Towards an all-orders calculation of the electroweak bubble wall velocity

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Gravitational Probes of Physics Beyond the Standard Model

First order electroweak phase transition

Within the Standard Model the EWPT is a crossover D'Onofrio & Rummukainen (2015)

Minimal new physics → first order PT Anderson & Hall (1992)

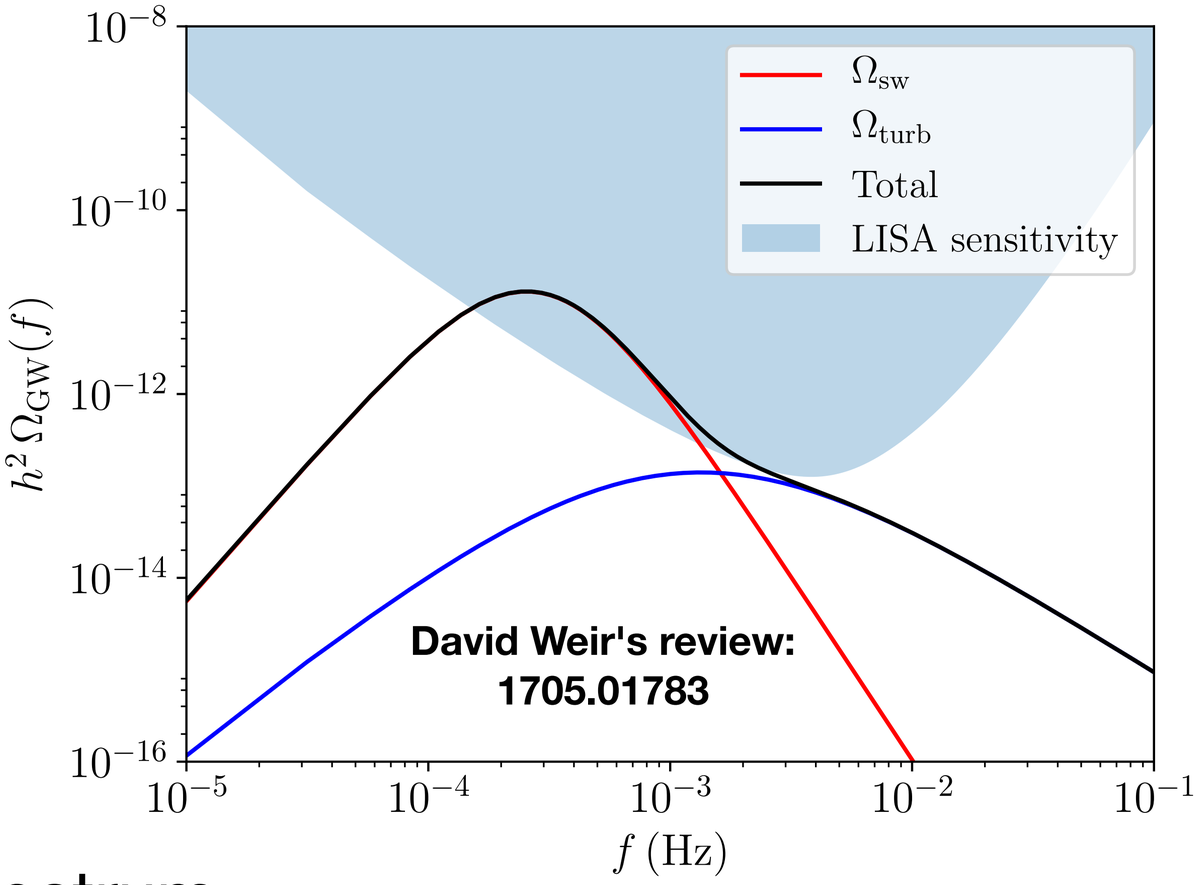
See overview by Caprini

- Matter-antimatter asymmetry Kuzmin, Rubakov & Shaposhnikov (1985)
- Topological defects Achucarro & Vachaspati (2000)
- Primordial magnetic fields Vachaspati (1991)
- Stochastic gravitational wave background Kamionkowski, Kosowsky & Turner (1993)

See talks by Weir, Lewicki, de Vis & Pol

First order electroweak phase transition

- Collision bubble walls: $\Omega_{\rm env}$
- Sound waves: $\Omega_{\rm sw}$
- Turbulence: $\Omega_{\rm turb}$



- Velocity affects gravitational wave spectrum
- Velocity affects efficiency of electroweak baryogenesis. But EWBG still viable with fast walls Cline & Kainulainen (2007.10935)
  see also Dorsch et al (2106.06547)

I will discuss relativistic bubble walls so the following calculations would not be valid for subsonic bubble wall

Vacuum pressure from symmetry breaking Higgs potential

Scales γ^0

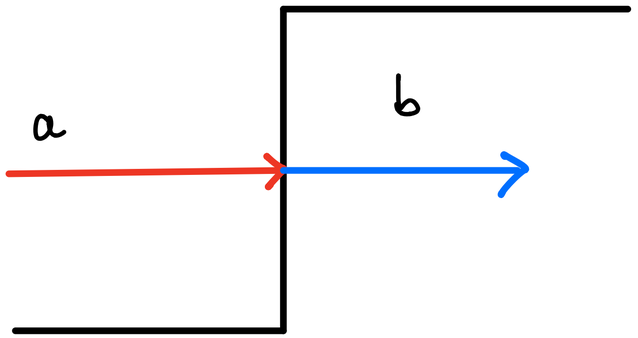
Thermal pressure, resulting from interactions of wall with the plasma particles

Non-trivial γ-dependence

Terminal velocity determined from balancing the force which drives the bubble expansion with the frictional pressure from the plasma on the wall

Kinematics and Assumptions

z-translation invariance broken by bubble wall



$$p_{a,z,s} = \sqrt{E_a^2 - |p_{a,\perp}|^2 - m_{a,s}^2}$$
$$p_{a,z,h} = \sqrt{E_a^2 - |p_{a,\perp}|^2 - m_{a,h}^2}$$

$s \equiv$ symmetric phase

$h \equiv$ broken phase

Throughout we assume infinitely thin walls: $L_w \to 0$

Transverse momentum conserved:

$$p_{a,\perp} = p_{b,\perp}$$

Energy conserved:

$$E_a = E_b$$

1-to-1 pressure calculation for relativistic bubble walls

1-to-1 calculation argument

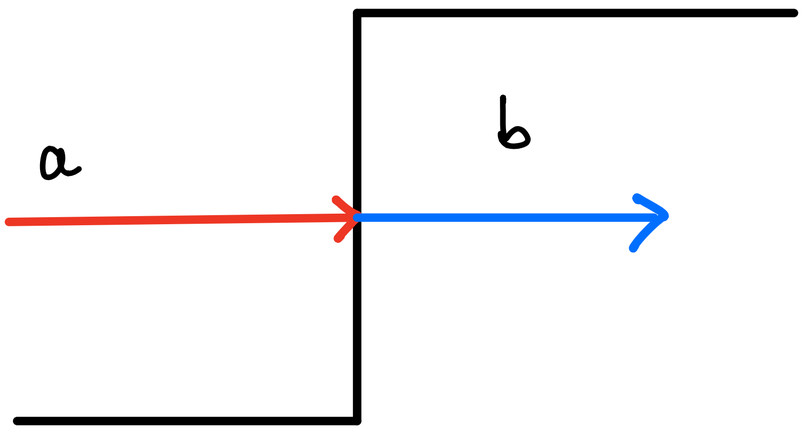
Arnold (1993)

Bodeker & Moore (2009)

Friction → scattering particles that couple to Higgs condensate

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

Lorentz factor of the wall



$$\Delta p_{\text{wall}} \equiv p_{a,z,s} - p_{b,z,h} = \sqrt{E_a^2 - m_{a,s}^2 - p_{a,\perp}^2} - \sqrt{E_b^2 - m_{b,h}^2 - p_{b,\perp}^2}$$

$$E_a^2 \sim p_{a,z,s}^2 \sim \gamma^2 T^2 \gg m_{a,s}^2, m_{b,h}^2, p_{a,\perp}^2 \Longrightarrow \Delta p_{\text{wall}} \approx \frac{m_{b,h}^2 - m_{a,s}^2}{2E_a} = \frac{m_{b,h}^2 - m_{a,s}^2}{2\gamma T}$$

$$\mathcal{P}_{1\to 1} = \frac{[\text{ force }]}{[\text{ area }]} = \frac{\Delta[\text{ momentum }]}{[\text{ area }] \times [\text{ time }]} = [\text{ flux }] \times \Delta[\text{ momentum }]$$

$$\sim \gamma T^3 \times \frac{\Delta m^2}{2\gamma T} = \frac{\gamma^0 T^2 \Delta m^2}{2}$$

Bodeker & Moore (2009)

- 1-to-1: no flavour change. Assume no particles reflected

Flux incoming particle

Momentum transfer "observable"

$$\mathcal{P}_{1\to1} = \sum_a \int \mathrm{d}\mathcal{F}_a \sum_b \int \mathrm{d}\mathbb{P}_{a\to b} \Delta p_z \left(1 \pm f_b\right)$$

