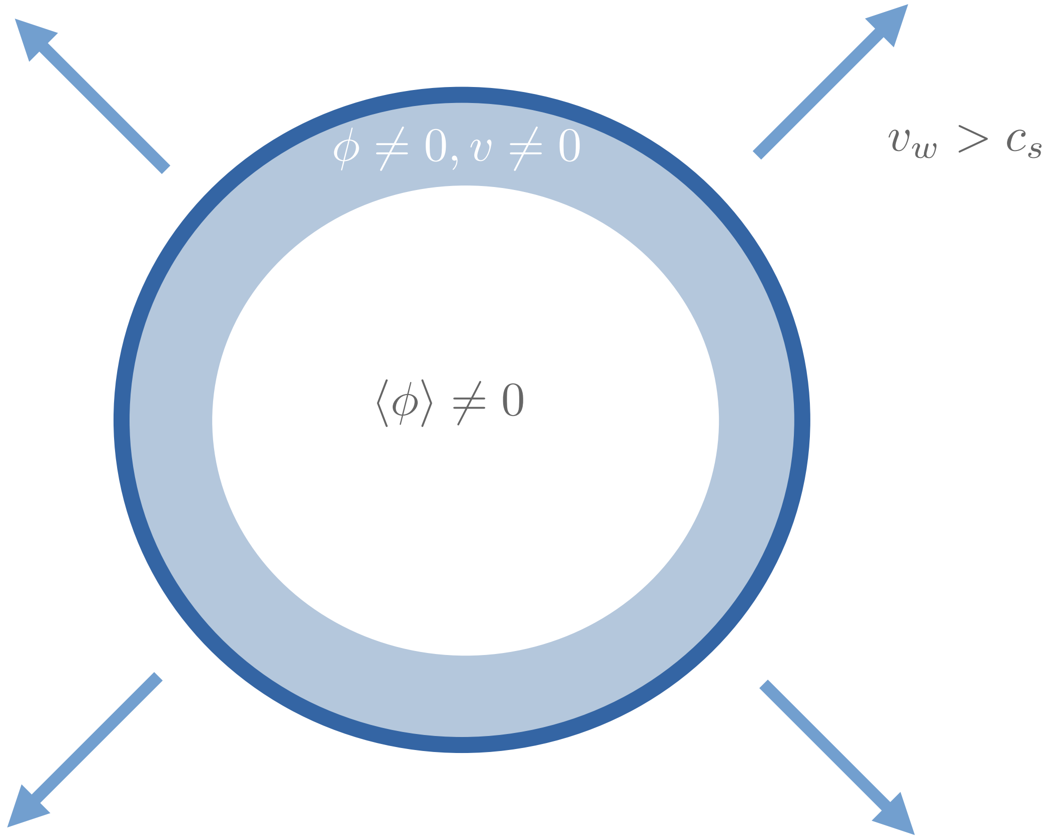
$\langle\phi\rangle = 0, \quad T = T_{\text{nuc}}$

Bubble basics: deflagration



$\langle \phi \rangle = 0, \quad T = T_{\rm nuc}$

Bubble basics: detonation



$\phi \neq 0, v \neq 0$

$v_w > c_s$

$\langle\phi\rangle \neq 0$

$\langle\phi\rangle = 0, \quad T = T_{\rm nuc}$

Bubble basics

- **Without friction** effects, latent heat converts to bulk motion and bubbles are expected to **accelerate** towards luminal speeds (**runaway** bubbles)

- **Colliding bubbles** source **gravitational waves**

Large bubble velocities imply more energy available for conversion into gws

- **Higgs bubbles** could lead to **electroweak baryogenesis**, usually requiring **low speeds**

It is crucial to understand friction!

Friction in the scalar equation of motion

- The usual treatment is based on the scalar equation of motion, averaged in plasma

$$\Box\phi + \frac{\partial V(\phi)}{\partial\phi} + \sum_i \frac{dm_i^2}{d\phi}\int\frac{d^3\mathbf{p}}{(2\pi)^3 2E_i} f_i(\mathbf{p},x) = 0$$

[Prokopec-Moore '95]

- For particles in equilibrium one recovers the finite T effective potential

$$f_i(\mathbf{p},x) = f_i^{\mathrm{eq}}(\mathbf{p}) + \delta f_i(\mathbf{p},x)$$

$$\Box\phi + \frac{\partial V_T(\phi,T)}{\partial\phi} + \sum_i \frac{dm_i^2}{d\phi}\int\frac{d^3\mathbf{p}}{(2\pi)^3 2E_i} \boldsymbol{\delta \mathbf{f}_\mathbf{i}(\mathbf{p},\mathbf{x})} = 0$$

Friction effect from deviations of equilibrium

Friction force per unit area

$$\left(\Box\phi + \frac{\partial V_T(\phi,T)}{\partial\phi} + \sum_i \frac{dm_i^2}{d\phi}\int\frac{d^3\mathbf{p}}{(2\pi)^3 2E_i}\delta f_i(\mathbf{p},x)\right) = 0$$

Friction force per unit area

$$\int dz \frac{d\phi}{dz}\left(\Box\phi + \frac{\partial V_T(\phi,T)}{\partial\phi} + \sum_i \frac{dm_i^2}{d\phi}\int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_i}\delta f_i(\mathbf{p},x)\right) = 0$$

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(In the static wall frame)

$$\Delta V_T = -\sum_i \int d\phi \frac{dm_i^2}{d\phi} \int \frac{d^3\mathbf{p}}{(s\pi)^3 2E_i} \delta f_i(\mathbf{p}, x)$$

Driving force (pressure from change in potential energy at equilibrium)

Friction per unit area, out of eq. [Bödeker-Moore]

Friction force per unit area

$$\int dz \frac{d\phi}{dz}\left(\Box\phi + \frac{\partial V_T(\phi,T)}{\partial\phi} + \sum_i \frac{dm_i^2}{d\phi}\int\frac{d^3\mathbf{p}}{(2\pi)^3 2E_i}\delta f_i(\mathbf{p},x)\right) = 0$$

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Driving force (pressure from change in potential energy at equilibrium)

Friction per unit area, out of eq. [Bödeker-Moore]

Alternatively, assuming an **ultrarrelativistic wall**, f does not change (no reflection)

$$\Delta V_{\rm vac} = -\sum_i \int \frac{d^3\mathbf{p}}{(s\pi)^3} f_i(\mathbf{p},z)\Delta p_z$$

Vacuum driving force

Total force from plasma, incl. friction [Bödeker-Moore] c.f. Jessica Turner's talk

Friction in local equilibrium?

