Model-independent energy budget of gravitational waves from a cosmological first-order phase transition

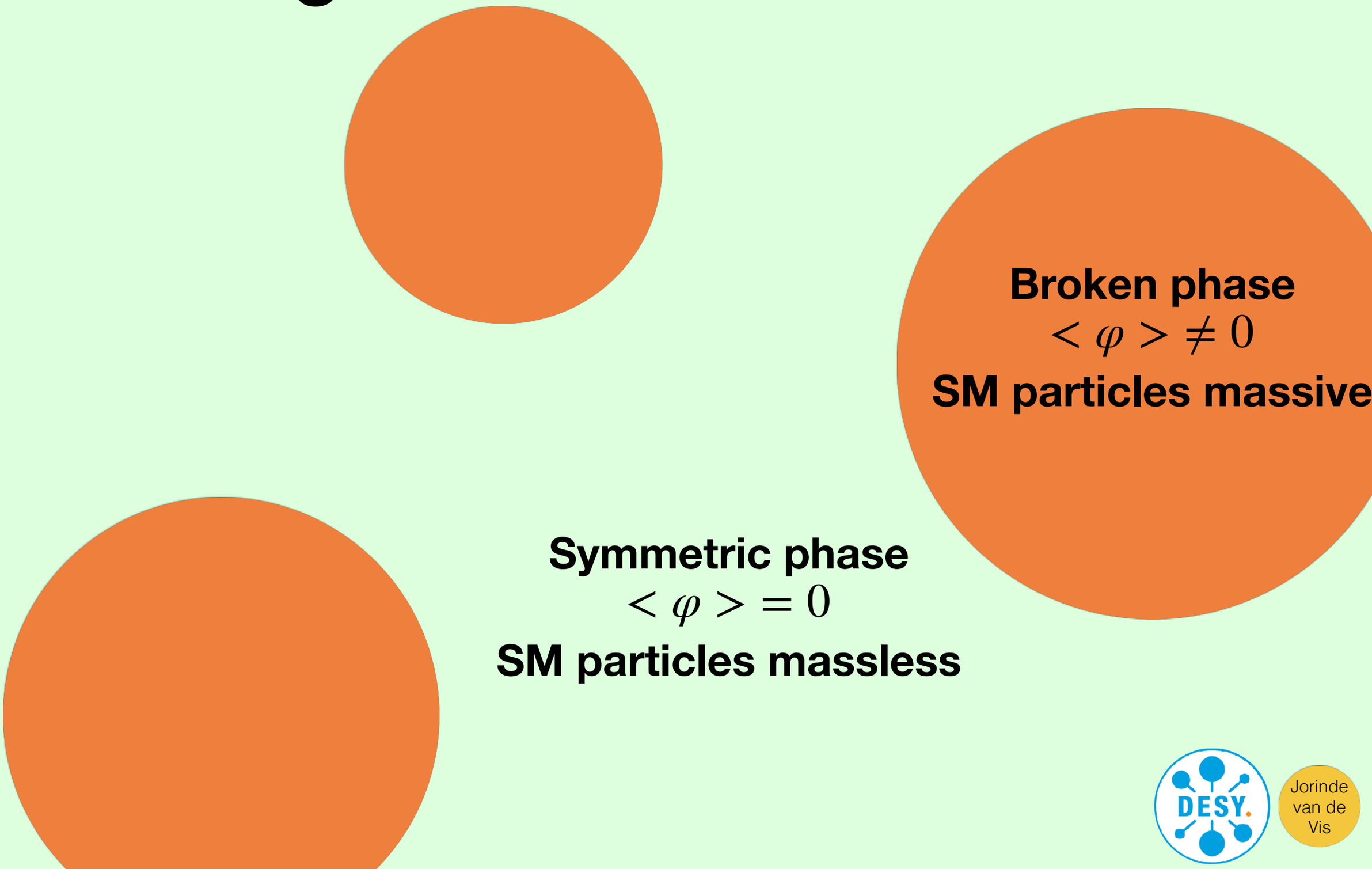
Jorinde van de Vis

F. Giese, T. Konstandin, JvdV, **2004.06995**
JCAP 07 (2020) 07, 057

F. Giese, T. Konstandin, K. Schmitz, JvdV, **2010.09744**
JCAP 01 (2021), 072

Gravitational Wave Probes of Physics Beyond Standard Model

First order phase transition: Colliding bubbles of true vacuum



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3 sources of gravitational waves

- Scalar field contribution
- Sound waves
- Turbulence

How to predict gravitational wave signal in sound waves?

- $\Omega_{tot} = \min\{1, H_* \tau_{\rm sh}\}\, 3F\tilde{\Omega} R_* H_* K^2$

- F : Redshift

- H_* : Hubble parameter at percolation

- R_* : mean bubble separation

M. Hindmarsh, S. Huber,
K. Rummukainen, D.Weir 2015, 2017
LISA Cosmology working group 2015, 2019

Particle physics at finite T
Bubble wall velocity

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How to predict gravitational wave signal in sound waves?