Differential probability

Quantum Statistical factors

$$\mathrm{d}\mathbb{P}_{a\to b} = \frac{d^3 p_b}{(2\pi)^3} \frac{1}{2E_b} \times (2\pi)^3 \delta^2 \left(p_{a,\perp} - p_{b,\perp}\right) \delta\left(E_a - E_b\right) \left(2p_{b,z,h}\right) \delta_{ab}$$

$$\mathcal{P}_{1\to1} = \sum_a \nu_a \int \frac{d^3 p_a}{(2\pi)^3} \frac{d^3 p_b}{(2\pi)^3} \frac{p_{b,z,s}}{E_b} f_a \left(1 \pm f_b\right) \left(p_{a,z,s} - p_{b,z,h}\right) (2\pi)^3 \delta^2 \left(p_{a,\perp} - p_{b,\perp}\right) \delta\left(E_a - E_b\right)$$

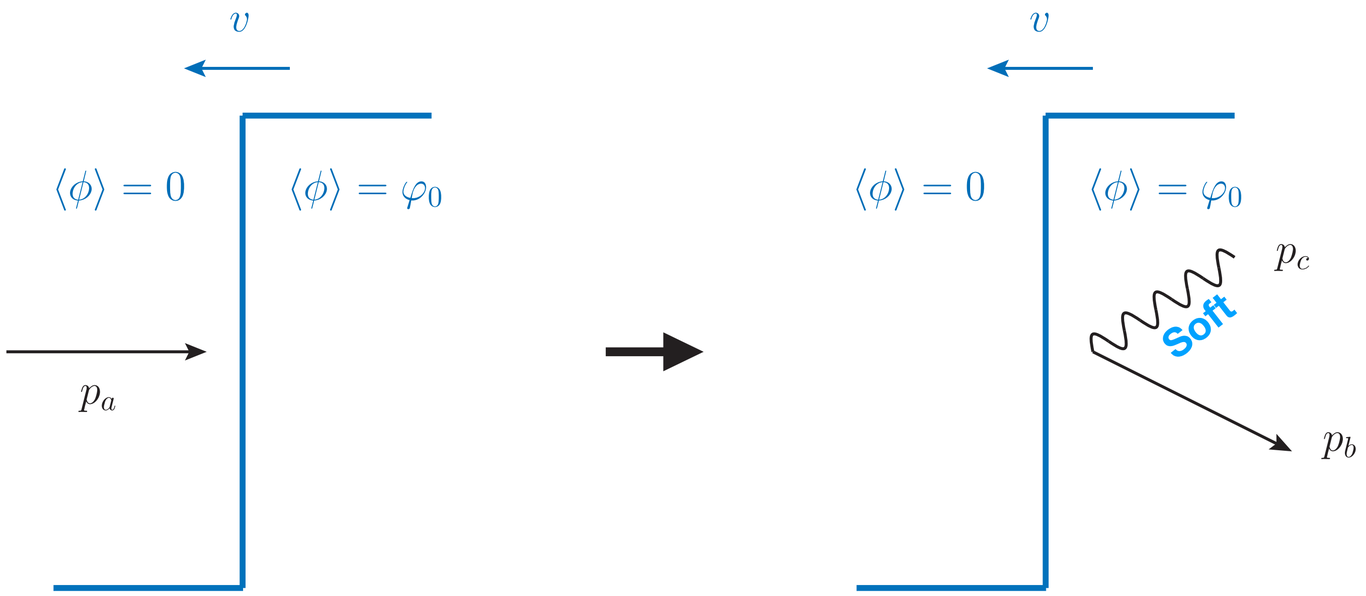
$$\frac{d^3 p_b}{(2\pi)^3 2E_b} = \frac{d^2 p_{b,\perp}}{(2\pi)^3} \frac{dE_b}{2E_b} \frac{E_b}{p_{b,z,s}}$$

$$\mathcal{P}_{1\to1} = \sum_a \nu_a \int \frac{d^3 p_a}{(2\pi)^3} f_a \left(1 \pm f_b\right) \Delta p_{\mathrm{wall}} \approx \gamma^0 T^2 \Delta m^2$$

1-to-2 pressure calculation for relativistic bubble walls

1-to-2 calculation

Bodeker & Moore (2017)



Incident particle's energy → mass second particle + transverse momentum

$\Delta p_{1\to1} < \Delta p_{1\to2}$ unless $p_\perp = 0$ and $m_c = 0$

1-to-2 calculation

Bodeker & Moore (2017)

Flux incoming particle

Quantum Statistical factors

$$\mathcal{P}_{1\to 2} = \sum \nu_{a,b,c} \int [dp_a]\,[dp_b]\,[dp_c]\, f\,(p_a, p_b, p_c)\, \Delta p_z (2\pi)^3 \delta^2\,(p_{a,\perp} - p_{c,\perp} - p_{b,\perp})\, \delta\,(E_a - E_c - E_b)\, |\mathcal{M}|^2$$

Momentum transfer "observable"

Combination of PS + "observable"

$$\mathcal{P}_{1\to 2} = \sum_{abc} \nu_a \int \frac{d^3 p_a}{(2\pi)^3 2E_a} \int \frac{d^2 p_{c,\perp}}{(2\pi)^2} \frac{dE_c}{(2\pi) 2E_c} f(p_a, p_b, p_c) \Delta p_z \frac{1}{2p_{b,z,s}} \frac{E_c}{p_{c,z,s}} |\mathcal{M}|^2$$

B&M region of interest:

Ingoing hard	$p_{a,\perp} \sim T$	$p_{a,z,s} \sim E_a \sim \gamma_w T$	$m_{a,s}, m_{a,h} \ll \gamma_w T$
Outgoing hard	$p_{b,\perp} \sim \max[T, m_c]$	$p_{b,z,s} \sim E_b \sim \gamma_w T$	$m_{b,s}, m_{b,h} \ll \gamma_w T$
Outgoing soft	$p_{c,\perp} \sim \max[T, m_c]$	$p_{c,z,s} \sim \max[T, m_c]$	$m_{c,s}, m_{c,h} \ll \gamma_w T$

1-to-2 calculation

Bodeker & Moore (2017)

$$\mathcal{P}_{1\to 2} = \sum_{abc} \nu_a \int \frac{d^3 p_a}{(2\pi)^3 2E_a} \int \frac{d^2 p_{c,\perp}}{(2\pi)^2} \frac{dE_c}{(2\pi) 2E_c} f(p_a, p_b, p_c) \Delta p_z \frac{1}{2p_{b,z,s}} \frac{E_c}{p_{c,z,s}} |\mathcal{M}|^2$$

B&M region of interest:

$$p_{c,z}(z) = E_c \sqrt{1 - \underbrace{\frac{m_{c,z}^2 + p_{c,\perp}^2}{E_c^2}}_{\epsilon}} \approx E_c \left(1 - \frac{\epsilon}{2} + \mathcal{O}(\epsilon^2) + \cdots\right)$$

Collinearity: ϵ

$$p_{b,z} = \sqrt{E_b^2 - p_{\perp,b}^2 - m_b^2(z)} \approx E_a(1 - x)$$

Softness: $x = \frac{E_c}{E_a}$

Bodeker & Moore (2017)

From PS

"Observable"

$$(p_{a,z,s} - p_{c,z,h} - p_{b,z,h})\,\frac{1}{2p_{b,z,s}}\,\frac{E_c}{p_{c,z,s}} \approx \underbrace{\frac{1}{2E_a}\,\frac{m_c^2(z) + p_{c,\perp}^2}{2E_c}}_{\sim x\epsilon + O(\epsilon^2 x) + \ldots}$$

B&M 1-to-2 master equation:

$$\mathcal{P}_{1\to 2} = \sum \nu_a \int \frac{d^3 p_a}{(2\pi)^3 2E_a} \int \frac{d^2 p_{c,\perp}}{(2\pi)^2} \frac{dE_c}{(2\pi) 2E_c} f(p_a, p_b, p_c) \frac{m_{c,h}^2(z) + p_{c,\perp}^2}{2E_c} |\mathcal{M}|^2$$

Still need to determine matrix element squared

The bubble wall is invariant in time and the transverse directions

Bodeker & Moore (2017)

$$\langle p_c p_b | T | p_a \rangle = \int d^4x \, \langle p_c p_b \, | H_{\rm int} | \, p_a \rangle = (2\pi)^3 \delta^2 \left(p_{a,\perp} - p_{b,\perp} - p_{c,\perp} \right) \delta \left(E_a - E_b - E_c \right) \mathcal{M}$$

$$\mathcal{M} \equiv \int dz \, \chi^*_{p_c}(z) \chi^*_{p_b}(z) V(z) \chi_{p_a}(z)$$

Mode functions are treated using the WKB approximation:

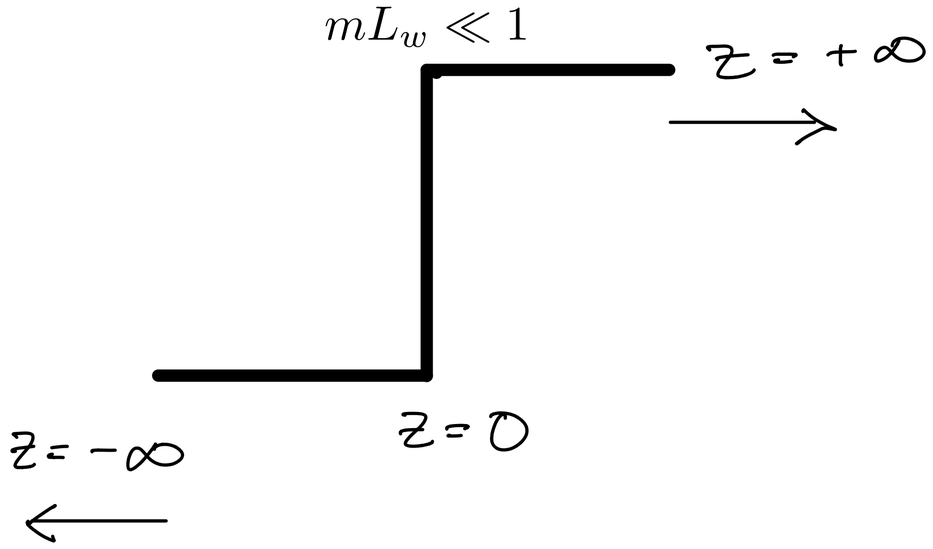
$$\chi_{p_c}(z) \simeq \sqrt{\frac{p_{c,z,s}}{p_{c,z}(z)}} \exp\left(i \int_0^z p_{c,z}\left(z'\right) dz' \right)$$

$$\mathcal{M} = \int dz\, \chi_b\left(p_{b,z,s}, z\right)^* \chi_c\left(p_{c,z,s}, z\right)^* \chi_a\left(p_{a,z,s}, z\right) V(z)$$

Bodeker & Moore (2017)

$$\mathcal{M} = V_s \int_{-\infty}^{0} dz \exp\left[iz\frac{A_s}{2E_a}\right] + V_h \int_{0}^{\infty} dz \exp\left[iz\frac{A_h}{2E_a}\right] = 2iE_a\left(\frac{V_h}{A_h} - \frac{V_s}{A_s}\right)$$

$$A_s = E_a\left(p_{a,z,s} - p_{b,z,s} - p_{c,z,s}\right)$$
$$A_h = E_a\left(p_{a,z,h} - p_{b,z,h} - p_{c,z,h}\right)$$



1/A terms resemble propagators, but they only propagate in the z-direction!

Vertex Function

Bodeker & Moore (2017)

$$|\mathcal{M}|^2 \simeq 4E_a^2|V|^2\frac{(A_h - A_s)^2}{A_h^2 A_s^2}$$

$a(p) \to b(k)c(p-k)$	V^2
$S \to V_T S$ $F \to V_T F$ $V \to V_T V$	$4g^2 C_2[R]\frac{1}{x^2}k_\perp^2$
$S \to V_L S$ $F \to V_L F$ $V \to V_L V$	$4g^2 C_2[R]\frac{1}{x^2}m^2$
$F \to F V_T$	$2g^2 C_2[R]\frac{1}{x}\left(k_\perp^2 + m_b^2\right)$
$V \to FF$	$2g^2 T[R]\frac{1}{x}\left(k_\perp^2 + m_b^2\right)$
$S \to S V_T$	$4g^2 C_2[R]k_\perp^2$
$F \to SF$	$y^2\left(k_\perp^2 + 4m_a^2\right)$
$S \to SS$	$\lambda^2\varphi^2$

$$k_\perp \equiv p_{c,\perp}$$

These are splitting functions up to the normalisation $P_{b\leftarrow a}(x) = |V|^2 x(1-x)/16\pi^2 k_\perp^2$

$$A_s \approx 2E_a \times \left(-\frac{|p_{a,\perp}|^2 + m_{a,s}^2}{2E_a} + \frac{|p_{b,\perp}|^2 + m_{b,s}^2}{2E_b} + \frac{|p_{c,\perp}|^2 + m_{c,s}^2}{2E_c} \right) \approx \frac{|p_{c,\perp}|^2 + m_{c,s}^2}{E_c/E_a}$$

$$A_h \approx 2E_a \times \left(-\frac{|p_{a,\perp}|^2 + m_{a,h}^2}{2E_a} + \frac{|p_{b,\perp}|^2 + m_{b,h}^2}{2E_b} + \frac{|p_{c,\perp}|^2 + m_{c,h}^2}{2E_c} \right) \approx \frac{|p_{c,\perp}|^2 + m_{c,h}^2}{E_c/E_a}$$

$$|\mathcal{M}|^2 = 4E_a^2 \, |V_s|^2 \, \frac{(A_s - A_h)^2}{A_s^2 A_h^2}$$

$$\approx 4E_a \frac{|V_s|^2}{\underbrace{(E_c/E_a)}_{x}{}^{-2}} \frac{\left(m_{c,h}^2 - m_{c,s}^2\right)^2}{\left(|p_{c,\perp}|^2 + m_{c,s}^2\right)^2 \left(|p_{c,\perp}|^2 + m_{c,h}^2\right)^2}$$

Lets keep track of what cancels where…

$$m_{c,s} \ll m_{c,h}$$

$$\begin{aligned}
|\mathcal{M}|^2 &= 4E_a^2 \left|V_s\right|^2 x^2 \frac{m_{c,h}^4}{\left|p_{c,\perp}\right|^4 \left(\left|p_{c,\perp}\right|^2 + m_{c,h}^2\right)^2} \\
&= 16 g^2 C_2[R] \frac{\left|p_{c,\perp}\right|^2}{\cancel{x^2}} \cancel{x^2} \frac{m_{c,h}^4}{\left|p_{c,\perp}\right|^4 \left(\left|p_{c,\perp}\right|^2 + m_{c,h}^2\right)^2} \\
&= 16 g^2 C_2[R] E_a^2 \frac{m_{c,h}^4}{\left|p_{c,\perp}\right|^2 \left(\left|p_{c,\perp}\right|^2 + m_{c,h}^2\right)^2}
\end{aligned}$$

Combine matrix element squared with momentum transfer observable:

$$\begin{aligned}
|\mathcal{M}|^2 \times \Delta p_z &\approx 16 E_a^2 g^2 C_2[R] \frac{m_{c,h}^4}{\left|p_{c,\perp}\right|^2 \left(\left|p_{c,\perp}\right|^2 + m_{c,h}^2\right)^2} \times \frac{m_{c,h}^2 + p_{c,\perp}^2}{2E_c} \\
&\approx \frac{8 g^2 E_a}{x} C_2[R] \frac{m_{c,h}^4}{\left|p_{c,\perp}\right|^2 \left(\left|p_{c,\perp}\right|^2 + m_{c,h}^2\right)}
\end{aligned}$$

Two pieces left: the integration of PS of incoming “a” gives the flux. We also need to integrate over phase space of particle “c” our soft emission

$$\int_{g^2T^2}^{m^2} \frac{dp_{c,\perp}^2}{(2\pi)^2 \left|p_{c,\perp}\right|^2 \left(\left|p_{c,\perp}\right|^2 + m_{c,h}^2\right)} \sim \frac{1}{24\pi m_{c,h}^2}$$

B&M assume $g^2T^2 \ll m^2$

The thermal mass would cut of the integral and you’d get some (possibly large) $\log\left(m_{c,h}^2/m_{c,s}^2\right)$ see Azatov et al (2010.02590)

$$\int \frac{dE_c}{E_c^2} \sim \frac{1}{m_{c,h}}$$

Gamma dependence enters from flux

$$\mathcal{P}_{1\to 2} \sim \gamma T^3 \frac{1}{m^2} \frac{1}{m} m^4$$

Mode Function

Transverse momentum integration

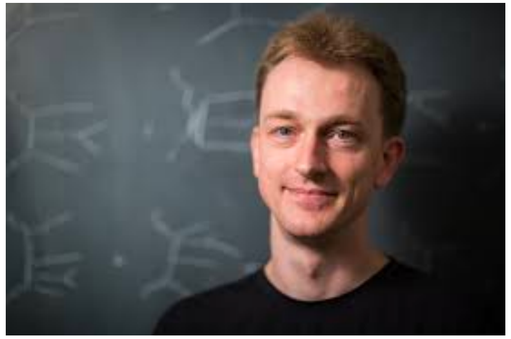
Energy integration

$$\mathcal{P}_{1\to 2} \sim m\gamma T^3$$

- $\mathscr{P}_{1\to 2} \propto \gamma$ while $\mathscr{P}_{1\to 1} \propto \gamma^0$. Since, vacuum pressure does not grow in λ, a terminal velocity will be reached.
- $\mathscr{P} \propto m^2 \implies$ no phase change pressure goes to zero. This comes from integrating over the mode functions
- $\mathscr{M}$: WKB and vertex. The vertex part is dominated in the soft regime.
- B&M cut off the $k_\perp$ and k^0 integration by gauge boson mass.
- This interaction looks like a collider/scattering experiment where the collision occurs between the ingoing particle and the wall $\implies$ the centre of mass energy will be large $\implies$ many soft emission.

