

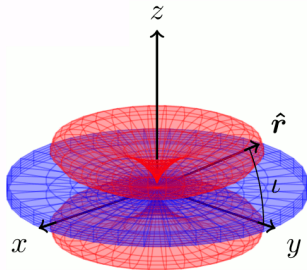
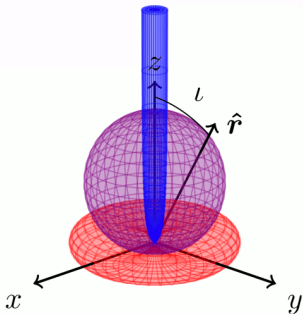
Nonlinear gravitational-wave memory from cusps and kinks on cosmic strings

Alex Jenkins

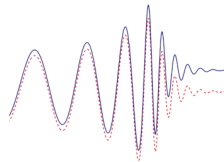
based on [arXiv:2102.12487](https://arxiv.org/abs/2102.12487)
with Mairi Sakellariadou
(to appear in CQG)

GWxBSM
12 July 2021

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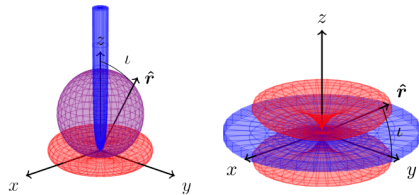
1 GW memory



2 Cosmic strings



3 Memory from strings



GW memory (linear)

- permanent GW strain offset
- well-known since 1970s
- typical of unbound systems
(gravitational scattering, supernovae, ...)

THE GENERATION OF GRAVITATIONAL WAVES. IV. BREMSSTRAHLUNG*†‡

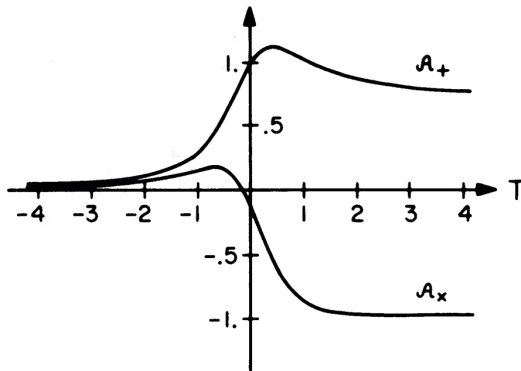
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Received 1977 October 21; accepted 1978 February 28



GW memory (nonlinear)

- discovered by Christodoulou '91
- nonlinear GR effect
- intuition (Thorne '92):
linear memory of unbound **gravitons**

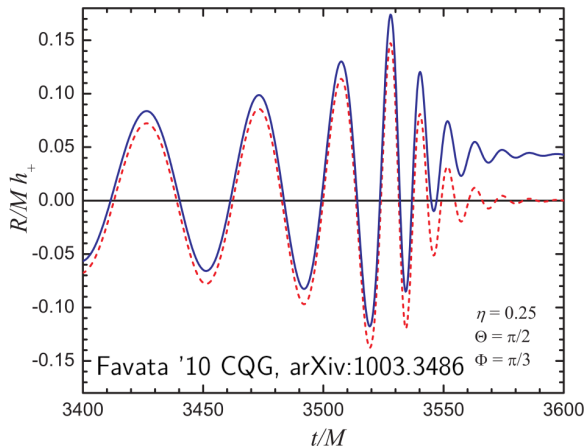
Nonlinear Nature of Gravitation and Gravitational-Wave Experiments

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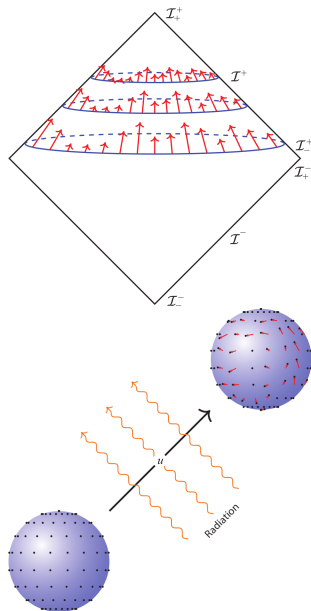
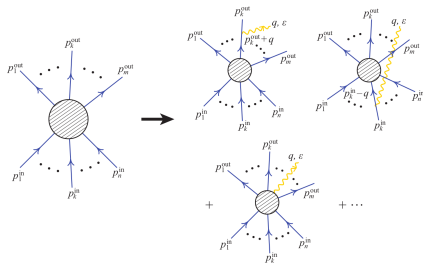
(Received 17 December 1990; revised manuscript received 20 June 1991)

It is shown that gravitational waves from astronomical sources have a nonlinear effect on laser interferometer detectors on Earth, an effect which has hitherto been neglected, but which is of the same order of magnitude as the linear effects. The signature of the nonlinear effect is a permanent displacement of test masses after the passage of a wave train.