- It would seem that **constant v_w** for $T < T_c$ requires **non-equilibrium effects.** For **relativistic bubbles:**
  - **Leading order** friction v_w-independent: allows **runaways** [Bödeker-Moore]
  - Higher order effects v_w-dependent: ultrarelativistic but **subluminal** speeds

    [Bödeker-Moore] [Höche, Kozaczuk, Long, Turner, Wang]

- It is **commonly assumed** that there is **no friction in local equilibrium**

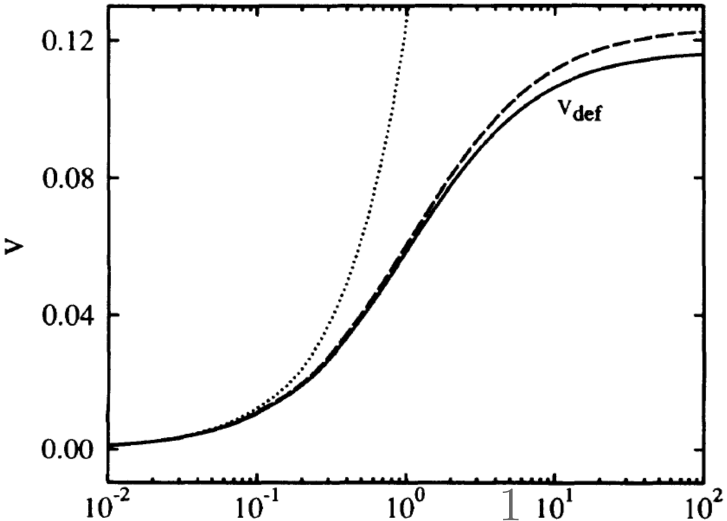
Local equilibrium: previous literature

Friction in local equilibrium?

Phenomenological friction term

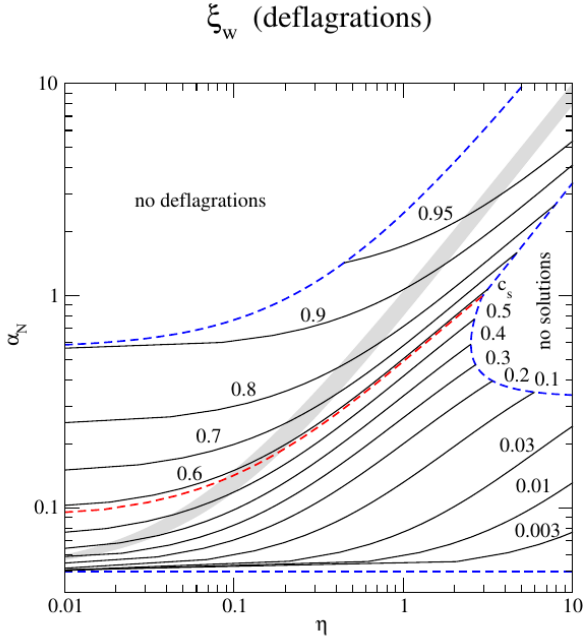
$$\Box\phi + \frac{\partial V_T(\phi, T)}{\partial \phi} + \eta\, u^\mu \partial_\mu \phi = 0$$

[Ignatius, Kajantie, Kurki-Suonio, Laine ‘93]



$\Gamma = \frac{1}{\eta}$

[Espinosa, Konstandin, No, Servant ‘10]



Subluminal velocity for **deflagrations without friction**?

Friction in local equilibrium

[Konstandin, No ‘10]

- First direct study of **bubble velocity** in **local equilibrium**.

- Subluminal velocities as a result of hydrodynamic equations causing the **fluid** to **heat up** in front of the bubbles, which **reduces driving force**

- Effect thought to happen **only** in **deflagrations**

Friction in local equilibrium

[Barroso Mancha, Prokopec, Świeżewska ‘20]

- **Stress-energy conservation** plus **Lorentz invariance**, away from bubble wall

$$\begin{aligned} T^{\mu\nu}_{\text{plasma}} &= (\rho + p)u^\mu u^\nu - p\eta^{\mu\nu} \\ &= Tsu^\mu u^\nu - p\eta^{\mu\nu} \end{aligned}$$

$$T^{\mu\nu}_{\phi} = \eta^{\mu\nu} V(\phi)$$

$$\langle \Delta T^{zz}_{\phi} \rangle + \langle \Delta T^{zz}_{\text{plasma}} \rangle = 0$$

$$\underbrace{-\Delta p + \Delta V_\phi}_{\text{Driving force}} = \underbrace{(\gamma^2 - 1)T\Delta s = \frac{F_{\text{fr}}}{A}}_{\text{Friction}}$$

- **No distinction** between **detonations** and **deflagrations**
- **Friction grows with** v_w: **no runaway** behaviour
- D.o.f. in local equilibrium lead to larger friction than usually expected

Open questions

- Is the **hydrodynamic obstruction** of [Konstandin, No] the **same** effect as the **friction force** of [Barroso Mancha, Prokopec, Świeżewska] ?

- If so, can one **extend results** of [Konstandin, No] to **detonations**?

- **Where is friction encoded** in the time-dependent, **differential equations** for the scalar and plasma?