1-to-n pressure calculation for relativistic bubble walls

2007.10343 (JCAP 2103 (2021) 009)



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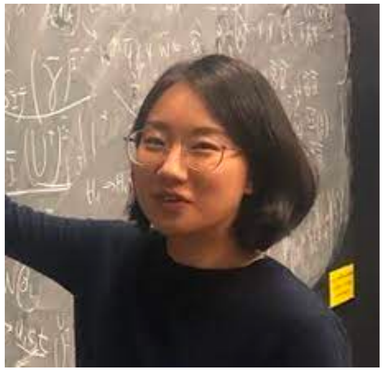

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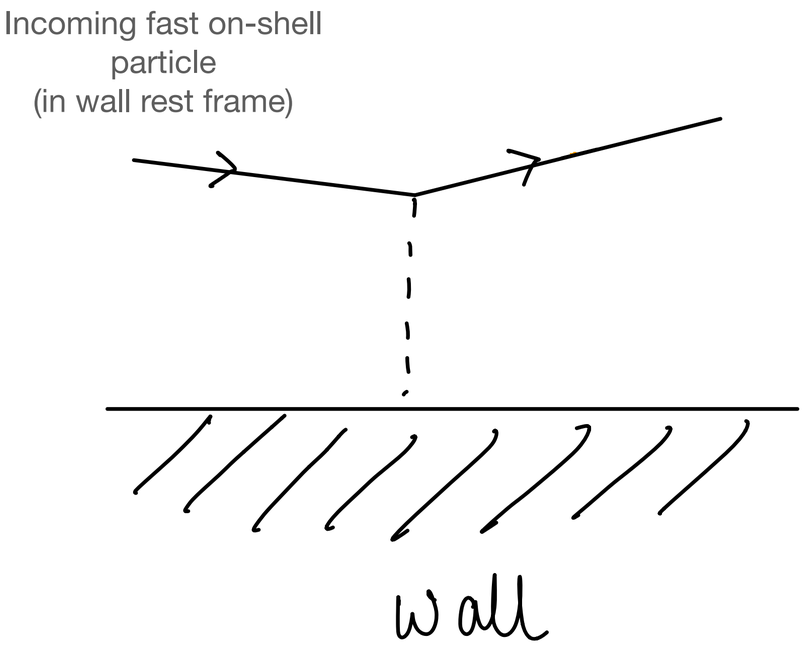
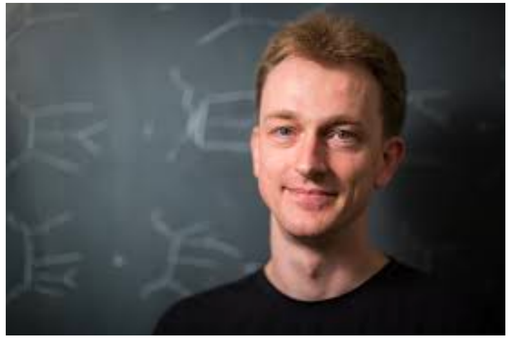


B&M investigated 1-to-1
which we will treat as leading order
process.



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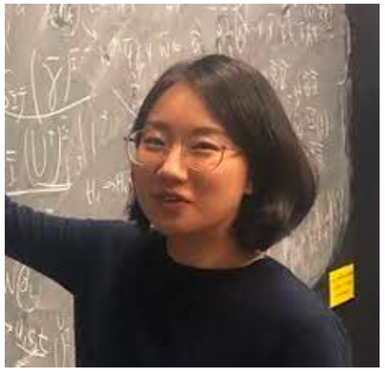

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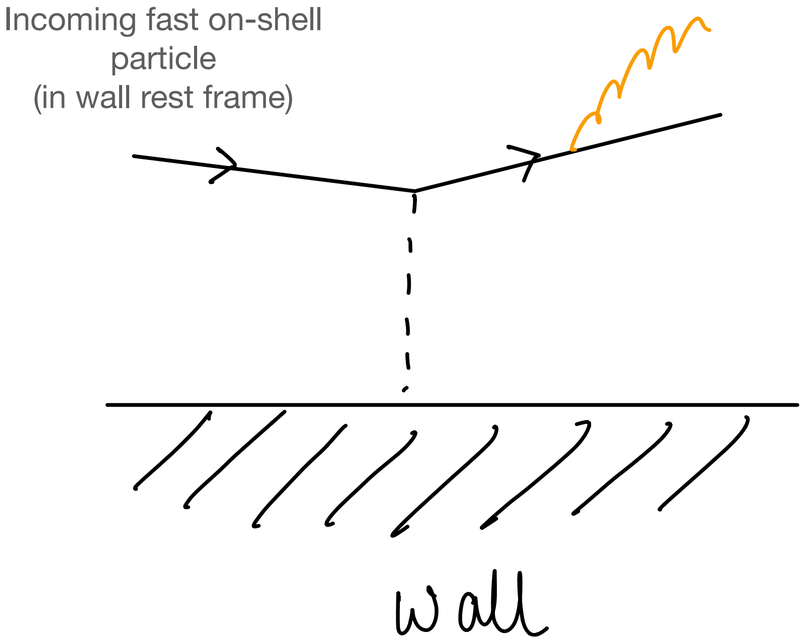
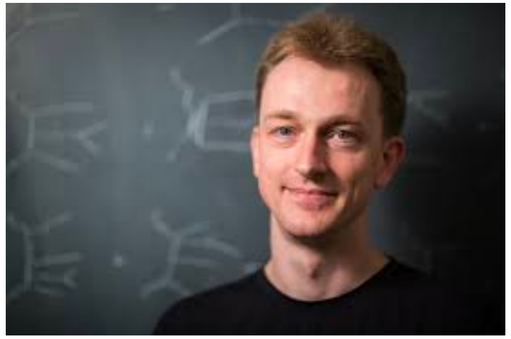


Treat 1-to-2 as quantum correction to 1-to-1 process.



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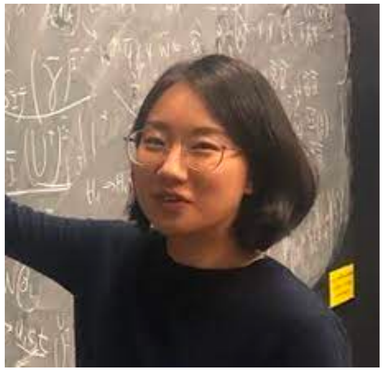

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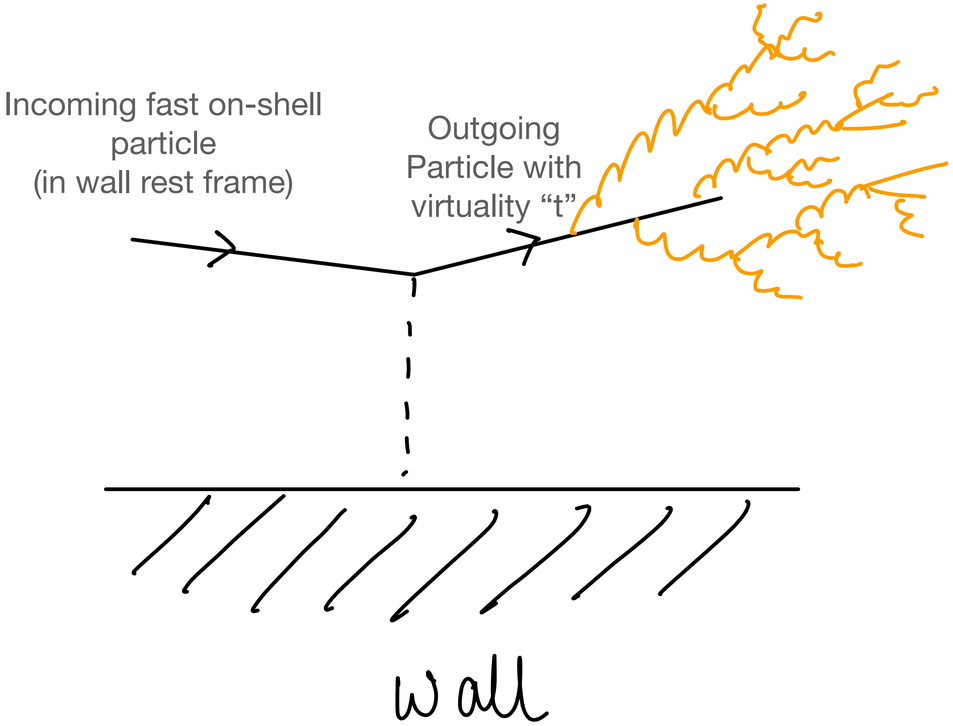


If 1-to-2 is important, will 1-to-n be even more important?

Reformulation of matrix element

B&M

$$A_s = -2E_a\left(p_{a,z,s} - p_{b,z,s} - p_{c,z,s}\right)$$
$$\approx p_{a,\perp}^2 + m_{a,s}^2 - \frac{p_{b,\perp}^2 + m_{b,s}^2}{E_b/E_a} - \frac{k_\perp^2 + m_{c,s}^2}{E_c/E_a}$$
$$= -\frac{k_\perp^2 + m_{b,s}^2}{(1-x)} - \frac{k_\perp^2 + m_{c,s}^2}{x}$$
$$\approx -\frac{k_\perp^2 + m_{c,s}^2}{x} \quad (x \ll 1)$$

Ultrarelativistic incoming particle

Thermal mass incoming particle negligible. Frame where incoming particle is "headon"

Soft limit

$$A_h = -2E_a\left(p_{a,z,h} - p_{b,z,h} - p_{c,z,h}\right)$$
$$\approx p_{a,\perp}^2 + m_{a,h}^2 - \frac{p_{b,\perp}^2 + m_{b,h}^2}{E_b/E_a} - \frac{k_\perp^2 + m_{c,h}^2}{E_c/E_a}$$
$$= -\frac{k_\perp^2 + m_{b,h}^2}{(1-x)} - \frac{k_\perp^2 + m_{c,h}^2}{x}$$
$$\approx -\frac{k_\perp^2 + m_{c,h}^2}{x} \quad (x \ll 1)$$