Why is nonlinear memory interesting?

- prediction of **nonlinear, dynamical** GR
- **observable** with GW observatories
- deep connections (Strominger et al):
 - ▶ global structure of spacetime (BMS)
 - ▶ IR quantum gravity (soft theorems)



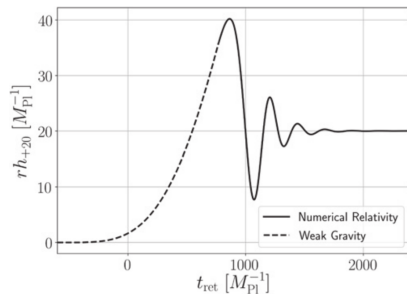
Existing results

focus is usually on BBHs

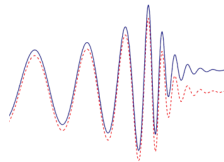
- memory from SMBBH mergers is a promising target for PTAs (e.g. Aggarwal et al, '20 ApJ)
- should be detectable by LIGO/Virgo after $\mathcal{O}(2000)$ BBHs (Hübner et al, '20 PRD)

other sources?

- scattering, SNe, GRBs, ...
are good sources of *linear* memory
- Aurrekoetxea et al, '20 CQG
studied linear+nonlinear memory from collapsing cosmic string loops



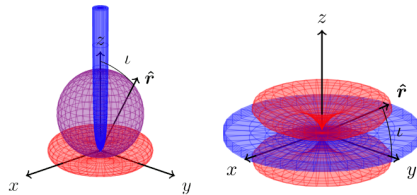
1 GW memory



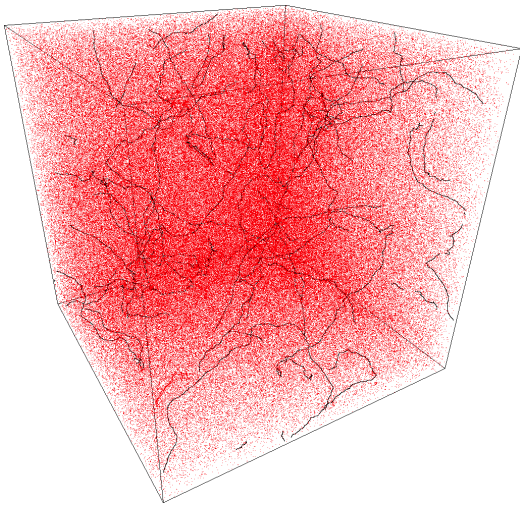
2 Cosmic strings



3 Memory from strings



What are cosmic strings?



- 1-dimensional topological defects
- generic in many theories beyond SM



- ~ 1 long string per horizon, chops off many loops
- loops decay through GWs

Ringeval, Sakellariadou, & Bouchet '07 JCAP,
[arXiv:astro-ph/0511646](https://arxiv.org/abs/astro-ph/0511646)

Key assumptions

① **Nambu-Goto approximation:**

string width \ll loop size

no non-gravitational long-range interactions

single parameter: string tension $G\mu$

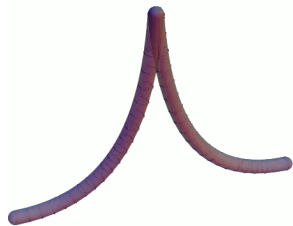
② **Linearised gravity:**

loop evolves on a flat background,

generates weak GWs suppressed by $G\mu \ll 1$

GW bursts from loops

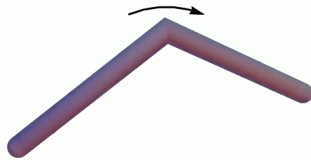
three main mechanisms for loop GW emission:



cusp

$$\tilde{h}(f) \sim f^{-4/3}$$

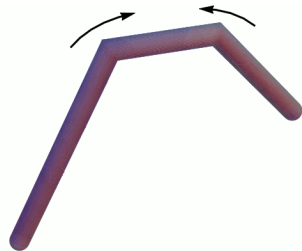
beamed



kink

$$\tilde{h}(f) \sim f^{-5/3}$$

beamed



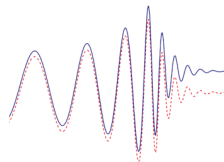
kink-kink collision

$$\tilde{h}(f) \sim f^{-2}$$

isotropic

illustrations from [Long, Hyde, & Vachaspati '14 JCAP, arXiv:1405.7679](#)

