Reliable predictions for cosmological phase transitions

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Gravitational Wave Probes of Physics Beyond Standard Model
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Collaborators

Many important contributions for today’s talk due to:

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Cosmological first-order phase transitions

Figure: Cutting et al. arXiv:1906.00480.

- Transition dynamics
  - Bubbles nucleate, expand and collide
  - This creates long-lived fluid flows, and gravitational waves

- Observable remnants
  Such as \((n_B - n_{\bar{B}})/s\), stochastic gravitational wave backgrounds, topological defects, magnetic fields, 
  \[
  \Rightarrow \text{ new probe of particle physics}
  \]
Gravitational waves versus colliders

- LISA (2034)
- Taiji (early 2030s)
- DECIGO (>2030s)
- . . .

- LHC Run 3 (2022)
- High Lumi LHC (2027)
- Future Higgs factory (?)
- . . .
Gravitational waves from phase transitions: the pipeline

Figure: The Light Interferometer Space Antenna (LISA) pipeline $\mathcal{L} \rightarrow \text{SNR}(f)$, Caprini et al. arXiv:1910.13125
Sound predictions

- How reliable are current predictions?
- Where do uncertainties come from?
- How to overcome them?
Perturbative sensitivity

- GW spectra of first-order phase transitions in any given specific model are very sensitive to details of calculation.

Figure: Renormalisation scale dependence of GW spectrum at one parameter point in SMEFT, Croon et al. arXiv:2009.10080.
Unwrapping perturbative sensitivity

- $\Omega_{GW}$ depends very strongly on the phase transition parameters,

$$\Omega_{GW} \propto \frac{\Delta \theta^2}{T^8_*}.$$  

- Uncertainties in these parameters are themselves quite large

**Figure:** Theoretical uncertainties for $T_c$ at one benchmark point in the 2HDM, Niemi et al. arXiv:1904.01329.
Origins of theoretical uncertainties

- **Infrared enhancements at high-$T$**
  Due to the high occupancy of infrared bosons, the effective expansion parameter $\alpha_{\text{eff}}$ grows

  \[
  \alpha_{\text{eff}} \sim g^2 \frac{1}{1 - e^{p/T}} \approx g^2 \frac{T}{p},
  \]

  so lighter modes are more strongly coupled:

  - **hard** : $p \sim \pi T \Rightarrow \alpha_{\text{eff}} \sim g^2$
  - **soft** : $p \sim gT \Rightarrow \alpha_{\text{eff}} \sim g$
  - **ultrasoft** : $p \sim g^2 T \Rightarrow \alpha_{\text{eff}} \sim 1$

- **Effective field theory**
  3d EFTs provide a means to organise calculations involving these different modes and couplings. (See Weir’s talk.)

  Farakos et al. ’94, Braaten & Nieto ’95, Kajantie et al. ’95
Lattice vs perturbation theory
The theory

- Real, singlet scalar extension of the SM ($x$SM):

\[ \mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{\text{singlet}} + \mathcal{L}_{\text{portal}}, \]

\[ \mathcal{L}_{\text{singlet}} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi), \]

\[ V(\phi) = \sigma \phi + \frac{1}{2}m^2 \phi^2 + \frac{1}{3!}g\phi^3 + \frac{1}{4!}\lambda\phi^4. \]

Focus on phase transition in the singlet direction.

- The 3d EFT:

\[ \mathcal{L}_3 = \frac{1}{2}(\partial_i \phi_3)^2 + V_3(\phi_3), \]

\[ V_3(\phi_3) = \sigma_3 \phi_3 + \frac{1}{2}m^2_3 \phi_3^2 + \frac{1}{3!}g_3 \phi_3^3 + \frac{1}{4!}\lambda_3 \phi_3^4. \]

Can think of $\phi_3$ as the zero Matsuyura mode.
Lattice simulations

- Monte-Carlo simulations of 3d EFT sample the thermal distribution of field configurations, $p \propto e^{-H[\phi]/T}$.

- Efficient update algorithms known. \textcite{kajantieetal95}

- Superrenormalisability $\Rightarrow$ exact lattice-continuum relations. \textcite{laine95}

\begin{itemize}
  \item $g_3/\lambda_3^{3/2} = -0.3$, $a\lambda_3 = 2$
  \item $g_3/\lambda_3^{3/2} = -0.3$, $L\lambda_3 = 48$
\end{itemize}
Perturbative expansion in 3d EFT

- In general loops within the 3d EFT are suppressed by
  \[ \frac{\lambda_3}{m_3}, \quad \frac{g_3^2}{m_3^3}. \]

- Near $T_c$, the effective mass is $m_3 \sim |g_3|/\sqrt{\lambda_3}$, and hence the 3d loop expansion parameter is
  \[ \alpha_3 = \frac{\hbar}{(4\pi)} \frac{\lambda_3^{3/2}}{|g_3|}. \]

