Towards an all-orders calculation of the electroweak bubble wall velocity

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Gravitational Probes of Physics Beyond the Standard Model
First order electroweak phase transition

Within the Standard Model the EWPT is a crossover

Minimal new physics $\rightarrow$ first order PT

- Matter-antimatter asymmetry
  
- Topological defects
  
- Primordial magnetic fields
  
- Stochastic gravitational wave background

See talks by Weir, Lewicki, de Vis & Pol

See overview by Caprini

D'Onofrio & Rummukainen (2015)

Anderson & Hall (1992)

Kuzmin, Rubakov & Shaposhnikov (1985)

Achucarro & Vachaspati (2000)

Vachaspati (1991)

Kamionkowski, Kosowsky & Turner (1993)
• Collision bubble walls: $\Omega_{\text{env}}$

• Sound waves: $\Omega_{\text{sw}}$

• Turbulence: $\Omega_{\text{turb}}$

• Velocity affects gravitational wave spectrum

• Velocity affects efficiency of electroweak baryogenesis. But EWBG still viable with fast walls Cline & Kainulainen (2007.10935) see also Dorsch et al (2106.06547)
I will discuss relativistic bubble walls so the following calculations would not be valid for subsonic bubble wall.

Vacuum pressure from symmetry breaking Higgs potential

Scales $\gamma^0$

Thermal pressure, resulting from interactions of wall with the plasma particles

Non-trivial $\gamma$-dependence

Terminal velocity determined from balancing the force which drives the bubble expansion with the frictional pressure from the plasma on the wall.
Kinematics and Assumptions

z-translation invariance broken by bubble wall

\[ p_{a,z,s} = \sqrt{E_{a}^{2} - |p_{a,\perp}|^{2} - m_{a,s}^{2}} \]

\[ p_{a,z,h} = \sqrt{E_{a}^{2} - |p_{a,\perp}|^{2} - m_{a,h}^{2}} \]

\( s \equiv \) symmetric phase

\( h \equiv \) broken phase

Throughout we assume infinitely thin walls:

\( L_{w} \rightarrow 0 \)

Transverse momentum conserved:

\[ p_{a,\perp} = p_{b,\perp} \]

Energy conserved:

\[ E_{a} = E_{b} \]
1-to-1 pressure calculation for relativistic bubble walls
1-to-1 calculation argument

Friction → scattering particles that couple to Higgs condensate

\[ \Delta p_{\text{wall}} \equiv p_{a,z,s} - p_{b,z,h} = \sqrt{E_a^2 - m_{a,s}^2 - p_{a,\perp}^2} - \sqrt{E_b^2 - m_{b,h}^2 - p_{b,\perp}^2} \]

\[ E_a^2 \sim p_{a,z,s}^2 \sim \gamma^2 T^2 \gg m_{a,s}^2, m_{b,h}^2, p_{a,\perp}^2 \implies \Delta p_{\text{wall}} \approx \frac{m_{b,h}^2 - m_{a,s}^2}{2E_a} = \frac{m_{b,h}^2 - m_{a,s}^2}{2\gamma T} \]

\[ \mathcal{P}_{1\rightarrow1} = \left[ \text{force} \right] \left[ \frac{\text{area}}{\text{area}} \right] \times \left[ \text{time} \right] = \left[ \text{flux} \right] \times \Delta \left[ \text{momentum} \right] \]

\[ \sim \gamma T^3 \times \frac{\Delta m^2}{2\gamma T} = \frac{\gamma^2 T^2 \Delta m^2}{2} \]
• 1-to-1: no flavour change. Assume no particles reflected

\[ \mathcal{P}_{1\rightarrow 1} = \sum_a \int d\mathcal{F}_a \sum_b \int d\mathbb{P}_{a\rightarrow b} \Delta p_z (1 \pm f_b) \]

\[
d\mathbb{P}_{a\rightarrow b} = \frac{d^3 p_b}{(2\pi)^3} \frac{1}{2E_b} \times (2\pi)^3 \delta^2 (p_{a,\perp} - p_{b,\perp}) \delta (E_a - E_b) (2p_{b,z,h}) \delta_{ab} \]

\[ \mathcal{P}_{1\rightarrow 1} = \sum_a \nu_a \int \frac{d^3 p_a}{(2\pi)^3} \frac{d^3 p_b}{(2\pi)^3} \frac{p_{b,z,s}}{E_b} f_a (1 \pm f_b) (p_{a,z,s} - p_{b,z,h}) (2\pi)^3 \delta^2 (p_{a,\perp} - p_{b,\perp}) \delta (E_a - E_b) \]

\[
\frac{d^3 p_b}{(2\pi)^3 2E_b} = \frac{d^2 p_{b,\perp}}{(2\pi)^3} \frac{dE_b}{2E_b} \frac{E_b}{p_{b,z,s}}
\]

\[ \mathcal{P}_{1\rightarrow 1} = \sum_a \nu_a \int \frac{d^3 p_a}{(2\pi)^3} f_a (1 \pm f_b) \Delta p_{\text{wall}} \approx \gamma^0 T^2 \Delta m^2 \]
1-to-2 pressure calculation for relativistic bubble walls
1-to-2 calculation

\[ \langle \phi \rangle = 0 \quad \langle \phi \rangle = \phi_0 \]

\[ \langle \phi \rangle = 0 \quad \langle \phi \rangle = \phi_0 \]