Friction in equilibrium from local stress-energy conservation

Local stress-energy conservation

$$T^{\mu\nu} = T^{\mu\nu}_{\phi} + T^{\mu\nu}_{p}$$

$$T^{\mu\nu}_{\phi} = \partial^{\mu}\phi\partial^{\nu}\phi - \eta^{\mu\nu}\left(\frac{1}{2}\partial_{\rho}\phi\partial^{\rho}\phi - V(\phi)\right)$$

$$T^{\mu\nu}_{p} = (\rho + p)u^{\mu}u^{\mu} - \eta^{\mu\nu}p = \omega\, u^{\mu}u^{\mu} - \eta^{\mu\nu}p$$

$$\nabla_{\mu}T^{\mu\nu} = 0$$

$$\Box\phi + \frac{\partial}{\partial\phi}(V(\phi) - p) = 0,$$

$$\partial_{\mu}(\omega u^{\mu}u^{\nu} - \eta^{\mu\nu}p) + \frac{\partial p}{\partial\phi}\partial^{\nu}\phi = 0.$$

[Ignatius, Kajantie, Kurki-Suonio, Laine '93]

Friction-like behaviour comes from field-dependence of $\omega = Ts$

It is all about pressure

- **Pressure** ⟷ free-energy density ⟷ **finite T corrections to potential**

$$p = -\Delta_T V \equiv -(V_T(\phi, T) - V(\phi))$$

- Calculable in arbitrary model from finite *T* field theory

- Standard **thermodynamical identities relate entropy/enthalpy to pressure**

$$\omega(\phi, T) = T\,s = T\,\frac{\partial p}{\partial T} = -T\,\frac{\partial V_T(\phi, T)}{\partial T}$$

- **Everything follows from the thermal effective potential!**

Matches direct computations of [Barroso Mancha, Prokopec, Świeżewska]

Total entropy conservation

$$\left(\partial_\mu(\omega u^\mu u^\nu - \eta^{\mu\nu} p) + \frac{\partial p}{\partial \phi}\partial^\nu \phi\right) = 0$$

Total entropy conservation

$$u_\nu \left(\partial_\mu (\omega u^\mu u^\nu - \eta^{\mu\nu} p) + \frac{\partial p}{\partial \phi} \partial^\nu \phi \right) = 0 \qquad \Rightarrow \partial_\mu (s u^\mu) = 0$$

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- Integrate over spatial volume with fluid at rest at the boundary:

$$\frac{dS}{dt} = 0, \quad S = \int d^3 x \, s \, \gamma(v)$$

Total entropy conservation

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- Integrate over spatial volume with fluid at rest at the boundary:

$$\frac{dS}{dt} = 0, \quad S = \int d^3x \, s \, \gamma(v)$$

- **Entropy density** dominated by **relativistic degrees of freedom**

$$s = \frac{2\pi^2}{45} g_{\star s} T^4$$

- **Phase transition** makes some d.o.f heavy: **local decrease in entropy density** from decrease in $g_{\star s}$

- This has to be **compensated by a heating effect** in front or behind the bubble wall

→ Connection to [Konstandin, No], but **should also apply to detonations**

Planar wall frame

Assuming stationary regime in the wall frame $v^z \equiv \mathfrak{v}$

$$-\phi''(z) + \frac{\partial}{\partial\phi}(V(\phi,T)) = 0,$$

$$\omega\gamma^2\mathfrak{v}^2 + \frac{1}{2}(\phi'(z))^2 - V(\phi,T) = c_1,$$

$$\omega\gamma^2\mathfrak{v} = c_2,$$

Also solved in [Konstandin, No]
cf [Espinosa, Konstandin, No, Servant]

From the second equation, comparing 2 sides of the wall where $\phi' = 0$

$$\boxed{\Delta V(\phi,T) = -\Delta p + \Delta V(\phi) = \Delta((\gamma^2-1)Ts) = \frac{F_{\text{fr}}}{V}}$$

Friction force of [Barroso Mancha, Prokopec, Świeżewska] **recovered** when **assuming constant** v, T **across wall**

→ Same effect as hydrodynamic obstruction of [Konstandin, No]

Reduction to single scalar equation

$$-\phi''(z) + \frac{\partial}{\partial\phi}(V(\phi, T)) = 0,$$

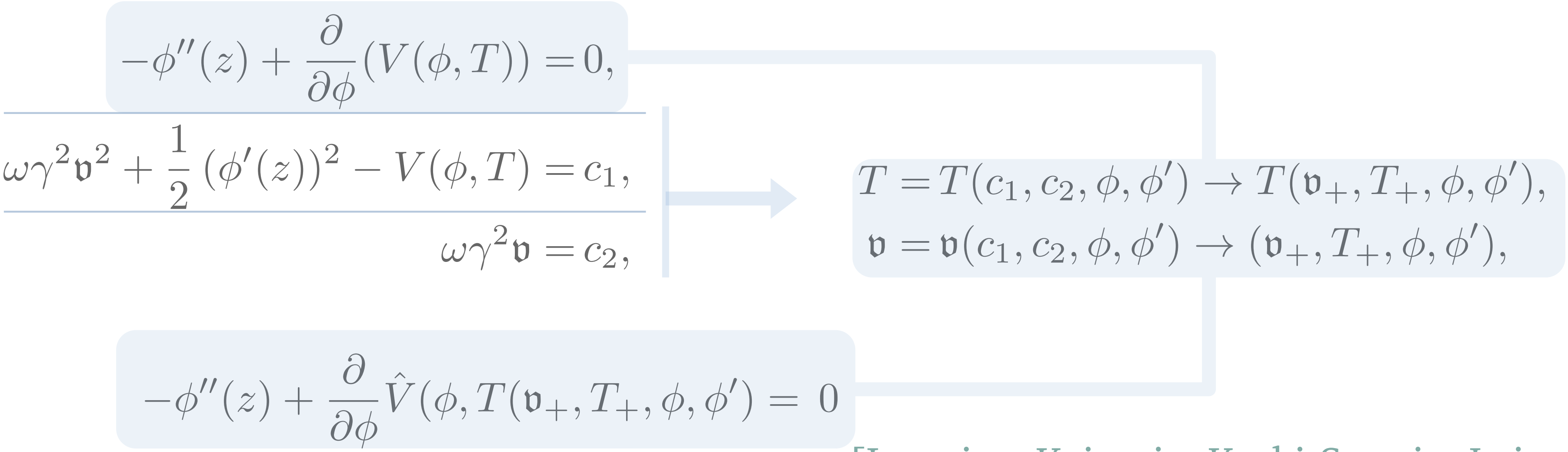
$$\omega\gamma^2\mathfrak{v}^2 + \frac{1}{2}(\phi'(z))^2 - V(\phi, T) = c_1,$$