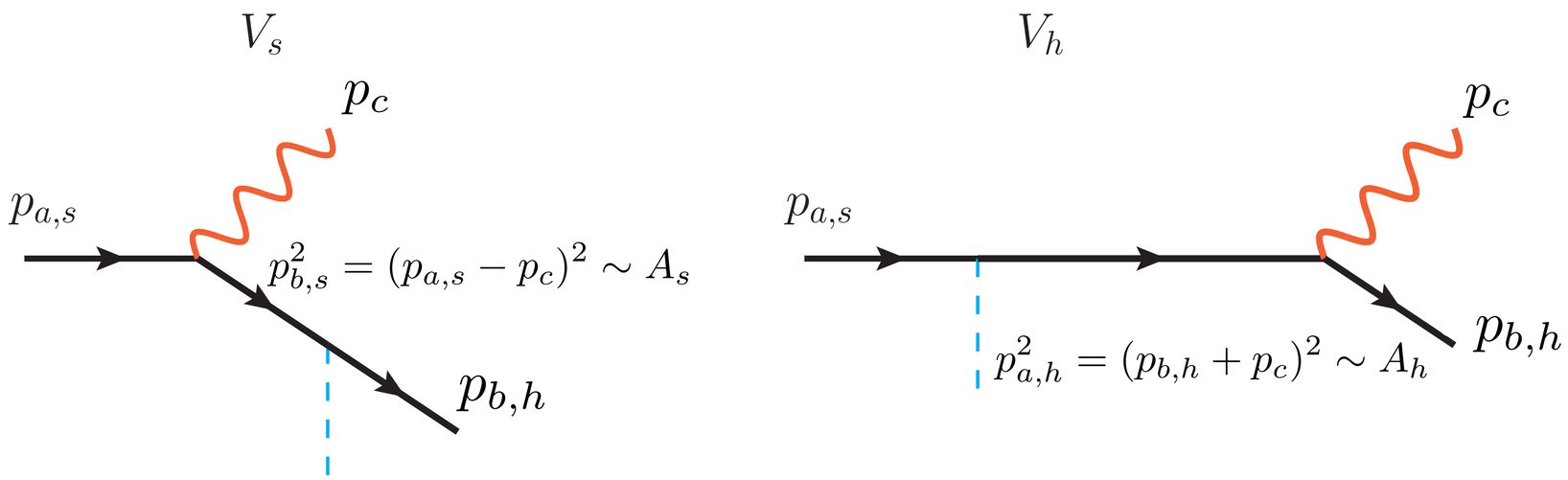
JT & co

$$A_s = -2p_{a,s}\cdot p_{c,s}$$
$$\approx -\frac{k_\perp^2 + m_{c,s}^2}{x(1-x)} - x m_{a,s}^2$$
$$\approx -\frac{k_\perp^2 + m_{c,s}^2}{x} \quad x \ll 1$$
$$A_h = -2p_{b,h}\cdot p_{c,h}$$
$$\approx -\frac{k_\perp^2}{x(1-x)} - \frac{1-x}{x}m_{c,h}^2 - \frac{x}{1-x}m_{b,h}^2$$
$$\approx -\frac{k_\perp^2}{x(1-x)} - \frac{1-x}{x}m_{c,h}^2 - \frac{x}{1-x}m_{b,h}^2$$
$$\approx -\frac{k_\perp^2 + m_{c,h}^2}{x} \quad x \ll 1$$

Two expressions same in the soft-limit but not same outside of soft limit.

Why the redefinition of A's?

Treat A's as covariant propagators. Wall provides spatial discontinuity off of which incoming particle scatters.



$$A_s = -2p_{a,s} \cdot p_{c,s} \approx 2p_{a,\perp} \cdot k_\perp - \frac{p_{a,\perp}^2 + m_{a,s}^2}{E_c/E_c} - \frac{k_\perp^2 + m_{c,s}^2}{E_c/E_c} \xrightarrow{x \ll 1} -\frac{k_\perp^2 + m_{c,s}^2}{x}$$

$$A_h = -2p_{b,h} \cdot p_{c,h} \approx 2p_{b,\perp} \cdot k_\perp - \frac{p_{b,\perp}^2 + m_{b,h}^2}{E_b/E_c} - \frac{k_\perp^2 + m_{c,h}^2}{E_c/E_b} \xrightarrow{x \ll 1} -\frac{k_\perp^2 + m_{c,h}^2}{x}$$

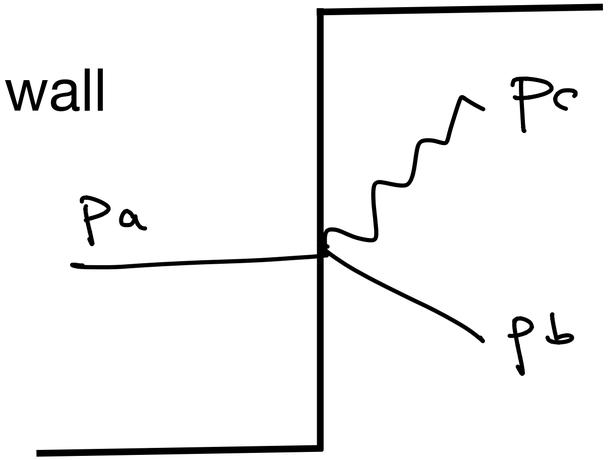
Reformulation of the Matrix Element

Our matrix element has the following form:

$$\left|M^{(0)}_{a\to bc}\right|^2 = 4E_a^2|g|^2\left(\frac{2p_{a,s}p_{b,h}}{p_{a,s}p_c p_{b,h}p_c} - \frac{m^2_{a,s}}{(p_{a,s}p_c)^2} - \frac{m^2_{b,h}}{(p_{b,h}p_c)^2}\right)$$

This is a gauge invariant squared matrix element that corresponds to decelerated charges emitting soft gauge field quanta.

Here, consider massless radiation with radiator acquiring mass across wall



$$\frac{1}{\left|\mathcal{M}^{(0)}_{a\to b}\right|^2}\int \frac{d^3p_c}{(2\pi)^3 2E_c}\left|M^{(0)}_{a\to bc}\right|^2 \approx \begin{cases} \dfrac{\alpha}{2\pi}\displaystyle\int \frac{dk_\perp^2}{k_\perp^2} C_{abc}\log\frac{m^2_{b,h}}{k_\perp^2} & \text{if} \quad m^2_{a,s} \ll k_\perp^2 \ll m^2_{b,h} \\ \dfrac{\alpha}{2\pi}\displaystyle\int \frac{dk_\perp^2}{k_\perp^2} C_{abc} & \text{if} \quad m^2_{a,s}, m^2_{b,h} \ll k_\perp^2 \end{cases}$$

Small $k_\perp$ →

Large $k_\perp$ →

Analytic Resummation

Banfi, Salam & Zanderighi (2005)

$R(v)$ probability for decay $a \to bc$. For this splitting to produce momentum transfer of v we require that it did not produce a large momentum transfer before.

$$R(v) = \int [dk] \left| M^2(k) \right| \Theta[V(\{p\}, k) - v]$$

$$V(p_a, p_b, p_c) = \frac{\Delta p_z}{\gamma T} \approx \frac{k_\perp^2 / \left(2E_a^2\right)}{x(1-x)}$$

$$R_{abc}(V) = C_{abc}|g|^2 \int \frac{d^3 p_c}{(2\pi)^3 2E_c} \left(\frac{2p_{b,h}p_{a,s}}{p_{a,s}p_c p_{b,h}p_c} + O\left(\frac{m_{a,s}^2}{k_\perp^2}, \frac{m_{b,h}^2}{k_\perp^2} \right) \right) \Theta\left(V\left(p_a, p_b, p_c\right) - V\right) \Theta\left(p_{b,z,h}\right) \Theta\left(p_{c,z,h}\right)$$

Rewrite phase space and matrix element squared in terms of observable V

$$R_{abc}(V) = C_{abc} \frac{\alpha}{2\pi} \int_V^1 \frac{\mathrm{d}V'}{V'} \int_0^1 \mathrm{d}x 2x \Theta\left(\frac{1}{1+V'} - x \right) \Theta\left(x - \frac{V'}{1+V'} \right)$$

$$R_{abc}(V) = \frac{\alpha}{2\pi} C_{abc} \left(L + 2\log\left(1 + e^{-L}\right) \right) \text{ where } L = \log \frac{1}{V}$$

Banfi, Salam & Zanderighi (2005)

Sudakov form factor

$$\Delta_a(V) = \exp\left\{-\sum_b R_{ab}(V)\right\}, \qquad \text{where} \qquad R_{ab}(V) = \sum_c R_{abc}(V)$$