Incident particle’s energy → mass second particle + transverse momentum

\[ \Delta p_{1\rightarrow 1} < \Delta p_{1\rightarrow 2} \text{ unless } p_{\perp} = 0 \text{ and } m_c = 0 \]
1-to-2 calculation

\[ \mathcal{P}_{1\to2} = \sum_{abc} \nu_a \int \frac{d^3p_a}{(2\pi)^3 2E_a} \int \frac{d^2p_{c,\perp}}{(2\pi)^2} \frac{dE_c}{(2\pi)2E_c} f(p_a, p_b, p_c) \Delta p_z (2\pi)^3 \delta^2 (p_{a,\perp} - p_{c,\perp} - p_{b,\perp}) \delta (E_a - E_c - E_b) |\mathcal{M}|^2 \]

B&M region of interest:

**Ingoing hard**

\[ p_{a,\perp} \sim T \quad p_{a,z,s} \sim E_a \sim \gamma_w T \quad m_{a,s}, m_{a,h} \ll \gamma_w T \]

**Outgoing hard**

\[ p_{b,\perp} \sim \max [T, m_c] \quad p_{b,z,s} \sim E_b \sim \gamma_w T \quad m_{b,s}, m_{b,h} \ll \gamma_w T \]

**Outgoing soft**

\[ p_{c,\perp} \sim \max [T, m_c] \quad p_{c,z,s} \sim \max [T, m_c] \quad m_{c,s}, m_{c,h} \ll \gamma_w T \]
1-to-2 calculation

\[ \mathcal{P}_{1 \rightarrow 2} = \sum_{abc} \nu_a \int \frac{d^3 p_a}{(2\pi)^3 2E_a} \int \frac{d^2 p_{c,\perp}}{(2\pi)^2} \frac{dE_c}{(2\pi)2E_c} f(p_a, p_b, p_c) \Delta p_z \frac{1}{2p_{b,z,s}} \frac{E_c}{p_{c,z,s}} |\mathcal{M}|^2 \]

B&M region of interest:

\[ p_{c,z}(z) = E_c \sqrt{1 - \frac{m_{c,z}^2 + p_{c,\perp}^2}{E_c^2}} \approx E_c \left(1 - \frac{\epsilon}{2} + \mathcal{O}(\epsilon^2) + \cdots\right) \]

Collinearity: \(\epsilon\)

\[ p_{b,z} = \sqrt{E_b^2 - p_{\perp,b}^2 - m_b^2(z)} \approx E_a (1 - x) \]

Softness: \(x = \frac{E_c}{E_a}\)
\[
\text{B&M 1-to-2 master equation:}
\]

\[
P_{1\rightarrow2} = \sum \nu_a \left( \frac{d^3p_a}{(2\pi)^3 2E_a} \int \frac{d^2p_{c,\perp}}{(2\pi)^2} \frac{dE_c}{2E_c} f(p_a, p_b, p_c) \frac{m_{c,h}^2(z) + p_{c,\perp}^2}{2E_c} |M|^2 \right)
\]

Still need to determine matrix element squared
The bubble wall is invariant in time and the transverse directions (Bodeker & Moore (2017))

\[
\langle p_c p_b | T | p_a \rangle = \int d^4x \, \langle p_c p_b | H_{\text{int}} | p_a \rangle = (2\pi)^3 \delta^2 (p_{a,\perp} - p_{b,\perp} - p_{c,\perp}) \delta (E_a - E_b - E_c) \mathcal{M}
\]

\[
\mathcal{M} \equiv \int dz \, \chi_{p_c}^* (z) \chi_{p_b}^* (z) V(z) \chi_{p_a} (z)
\]

Mode functions are treated using the WKB approximation:

\[
\chi_{p_c}(z) \simeq \sqrt{\frac{p_{c,z,s}}{p_{c,z}(z)}} \exp \left( i \int_0^z p_{c,z} (z') \, dz' \right)
\]
\[
\mathcal{M} = \int d\zeta \chi_b (p_{b,z,s}, \zeta)^* \chi_c (p_{c,z,s}, \zeta)^* \chi_a (p_{a,z,s}, \zeta) V(\zeta)
\]

\[
\mathcal{M} = V_s \int_{-\infty}^{0} d\zeta \exp \left[ i\zeta \frac{A_s}{2E_a} \right] + V_h \int^{\infty}_{0} d\zeta \exp \left[ i\zeta \frac{A_h}{2E_a} \right] = 2iE_a \left( \frac{V_h}{A_h} - \frac{V_s}{A_s} \right)
\]

\[
A_s = E_a \left( p_{a,z,s} - p_{b,z,s} - p_{c,z,s} \right)
\]

\[
A_h = E_a \left( p_{a,z,h} - p_{b,z,h} - p_{c,z,h} \right)
\]

\[m L_w \ll 1 \quad \text{as} \quad \zeta = +\infty\]