$$\omega\gamma^2\mathfrak{v} = c_2,$$

$$\Longrightarrow$$

$$\begin{aligned} T &= T(c_1, c_2, \phi, \phi') \rightarrow T(\mathfrak{v}_+, T_+, \phi, \phi'), \\ \mathfrak{v} &= \mathfrak{v}(c_1, c_2, \phi, \phi') \rightarrow (\mathfrak{v}_+, T_+, \phi, \phi'), \end{aligned}$$

Reduction to single scalar equation



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$$\omega\gamma^2\mathfrak{v} = c_2,$$

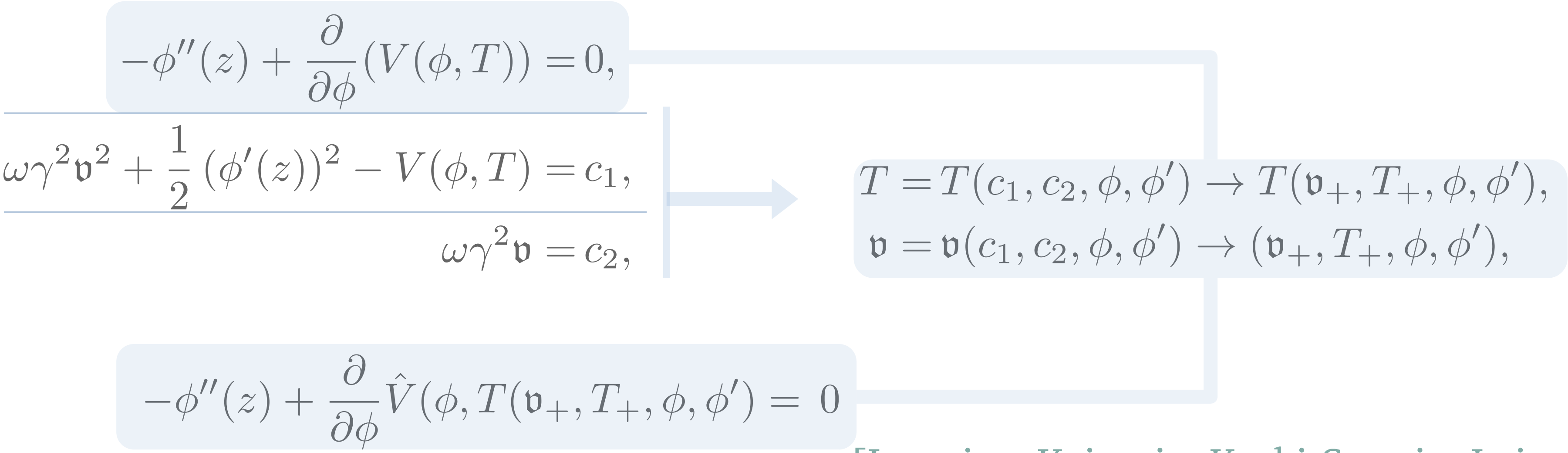
$$T = T(c_1, c_2, \phi, \phi') \to T(\mathfrak{v}_+, T_+, \phi, \phi'),$$

$$\mathfrak{v} = \mathfrak{v}(c_1, c_2, \phi, \phi') \to (\mathfrak{v}_+, T_+, \phi, \phi'),$$

$$-\phi''(z) + \frac{\partial}{\partial\phi}\hat{V}(\phi, T(\mathfrak{v}_+, T_+, \phi, \phi') = 0$$

[Ignatius, Kajantie, Kurki-Suonio, Laine]

Reduction to single scalar equation



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$$-\phi''(z) + \frac{\partial}{\partial\phi}\hat{V}(\phi, T(\mathfrak{v}_+, T_+, \phi, \phi') = 0$$

[Ignatius, Kajantie, Kurki-Suonio, Laine]

Boundary conditions

$$\phi(z) \rightarrow 0, \quad z \rightarrow \infty, \quad \phi'(z) \rightarrow 0, \quad |z| \rightarrow \infty,$$

Additionally, expect that in **broken phase** field goes to a **minimum** [Konstandin, No]

$$\phi''(z) \rightarrow 0, \quad z \rightarrow -\infty$$

These conditions fix $\mathfrak{v}_+$ in terms of T_+. Latter fixed by nucleation temperature away from wall (accounting from extra hydrodynamic profile for deflagrations)

Numerical studies

Example model

SM extension by N additional **complex singlet**s allowing for **first order phase transition** for the Higgs

$$\mathcal{L} \supset -m_H^2 \Phi^\dagger \Phi - \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 - m_\chi^2 \chi^\dagger \chi - \frac{\lambda_\chi}{2}(\chi^\dagger\chi)^2 - \lambda_{H\chi}\Phi^\dagger\Phi\chi^\dagger\chi \, .$$