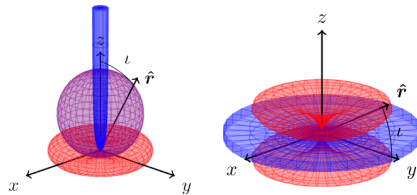
1 GW memory



2 Cosmic strings



3 Memory from strings

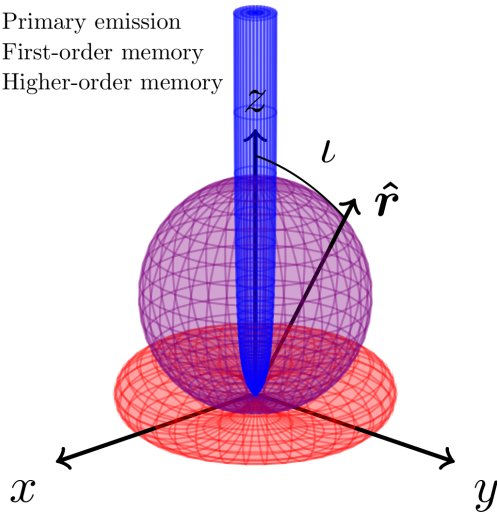


Why memory + cosmic strings?

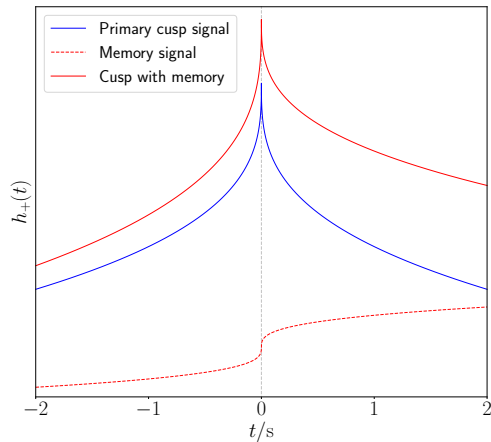
- ① memory literature has focused on BBHs
- ② strings are a promising GW source for LVK/LISA/PTAs
- ③ strong high-frequency emission
(memory pushes things to lower frequencies)
- ④ memory could “leak” out of GW beam

Leading-order cusp memory

- Primary emission
- First-order memory
- Higher-order memory

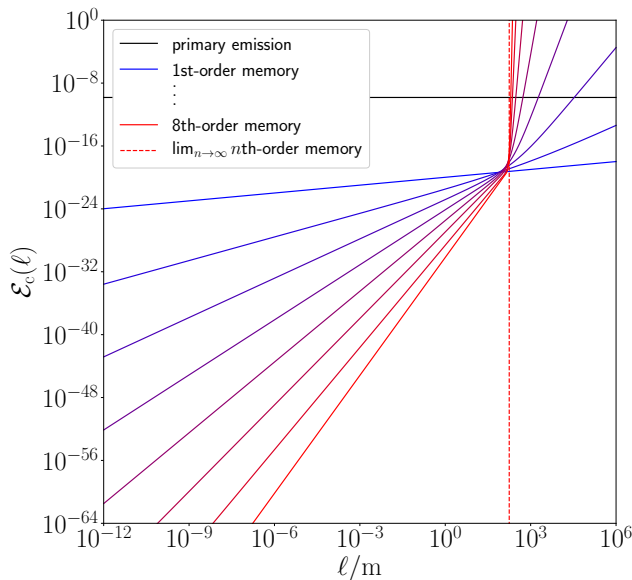


- memory “leaks” out of beam
- same $\sim f^{-4/3}$ as original waveform

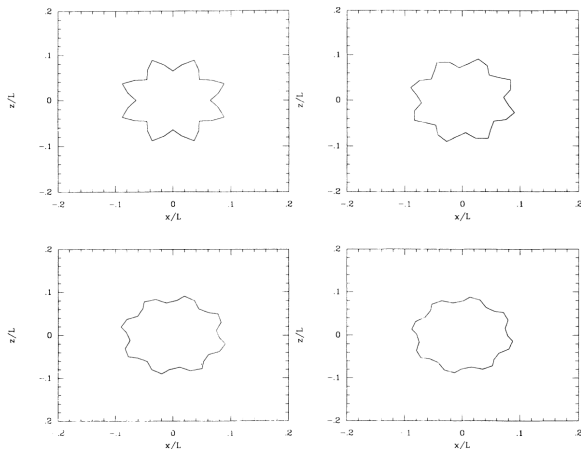


Higher-order memory

- the memory GWs are themselves sources of memory (“memory of the memory”)
- need to iterate to all orders to get complete memory signal
- higher-order contributions *diverge* for large enough loops
- is the weak-field description of cusps unphysical?