1/A terms resemble propagators, but they only propagate in the z-direction!
Vertex Function

\[ |\mathcal{M}|^2 \sim 4E_a^2 |V|^2 \left( \frac{A_h - A_s}{A_h^2 A_s^2} \right)^2 \]

\[ k_\perp \equiv p_{c,\perp} \]

These are splitting functions up to the normalisation

\[ P_{b\leftarrow a}(x) = |V|^2 x(1-x)/16\pi^2 k_\perp^2 \]

<table>
<thead>
<tr>
<th>(a(p) \to b(k)c(p-k))</th>
<th>(V^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S \to V_T S)</td>
<td>(4g^2 C_2[R] \frac{1}{x^2} k_\perp^2)</td>
</tr>
<tr>
<td>(F \to V_T F)</td>
<td>(4g^2 C_2[R] \frac{1}{x^2} m_b^2)</td>
</tr>
<tr>
<td>(V \to V_T V)</td>
<td>(2g^2 C_2[R] \frac{1}{x} (k_\perp^2 + m_b^2))</td>
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<tr>
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<td>(2g^2 T[R] \frac{1}{x} (k_\perp^2 + m_b^2))</td>
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<td>(F \to V_L F)</td>
<td>(4g^2 C_2[R] k_\perp^2)</td>
</tr>
<tr>
<td>(V \to V_L V)</td>
<td>(y^2 (k_\perp^2 + 4m_a^2))</td>
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<tr>
<td>(F \to F V_T)</td>
<td>(\lambda^2 \varphi^2)</td>
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<tr>
<td>(S \to S V_T)</td>
<td>(\lambda^2 \varphi^2)</td>
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<td>(F \to S F)</td>
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</tr>
<tr>
<td>(S \to S S)</td>
<td>(\lambda^2 \varphi^2)</td>
</tr>
</tbody>
</table>
\[ A_s \approx 2E_a \times \left( -\frac{|p_a, \perp|^2 + m_{a,s}^2}{2E_a} + \frac{|p_b, \perp|^2 + m_{b,s}^2}{2E_b} + \frac{|p_c, \perp|^2 + m_{c,s}^2}{2E_c} \right) \approx \frac{|p_c, \perp|^2 + m_{c,s}^2}{E_c / E_a} \]

\[ A_h \approx 2E_a \times \left( -\frac{|p_a, \perp|^2 + m_{a,h}^2}{2E_a} + \frac{|p_b, \perp|^2 + m_{b,h}^2}{2E_b} + \frac{|p_c, \perp|^2 + m_{c,h}^2}{2E_c} \right) \approx \frac{|p_c, \perp|^2 + m_{c,h}^2}{E_c / E_a} \]

\[ |\mathcal{M}|^2 = 4E_a^2 |V_s|^2 \frac{(A_s - A_h)^2}{A_s^2 A_h^2} \]

\[ \approx 4E_a \frac{|V_s|^2}{(E_c / E_a)^2} x \left( |p_c, \perp|^2 + m_{c,s}^2 \right)^2 \left( |p_c, \perp|^2 + m_{c,h}^2 \right)^2 \]
Let's keep track of what cancels where...

\[ |\mathcal{M}|^2 = 4 E_a^2 |V_s|^2 x^2 \frac{m_{c,h}^4}{|p_{c,\perp}|^4 \left( |p_{c,\perp}|^2 + m_{c,h}^2 \right)^2} \]

\[ = 16 g^2 C_2[R] \frac{|p_{c,\perp}|^2}{x^2} \frac{m_{c,h}^4}{|p_{c,\perp}|^4 \left( |p_{c,\perp}|^2 + m_{c,h}^2 \right)^2} \]

\[ = 16 g^2 C_2[R] E_a \frac{m_{c,h}^4}{|p_{c,\perp}|^2 \left( |p_{c,\perp}|^2 + m_{c,h}^2 \right)^2} \]

Combine matrix element squared with momentum transfer observable:

\[ |\mathcal{M}|^2 \times \Delta p_z \approx 16 E_a^2 g^2 C_2[R] \frac{m_{c,h}^4}{|p_{c,\perp}|^2 \left( |p_{c,\perp}|^2 + m_{c,h}^2 \right)^2} \times \frac{m_{c,h}^2 + p_{c,\perp}^2}{2 E_c} \]

\[ \approx \frac{8 g^2 E_a}{x} C_2[R] \frac{m_{c,h}^4}{|p_{c,\perp}|^2 \left( |p_{c,\perp}|^2 + m_{c,h}^2 \right)} \]

\[ m_{c,s} \ll m_{c,h} \]
Two pieces left: the integration of PS of incoming “a” gives the flux. We also need to integrate over phase space of particle “c” our soft emission

\[ \int \frac{m^2}{g^2 T^2} \frac{dp^2_{c,\perp}}{(2\pi)^2 |p_{c,\perp}|^2 \left( |p_{c,\perp}|^2 + m^2_{c,h} \right)} \sim \frac{1}{24\pi m^2_{c,h}} \]

\[ \int \frac{dE_c}{E_c^2} \sim \frac{1}{m_{c,h}} \]