M. Hindmarsh, S. Huber,
K. Rummukainen, D.Weir 2015, 2017
LISA Cosmology working group 2015, 2019

- $\Omega_{tot} = \min\{1, H_* \tau_{\rm sh}\}\, 3F\tilde{\Omega} R_* H_* K^2$
- $\tilde{\Omega}$: Numerical factor ~ 0.01
- $\tau_{\rm sh}$: Onset of shock formation ~ $\sqrt{K/\Gamma}$
- K : Kinetic energy fraction

Kinetic energy fraction

$$K = \rho_{fl} / e_n$$

- Determined by hydrodynamics of single expanding bubble
- Depends on the phase transition strength and speed of sound in the plasma
- Depends on bubble wall velocity - which is treated as external parameter

Goal

Determine K as a function of the phase transition strength, wall velocity and the speed of sound, without further model-dependence

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Determine K as a function of the phase transition strength, wall velocity and the speed of sound, without further model-dependence

Result should be reusable without solving the hydrodynamics

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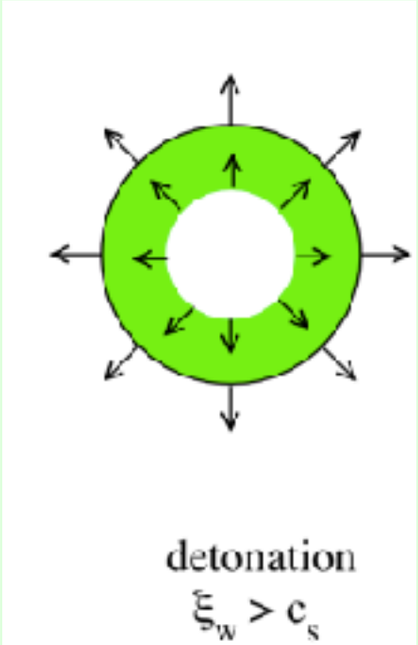
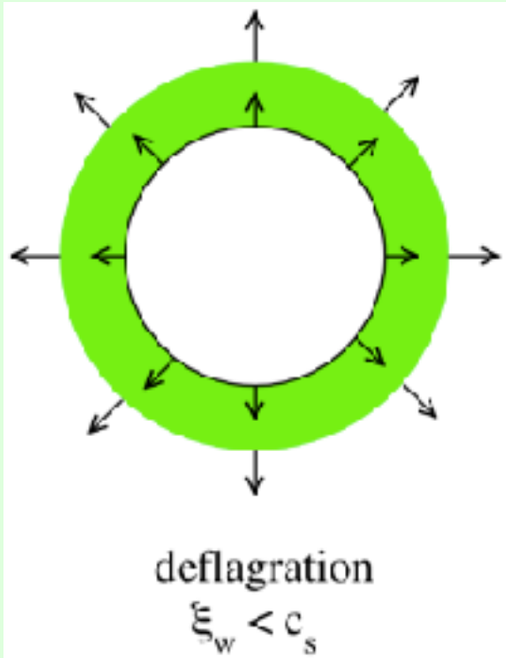
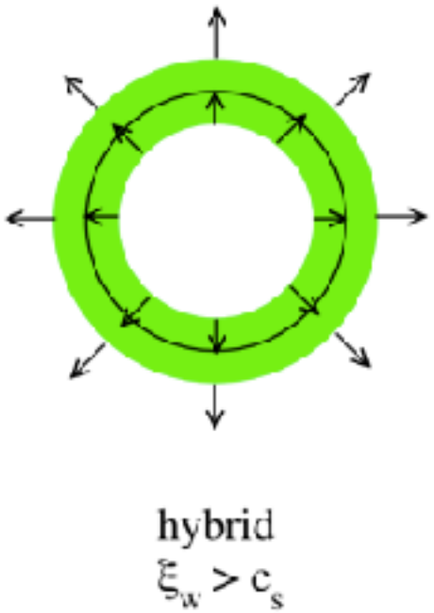
Hydrodynamics of a single bubble

- Plasma is described by a perfect fluid: $T_{\mu\nu} = \underset{\substack{\uparrow \\ \text{enthalpy}}}{w} u_\mu u_\nu - \underset{\substack{\uparrow \\ \text{pressure}}}{p} g_{\mu\nu}$
- Hydrodynamic equations from continuity equations: $\partial_\mu T^{\mu\nu} = 0$
- Matching equations:

$$v_+ v_- = \frac{p_+ - p_-}{\underset{\substack{\uparrow \\ \text{energy density}}}{e_+} - e_-}, \qquad \frac{v_+}{v_-} = \frac{e_- + p_+}{e_+ + p_-}$$