Backreaction?

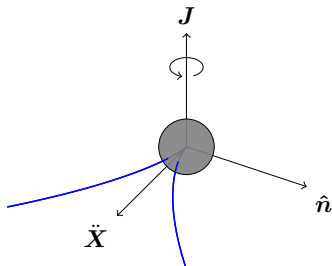


(figure from Quashnock & Spergel, '90 PRD)

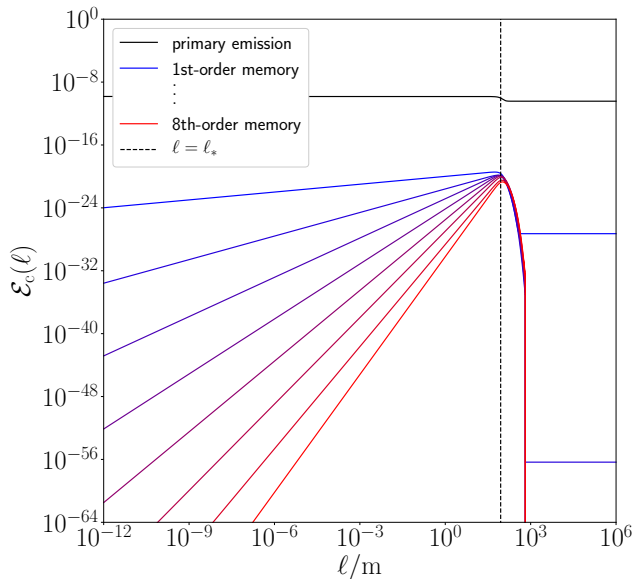
- gravitational backreaction could smooth out the cusp?
- see e.g. Chernoff et al, '18 PRD or Blanco-Pillado et al, '19 PRD
- typical timescale too long, $\tau \sim \mathcal{O}(1/G\mu)$

One possible cure: PBHs

- predicted for the same range of loop lengths!
- horizon forms, “traps” GWs
- higher-order memory converges

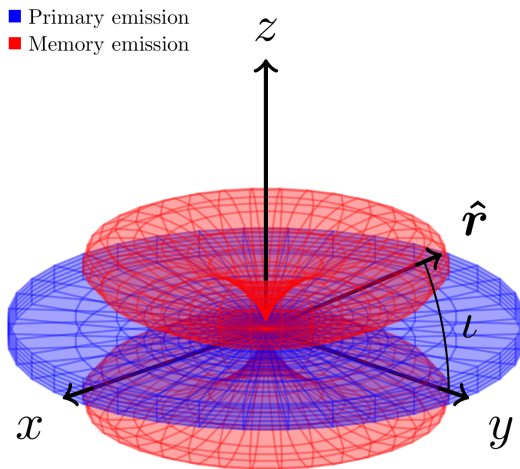


ACJ & Sakellariadou, arXiv:2006.16249



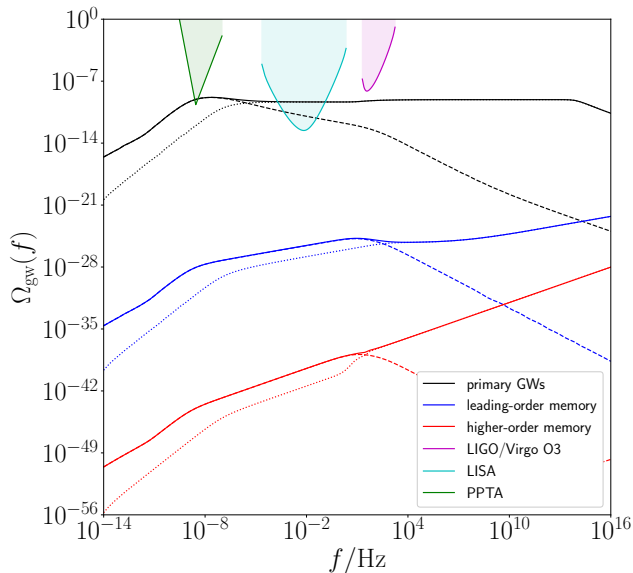
Memory from kinks

- memory suppressed at high frequencies due to interference
- higher-order memory always converges
- PBHs not predicted for kinks—makes sense that the memory is well-behaved here



Observable implications?