B&M assume \( g^2 T^2 \ll m^2 \)

The thermal mass would cut of the integral and you’d get some (possibly large) \( \log \left( \frac{m^2_{c,h}}{m^2_{c,s}} \right) \) see Azatov et al (2010.02590)

Gamma dependence enters from flux

\[ \mathcal{P}_{1 \rightarrow 2} \sim \sqrt{T}^3 \frac{1}{m^2} \frac{1}{m} m^4 \]

Transverse momentum integration Energy integration
\[ \mathcal{P}_{1 \rightarrow 2} \sim m \gamma T^3 \]

- \( \mathcal{P}_{1 \rightarrow 2} \propto \gamma \) while \( \mathcal{P}_{1 \rightarrow 1} \propto \gamma^0 \). Since, vacuum pressure does not grow in \( \lambda \), a terminal velocity will be reached.
- \( \mathcal{P} \propto m^2 \implies \) no phase change pressure goes to zero. This comes from integrating over the mode functions.
- \( \mathcal{M} \): WKB and vertex. The vertex part is dominated in the soft regime.
- B&M cut off the \( k_\perp \) and \( k^0 \) integration by gauge boson mass.
- This interaction looks like a collider/scattering experiment where the collision occurs between the ingoing particle and the wall \( \implies \) the centre of mass energy will be large \( \implies \) many soft emission.
1-to-n pressure calculation for relativistic bubble walls

2007.10343 (JCAP 2103 (2021) 009)
Incoming fast on-shell particle (in wall rest frame)

B&M investigated 1-to-1 which we will treat as leading order process.
Incoming fast on-shell particle (in wall rest frame)

Treat 1-to-2 as quantum correction to 1-to-1 process.
If 1-to-2 is important, will 1-to-n be even more important?
Reformulation of matrix element

Two expressions same in the soft-limit but not same outside of soft limit.

\[ A_s = -2E_a (p_{a,z,s} - p_{b,z,s} - p_{c,z,s}) \]

\[ \approx p_{a,\perp}^2 + m_{a,s}^2 - \frac{p_{b,\perp}^2 + m_{b,s}^2}{E_b/E_a} - \frac{k_{\perp}^2 + m_{c,s}^2}{E_c/E_a} \]

\[ = -\frac{k_{\perp}^2 + m_{b,s}^2}{(1-x)} - \frac{k_{\perp}^2 + m_{c,s}^2}{x} \approx -\frac{k_{\perp}^2 + m_{c,s}^2}{x} \quad (x \ll 1) \]

\[ A_h = -2E_a (p_{a,z,h} - p_{b,z,h} - p_{c,z,h}) \]

\[ \approx p_{a,\perp}^2 + m_{a,h}^2 - \frac{p_{b,\perp}^2 + m_{b,h}^2}{E_b/E_a} - \frac{k_{\perp}^2 + m_{c,h}^2}{E_c/E_a} \]

\[ = -\frac{k_{\perp}^2 + m_{b,h}^2}{(1-x)} - \frac{k_{\perp}^2 + m_{c,h}^2}{x} \approx -\frac{k_{\perp}^2 + m_{c,h}^2}{x} \quad (x \ll 1) \]
Why the redefinition of A’s?

Treat A’s as covariant propagators. Wall provides spatial discontinuity off of which incoming particle scatters.

\[
A_s = -2p_{a,s} \cdot p_{c,s} \approx 2p_{a,\perp} \cdot k_\perp - \frac{p_{a,\perp}^2 + m_{a,s}^2}{E_c/E_c} - \frac{k_\perp^2 + m_{c,s}^2}{E_c/E_c} \quad x \ll 1 \quad - \frac{k_\perp^2 + m_{c,s}^2}{E_c/E_c} \quad x \ll 1
\]

\[
A_h = -2p_{b,h} \cdot p_{c,h} \approx 2p_{b,\perp} \cdot k_\perp - \frac{p_{b,\perp}^2 + m_{b,h}^2}{E_b/E_c} - \frac{k_\perp^2 + m_{c,h}^2}{E_c/E_b} \quad x \ll 1 \quad - \frac{k_\perp^2 + m_{c,h}^2}{E_c/E_b} \quad x \ll 1
\]
Reformulation of the Matrix Element

Our matrix element has the following form:

$$ \left| M^{(0)}_{a \to bc} \right|^2 = 4E_a^2|g|^2 \left( \frac{2p_{a,s}p_{b,h}}{p_{a,s}p_{c,b,h}p_c} - \frac{m_{a,s}^2}{(p_{a,s}p_c)^2} - \frac{m_{b,h}^2}{(p_{b,h}p_c)^2} \right) $$

