

Turbulent production of gravitational radiation from cosmological phase transitions

Gravitational Wave Probes of Physics Beyond Standard Model
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ARP *et al.*, *Geophys. Astrophys. Fluid Dyn.* **114**, 130 (2020), arXiv:1807.05479

ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020), arXiv:1903.08585

T. Kahniashvili *et al.*, *Phys. Rev. Res.* **3**, 013193 (2021), arXiv:2011.0556

ARP *et al.*, arXiv:2107.05356 (2021).

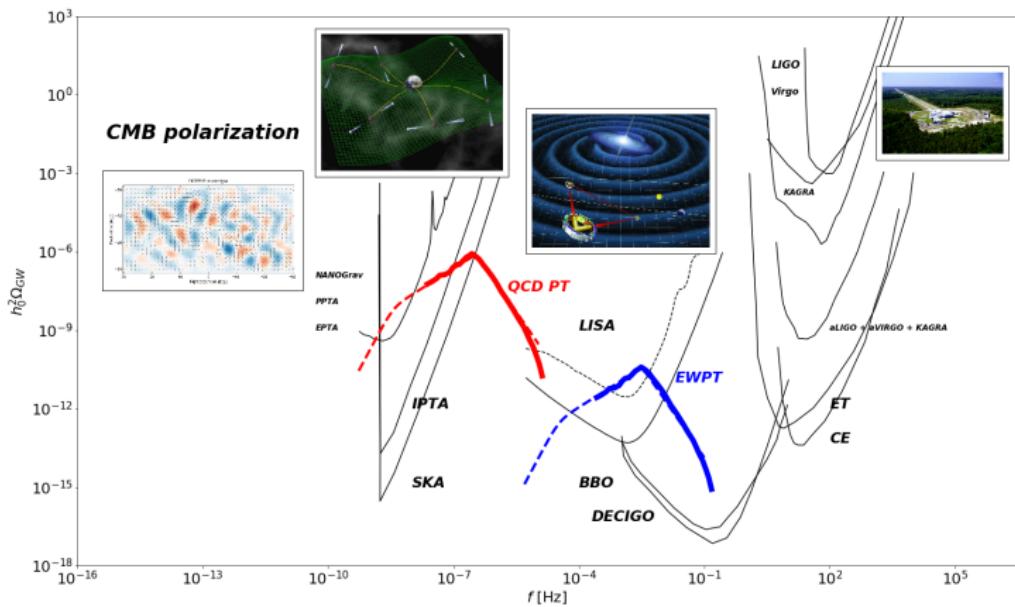
Overview

- 1 Introduction and Motivation
- 2 Primordial magnetic fields
- 3 Magnetohydrodynamics
- 4 Gravitational waves
- 5 Numerical Simulations

Introduction and Motivation

- Generation of cosmological GWs during phase transitions
 - LIGO-Virgo frequencies are 10–1000 Hz ($T \sim 10^7$ GeV)
Peccei-Quinn, B-L, left-right symmetries, ...
(untested physics, SM extensions)
 - **LISA** frequencies are 10^{-5} – 10^{-2} Hz
Electroweak phase transition ~ 100 GeV ($f_c \sim 10^{-5}$ Hz)
 - Pulsar Timing Array (**PTA**) frequencies are 10^{-9} – 10^{-7} Hz
Quantum chromodynamic (QCD) phase transition ~ 100 MeV ($f_c \sim 10^{-9}$ Hz)

Gravitational Spectrum



Introduction and Motivation

- Magnetohydrodynamic (MHD) sources of GWs
 - Hydrodynamic turbulence from first-order phase transitions
 - **Primordial magnetic fields**
- Other sources of GWs include
 - True vacuum bubble collisions
 - Sound waves
 - Cosmic topological defects (cosmic strings)
 - Primordial black holes

Introduction and Motivation

- Direct numerical simulations using the PENCIL CODE¹ to solve:
 - Relativistic MHD equations adapted for radiation-dominated era (after electroweak symmetry is broken)
 - Gravitational waves equation

¹Pencil Code Collaboration, JOSS **6**, 2807 (2020),
<https://github.com/pencil-code/>

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Primordial magnetic fields

- Magnetic fields can either be produced at or present during cosmological phase transitions
- The magnetic fields are strongly coupled to the primordial plasma and inevitably lead to MHD turbulence²
- Present magnetic fields can be reinforced by primordial turbulence or generated via dynamo³

²J. Ahonen and K. Enqvist, *Phys. Lett. B* **382**, 40 (1996).

³A. Brandenburg, T. Kahniashvili, S. Mandal, A. Roper Pol, A. G. Tevzadze and T. Vachaspati *Phys. Rev. Fluids* **4**, 024608 (2019).

Hints for primordial magnetic fields

There are different astrophysical evidences that indicate the presence of large scale coherent magnetic fields.⁴

Fermi blazar observations

- Gamma rays from blazars (~ 1 TeV) interact with extragalactic background light
- Generation of electron - positron beam
- Observed power removal from gamma-ray beam

⁴L. M. Widrow, *Rev. of Mod. Phys.* **74** 775 (2002).