- key GW observables:
 - ① individual bursts
 - ② combined stochastic background
- assuming PBHs, memory is unobservable
- other scenarios could be more interesting?

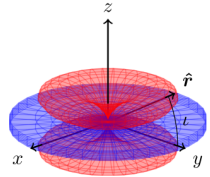
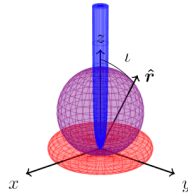
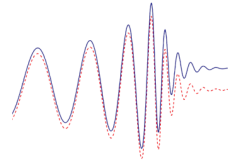


Wrapping up

- nonlinear GW memory is interesting and worth studying beyond BBHs
- memory from cusps *diverges* using the standard waveform
- need some (unknown) strong-gravity physics to fix this—PBH formation is one possible resolution
- assuming PBH formation, observational prospects are poor. . .
...but nature might surprise us

thanks for listening!

Backup Slides



How do we observe GW memory?

given linear GW signal $h^{(0)}$, can compute memory signal

$$h^{(1)}(t, \mathbf{r}) = \frac{1}{2r} \int_{-\infty}^t dt' \int_{\hat{\mathbf{r}}'} |r \dot{h}^{(0)}(t, \mathbf{r}')|^2$$

① “late-time memory”

$$\Delta h^{(1)} \equiv \lim_{t \rightarrow \infty} h^{(1)}(t)$$

unobservable with
ground-based interferometers

② frequency domain waveform

$$\tilde{h}^{(1)}(f) = - \int_{\mathbb{R}} dt \int_{\hat{\mathbf{r}}'} \frac{ie^{-2\pi i f t}}{4\pi f r} |r \dot{h}^{(0)}|^2$$

this is our best bet

Higher-order memory

- the memory GWs can themselves act as a source
“memory of the memory”

$$\tilde{h}^{(n)}(f) = -\frac{4G}{fr} \int_{\mathbb{R}} dt \int_{\hat{r}'} ie^{-2\pi ift} \frac{d^3 E_{\text{gw}}^{(n-1)}}{dt d^2 \hat{r}'}$$

- for cusps, this introduces factors of ℓ/ℓ_* , where

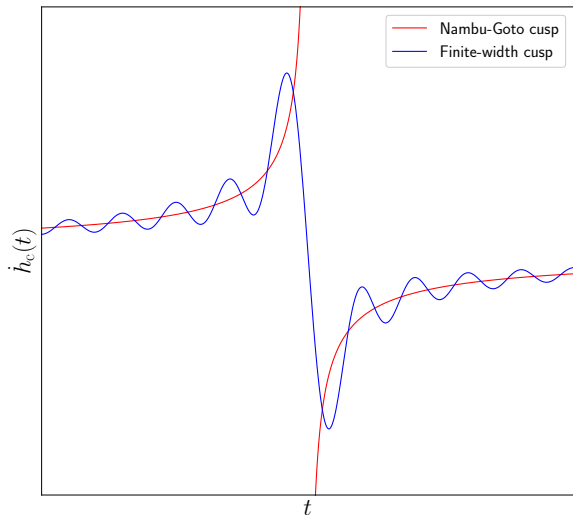
$$\ell_* \sim \delta/(G\mu)^3 \sim 90 \text{ m} \times \left(\frac{G\mu}{10^{-11}} \right)^{-7/2}$$

Finite-width regularisation

- original waveform assumes string has zero width (Nambu Goto approximation)
- UV divergence due to $\dot{h}^{(0)} \rightarrow \infty$
- “hidden” by narrow beam, but leaks out due to GR nonlinearity
- natural solution: high-frequency cutoff at string width scale

$$\delta \sim \ell_{\text{Pl}} / \sqrt{G\mu}$$

$$f < 1/\delta \sim 10^{38} \text{ Hz} \times \left(\frac{G\mu}{10^{-11}} \right)^{1/2}$$



Understanding the divergence

- toy model suggests a necessary (but not sufficient) condition:

$$\max_t |r\dot{h}| \gg 1, \quad \max_t |\dot{E}_{\text{gw}}| \gg \frac{1}{G} = \frac{m_{\text{Pl}}}{t_{\text{Pl}}}$$

- for BBHs,

$$\max_t |r\dot{h}| \sim \frac{m_1 m_2}{(m_1 + m_2)^2} \lesssim 1$$

(which makes sense)