This is a gauge invariant squared matrix element that corresponds to decelerated charges emitting soft gauge field quanta.
Here, consider massless radiation with radiator acquiring mass across wall

\[
\frac{1}{|M_{a \rightarrow b}^{(0)}|^2} \int \frac{d^3 p_c}{(2\pi)^3 2E_c} |M_{a \rightarrow bc}^{(0)}|^2 \approx \begin{cases} 
\frac{\alpha}{2\pi} \int \frac{dk^2_{\perp}}{k^2_{\perp}} C_{abc} \log \frac{m^2_{b,h}}{k^2_{\perp}} & \text{if } m^2_{a,s} \ll k^2_{\perp} \ll m^2_{b,h} \\
\frac{\alpha}{2\pi} \int \frac{dk^2_{\perp}}{k^2_{\perp}} C_{abc} & \text{if } m^2_{a,s}, m^2_{b,h} \ll k^2_{\perp}
\end{cases}
\]
Analytic Resummation

Banfi, Salam & Zanderighi (2005)

$R(v)$ probability for decay $a \to bc$. For this splitting to produce momentum transfer of $v$ we require that it did not produce a large momentum transfer before.

$$R(v) = \int [dk] \left| M^2(k) \right| \Theta[V(\{p\}, k) - v]$$

$$V(p_a, p_b, p_c) = \frac{\Delta p_z}{\gamma T} \approx \frac{k_{\perp}^2}{(2E_a^2)} x(1-x)$$

$$R_{abc}(V) = C_{abc}|g|^2 \int \frac{d^3p_c}{(2\pi)^3 2E_c} \left( \frac{2p_{b,h}p_{a,s}}{p_{a,s}p_cp_{b,h}p_c} + O\left( \frac{m_{a,s}^2}{k_{\perp}^2}, \frac{m_{b,h}^2}{k_{\perp}^2} \right) \right) \Theta(V(p_a, p_b, p_c) - V) \Theta(p_{b,z,h}) \Theta(p_{c,z,h})$$

Rewrite phase space and matrix element squared in terms of observable $V$

$$R_{abc}(V) = C_{abc} \frac{\alpha}{2\pi} \int_V^1 \frac{dV'}{V'} \int_0^1 dx 2x \Theta \left( \frac{1}{1 + V'} - x \right) \Theta \left( x - \frac{V'}{1 + V'} \right)$$

$$R_{abc}(V) = \frac{\alpha}{2\pi} C_{abc} \left( L + 2\log (1 + e^{-L}) \right) \text{ where } L = \log \frac{1}{V}$$
\[ \Delta_a(V) = \exp \left\{ - \sum_b R_{ab}(V) \right\}, \quad \text{where} \quad R_{ab}(V) = \sum_c R_{abc}(V) \]

\[ \langle \frac{\Delta p_z}{\gamma T} \rangle = \int_0^1 dV \frac{d}{dV} \prod_{a \in S} \Delta_a(V) \]

\[ \langle \frac{\Delta p_z}{\gamma T} \rangle_{FC} = \int_0^\infty dLe^{-L} \frac{(\alpha C)\Sigma}{2\pi} \frac{e^L - 1}{e^L + 1} \exp \left\{ -\frac{(\alpha C)\Sigma}{2\pi} \left( L + 2 \log \left( 1 + e^{-L} \right) \right) \right\} \approx \zeta (\log 4 - 1) \]

\[ \zeta = \frac{(\alpha C)\Sigma}{2\pi} \]

\[ \langle \Delta p_z \rangle \sim \gamma T \]
\[ \langle \frac{\Delta p_z}{\gamma T} \rangle = 0.89(17)\% - 0.14(3)\% \log_{10} \gamma \quad \Rightarrow \quad P \propto \gamma^2 T^4 \]
### Comparing analytic and numerics

<table>
<thead>
<tr>
<th>Particle</th>
<th>$n_f$</th>
<th>$\nu_a$</th>
<th>analytic ($\langle \Delta p_z/(\gamma T) \rangle_{\text{FC}}$)</th>
<th>numeric ($\langle \Delta p_z/(\gamma T) \rangle_{\text{FC}}$)</th>
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</thead>
<tbody>
<tr>
<td>$l^\pm$</td>
<td>$2 \times 3$</td>
<td>2</td>
<td>0.17%</td>
<td>0.17%</td>
</tr>
<tr>
<td>$u$</td>
<td>$2 \times 3$</td>
<td>$2 \times 3$</td>
<td>0.46%</td>
<td>0.44%</td>
</tr>
<tr>
<td>$d$</td>
<td>$2 \times 3$</td>
<td>$2 \times 3$</td>
<td>0.46%</td>
<td>0.44%</td>
</tr>
<tr>
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<td>0.52%</td>
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</tr>
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<td>0.21%</td>
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<tr>
<td>$G_Z$</td>
<td>1</td>
<td>1</td>
<td>0.22%</td>
<td>0.21%</td>
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</table>

Good agreement between analytics and numerics. Largest pressure contribution from QCD processes.
Summary

• 1st order EWPT has many interesting physical consequences such as baryogenesis & GW production. Both quantitatively depend on the velocity of the bubble wall.

• Bubble wall velocity is a force balancing exercise: pressure from Higgs potential versus frictional pressure from plasma.

• We reformulated the calculation of the latter in a GI way and calculated the average pressure to all orders. This has a physical correspondence with a charged particle decelerating and emitting a soft radiation pattern.

• We find pressure $\propto \gamma^2$ also massless GB contribute the largest pressure of all SM. NB phase change is still required by fermion across wall. Numerical and analytic resumption agree to 10% level.