Hints for primordial magnetic fields

Solution

- Large scale (intergalactic) magnetic fields could deviate the electron-positron from beam in opposite directions
- Recombination does not happen leading to loss of energy
- Strength $\sim 10^{-16}$ G, scale ~ 100 kpc⁵

Origin

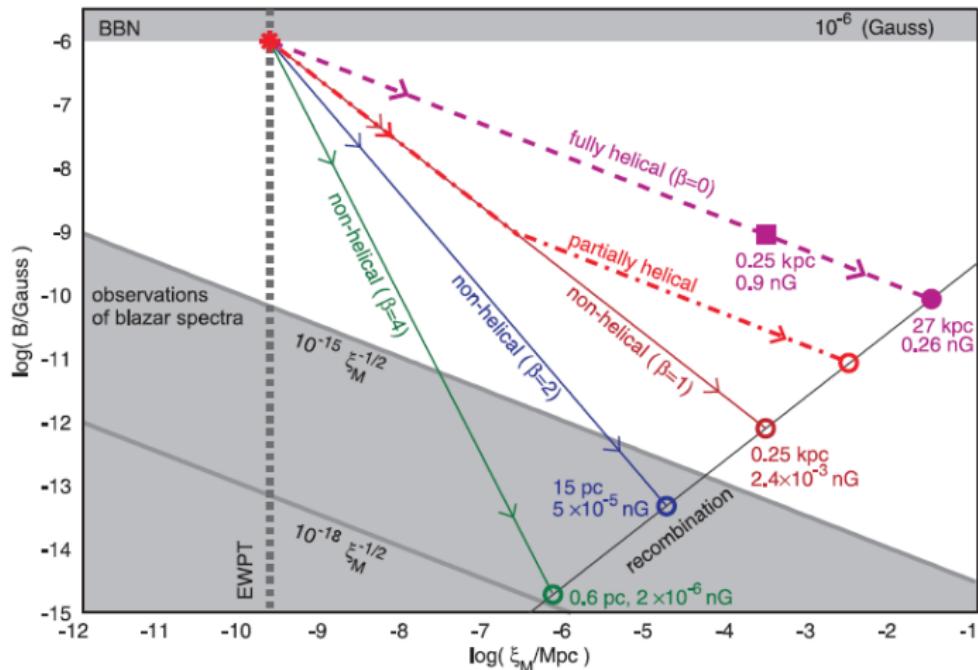
Intergalactic magnetic fields could have been originated from:

- Astrophysical or
- **Cosmological** seed fields

subsequently amplified during structure formation

⁵A. Neronov and I. Vovk, *Science* **328**, 73 (2010)

Evolution of magnetic strength and correlation length⁶



⁶A. Brandenburg, T. Kahnashvili, S. Mandal, A. Roper Pol, A. Tevzadze and T. Vachaspati, *Phys. Rev. D* **96**, 123528 (2017)

Generation of primordial magnetic fields

- Velocity fields induced by first-order phase transitions can generate magnetic fields
- Parity-violating processes during the EWPT can produce helical magnetic fields:
 - Sphaleron decay (non-helical⁷ and helical⁸)
 - Generation of Chern-Simons number through B+L anomalies⁹
 - Inhomogeneities in the Higgs field in low-scale electroweak hybrid inflation¹⁰

⁷ T. Vachaspati, *Phys. Rev. B* **265**, 258 (1991).

⁸ T. Vachaspati, *Phys. Rev. Lett.* **87**, 251302 (2001).

⁹ J. M. Cornwall, *Phys. Rev. D* **56**, 6146 (1997).

¹⁰ M. Joyce and M. E. Shaposhnikov, *Phys. Rev. Lett.* **79**, 1193 (1997).
J. García-Bellido *et al.*, *Phys. Rev. D* **60**, 123504 (1999).

Generation of primordial magnetic fields

- Axion fields can generate helicity in an already existing magnetic field¹¹
- Magnetic fields from inflation can be present during phase transitions (non-helical¹² and helical¹³)

¹¹ M. M. Forbes and A. R. Zhitnitsky, *Phys. Rev. Lett.* **85**, 5268 (2000).

¹² M. S. Turner and L. M. Widrow, *Phys. Rev. D* **37**, 2743 (1988).

¹³ M. Giovannini, *Phys. Rev. D* **58**, 124027 (1998).