J. Espinosa, T. Konstandin, J. No, G. Servant, 2010

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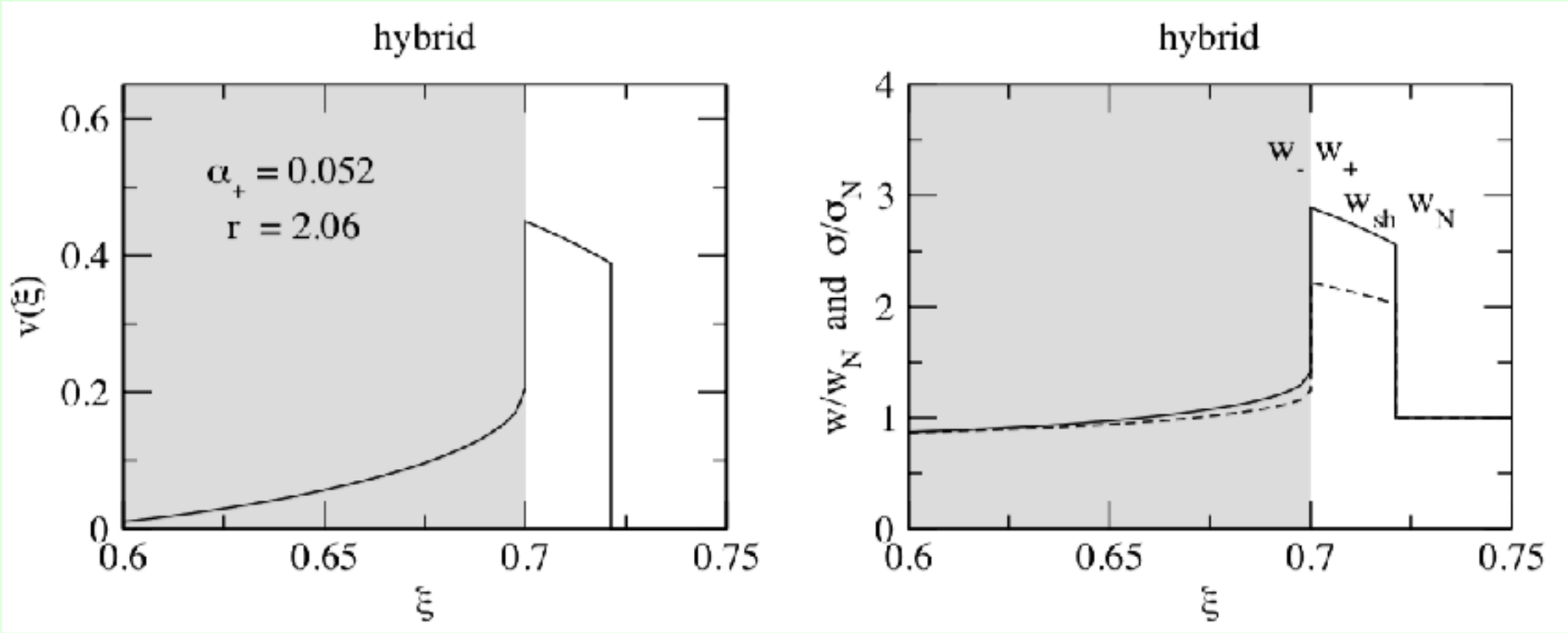
Velocity and enthalpy profile