• Still many interesting avenues to explore.
Thank you for your time
Mode functions
Mode function quick summary

scalar interacting with wall

\[ \mathcal{L} = \sum_{f=a,b,c} \left[ \frac{1}{2} (\partial_\mu \phi_f)^2 - \frac{1}{2} m_f^2(z) \phi_f^2 \right] \]

KG field equation

\[ \Box \phi_f + m_f^2(z) \phi_f = 0 \]

solve with an **homogeneous** mass parameter, solutions can be can labeled by a 3-vector \( \vec{p} \)

\[ \chi_f(\vec{p}, x) = e^{-iE_f(\vec{p}) t} e^{i\vec{p} \cdot \vec{x}} \quad \text{with} \quad E_f(\vec{p}) \equiv \sqrt{|\vec{p}|^2 + m_f^2} \]

\[ \phi_f(x) = \int \frac{d^3 p}{(2\pi)^3} \tilde{\phi}_f(\vec{p}) \chi_f(\vec{p}, x) \]
But we have **inhomogeneous** mass term, make Ansatz for solution to KG equation

\[
\phi_f(x) = \int \frac{d^2 \vec{p}_\perp}{(2\pi)^2} \frac{dp_{z,s}}{(2\pi)} \phi_f(\vec{p}_\perp, p_{z,s}) \chi_f(p_{z,s}, z) e^{-iE_f(\vec{p}_\perp, p_{z,s})t} e^{i\vec{p}_\perp \cdot \vec{x}_\perp}
\]

Substitute into Klein Gordon → WKB solution for a particle with inhomogeneous mass

\[
\chi_p(z) = \sqrt{\frac{p_{z,s}}{p_z(z)}} \exp \left( i \int_{0}^{z} p_z(z') \, dz' \right)
\]

Analogous 1D-scattering off a potential well. Normally there would be a wave function with a negative phase (reflected) but here all particles transmitted.
Splitting function
Quick Recap on splitting functions

$P_{CA}$ is probability that $A$ emits (collinear) $C$
which carries $x = E_C/E_A$ energy fraction of
parent particle

\[ P_{BA}(z) = \frac{1}{2} z (1 - z) \sum_{\text{spins}} \frac{|V_{A \to B+C}|^2}{p_\perp^2} \]

\[ \sum_{\text{spin}} |V_{A \to B+C}|^2 = \frac{1}{2} C_2(R) \text{Tr} \left( k_B^\gamma \gamma_\mu k_A^\gamma \gamma_\nu \right) \sum_{\text{pol}} \epsilon^*_\mu \epsilon_\mu \]

\[ \sum_{\text{spin}} |V_{A \to B+C}|^2 = \frac{2 p_\perp^2}{x(1-x)} \frac{1 + (1-x)^2}{x} C_2(R) \]

Sudakov parametrisation
used in Altarelli & Parisi's paper

\[ k_A = (P, P, 0) \]
\[ k_B = \left( (1-x)P + \frac{p_\perp^2}{2(1-x)P}, (1-x)P, -p_\perp \right) \]
\[ k_C = \left( xP + \frac{p_\perp^2}{2xP}, xP, p_\perp \right) \]

Calculated in light-like Axial gauge
in Altarelli Parisi (AP) paper
Matrix element reformulation
Reformulation of the Matrix Element

\[ V_s = (-ig) \bar{u}_s (p_{b,s}) \gamma^\mu \varepsilon^*_\mu (p_{c,s}) u_s (p_{a,s}) \]

\[ V_h = (-ig) \bar{u}_h (p_{b,h}) \gamma^\mu \varepsilon^*_\mu (p_{c,h}) u_h (p_{a,h}) \]

\[ \sum \epsilon \epsilon^* = -g^{\mu \nu} + \zeta \left( -\frac{n^2 p^\mu_c p'^\nu_c}{(n \cdot p_c)^2} + \frac{n^\nu p^\mu_c + n^\mu p'^\nu_c}{n \cdot p_c} \right) \]

\[ V_s^* V_s = 2g^2 \left( \left( 1 + \frac{2\zeta}{x^2} \right) k^2_\perp - 2(1 - \zeta)m^2_{b,s} + 2\zeta m^2_c \frac{1-x}{x^2} \right) \]

\[ V_s^* V_h = 2g^2 \left( \left( 1 + \frac{2\zeta}{x^2} \right) k^2_\perp - (1 - \zeta) \left( m^2_{b,s} + m^2_{b,h} \right) + 2\zeta m^2_c \frac{1-x}{x^2} \right) \]

\[ V_h^* V_s = 2g^2 \left( \left( 1 + \frac{2\zeta}{x^2} \right) k^2_\perp - (1 - \zeta) \left( m^2_{b,s} + m^2_{b,h} \right) + 2\zeta m^2_c \frac{1-x}{x^2} \right) \]

\[ V_h^* V_h = 2g^2 \left( \left( 1 + \frac{2\zeta}{x^2} \right) k^2_\perp - 2(1 - \zeta)m^2_{b,h} + 2\zeta m^2_c \frac{1-x}{x^2} \right) \]

\[ \zeta = 1 \implies |V|^2 \propto \frac{k^2_\perp}{x^2} \]

from A&P and same as B&M
Reformulation of the Matrix Element

Feynman gauge: $\zeta = 0$

\[
\begin{align*}
V_s^*V_s &= 2g^2 k_{\perp}^2 \\
V_s^*V_h &= 2g^2 \left( k_{\perp}^2 - m_{b,h}^2 \right) \\
V_h^*V_s &= 2g^2 \left( k_{\perp}^2 - m_{b,h}^2 \right) \\
V_h^*V_h &= 2g^2 \left( k_{\perp}^2 - 2m_{b,h}^2 \right)
\end{align*}
\]