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MHD description

Right after the electroweak phase transition we can model the plasma using continuum MHD

- Quark-gluon plasma (above QCD phase transition)
- Charge-neutral, electrically conducting fluid
- Relativistic magnetohydrodynamic (MHD) equations
- Ultrarelativistic equation of state

$$p = \rho c^2 / 3$$

- Friedmann–Lemaître–Robertson–Walker model

$$g_{\mu\nu} = \text{diag}\{-1, a^2, a^2, a^2\}$$

Contributions to the stress-energy tensor

$$T^{\mu\nu} = (\textcolor{red}{p/c^2} + \rho) U^\mu U^\nu + p g^{\mu\nu} + F^{\mu\gamma} F_\gamma^\nu - \frac{1}{4} g^{\mu\nu} F_{\lambda\gamma} F^{\lambda\gamma},$$

- From fluid motions

$$T_{ij} = (\textcolor{red}{p/c^2} + \rho) \gamma^2 u_i u_j + p \delta_{ij}$$

Relativistic equation of

state: $p = \rho c^2 / 3$

- From magnetic fields:

$$T_{ij} = -B_i B_j + \delta_{ij} B^2 / 2$$

- 4-velocity $U^\mu = \gamma(c, u^i)$

- 4-potential $A^\mu = (\phi/c, A^i)$

- 4-current $J^\mu = (c\rho_e, J^i)$

- Faraday tensor

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

Conservation laws

$$T^{\mu\nu}_{;\nu} = 0$$

Relativistic MHD equations are reduced to¹⁴

$$\frac{\partial \ln \rho}{\partial t} = -\frac{4}{3}(\nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla \ln \rho) + \frac{1}{\rho} [\mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) + \eta \mathbf{J}^2],$$

$$\frac{D\mathbf{u}}{Dt} = \frac{1}{3}\mathbf{u}(\nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla \ln \rho) - \frac{\mathbf{u}}{\rho} [\mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) + \eta \mathbf{J}^2]$$

$$-\frac{1}{4}\nabla \ln \rho + \frac{3}{4\rho}\mathbf{J} \times \mathbf{B} + \frac{2}{\rho}\nabla \cdot (\rho \nu \mathbf{S}) + \mathcal{F},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \mathbf{J} + \mathcal{E}),$$

for a flat expanding universe with comoving and normalized

$p = a^4 p_{\text{phys}}$, $\rho = a^4 \rho_{\text{phys}}$, $B_i = a^2 B_{i,\text{phys}}$, u_i , and conformal time t .

¹⁴A. Brandenburg, et al., *Phys. Rev. D* **54**, 1291 (1996)

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GWs equation for an expanding flat Universe

- Assumptions: isotropic and homogeneous Universe
- Friedmann–Lemaître–Robertson–Walker (FLRW) metric
$$\gamma_{ij} = a^2 \delta_{ij}$$
- Tensor-mode perturbations above the FLRW model:

$$g_{ij} = a^2 \left(\delta_{ij} + h_{ij}^{\text{phys}} \right)$$

- GWs equation is¹⁵

$$\left(\partial_t^2 - \cancel{\frac{a''}{a}} - c^2 \nabla^2 \right) h_{ij} = \frac{16\pi G}{ac^2} T_{ij}^{\text{TT}}$$

- h_{ij} are rescaled $h_{ij} = ah_{ij}^{\text{phys}}$
- Comoving spatial coordinates $\nabla = a\nabla^{\text{phys}}$
- Conformal time $dt = a dt^{\text{phys}}$
- Comoving stress-energy tensor components $T_{ij} = a^4 T_{ij}^{\text{phys}}$
- Radiation-dominated epoch such that $a'' = 0$

¹⁵L. P. Grishchuk, Sov. Phys. JETP 40, 409 (1974).

Normalized GW equation¹⁶

$$\left(\partial_t^2 - \nabla^2\right) h_{ij} = 6 T_{ij}^{\text{TT}} / t$$

Properties

- All variables are normalized and non-dimensional
- Conformal time is normalized with t_*
- Comoving coordinates are normalized with c/H_*
- Stress-energy tensor is normalized with $\mathcal{E}_{\text{rad}}^* = 3H_*^2 c^2 / (8\pi G)$
- Scale factor is $a_* = 1$, such that $a = t$

¹⁶A. Roper Pol *et al.*, *Geophys. Astrophys. Fluid Dyn.* **114**, 130 (2020).
arXiv:1807.05479.

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Numerical results for decaying MHD turbulence¹⁷

Initial conditions

- Initial stochastic magnetic field with fractional helicity
 $\mathcal{P}_M = 2\sigma/(1 + \sigma^2)$
- Batchelor spectrum, i.e., $E_M \propto k^4$ for small k
- Kolmogorov spectrum in the inertial range, i.e., $E_M \propto k^{-5/3}$

$$kB_i = \left(P_{ij} - i\sigma\varepsilon_{ijl} \hat{k}_l \right) g_j \sqrt{2E_M(k)},$$

$$E_M(k) = \frac{1}{2}B_0^2 k_*^{-1/2} \frac{(k/k_*)^4}{(1 + (k/k_*)^{34/3})^{1/2}}$$

¹⁷ A. Brandenburg *et al.* *Phys. Rev. D* **96**, 123528 (2017)