Higgs

Extra scalars

Pressure from **thermal corrections to potential** in high-T expansion

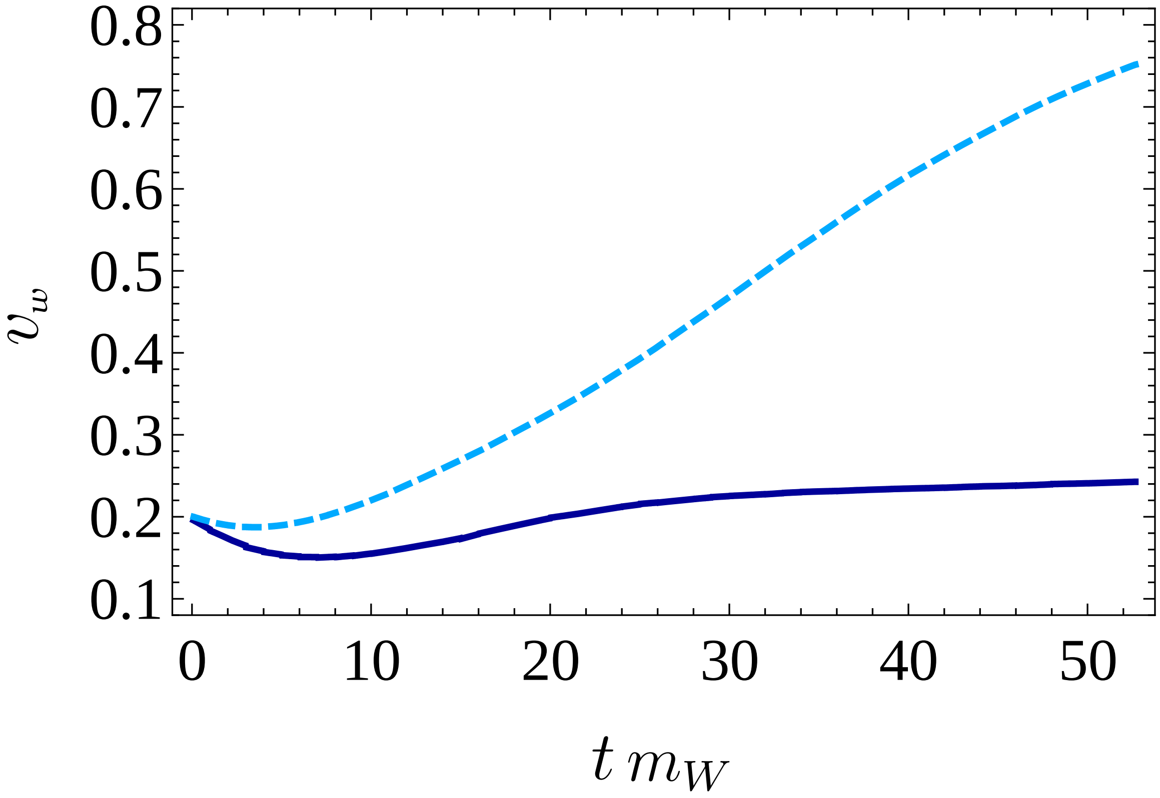
$$p(h,T) =$$

$$\frac{\pi^2 T^4}{90}(g_{*,\mathrm{SM}} + 2N) - T^2 \left(h^2 \left(\frac{y_b^2}{8} + \frac{3g_1^2}{160} + \frac{3g_2^2}{32} + \frac{\lambda}{8} + \frac{N\lambda_{H\chi}}{24} + \frac{y_t^2}{8} \right) + \frac{m_H^2}{6} + \frac{N m_\chi^2}{12} \right)$$

$$- \frac{T}{12\pi} \left(-\frac{3}{4}(g_2 h)^3 - \frac{3h^3}{8}\left(\frac{3g_1^2}{5} + g_2^2 \right)^{3/2} - 3\left(\frac{h^2\lambda}{2} + m_H^2 \right)^{3/2} - \left(\frac{3h^2\lambda}{2} + m_H^2 \right)^{3/2} \right.$$

$$\left. -2N \left(\frac{h^2 \lambda_{H\chi}}{2} + m_\chi^2 \right)^{3/2} \right)$$

Time-dependent deflagrations



Ignoring $\frac{\partial \omega}{\partial \phi} = \frac{\partial (Ts)}{\partial \phi}$

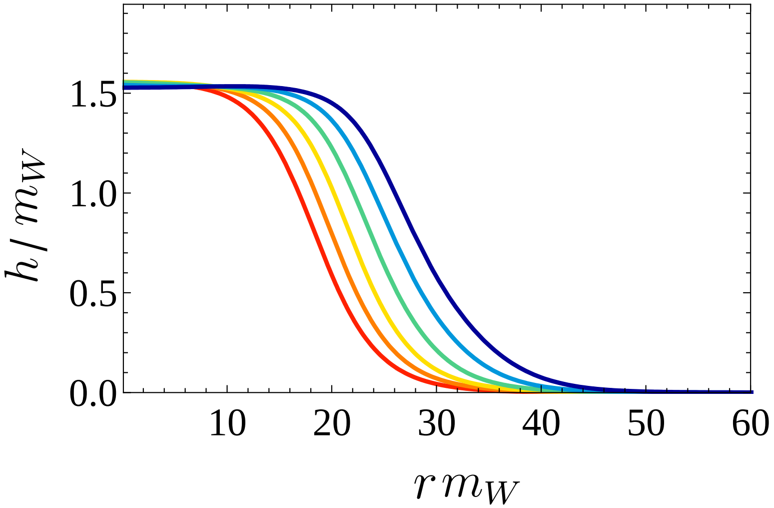
Accounting for $\frac{\partial \omega}{\partial \phi} = \frac{\partial (Ts)}{\partial \phi}$