J. Espinosa, T. Konstandin,
J. No, G. Servant, 2010

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Bag equation of state

- $p_s = \frac{1}{3} a_+ T^4 - \epsilon$ $\qquad e_s = a_+ T^4 + \epsilon$

  $p_b = \frac{1}{3} a_- T^4$ $\qquad e_b = a_- T^4$

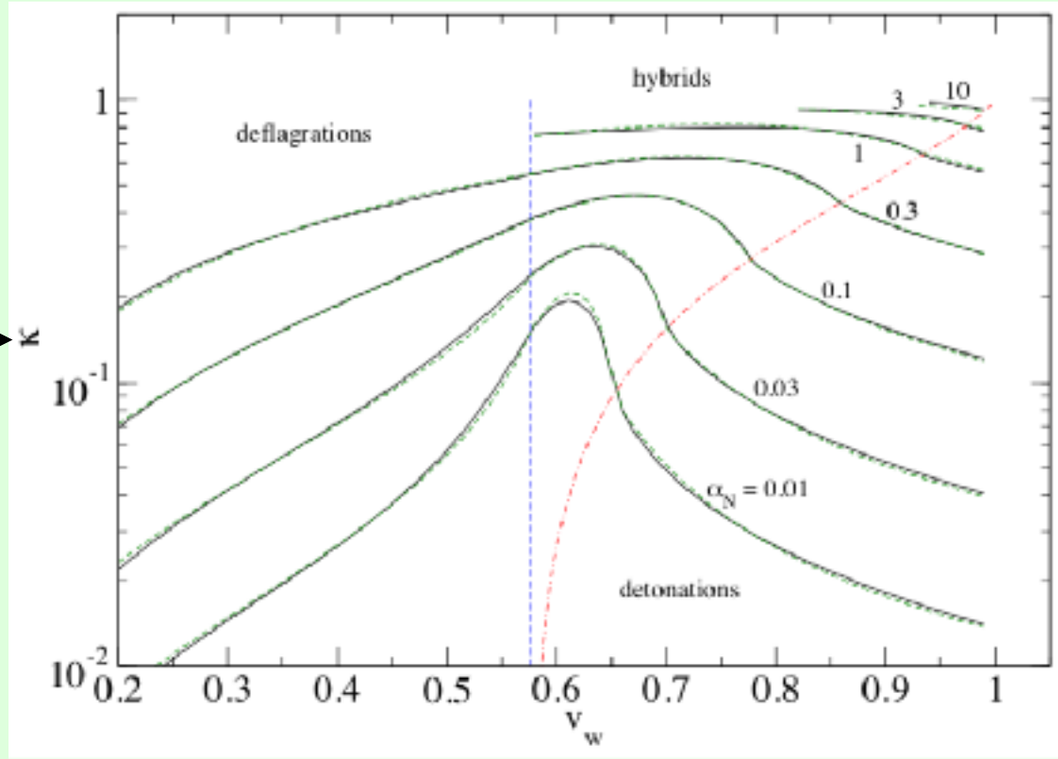
- Bag constant ϵ independent of temperature

- Speed of sound $\quad c_s^2 = \frac{dp/dT}{de/dT} = \frac{1}{3}$

- Phase transition strength $\quad \alpha_\epsilon = \frac{4\epsilon}{3w_n}$

- $K = \frac{\alpha_\epsilon \kappa_\epsilon}{\alpha_\epsilon + 1}$ completely determined by α_ϵ and ξ_w

$\kappa_\epsilon = \frac{\rho_{fl}}{\epsilon}$

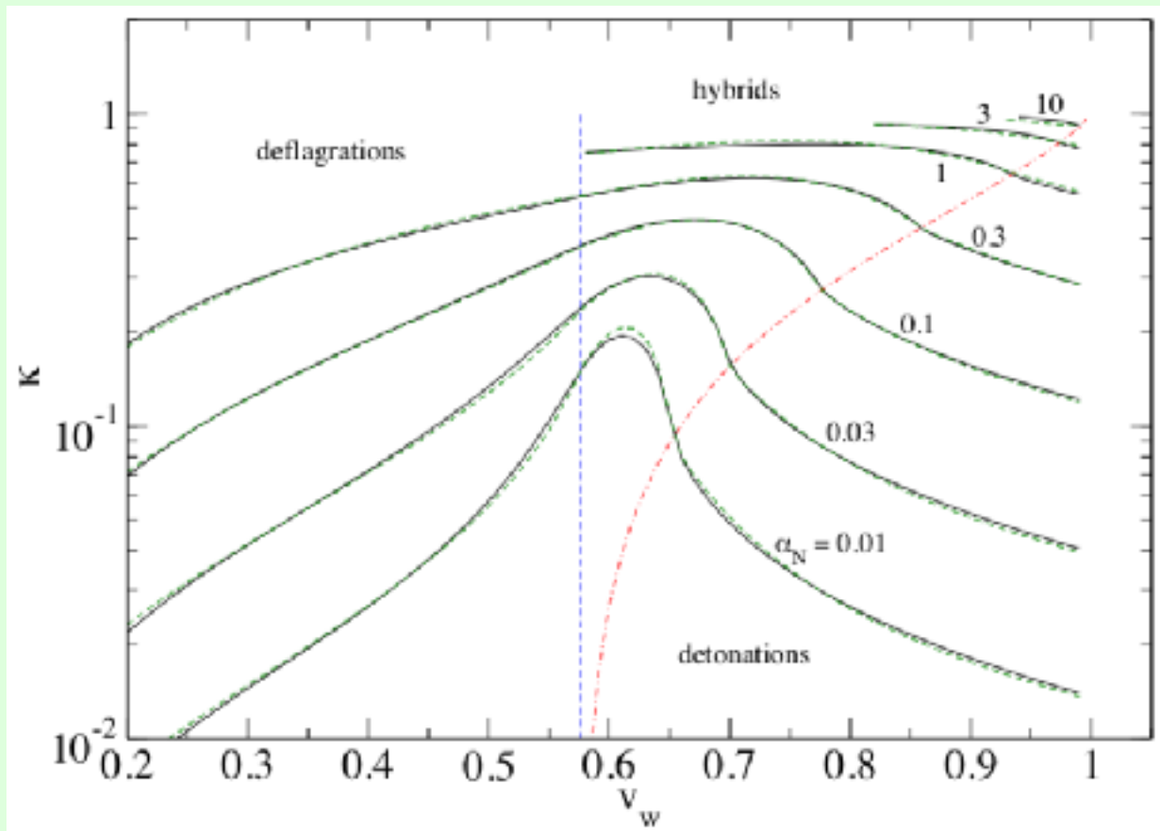


J. Espinosa, T. Konstandin, J. No, G. Servant, 2010

Generalization to other models

- Compute phase transition strength α_n and use fit for bag equation of state from **J. Espinosa, T. Konstandin, J. No, G. Servant, 2010**