A. Roper Pol *et al.* *Phys. Rev. D* **102**, 083512 (2020)

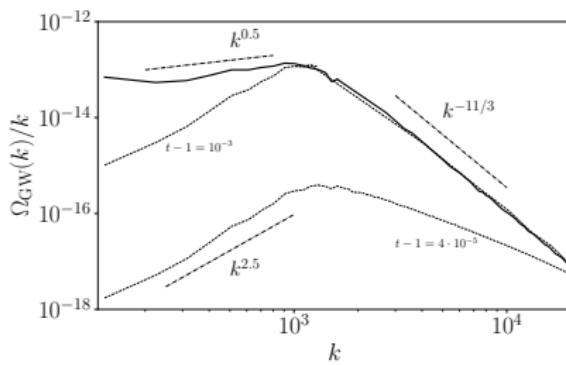
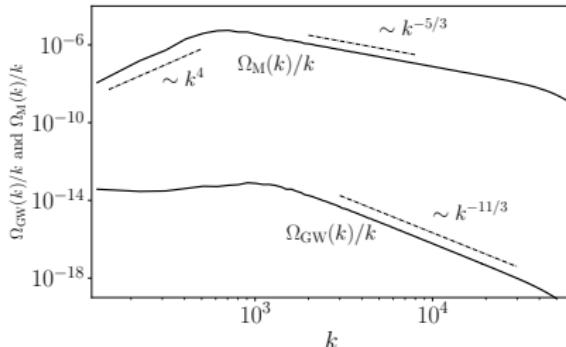
A. Roper Pol *et al.* arXiv:2107.05356

Numerical results for decaying MHD turbulence

Initial conditions

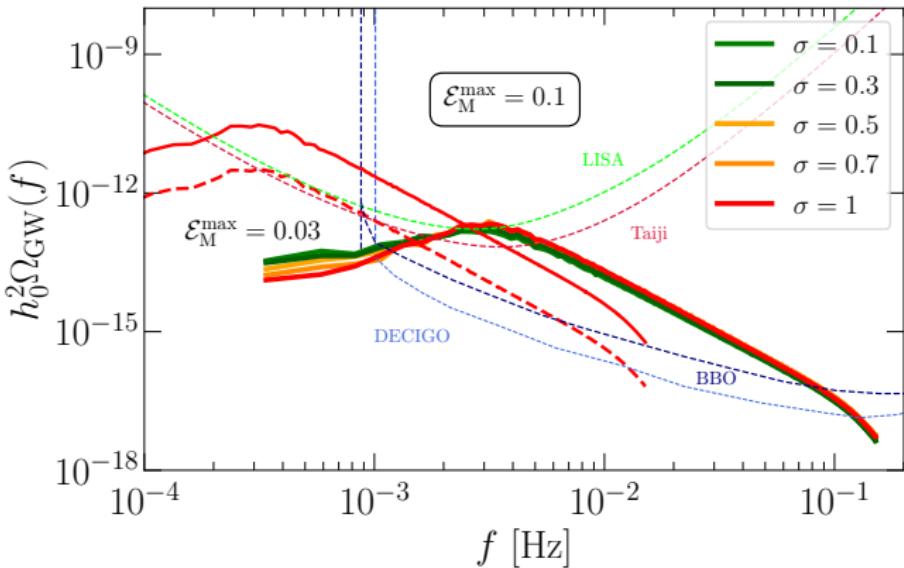
- Magnetic energy density at t_* is a fraction of the radiation energy density, $\mathcal{E}_M/\mathcal{E}_{\text{rad}}^* = \frac{1}{2}B_0^2 \leq 0.1$ (BBN limit).
- Spectral peak $k_* = N_* \times 2\pi$, normalized by H_*/c is given by the characteristic scale of the sourcing turbulence (as a fraction of the Hubble radius).

Numerical results for decaying MHD turbulence for $N_* = 100$, $\mathcal{E}_M \sim 10^{-2}$



- **Novel k^0 scaling in the subinertial range, i.e., $\Omega_{\text{GW}}(f) \sim f$**
- k^2 is expected for larger scales, i.e., $\Omega_{\text{GW}}(f) \sim f^3$
- Further investigation on the k^0 development

Detectability of the SGWB from the EWPT with LISA (for decaying MHD turbulence with initial magnetic field)¹⁸



¹⁸A. Roper Pol, et al. Phys. Rev. D **102**, 083512 (2020)

Numerical results for decaying MHD turbulence¹⁹

Driven magnetic field

- Initial magnetic and velocity are zero
- Magnetic field is built-up for a short duration ($\sim 0.1 H_*^{-1}$) via the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \mathbf{J} + \mathcal{F}).$$