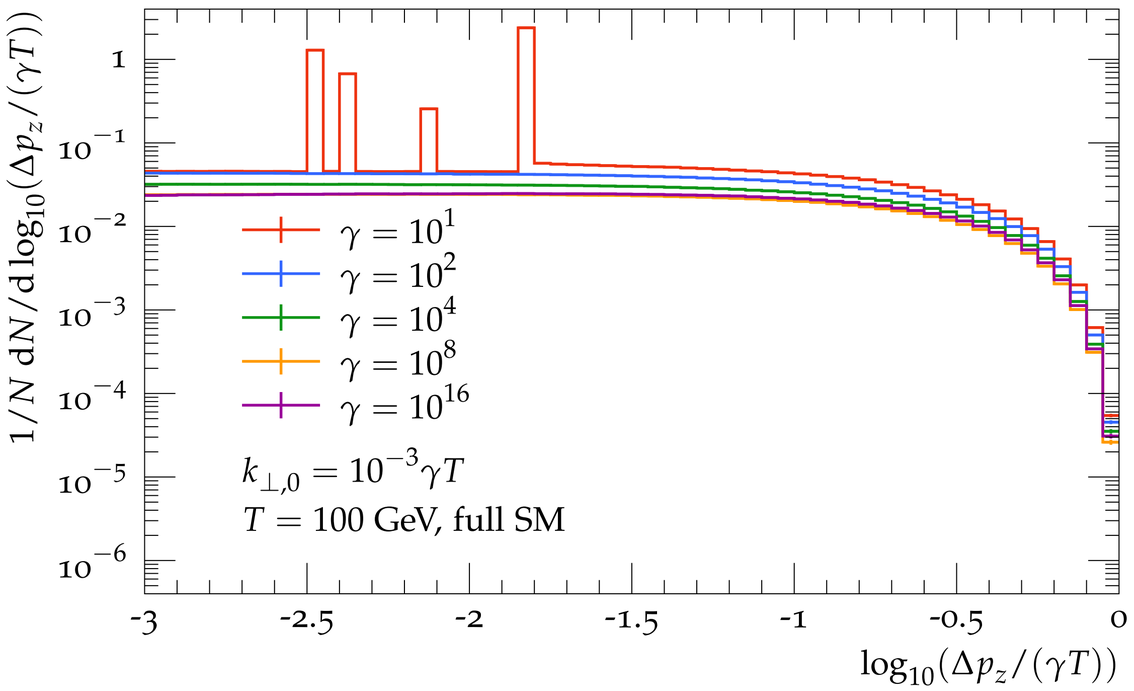
Average momentum transfer from all branchings

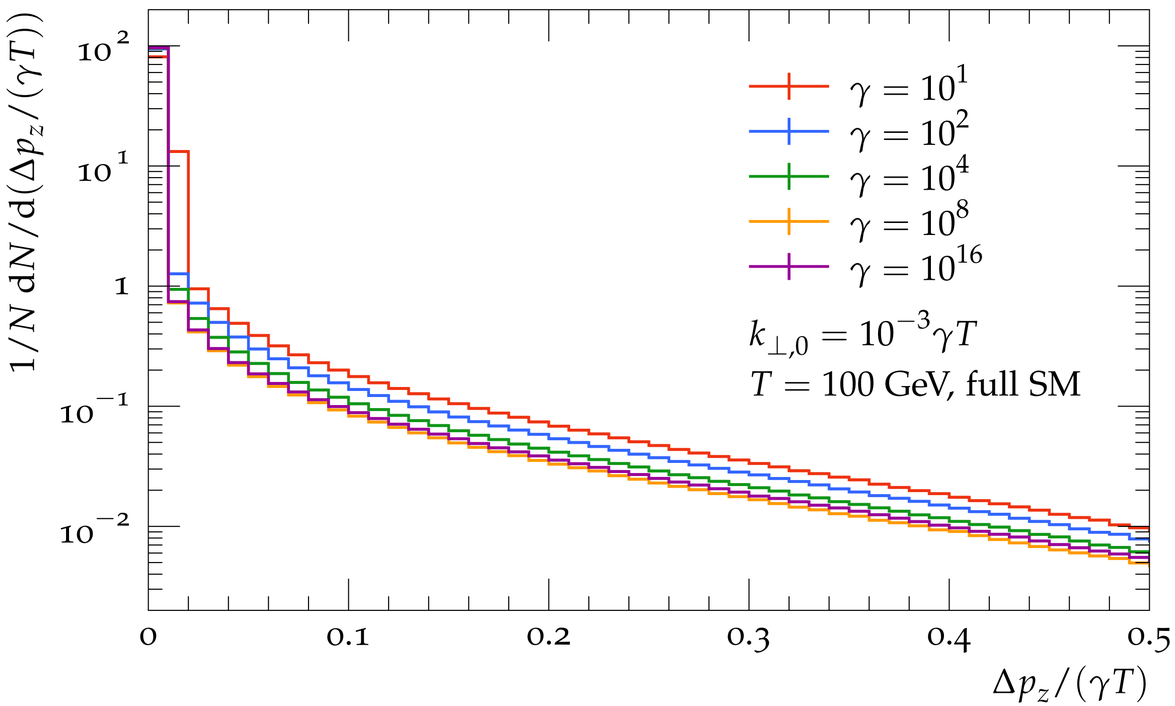
$$\left\langle \frac{\Delta p_z}{\gamma T} \right\rangle = \int_0^1 \mathrm{d}V V \frac{\mathrm{d}}{\mathrm{d}V} \prod_{a\in\mathcal{S}} \Delta_a(V)$$

$$\left\langle \frac{\Delta p_z}{\gamma T} \right\rangle_{\mathrm{FC}} = \int_0^\infty \mathrm{d}L e^{-L} \frac{(\alpha C)_\Sigma}{2\pi} \frac{e^L - 1}{e^L + 1} \exp\left\{-\frac{(\alpha C)_\Sigma}{2\pi}\left(L + 2\log\left(1 + e^{-L}\right)\right)\right\} \approx \zeta(\log 4 - 1) \qquad \zeta = \frac{(\alpha C)_\Sigma}{2\pi}$$

$$\boxed{\langle \Delta p_z \rangle \sim \gamma T}$$

Numerical Resummation





$$\left\langle \frac{\Delta p_z}{\gamma T} \right\rangle = 0.89(17)\,\% - 0.14(3)\,\% \log_{10}\gamma \implies P \propto \gamma^2 T^4$$

Comparing analytic and numerics

Particle	n_f	ν_a	analytic $(\langle \Delta p_z/(\gamma T)\rangle_{\rm FC})$	numeric $(\langle \Delta p_z/(\gamma T)\rangle_{\rm FC})$
$l^\pm$	2×3	2	0.17%	0.17%
u	2×3	2×3	0.46%	0.44%
d	2×3	2×3	0.46%	0.44%
$W^\pm$	2	2	0.52%	0.52%
Z	1	2	0.41%	0.40%
h	1	1	0.22%	0.21%
$G_{W^\pm}$	2	1	0.22%	0.21%
G_Z	1	1	0.22%	0.21%

Good agreement between analytics and numerics. Largest pressure contribution From QCD processes

Summary

- 1st order EWPT has many interesting physical consequences such as baryogenesis & GW production. Both quantitatively depend on the velocity of the bubble wall.

- Bubble wall velocity is a force balancing exercise: pressure from Higgs potential versus frictional pressure from plasma.

- We reformulated the calculation of the latter in a GI way and calculated the average pressure to all orders. This has a physical correspondence with a charged particle decelerating and emitting a soft radiation pattern.

- We find pressure $\propto \gamma^2$ also massless GB contribute the largest pressure of all SM. NB phase change is still required by fermion across wall. Numerical and analytic resumption agree to 10% level.

- Still **many** interesting avenues to explore.

Thank you for your time

Mode functions

Mode function quick summary

Bodeker & Moore (2017)

scalar interacting with wall

$$\mathcal{L} = \sum_{f=a,b,c} \left[\frac{1}{2} \left(\partial_\mu \phi_f \right)^2 - \frac{1}{2} m_f^2(z) \phi_f^2 \right]$$

Z-varying mass parametrises spatial inhomogeneity

KG field equation

$$\Box \phi_f + m_f^2(z) \phi_f = 0$$

solve with an **homogeneous** mass parameter,
solutions can be can labeled by a 3-vector $\overrightarrow{p}$

$$\chi_f(\vec{p}, x) = e^{-iE_f(\vec{p})t} e^{i\vec{p}\cdot\vec{x}} \quad \text{with} \quad E_f(\vec{p}) \equiv \sqrt{|\vec{p}|^2 + m_f^2}$$

$$\phi_f(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \tilde{\phi}_f(\vec{p}) \chi_f(\vec{p}, x)$$

Mode function quick summary

Bodeker & Moore (2017)

But we have **inhomogeneous** mass term, make
Ansatz for solution to KG equation

$$\phi_f(x) = \int \frac{d^2\vec{p}_\perp}{(2\pi)^2} \frac{dp_{z,s}}{(2\pi)} \tilde{\phi}_f\left(\vec{p}_\perp, p_{z,s}\right) \chi_f\left(p_{z,s}, z\right) e^{-iE_f(\vec{p}_\perp, p_z, s)t} e^{i\vec{p}_\perp \cdot \vec{x}_\perp}$$

Substitute into Klein Gordon → WKB solution for a particle with inhomogeneous mass

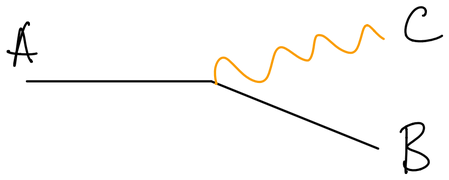
$$\chi_p(z) = \underbrace{\sqrt{\frac{p_{z,s}}{p_z(z)}}}_{\text{Amplitude}} \exp\underbrace{\left(i \int_0^z p_z\left(z'\right) dz'\right)}_{\text{Phase}}$$

Analogous 1D-scattering off a potential well. Normally there would be a wave function with a negative phase (reflected) but here all particles transmitted

Splitting function

Quick Recap on splitting functions

Gribov & Lipatov (1972)
Dokshitzer (1977)
Altarelli & Parisi (1977)