$$\kappa_\epsilon = \frac{\rho_{fl}}{\epsilon}$$



- But how to compute α_n in a different model? And where is the sound speed dependence?

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Generalization to other models

- Only T_n is known
- $\alpha_p = -\dfrac{4Dp}{3w_s(T_n)}$ $\qquad \alpha_e = \dfrac{4De}{3w_s(T_n)}$ $\qquad DX \equiv X_s(T_n) - X_b(T_n)$
- $\alpha_\theta = \dfrac{D\theta}{3w_s(T_n)}$ $\qquad \theta \equiv e - 3p$
- The speed of sound never enters 

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Model-dependence in hydrodynamics

- Hydrodynamic equations
  - $$2\frac{v}{\xi} = \gamma^2(1 - v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v, \qquad \frac{\partial_v w}{w} = \left(\frac{1}{c_s^2} + 1\right)\gamma^2\mu$$
- Boundary conditions
  - $w(T_n) = w_n$
  - $$\frac{v_+}{v_-} = \frac{e_b(T_-) + p_s(T_+)}{e_s(T_+) + p_b(T_-)}, \qquad v_+ v_- = \frac{p_s(T_+) - p_b(T_-)}{e_s(T_+) - e_b(T_-)}$$

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Velocity matching

- $$\frac{v_+}{v_-} = \frac{e_b(T_-) + p_s(T_+)}{e_s(T_+) + p_b(T_-)}, \qquad v_+ v_- = \frac{p_s(T_+) - p_b(T_-)}{e_s(T_+) - e_b(T_-)}$$
- We assume that $T_+ \simeq T_-$.
- $$\frac{v_+}{v_-} \simeq \frac{(v_+ v_- / c_{s,b}^2 - 1) + 3\alpha_{\bar{\theta}}}{(v_+ v_- / c_{s,b}^2 - 1) + 3 v_+ v_- \alpha_{\bar{\theta}}}$$, either v_+ or v_- is known
- $$\boxed{\bar{\theta} \equiv e - \frac{p}{c_{s,b}^2} \qquad \alpha_{\bar{\theta}} \equiv \frac{D\bar{\theta}}{3 w_n}}$$

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Capture the model dependence in small number of parameters

- Model dependent parameters: $\alpha_{\bar{\theta}}$, $c_{s,\mathrm{broken}}$, $c_{s,\mathrm{symm}}$ + wall velocity ξ_w
- Can determine $\kappa_{\bar{\theta}} = \dfrac{4\rho_{fl}}{D\bar{\theta}} = \dfrac{4\rho_{\mathrm{fl}}}{3\alpha_{\bar{\theta}} w_n}$
- $K = \dfrac{D\bar{\theta}}{4e_n}\kappa_{\bar{\theta}}$ can now easily be obtained (but depends on the model)

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- $K = \dfrac{D\bar{\theta}}{4e_n}\kappa_{\bar{\theta}}$ can now easily be obtained (but depends on the model)

- Tricky detail: $\alpha_{\bar{\theta}}$ is defined at T_n, which is not T_+ for hybrids and deflagrations -> need shooting algorithm.

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Quick recap

- We want to find a model-independent expression/fit for $K = \frac{\rho_{fl}}{e_n}$
- We introduced $\bar{\theta} \equiv e - \frac{p}{c_s^2}$ $\alpha_{\bar{\theta}} \equiv \frac{D\bar{\theta}}{3w_n}$
- The velocity profile and $\kappa_{\bar{\theta}}$ only depend on $\alpha_{\bar{\theta}}$, $c_{s,\mathrm{broken}}, c_{s,\mathrm{symm}}$ and ξ_w
- K can easily be determined from $\kappa_{\bar{\theta}}$

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Make the result reusable

Template model with varying speed of sound

- $p_s = \frac{1}{3} a_+ T^\mu - \epsilon$ $\qquad e_s = a_+(\mu - 1)T^\mu + \epsilon$

  $p_b = \frac{1}{3} a_- T^\nu$ $\qquad e_b = \frac{1}{3} a_-(\nu - 1)T^\nu$

  $\nu = 1 + \frac{1}{c^2_{s,\mathrm{broken}}}, \quad \mu = 1 + \frac{1}{c^2_{s,\mathrm{symm}}}$

L. Leitao,
A. Megevand, 2015

- $\alpha_{\bar{\theta}} = \frac{1}{3}\left(1 - \frac{\nu}{\mu} + \frac{3\epsilon\nu}{a_+ \mu T_n^\mu}\right)$ $\qquad \frac{v_+}{v_-} = \frac{v_+ v_-(\nu - 1) - 1 + 3\alpha_{\bar{\theta}}}{v_+ v_-(\nu - 1) - 1 + 3 v_+ v_- \alpha_{\bar{\theta}}}$

NB!