- The forcing term is quasi-monochromatic with fractional magnetic helicity

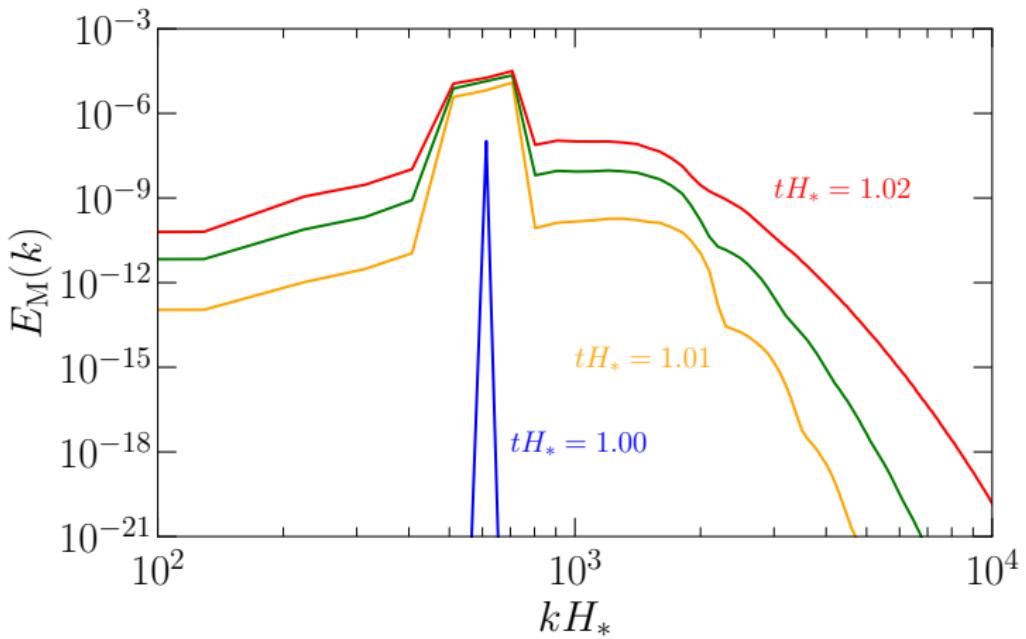
$$\mathcal{F} = \text{Re}(\mathcal{A}\mathbf{f}) \exp[i\mathbf{k} \cdot \mathbf{x} + i\phi], \quad k_* - \frac{1}{2}\delta k \leq |\mathbf{k}| \leq k_* + \frac{1}{2}\delta k$$

$$f_i = \left(\delta_{ij} - i\sigma\varepsilon_{ijl}\hat{k}_l \right) f_j^{(0)} / \sqrt{1 + \sigma^2}$$

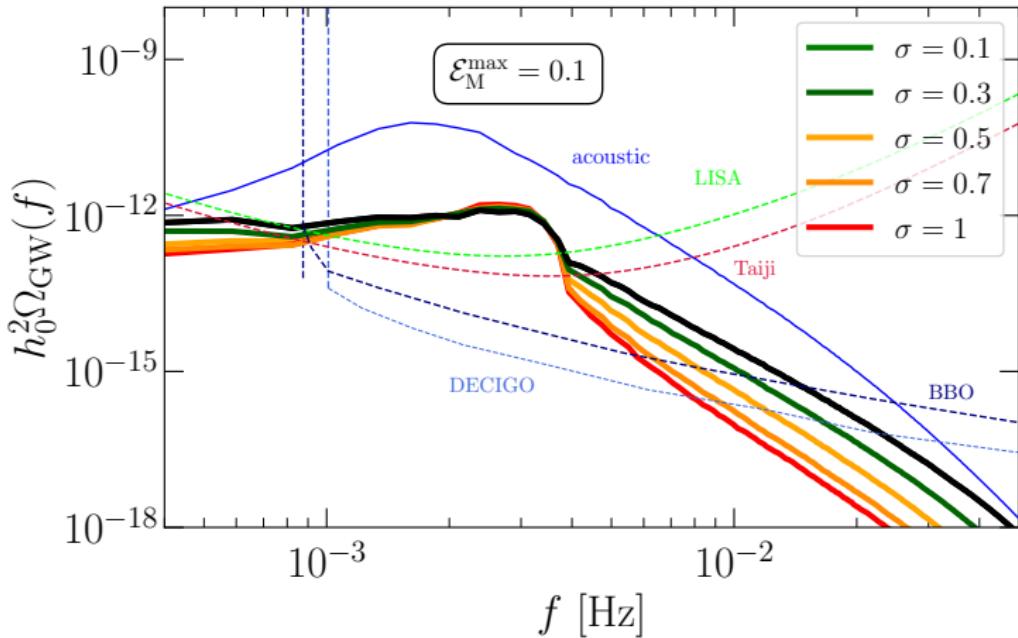
¹⁹A. Roper Pol, et al. *Phys. Rev. D* **102**, 083512 (2020)

A. Roper Pol, et al. arXiv:2107.05356

Driven magnetic field



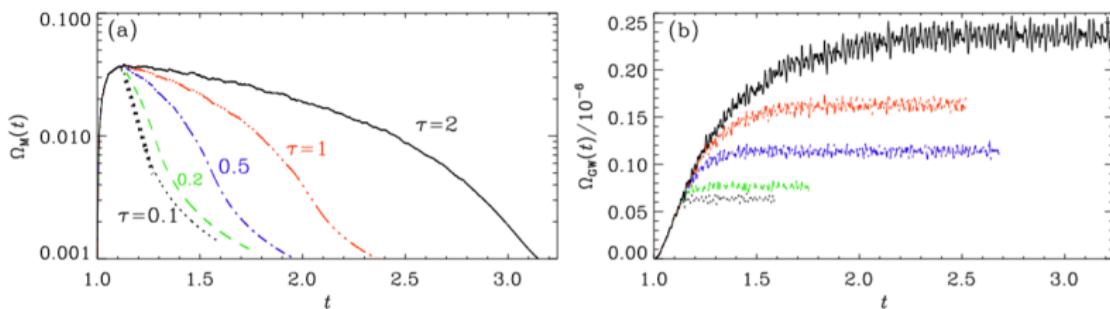
Detectability of the SGWB from the EWPT with LISA (for decaying
MHD turbulence with an initially forced magnetic field)²⁰