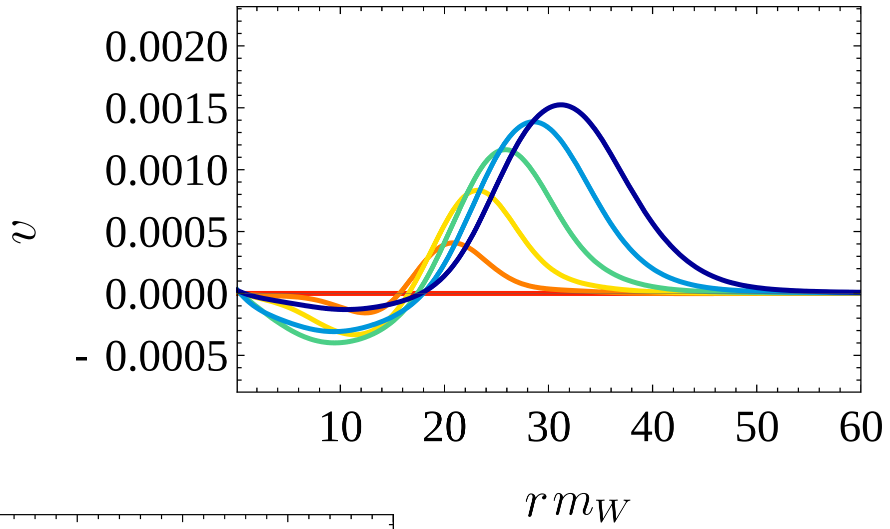
Friction-like behaviour!

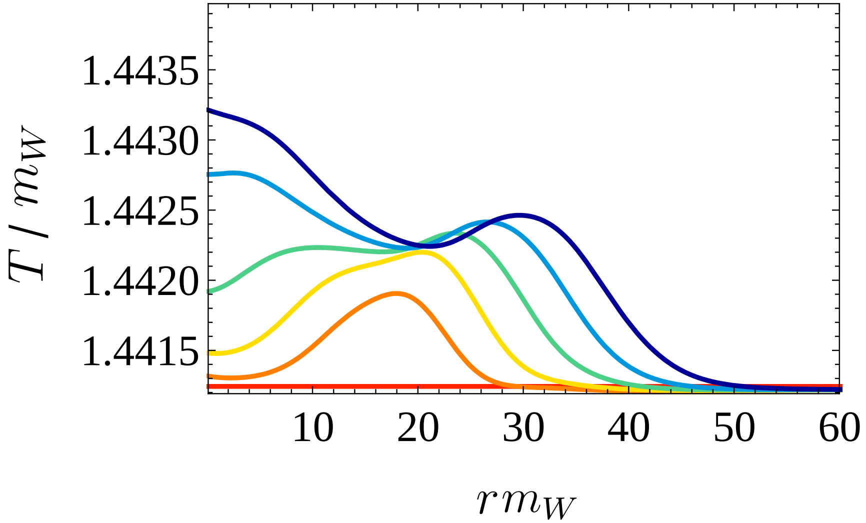
$$N = 4, \quad \frac{m_S^2}{m_W^2} = 0.0625, \quad \lambda_\chi = 0.085, \quad \lambda_{H\chi} = 0.85$$