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Python snippet computes $\kappa_{\bar{\theta}}$

2010.09744

Compute $\alpha_{\bar{\theta}}$, both c_ss and choose ξ_w

```
import numpy as np
from scipy.integrate import odeint
from scipy.integrate import simps

def mu(a,b):
  return (a-b)/(1.-a*b)

def getwow(a,b):
  return a/(1.-a**2)/b*(1.-b**2)

def getvm(al,vw,cs2b):
  if vw**2<cs2b:
    return (vw,0)
  cc = 1.-3.*al+vw**2*(1./cs2b+3.*al)
  disc = -4.*vw**2/cs2b+cc**2
  if (disc<0.)|(cc<0.):
    return (np.sqrt(cs2b), 1)
  return ((cc+np.sqrt(disc))/2.*cs2b/vw, 2)

def dfdv(xiw, v, cs2):
```

...

```
    Krf*= -wow*getwow(vp,vm)
  else:
    Krf = 0
  return (Ksh + Krf)/al
```

$\kappa_{\bar{\theta}}$

$$K = \frac{D\bar{\theta}}{4e_n}\kappa_{\bar{\theta}}$$

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Comparison of methods to compute K

M1	K
M2	$\left(\frac{D\bar{\theta}}{4e_+}\right)\kappa_{\bar{\theta}}(\alpha_{\bar{\theta}}, c_s)\|_\nu$
M3	$\left(\frac{D\theta}{4e_+}\right)\kappa_\epsilon(\alpha_\theta)$
M4	$\left(\frac{\alpha_\theta}{\alpha_\theta+1}\right)\kappa_\epsilon(\alpha_\theta)$
M5	$\left(\frac{\alpha_p}{\alpha_p+1}\right)\kappa_\epsilon(\alpha_p)$
M6	$\left(\frac{\alpha_e}{\alpha_e+1}\right)\kappa_\epsilon(\alpha_e)$

Full hydrodynamics

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Comparison of methods to compute K

M1	K
M2	$\left(\frac{D\bar{\theta}}{4e_+}\right)\kappa_{\bar{\theta}}(\alpha_{\bar{\theta}}, c_s)\big\|_{\nu}$
M3	$\left(\frac{D\theta}{4e_+}\right)\kappa_{\epsilon}(\alpha_{\theta})$
M4	$\left(\frac{\alpha_{\theta}}{\alpha_{\theta}+1}\right)\kappa_{\epsilon}(\alpha_{\theta})$
M5	$\left(\frac{\alpha_{p}}{\alpha_{p}+1}\right)\kappa_{\epsilon}(\alpha_{p})$
M6	$\left(\frac{\alpha_{e}}{\alpha_{e}+1}\right)\kappa_{\epsilon}(\alpha_{e})$

New approach

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Comparison of methods to compute K

M1	K
M2	$\left(\frac{D\bar{\theta}}{4e_+}\right)\kappa_{\bar{\theta}}(\alpha_{\bar{\theta}}, c_s)\|_\nu$
M3	$\left(\frac{D\theta}{4e_+}\right)\kappa_\epsilon(\alpha_\theta)$
M4	$\left(\frac{\alpha_\theta}{\alpha_\theta+1}\right)\kappa_\epsilon(\alpha_\theta)$
M5	$\left(\frac{\alpha_p}{\alpha_p+1}\right)\kappa_\epsilon(\alpha_p)$
M6	$\left(\frac{\alpha_e}{\alpha_e+1}\right)\kappa_\epsilon(\alpha_e)$

Methods used in the literature. Mapping onto bag equation of state.