Our A's

Soft and collinear limit

\[
\begin{align*}
k_{\perp} &\to \lambda k_{\perp}, \\
x &\to \lambda x
\end{align*}
\]

\[
\begin{align*}
\frac{V_s^*V_s}{A_s^2} &= O(1) \\
\frac{V_s^*V_h}{A_s A_n} &= -2g^2 \frac{1}{\lambda^2 k_{\perp}^2} \frac{x^2}{x^2 + k_{\perp}^2 / m_{b,h}^2} + O(1/\lambda) \\
\frac{V_h^*V_s}{A_h A_s} &= -2g^2 \frac{1}{\lambda^2 k_{\perp}^2} \frac{x^2}{x^2 + k_{\perp}^2 / m_{b,h}^2} + O(1/\lambda) \\
\frac{V_h^*V_h}{A_h^2} &= -2g^2 \frac{2}{\lambda^2 x^2 m_{b,h}^2} \left( \frac{x^2}{x^2 + k_{\perp}^2 / m_{b,h}^2} \right)^2 + O(1/\lambda)
\end{align*}
\]

\[
|M|^2 = 4g^2 \frac{1}{\lambda^2 k_{\perp}^2} \left( \frac{x^2}{x^2 + k_{\perp}^2 / m_{b,h}^2} \right)^2 + O(1/\lambda)
\]
Reformulation of the Matrix Element

Axial gauge: $\zeta = 1$ and choose auxiliary vector $n = p_{a,s}$

$$V_s = 0 \quad \implies \quad |V_h|^2 = \text{Tr}\left[ (p_{b,h} - m_{b,h}) \gamma^\mu (p_{b,h} - m_{b,h}) \gamma^\nu \right] \left( -g^{\mu\nu} + \frac{p_c^\mu p_{a,s}^\nu + p_{c}^\nu p_{a,s}^\mu}{p_{a,s} p_c} \right)$$

$$= 8 p_{b,h}^\mu p_{b,h}^\nu \left( -g^{\mu\nu} + \frac{p_c^\mu p_{a,s}^\nu + p_{c}^\nu p_{a,s}^\mu}{p_{a,s} p_c} \right)$$

$$= 8 \left( 2 p_{b,h} p_c \frac{p_{a,s} p_{b,h}}{p_{a,s} p_c} - m_{b,h}^2 \right)$$

$$\sum \frac{|V_h|^2}{A_h^2} = \frac{2 p_{a,s} p_{b,h}}{p_{a,s} p_c p_{b,h} p_c} - \frac{m_{b,h}^2}{(p_{b,h} p_c)^2} \longrightarrow 4 g^2 \frac{1}{\lambda^2 k_{\perp}^2} \left( \frac{x^2}{x^2 + k_{\perp}^2/m_{b,h}^2} \right)^2 + O(1/\lambda)$$

Same as Feynman gauge
Reformulation of the Matrix Element

See R Field’s “Application of Perturbative QCD” Chp 3

\[
\begin{align*}
S_{11} &= |B_R|^2 \propto \frac{(1-x)}{t} \\
S_{22} &= |A_R|^2 = 0 \\
S_{12} &= 2A_R^*B_R \propto \frac{x}{t}
\end{align*}
\]

Feynman gauge

\[
\begin{align*}
S_{11} &= |B_R|^2 \propto \frac{(1-x)}{t} \\
S_{22} &= |A_R|^2 = 0 \\
S_{12} &= 2A_R^*B_R \propto \frac{x}{t}
\end{align*}
\]

Axial gauge

Total matrix element gauge invariant
But certain gauge choices can make subamplitudes differ between gauges.
Phase change dependence
Points for discussion and future work

Where does the dependence in the phase change enter?