²⁰A. Roper Pol *et al.* *Phys. Rev. D* **102**, 083512 (2020)

Extended pumping of energy

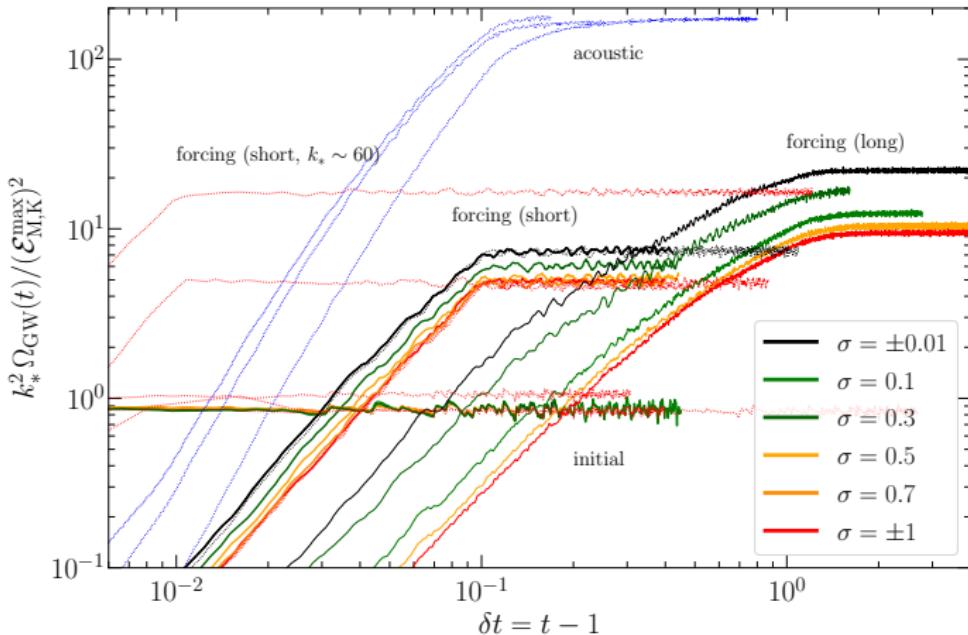
- The GW energy density becomes 'stationary' after the source energy density reaches its maximum in a time scale $\delta t = t - t_* \sim 1/k_*$.
- If we extend the pumping of energy, the difference is small (~ 4 times for $\tau = 2$, $\tau > 0.5$ is highly unrealistic)²¹



²¹T. Kahnashvili, A. Brandenburg, A. Kosowsky, S. Mandal, A. Roper Pol
Phys. Rev. Res. 3, 013193, arXiv:2009.14174 (2021)

Efficiency of GW production²²

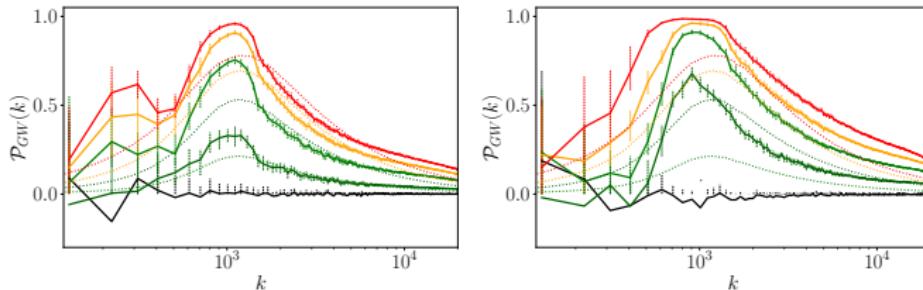
- The GW energy density is proportional to Ω_M^2 and k_*^{-2}



²²A. Roper Pol *et al.* arXiv:2107.05356

Polarization degree from stationary turbulence (long-time forcing)

- Helical magnetic fields induce circularly polarized GWs²³
- Kinetic turbulence
- Magnetic turbulence