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Two toy models

- SM-like

$$F(\phi,T) = -\frac{a_+}{3}T^4 + \lambda(\phi^4 - 2E\phi^3 T + \phi^2(E^2T_{\rm cr}^2 + c(T^2 - T_{\rm cr}^2))) + \frac{\lambda}{4}(c - E^2)^2 T_{\rm cr}^4$$

with $p_s = -F(0,T)$ and $p_b = -F(\phi_{min},T)$

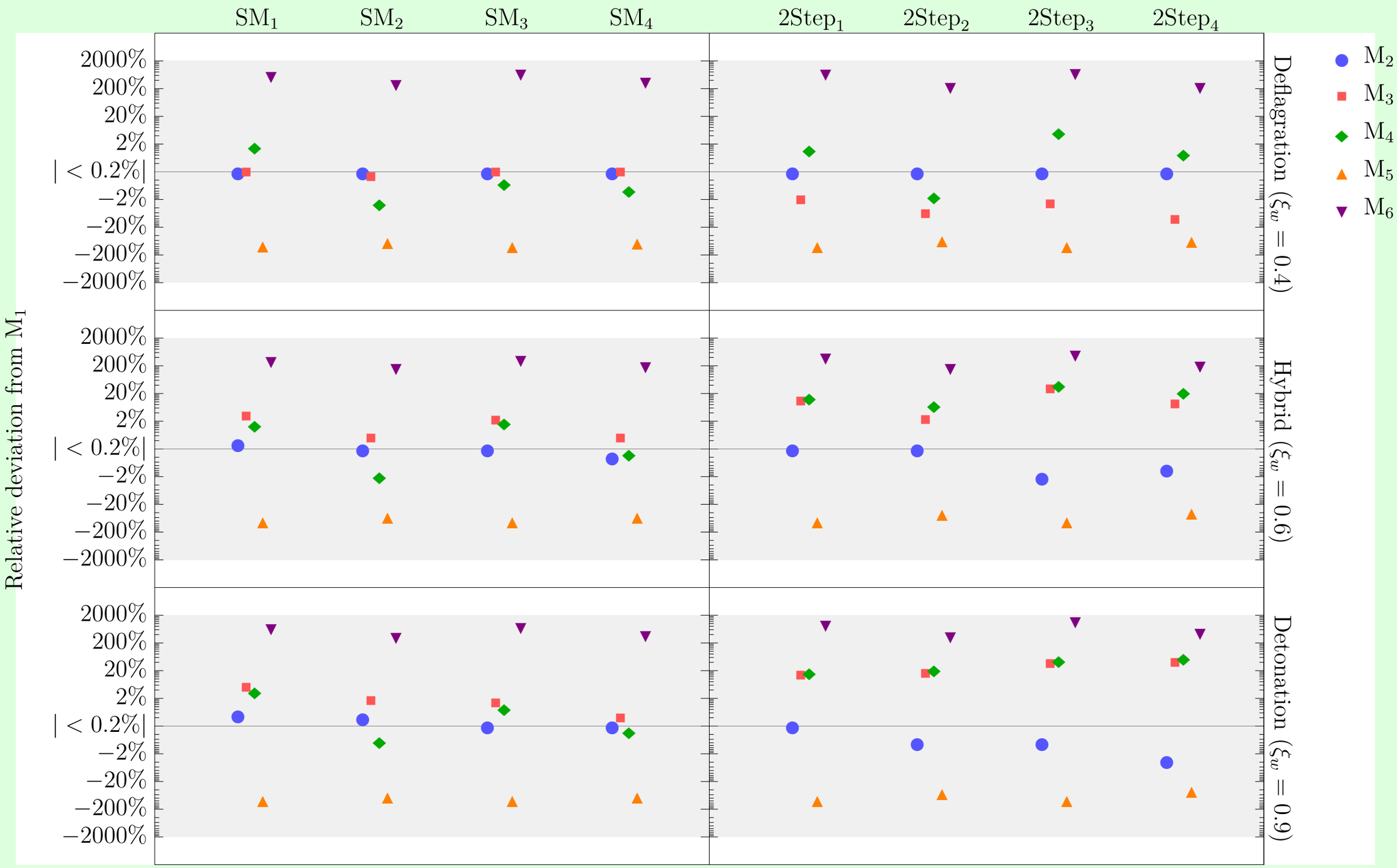
- Two-step

$$p_s(T) = \frac{1}{3}a_+T^4 + (b_+ - c_+T^2)^2 - b_-^2\,, \qquad p_b(T) = \frac{1}{3}a_+T^4 + (b_- - c_-T^2)^2 - b_-^2$$

Model	$3\lambda/a_+$	E	c	$T_+/T_{\rm cr}$	$\alpha_{\bar\theta}$	c_s^2
SM_1	10	0.3	0.2	0.9	0.0297	0.326
SM_2	10	0.3	0.2	0.8	0.0498	0.331
SM_3	3	0.3	0.2	0.9	0.00887	0.331
SM_4	3	0.3	0.2	0.8	0.0149	0.333