To perform a resummation or even define what the quantum correction is, we need to define what the leading order process is (aka Born-level process/hard function)

Lets consider our hard function to be a scalar interacting with the wall (fermions follow analogously). Leading order process is calculated from VEV-insertion:

\[
L_{\text{int}} \supset -\frac{\lambda}{4} S^2 \Phi^2
\]

\[
\Phi = \varphi(x, t) + \hat{\varphi}
\]

\[
\varphi(z) = \varphi_0 \Theta(z)
\]
\[
\mathcal{M}_{a \rightarrow b}^{(0)} = \frac{\lambda}{4} \left\langle p_b \left| T \int d^4 x S(x, t)^2 \varphi(x, t)^2 \right| p_a \right\rangle
\]

Put it all together

\[
\varphi(x, t)^2 = \varphi_0^2 \int \frac{dk_z}{2\pi} \int \frac{dk'_z}{2\pi} e^{i(k_z + k'_z)z} \left( \frac{\delta(k_z)}{2} - \frac{i}{2\pi} PV \frac{1}{k_z} \right) \left( \frac{\delta(k'_z)}{2} - \frac{i}{2\pi} PV \frac{1}{k'_z} \right)
\]

\[
S(x, t) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} (a_p e^{-ip \cdot x} + a_p^\dagger e^{ip \cdot x})
\]

Scalar quantised usual way

\[
\left\langle p_b \left| S(x, t)^2 \right| p_a \right\rangle = \langle p_b | S(x, t) | 0 \rangle \langle 0 | S(x, t) | p_a \rangle = e^{ip_a - p_b \cdot x} \quad (\Delta p_z = \frac{m_{b,h}^2 - m_{a,s}^2}{2E_a} \approx \frac{m_{b,h}^2}{2E_a} \quad (m_{a,s} \ll m_{b,h})}
\]

\[
E_a^2 \gg m^2
\]

\[
\Delta p_z = \frac{m_{b,h}^2 - m_{a,s}^2}{2E_a} \approx \frac{m_{b,h}^2}{2E_a} \quad (m_{a,s} \ll m_{b,h})
\]

\[
P_{a \rightarrow b} = \frac{\lambda \varphi_0^2}{8\pi^3} \delta(E_a - E_b) \delta^2(p_{a,\perp} - p_{b,\perp}) \frac{1}{\Delta p_z}
\]

\[
P_{a \rightarrow b} = \frac{\lambda \varphi_0^2}{8\pi^3} \delta(E_a - E_b) \delta^2(p_{a,\perp} - p_{b,\perp}) \frac{1}{\Delta p_z}
\]

\[
P_{a \rightarrow b} = \frac{\lambda \varphi_0^2}{8\pi^3} \delta(E_a - E_b) \delta^2(p_{a,\perp} - p_{b,\perp}) \frac{1}{\Delta p_z}
\]
Recall the B&M 1-to-1 calculation

\[
\begin{align*}
\mathcal{P}_{a \rightarrow b} & = \frac{d^3 p_b}{(2\pi)^3} \frac{1}{2 E_b} \times (2\pi)^3 \delta^2 (p_{a,\perp} - p_{b,\perp}) \delta (E_a - E_b) (2p_{b,z,h}) \delta_{ab} \\
\mathcal{P}_{1 \rightarrow 1} & = \sum_a \nu_a \int \frac{d^3 p_a}{(2\pi)^3} f_a (1 \pm f_b) \Delta p_{\text{wall}} \approx \gamma^0 T^2 \Delta m^2
\end{align*}
\]

**Summary:** treat 1-to-1 as leading order process using infinitely thin wall approximation. As long as the ingoing particles gains a mass it will interact with a probability of 1 following from VEV-insertion method. This hard function/LO process normalises our NLO process. This procedure of defining LO process that normalises NLO process is tried and tested (> 30 years of collider data) and follows from the universal factorisation properties of gauge theories in the soft limit.
Resummation
Radiative corrections as branching processes

Marchesini & Webber (1983)
Sjöstrand (1985)

\[
P_{\text{no em}} + P_{\text{em}} = 1
\]

Expression for change in population:

\[ dN = -\lambda N dt \]

Survival probability at time “t”: \( e^{-\lambda t} \) where \( t \leftrightarrow \) energy scale (\( \log(1/v) \))

Change in population analogous to boson emission probability

\[ \lambda N dt = \int [dk] M^2(k) \Theta(v - V(\{\tilde{p}\}, k)) \]

Probability of not emitting bosons above “v” (Sudakov)

\[ e^{-\int [dk] M^2(k) \Theta(V(\{\tilde{p}\}, k) - v)} \]

where

- \( N \) = population
- \( \lambda \) = decay constant

\( \tilde{p} \) represents unresolved emission.
Kinematics

\[ p_{\mu}^a = \left( E_a, \vec{0}, \sqrt{E_a^2 - m_a^2} \right) \approx \left( E_a, \vec{0}, E_a \left( 1 - \frac{m_a^2}{2E_a^2} \right) \right) \]

\[ p_{\mu}^b = \left( (1 - x)E_a, -\vec{k}_\perp, \sqrt{(1 - x)^2 E_a^2 - \vec{k}_\perp^2 - m_b^2} \right) \approx \left( (1 - x)E_a, -\vec{k}_\perp, (1 - x)E_a \left( 1 - \frac{\vec{k}_\perp^2 + m_b^2}{2(1-x)^2 E_a^2} \right) \right) \]

\[ p_{\mu}^c = \left( xE_a, \vec{k}_\perp, \sqrt{x^2 E_a^2 - \vec{k}_\perp^2 - m_c^2} \right) \approx \left( xE_a, \vec{k}_\perp, xE_a \left( 1 - \frac{\vec{k}_\perp^2 + m_c^2}{2x^2 E_a^2} \right) \right) \]