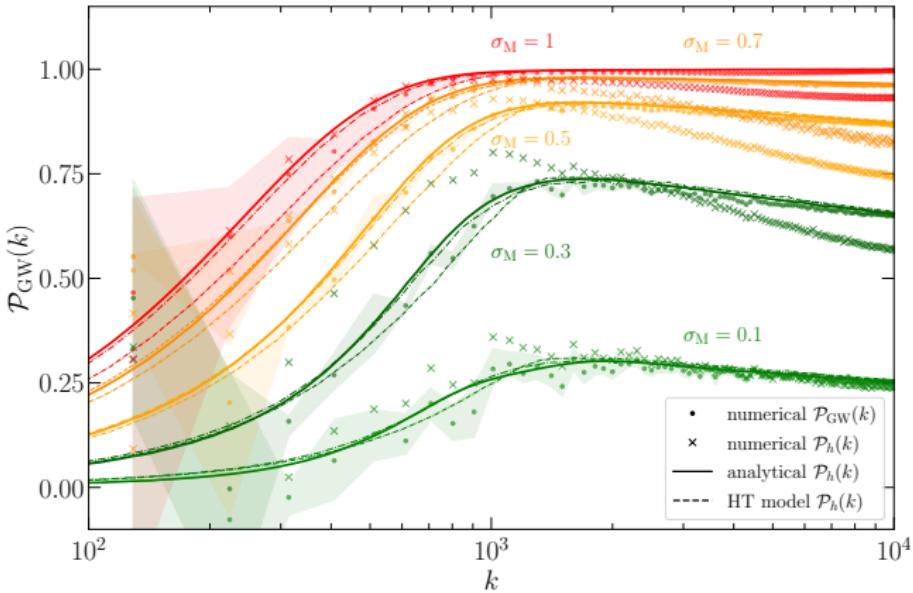


- Degree of circular polarization

$$\mathcal{P}_{\text{GW}}(k) = \frac{\Xi_{\text{GW}}(k)}{\Omega_{\text{GW}}(k)} = \frac{\left\langle \dot{\tilde{h}} \times \dot{\tilde{h}}_+^* - \dot{\tilde{h}}_+ \dot{\tilde{h}}_x^* \right\rangle}{\left\langle \dot{\tilde{h}}_+ \dot{\tilde{h}}_+^* + \dot{\tilde{h}}_x \dot{\tilde{h}}_x^* \right\rangle}$$

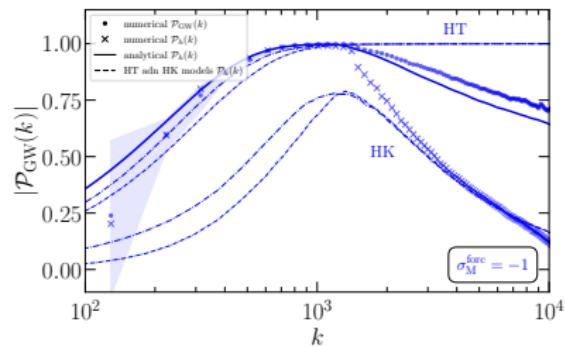
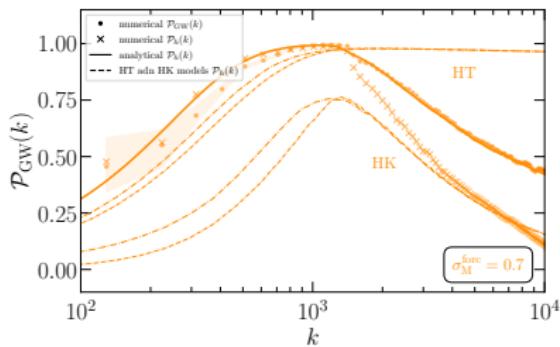
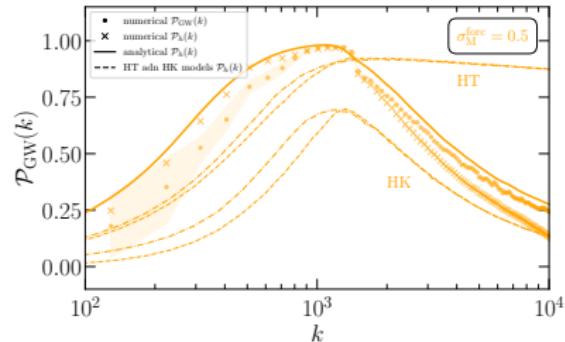
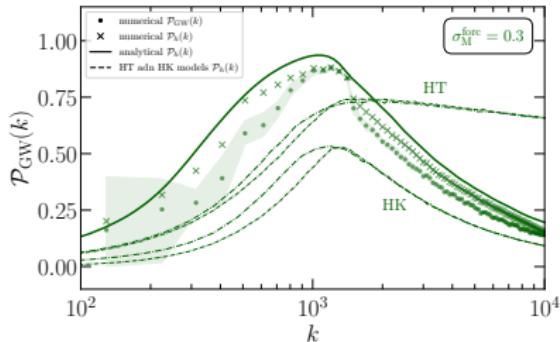
²³ L. Kisslinger and T. Kahnashvili, Phys. Rev. D **92**, (2015)
T. Kahnashvili, G. Gogoberidze and B. Ratra, Phys. Rev. Lett. **95**, 151301 (2005)
T. Kahnashvili, A. Brandenburg, A. Kosowsky, S. Mandal, A. Roper Pol,
Phys. Rev. Res. **3**, 013193 (2021)