P_{CA} is probability that A emits (collinear) C which carries $x = E_C/E_A$ energy fraction of parent particle

$$P_{BA}(z) = \frac{1}{2} z(1-z) \sum_{\text{spins}} \frac{|V_{A\to B+C}|^2}{p_\perp^2}$$

$$\sum_{\text{spin}} |V_{A\to B+C}|^2 = \frac{1}{2} C_2(R)\, \mathrm{Tr}\left(\not{k}_B \gamma_\mu \not{k}_A \gamma_\nu\right) \sum_{\text{pol}} \epsilon^{*\mu}\epsilon_\mu$$

$$\sum_{\text{spin}} |V_{A\to B+C}|^2 = \frac{2p_\perp^2}{x(1-x)} \frac{1+(1-x)^2}{x} C_2(R)$$

Calculated in light-like Axial gauge in Altarelli Parisi (AP) paper

Sudakov parametrisation used in Altarelli & Parisi's paper

$$k_A = (P, P, 0)$$

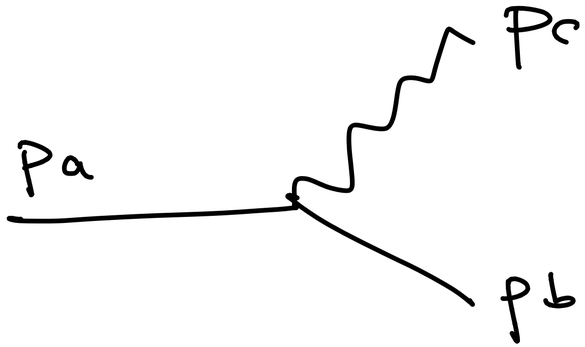
$$k_B = \left((1-x)P + \frac{p_\perp^2}{2(1-x)P}, (1-x)P, -p_\perp\right)$$

$$k_C = \left(xP + \frac{p_\perp^2}{2xP}, xP, p_\perp\right)$$

Matrix element reformulation

Reformulation of the Matrix Element

$$V_s = (-ig)\bar{u}_s\left(p_{b,s}\right)\gamma^\mu \varepsilon^*_\mu\left(p_{c,s}\right)u_s\left(p_{a,s}\right)$$
$$V_h = (-ig)\bar{u}_h\left(p_{b,h}\right)\gamma^\mu \varepsilon^*_\mu\left(p_{c,h}\right)u_h\left(p_{a,h}\right)$$



$$\sum_\lambda \epsilon\epsilon^* = -g^{\mu\nu} + \zeta\left(-\frac{n^2 p_c^\mu p_c^\nu}{\left(n\cdot p_c\right)^2} + \frac{n^\nu p_c^\mu + n^\mu p_c^\nu}{n\cdot p_c}\right)$$

$$V_s^* V_s = 2g^2\left(\left(1+\frac{2\zeta}{x^2}\right)k_\perp^2 - 2(1-\zeta)m_{b,s}^2 + 2\zeta m_c^2\frac{1-x}{x^2}\right)$$
$$V_s^* V_h = 2g^2\left(\left(1+\frac{2\zeta}{x^2}\right)k_\perp^2 - (1-\zeta)\left(m_{b,s}^2 + m_{b,h}^2\right) + 2\zeta m_c^2\frac{1-x}{x^2}\right)$$
$$V_h^* V_s = 2g^2\left(\left(1+\frac{2\zeta}{x^2}\right)k_\perp^2 - (1-\zeta)\left(m_{b,s}^2 + m_{b,h}^2\right) + 2\zeta m_c^2\frac{1-x}{x^2}\right)$$
$$V_h^* V_h = 2g^2\left(\left(1+\frac{2\zeta}{x^2}\right)k_\perp^2 - 2(1-\zeta)m_{b,h}^2 + 2\zeta m_c^2\frac{1-x}{x^2}\right)$$

$$\zeta = 1 \implies |V|^2 \propto \frac{k_\perp^2}{x^2}$$

from A&P and same as B&M

Reformulation of the Matrix Element

Feynman gauge: $\zeta = 0$

$$V_s^* V_s = 2g^2 k_\perp^2$$
$$V_s^* V_h = 2g^2 \left(k_\perp^2 - m_{b,h}^2\right)$$
$$V_h^* V_s = 2g^2 \left(k_\perp^2 - m_{b,h}^2\right)$$
$$V_h^* V_h = 2g^2 \left(k_\perp^2 - 2m_{b,h}^2\right)$$

Our A's

Soft and collinear limit

$k_\perp \to \lambda k_\perp$,

$x \to \lambda x$

$$\frac{V_s^* V_s}{A_s^2} = O(1)$$
$$\frac{V_s^* V_h}{A_s A_h} = -2g^2 \frac{1}{\lambda^2 k_\perp^2} \frac{x^2}{x^2 + k_\perp^2/m_{b,h}^2} + O(1/\lambda)$$
$$\frac{V_h^* V_s}{A_h A_s} = -2g^2 \frac{1}{\lambda^2 k_\perp^2} \frac{x^2}{x^2 + k_\perp^2/m_{b,h}^2} + O(1/\lambda)$$
$$\frac{V_h^* V_h}{A_h^2} = -2g^2 \frac{2}{\lambda^2 x^2 m_{b,h}^2} \left(\frac{x^2}{x^2 + k_\perp^2/m_{b,h}^2}\right)^2 + O(1/\lambda)$$

$$|\mathcal{M}|^2 = 4g^2 \frac{1}{\lambda^2 k_\perp^2} \left(\frac{x^2}{x^2 + k_\perp^2/m_{b,h}^2}\right)^2 + O(1/\lambda)$$

Reformulation of the Matrix Element

Axial gauge: $\zeta = 1$ and choose auxiliary vector $n = p_{a,s}$

$$V_s = 0 \quad \Longrightarrow \quad |V_h|^2 = \mathrm{Tr}\left[(\not{p}_{b,h} - m_{b,h})\,\gamma^\mu\,(\not{p}_{b,h} - m_{b,h})\,\gamma^\nu\right]\left(-g^{\mu\nu} + \frac{p_c^\mu p_{a,s}^\nu + p_c^\nu p_{a,s}^\mu}{p_{a,s}p_c}\right)$$

$$= 8p_{b,h}^\mu p_{b,h}^\nu \left(-g^{\mu\nu} + \frac{p_c^\mu p_{a,s}^\nu + p_c^\nu p_{a,s}^\mu}{p_{a,s}p_c}\right)$$

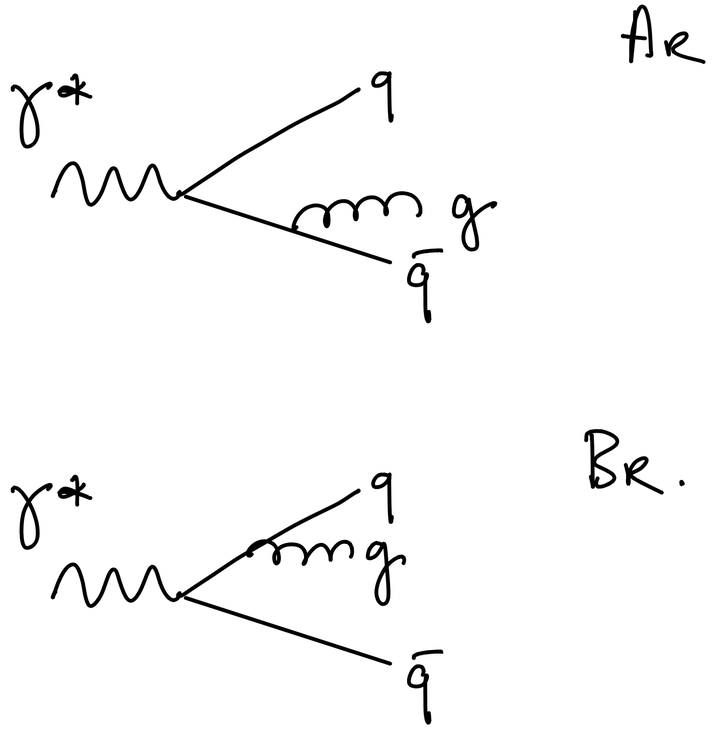
$$= 8\left(2p_{b,h}p_c\frac{p_{a,s}p_{b,h}}{p_{a,s}p_c} - m_{b,h}^2\right)$$

$$\frac{\sum |V_h|^2}{A_h^2} = \frac{2p_{a,s}p_{b,h}}{p_{a,s}p_c p_{b,h}p_c} - \frac{m_{b,h}^2}{(p_{b,h}p_c)^2} \longrightarrow 4g^2\frac{1}{\lambda^2 k_\perp^2}\left(\frac{x^2}{x^2 + k_\perp^2/m_{b,h}^2}\right)^2 + O(1/\lambda)$$

Same as Feynman gauge

Reformulation of the Matrix Element

See R Field's "Application of Perturbative QCD" Chp 3



$\gamma^* \to q\bar{q}g$

Feynman gauge

$$S_{11} = |B_R|^2 \propto \frac{(1-x)}{t}$$
$$S_{22} = |A_R|^2 = 0$$
$$S_{12} = 2A_R^* B_R \propto \frac{x}{t}$$

Axial gauge

$$S_{11} = |B_R|^2 \propto \frac{1}{t}$$
$$S_{12} = S_{22} = 0$$

Total matrix element gauge invariant

But certain gauge choices can make subamplitudes differ between gauges.