- This diverges as one approaches the $Z_2$-symmetric second-order transition $\Rightarrow$ perturbation theory breaks down completely.
Results: lattice versus perturbation theory

\[ \frac{1}{v_0} \Delta \langle \bar{\phi}_3 \rangle = 2 + \sqrt{3} \alpha_3 + \frac{1}{2} (1 + 2 \log \tilde{\mu}_3) \alpha_3^2 \]

\[ + \sqrt{3} \left[ -\frac{3}{8\sqrt{2}} \xi + \frac{21}{32} \text{Li}_2 \frac{1}{4} - \frac{7\pi^2}{128} - \frac{1}{2} + \frac{21}{64} \log^2 \frac{4}{3} + \frac{5}{8} \log \frac{4}{3} \right] \alpha_3^3 \]

\[ + O (\alpha_3^4) \]

OG arXiv:2101.05528
Implications

What does this teach us about this theory?

- (RG improved) perturbation theory is very accurate at high orders for $\alpha_3 \lesssim 1$
- EFT results can be applied to e.g. xSM

What about other 3d EFTs?

- Theories with two scale hierarchies? $\leftarrow$ typically $\alpha_3 \sim \lambda^{1/4}$
- Non-Abelian gauge theories? $\leftarrow$ high orders not computable

Neither of these are deal-breakers, so there is promise.
Perturbation theory and gravitational waves

Figure: Renormalisation scale dependence of GW spectrum at one parameter point in $Z_2$-xSM, OG & Tenkanen arXiv:2104.04399.
Conclusions

- Phase transitions may be observable by GW detectors
- Calculational developments necessary for reliable $\Omega_{GW}$ predictions
- Effective field theory provides suitable tools (see Weir's talk)
- For real scalar theory, high order perturbation theory agrees very well with lattice up to $\alpha_3 \lesssim 1$
- Promising for more difficult theories
Conclusions

- Phase transitions may be observable by GW detectors
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Thanks for listening!
Backup slides
QFT at high temperatures

Equilibrium thermodynamics can be formulated in $\mathbb{R}^3 \times S^1$.

Fields are expanded into Fourier (Matsubara) modes:

\[
\Phi(x, \tau) = \sum_{n \text{ even}} \phi_n(x) e^{i\pi T n \tau} \leftarrow \text{boson}
\]

\[
\Psi(x, \tau) = \sum_{n \text{ odd}} \psi_n(x) e^{i\pi T n \tau} \leftarrow \text{fermion}
\]

Effective masses of Matsubara modes are

\[
m_n^2 = m^2 + (n\pi T)^2
\]
Comparing uncertainties

▶ Renormalisation scale dependence appears to be the largest source of theoretical uncertainty, $\Delta \Omega_{GW}/\Omega_{GW} \sim 10^{2-3}$ in the SMEFT, and can be as large as $\sim 10^{10}$ in e.g. xSM.

▶ Some sources (e.g. inconsistencies) are hard to estimate.

Figure: Sources of theoretical uncertainty in $\Omega_{GW}$ for the SMEFT, Croon et al. arXiv:2009.10080. See also Guo et al. arXiv:2103.06933.
Loop versus coupling expansions

\[ V_{\text{eff}} = \#g^2 + \#g^3 + \#g^4 + \ldots \]

**Low temperature**

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<th>loop order</th>
<th>included</th>
<th>error</th>
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<td>five loop</td>
<td>(O(g^{12}))</td>
<td>(O(g^{14}))</td>
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<td>five loop*</td>
<td>(O(g^6 \ln g))</td>
<td>(O(g^6))</td>
</tr>
</tbody>
</table>

*resummed

Lowest order at which RG improvement is possible

OG & Tenkanen arXiv:2104.04399
Comparing orders

Dramatic improvements at $O(g^4)$

Figure: Unphysical renormalisation scale dependence of critical temperature at benchmark points in xSM, OG & Tenkanen arXiv:2104.04399.
Phase diagram of EFT

By making the following shift

$$\phi_3 \rightarrow -\frac{g_3}{\lambda_3} + \phi_3$$

the bare potential takes the form,

$$V_3 = \left( \sigma_3 + \frac{g_3^3}{3\lambda_3^2} - \frac{g_3 m_3^2}{\lambda_3} \right) \phi_3 + \frac{1}{2} \left( m_3^2 - \frac{g_3^2}{2\lambda_3} + \delta m_3^2 \right) \phi_3^2 + \frac{1}{4!} \lambda_3 \phi_3^4.$$
Results: lattice vs (unimproved) perturbation theory

\[ \alpha_3(\mu_3, L) = \frac{\bar{h}\lambda_3^{3/2}}{4\pi|g_3|} \]

\[ \Delta \langle \bar{\phi}_3 \rangle / \sqrt{\lambda_3} \]

Graph showing the comparison between lattice and perturbation theory results for the parameter \( \Delta \langle \bar{\phi}_3 \rangle / \sqrt{\lambda_3} \) as a function of \( \alpha_3(\mu_3, L) = \frac{\bar{h}\lambda_3^{3/2}}{4\pi|g_3|} \).

OG arXiv:2101.05528
Results: approaching the second-order phase transition