Model	$b_-/(\sqrt{a_+}T_{\rm cr}^2)$	$c_-/\sqrt{a_+}$	$c_+/\sqrt{a_+}$	$T_+/T_{\rm cr}$	$\alpha_{\bar\theta}$	c_s^2
$2\mathrm{step}_1$	$0.4/\sqrt{3}$	$0.2/\sqrt{3}$	$0.1/\sqrt{3}$	0.9	0.0156	0.312
$2\mathrm{step}_2$	$0.4/\sqrt{3}$	$0.2/\sqrt{3}$	$0.1/\sqrt{3}$	0.7	0.0704	0.297
$2\mathrm{step}_3$	$0.5/\sqrt{3}$	$0.4/\sqrt{3}$	$0.2/\sqrt{3}$	0.9	0.0254	0.282
$2\mathrm{step}_4$	$0.5/\sqrt{3}$	$0.4/\sqrt{3}$	$0.2/\sqrt{3}$	0.7	0.159	0.245

6 Methods to compute K



Summary

- Gravitational waves are a promising test of cosmological phase transitions
- $\alpha_{\bar{\theta}}$, c_s and ξ_w: model-independent parameterization of the hydrodynamics
- α_p and α_e give a very bad estimate of K
- Matching onto the template model gives most accurate approximation of K

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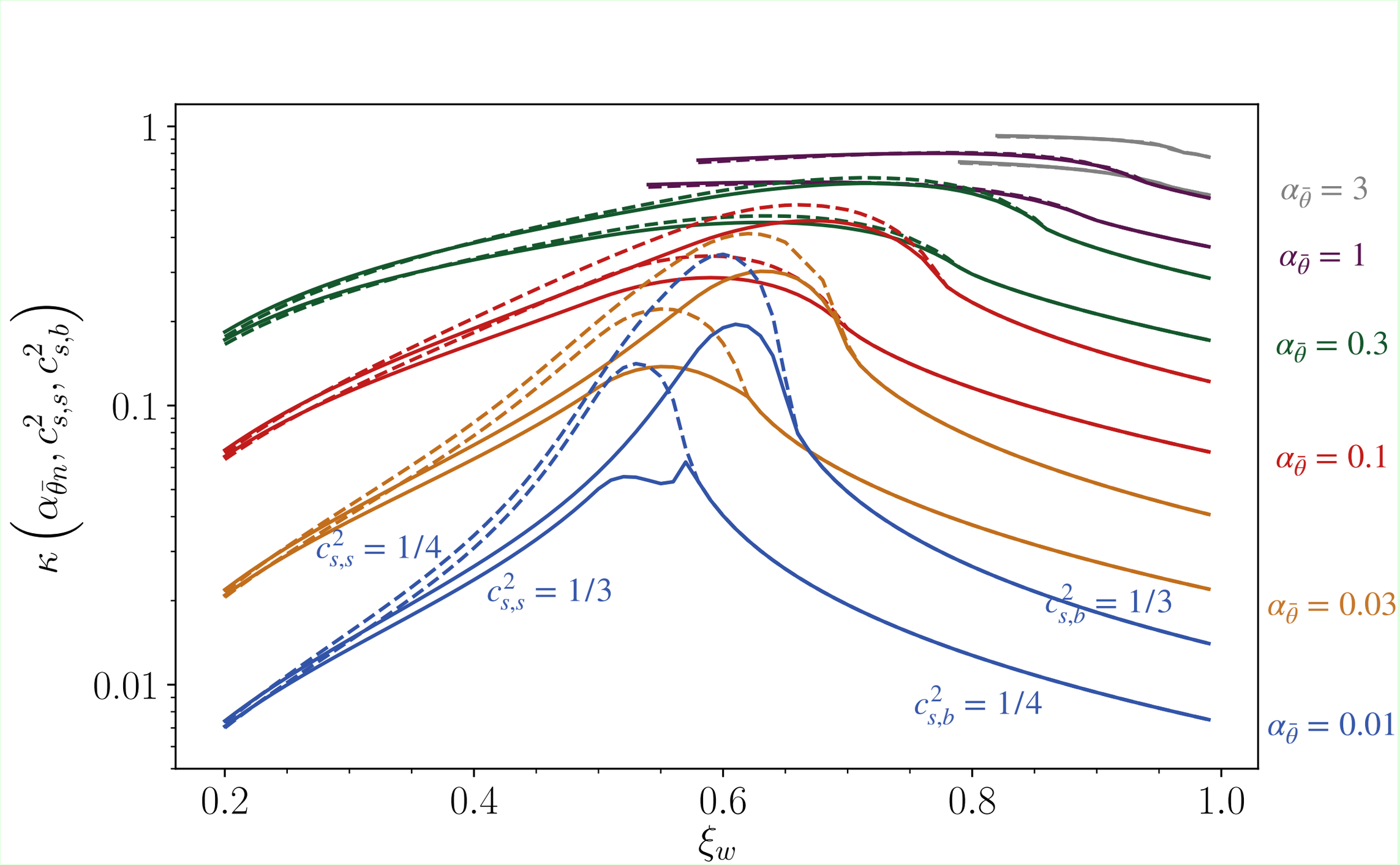

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