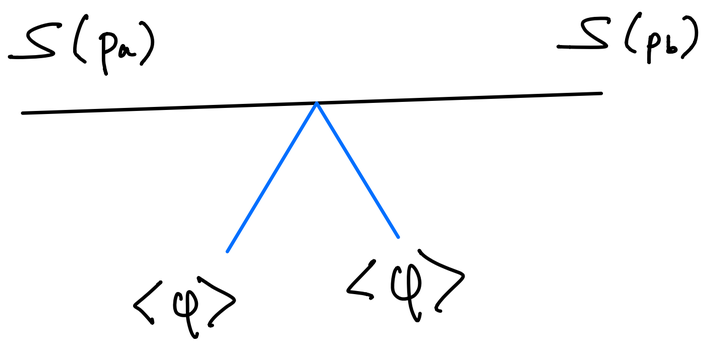
Phase change dependence

Points for discussion and future work

Where does the dependence in the phase change enter?

To perform a resummation or even define what the quantum correction is, we need to define what the leading order process is (aka Born-level process/hard function)

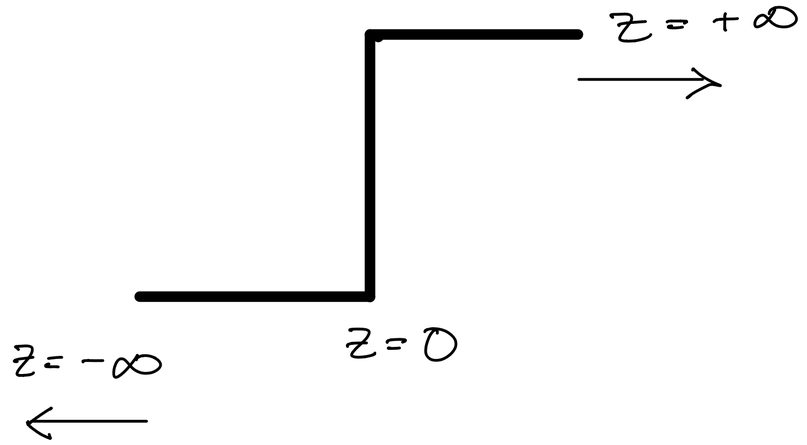
Lets consider our hard function to be a scalar interacting with the wall (fermions follow analogously). Leading order process is calculated from VEV-insertion:

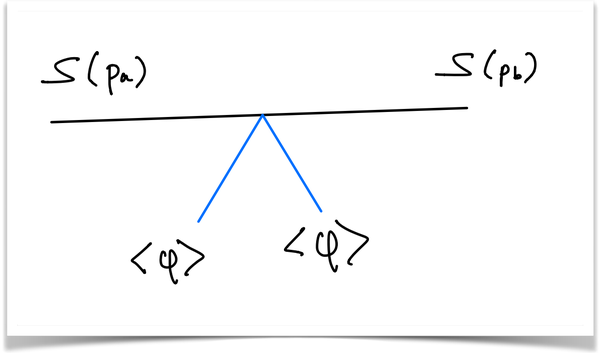


$$L_{\text{int}} \supset -\frac{\lambda}{4} S^2 \Phi^2$$

$$\Phi = \varphi(x,t) + \hat{\varphi}$$

$$\varphi(z) = \varphi_0 \Theta(z)$$





$$\mathcal{M}^{(0)}_{a\to b} = i\frac{\lambda}{4}\left\langle p_b \left| T\int d^4x S(x,t)^2\varphi(x,t)^2 \right| p_a \right\rangle$$

Fourier transform
step function profile of the wall

$$\varphi(x,t)^2 = \varphi_0^2 \int \frac{dk_z}{2\pi}\int \frac{dk_z'}{2\pi} e^{i(k_z+k_z')z}\left(\frac{\delta(k_z)}{2} - \frac{i}{2\pi}PV\frac{1}{k_z}\right)\left(\frac{\delta(k_z')}{2} - \frac{i}{2\pi}PV\frac{1}{k_z'}\right) \xrightarrow{\text{Perform integrations}} \frac{1}{4\pi^3}\frac{1}{\Delta p_z}$$

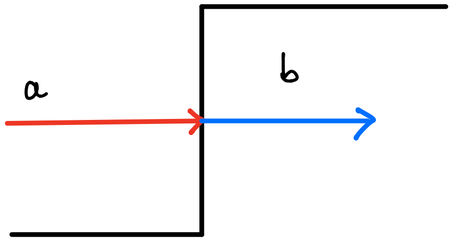
$$S(x,t) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\left(a_p e^{-ip\cdot x} + a_p^\dagger e^{ip\cdot x}\right) \xrightarrow{\text{Scalar quantised usual way}} \left\langle p_b \left| S(x,t)^2 \right| p_a \right\rangle = \langle p_b|S(x,t)|0\rangle\langle 0|S(x,t)|p_a\rangle = e^{i(p_a-p_b)\cdot x}$$

Put it all together

$$P_{a\to b} = \frac{\lambda\varphi_0^2}{8\pi^3}\delta(E_a - E_b)\,\delta^2(p_{a,\perp} - p_{b,\perp})\frac{1}{\Delta p_z} \xrightarrow{E_a^2 \gg m^2} \Delta p_z = \frac{m_{b,h}^2 - m_{a,s}^2}{2E_a} \approx \frac{m_{b,h}^2}{2E_a} \quad (m_{a,s} \ll m_{b,h})$$

$$P_{a\to b} = \frac{\lambda\cancel{\varphi_0^2}}{8\pi^3}\delta(E_a - E_b)\,\delta^2(p_{a,\perp} - p_{b,\perp})\frac{1}{\cancel{\Delta p_z}} \xrightarrow{m_{b,h}^2 \gg m_{a,s}} P_{a\to b} = \frac{\cancel{m_{b,h}^2}}{8\pi^3}\delta(E_a - E_b)\,\delta^2(p_{a,\perp} - p_{b,\perp})\frac{E_a}{\cancel{m_{b,h}^2}}$$

Recall the B&M 1-to-1 calculation



$$\mathrm{d}\mathbb{P}_{a\to b} = \frac{d^3p_b}{(2\pi)^3}\frac{1}{2E_b} \times (2\pi)^3\delta^2\left(p_{a,\perp} - p_{b,\perp}\right)\delta\left(E_a - E_b\right)\left(2p_{b,z,h}\right)\delta_{ab}$$

$$\mathcal{P}_{1\to 1} = \sum_a \nu_a \int \frac{d^3p_a}{(2\pi)^3} f_a\left(1 \pm f_b\right)\Delta p_{\mathrm{wall}} \approx \gamma^0 T^2 \Delta m^2$$

Mass dependence comes from observable not from matrix element

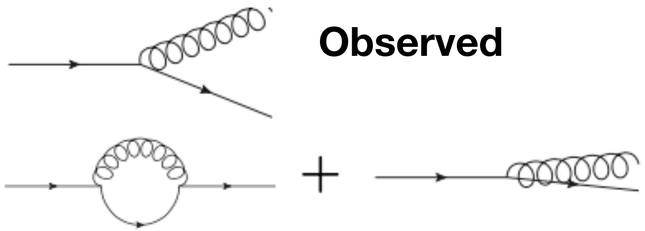
Summary: treat 1-to-1 as leading order process using infinitely thin wall approximation. As long as the ingoing particles gains a mass it will interact with a probability of 1 following from VEV-insertion method. This hard function/LO process normalises our NLO process. This procedure of defining LO process that normalises NLO process is tried and tested (> 30 years of collider data) and follows from the universal factorisation properties of gauge theories in the soft limit.

Resummation

Radiative corrections as branching processes

Marchesini & Webber (1983)
Sjöstrand (1985)

$$P_{\text{no em}} + P_{em} = 1$$



Unobserved processes captured by Sudakov factor

Expression for change in population:

$$dN = -\lambda N dt$$

$N = \text{population}$

$\lambda = \text{decay constant}$

Survival probability at time "t": $e^{-\lambda t}$ where $t \leftrightarrow$ energy scale $(\log(1/\text{v}))$

Change in population analogous to boson emission probability

$$\lambda N dt = \int [dk] M^2(k) \Theta(v - V(\{\tilde{p}\}, k)) \longrightarrow [\text{ virt. } + \text{ unres. }] = e^{-\int [dk] M^2(k) \Theta(V(\{\tilde{p}\},k) - v)}$$

Probability of not emitting bosons above "v" (Sudakov)

Kinematics

$$p_a^\mu = \left(E_a, \overrightarrow{0}, \sqrt{E_a^2 - m_a^2}\right) \approx \left(E_a, \overrightarrow{0}, E_a\left(1 - \frac{m_a^2}{2E_a^2}\right)\right)$$

$$p_b^\mu = \left((1-x)E_a, -\vec{k}_\perp, \sqrt{(1-x)^2 E_a^2 - \vec{k}_\perp^2 - m_b^2}\right) \approx \left((1-x)E_a, -\vec{k}_\perp, (1-x)E_a\left(1 - \frac{\vec{k}_\perp^2 + m_b^2}{2(1-x)^2 E_a^2}\right)\right)$$

$$p_c^\mu = \left(xE_a, \vec{k}_\perp, \sqrt{x^2 E_a^2 - \vec{k}_\perp^2 - m_c^2}\right) \approx \left(xE_a, \vec{k}_\perp, xE_a\left(1 - \frac{\vec{k}_\perp^2 + m_c^2}{2x^2 E_a^2}\right)\right)$$