- Obtained with neural network pre-trained with Mathematica solution

Time-dependent deflagrations



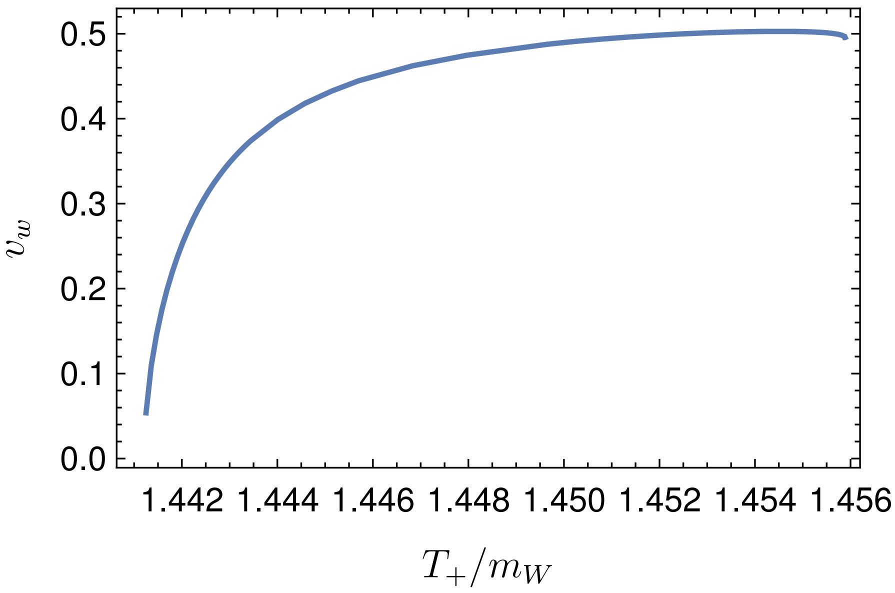


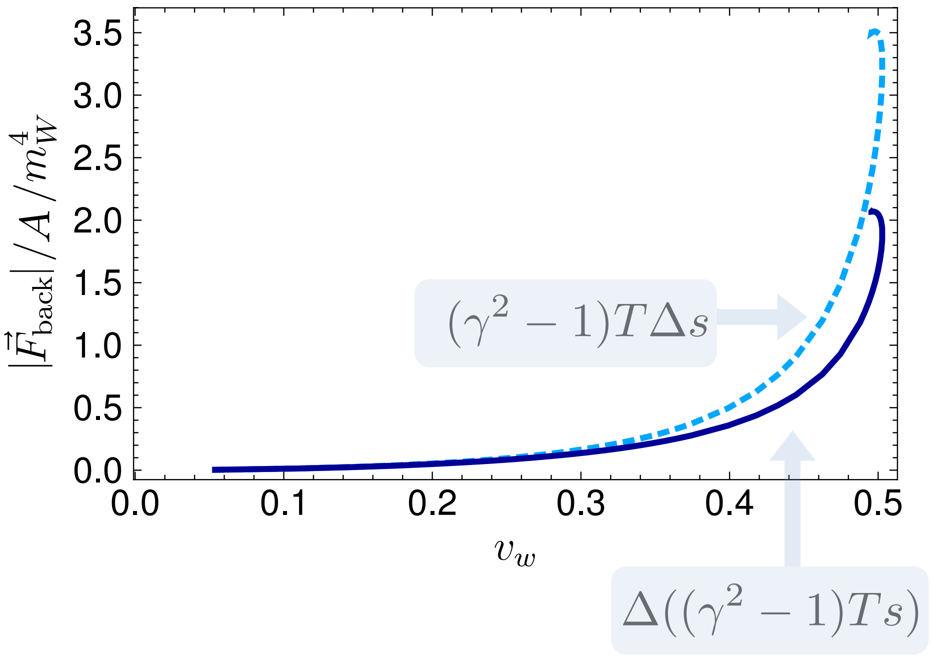


$$N = 4, \quad \frac{m_S^2}{m_W^2} = 0.0625, \quad \lambda_\chi = 0.085, \quad \lambda_{H\chi} = 0.85$$

Static deflagrations in wall frame

Family of solutions without necessarily imposing $\phi''(z) \to 0, \quad z \to -\infty$




$(\gamma^2-1)T\Delta s$

$\Delta((\gamma^2-1)Ts)$

Friction force grows with velocity!

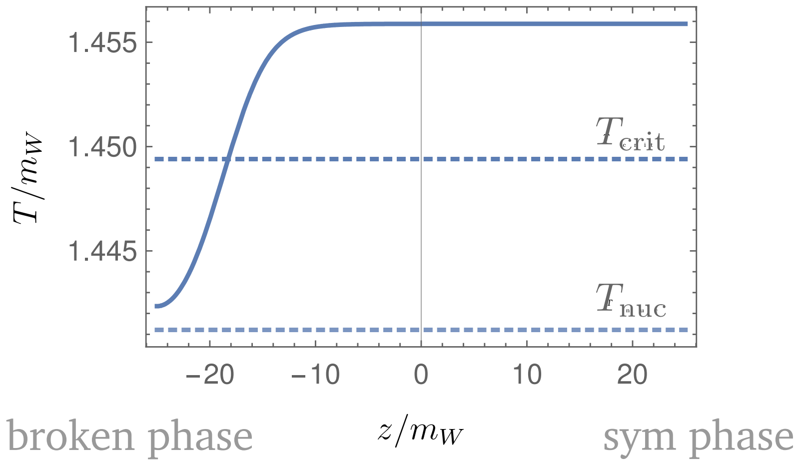
Physical case with $\phi''(-\infty) \to 0$ corresponds to right endpoint of curves

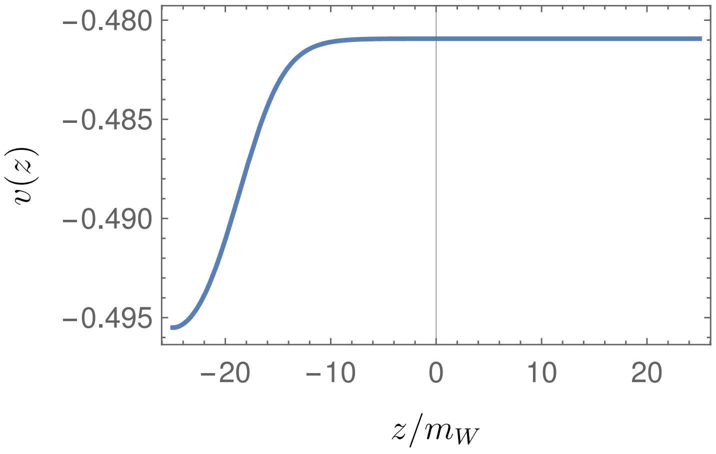
$$N = 4, \quad \frac{m_S^2}{m_W^2} = 0.0625, \quad \lambda_\chi = 0.085, \quad \lambda_{H\chi} = 0.85$$

Static deflagrations in wall frame

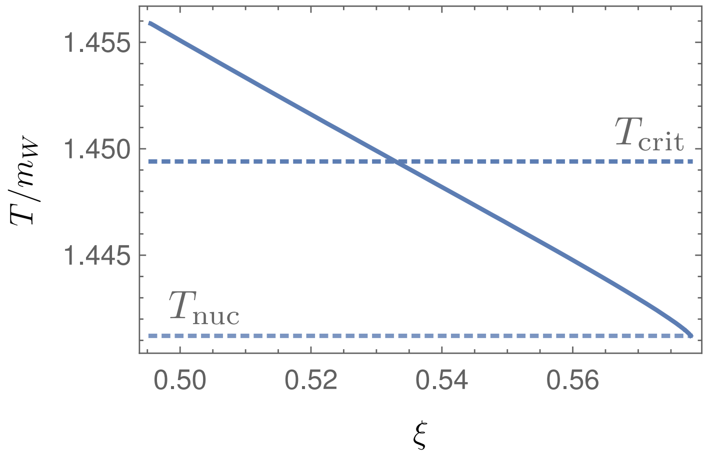
Physical solution:

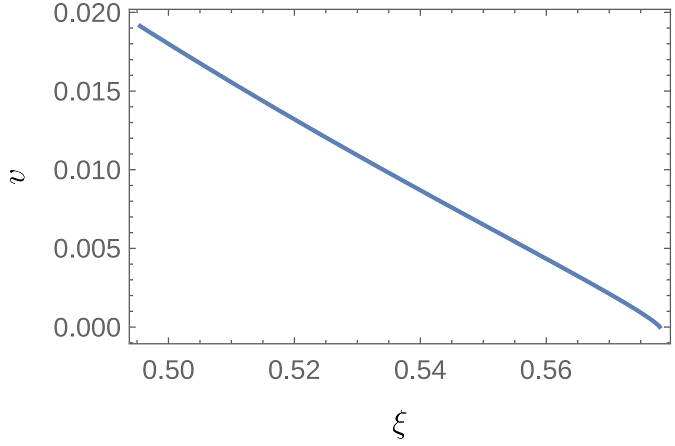
Static solution near wall





Self-similar hydrodynamic profile $(\xi = r/t)$





Novel static detonations in wall frame

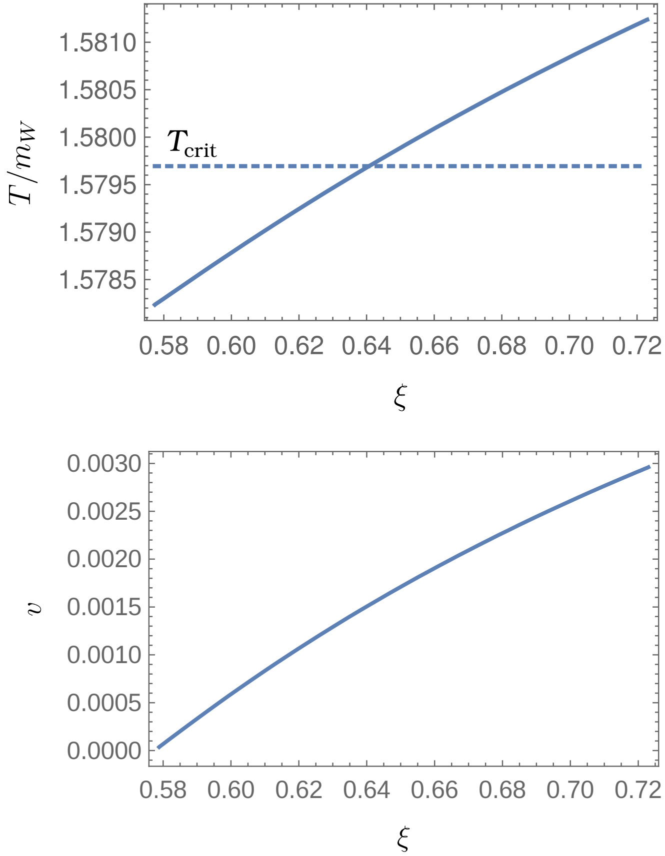
- The solutions $T(\mathfrak{v}_+, T_+, \phi, \phi'), \mathfrak{v}(\mathfrak{v}_+, T_+, \phi, \phi')$ are actually **multivalued,** and so is the "pseudopotential" $\hat{V}(\phi, T(\mathfrak{v}_+, T_+, \phi, \phi'))$

- We find that a branch of solutions with larger fluid velocities supports **static detonation solutions**

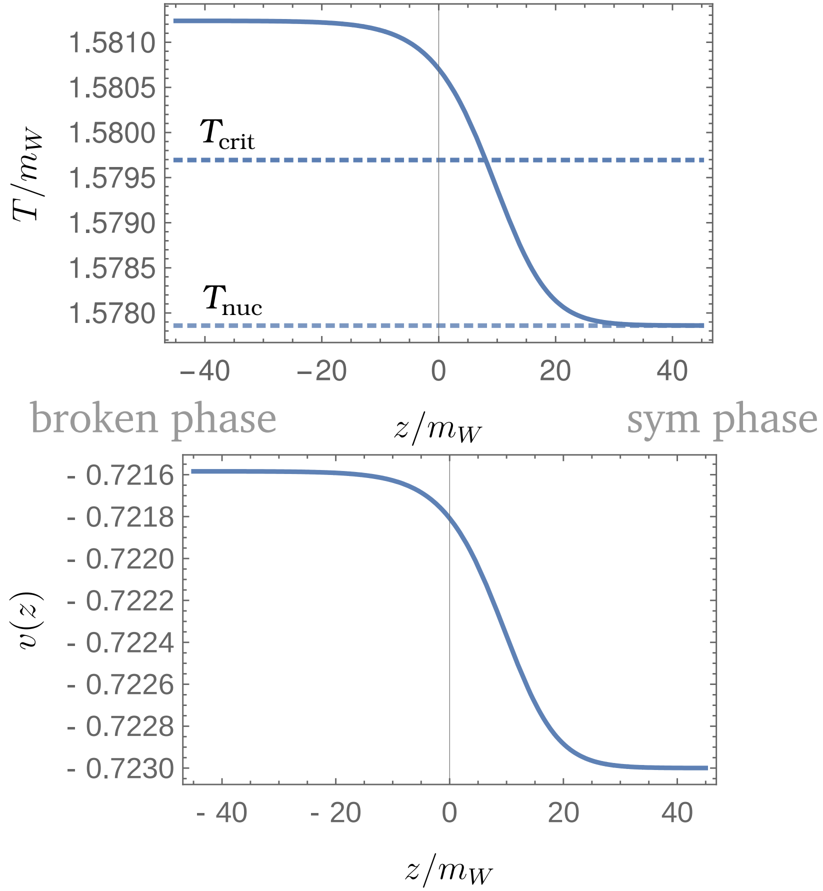
- We have found that the friction force can deviate from [Barroso Mancha et al] by a factor of 3

Static detonation solutions in wall frame

Self-similar hydrodynamic profile



Static solution near wall



$$N = 2, \quad \frac{m_S^2}{m_W^2} = 0.0625, \quad \lambda_\chi = 0.085, \quad \lambda_{H\chi} = 0.85$$

Conclusions

Even in **local equilibrium**, there is a **non-dissipative**, friction-like **backreaction effect**

This effect is behind the runaway obstruction of [Konstandin, No] and the friction force of [Barroso Mancha, Prokopec, Świeżewska]

We provided an **intuitive understanding** based on **entropy conservation**

By solving the time-dependent equations for bubble propagation, we showed that the **backreaction** is **generated locally** by the **field-derivatives of the enthalpy**

We showed that, as expected from the results of [Barroso Mancha et et al], the **backreaction exists for detonations**, yet **accurate estimates** of the friction force require **tracking changes** of $\mathfrak{v}, T$ across the bubble.

Thank you!