Polarization degree from decaying turbulence (initially given field)²⁴



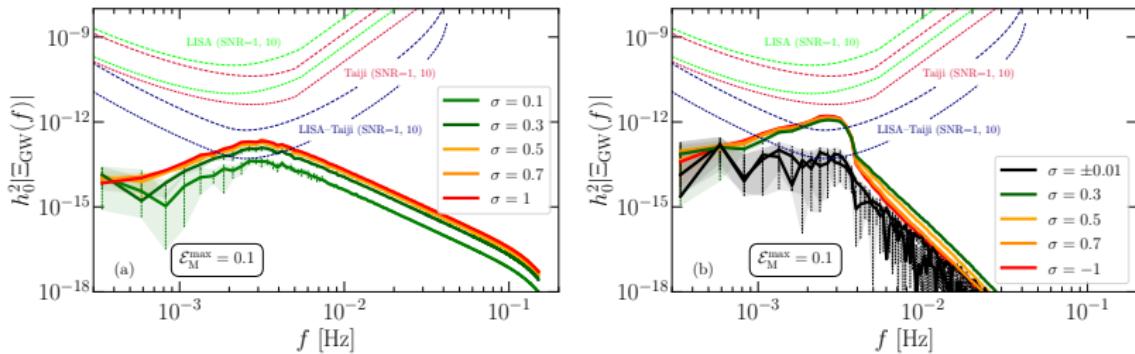
²⁴ A. Roper Pol et al. arXiv:2107.05356

Polarization degree from decaying turbulence (initially driven field)²⁵



Detectability of the polarized SGWB from the EWPT with LISA and Taiji²⁸

- LISA's dipole response function can provide us with a polarized gravitational wave background due to our proper motion²⁶
- Cross-correlation of LISA and an additional space-based GW detector can improve the detectability of a polarized GW background²⁷



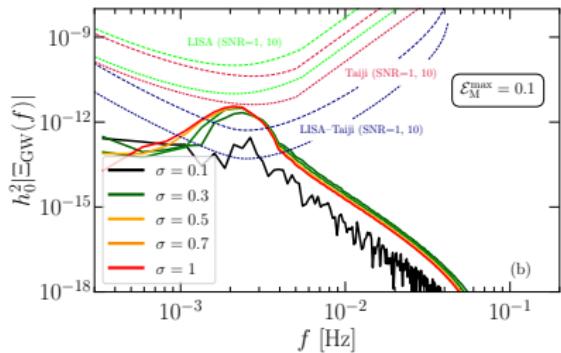
²⁶V. Domcke, et al., JCAP **05**, 028 (2020)

²⁷G. Orlando, M. Pieroni and A. Ricciardone, JCAP **03**, 069 (2021)

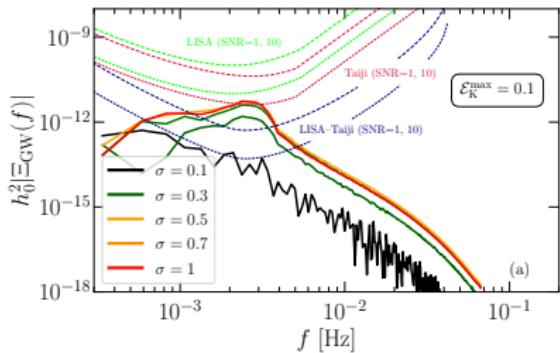
²⁸A. Roper Pol et al. arXiv:2107.05356

Detectability of the polarized SGWB from the EWPT with LISA and Taiji ²⁹

- Magnetic turbulence



- Kinetic turbulence



²⁹

T. Kahnashvili, A. Brandenburg, A. Kosowsky, S. Mandal, A. Roper Pol, *Phys. Rev. Res.* **3**, 013193 (2021)
A. Roper Pol et al. arXiv:2107.05356

Conclusions 1/3

- We started the implementation of the GW module of the PENCIL CODE in 2017 to allow to obtain background stochastic GW spectra from primordial magnetic fields and hydrodynamic turbulence.
- GW equation is normalized such that it can be easily scaled for different times within the radiation-dominated epoch.
- The PENCIL CODE provides an ideal set of tools to couple the MHD dynamical equations as the sources of GWs, as well as to adapt to other sources (e.g., only kinetic/magnetic sources) or couple to other modules (e.g., chiral MHD)

Conclusions 2/3

- Depending on the mechanism of turbulence generation and/or the initial energy density and characteristic scale, the GW signal from the EWPT is detectable by LISA.
- General f spectrum obtained for GWs in the low frequency range vs f^3 obtained from analytical estimates (above horizon scales).
- Detection of GW spectrum can provide *clean* information from the epoch of generation and the turbulence characteristics.
- The circular polarization of GWs produced by helical magnetic fields can be detected by LISA and improved by correlating LISA and additional space-based GW detectors (e.g., TianQin, Taiji)
- Polarization degree can provide information on magnetic helicity of the seed field, about its nature (kinetically or magnetically dominant), and formation process.

Conclusions 3/3

- A lot of interesting science has been and can be done since then in a very unique time for GW astronomy with future GW detectors (LISA, IPTA, SKA, CE, BBO, DECIGO, atomic interferometry, Gaia, CMB anisotropies with LiteBIRD, ...)
- Production of helical magnetic fields can be related to Chern-Simons violations and to production of particles, shedding light into the baryon-asymmetry problem
- The origin of magnetic fields in the largest scales of our Universe is still a big open question in cosmology



The End Thank You!



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