The gravitational memory of supernova neutrinos

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Mainak Mukhopadhyay, Carlos Cardona and Cecilia Lunardini; arXiv:2105.05862; accepted in JCAP.

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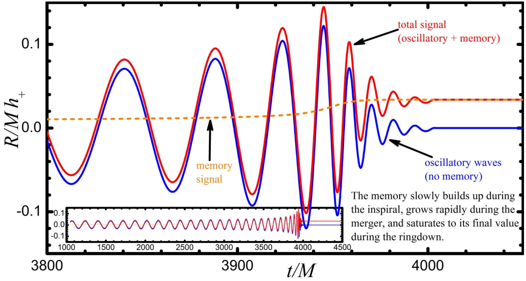
Structure of this talk

- Introduction
- Theoretical considerations
- A phenomenological model of the supernova neutrino memory.
- Detectability and physics potential
- Summary and discussion

Introduction

Gravitational waveforms with memory

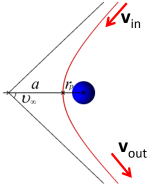


slide from M. Favata

- the GW strain converges to a non-zero value: *memory* is present

Memory from General Relativity

- *permanent* distortion of the local space time metric
- Due to gravitationally *unbound* systems:
  - anisotropic emission of energy (mass/radiation)
- appears as a permanent change in the distance between two free falling masses: signal at GW interferometers!



figures from M. Favata

The memory of supernova neutrinos

- the memory has never been observed
- observation requires: (i) very powerful emitter and (ii) some anisotropy
- ideal candidate: a core collapse supernova!
  - $E_{tot} \sim 3\ 10^{53}$ ergs, *most as neutrinos*
  - anisotropy at $\sim 10^{-3} - 10^{-2}$ level
  - neutrino emission timescale $\Delta t \sim O(10)$ s $\rightarrow$ sub-Hz scale

The SN ν memory: a signal for Deci-Hz interferometers

- simulations are costly, limited to ~ 1 s.
- Must use phenomenology, to describe long term emission, diversity of scenarios

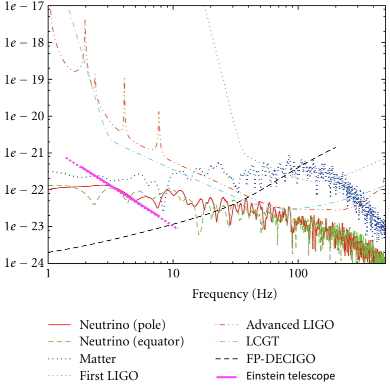


fig. from Kotake, Adv. Astron. (2012), 428757

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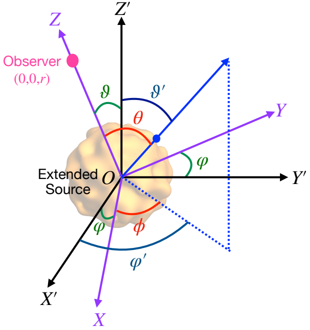
Theoretical considerations

How to calculate the memory



- solving Einstein's equation, in weak-field approximation: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- longitudinal polarization ($h^{xx}_{TT} = -h^{yy}_{TT} = -h^{+}_{TT}$):

$$h^{xx}_{TT} = \frac{2G}{rc^4} \int_{-\infty}^{t-r/c} dt' \int_{4\pi} (1+\cos\theta)\cos 2\phi \frac{dL_\nu(\Omega', t')}{d\Omega'} d\Omega'.$$

- Change of separation of free-falling masses: $\delta l_j = \frac{1}{2} h^{TT}_{jk} l^k$

The anisotropy parameter

- For convenience, the angular dependence can be lumped into the *anisotropy parameter*:

$$\alpha(t) = \frac{1}{L_\nu(t)} \int_{4\pi} d\Omega' \; \Psi(\vartheta', \varphi') \; \frac{dL_\nu(\Omega', t)}{d\Omega'} \; ,$$

- Final form:

$$\boxed{h^{xx}_{TT} = h(t) = \frac{2G}{rc^4} \int_{-\infty}^{t-r/c} dt' L_\nu(t')\alpha(t') \, .}$$

Phenomenology: upper bounds

- In the time domain ($\Delta h = h(+\infty) - h(-\infty)$):

$$
\begin{aligned}
|\Delta(h)| &\leq \frac{2G}{rc^4}|\alpha|_{max} E_{tot} \\
&\simeq 6.41\ 10^{-20} \left(\frac{|\alpha|_{max}}{0.04}\right) \left(\frac{E_{tot}}{3\ 10^{53}\ \mathrm{ergs}}\right) \left(\frac{r}{10\ \mathrm{kpc}}\right)^{-1} .
\end{aligned}
$$

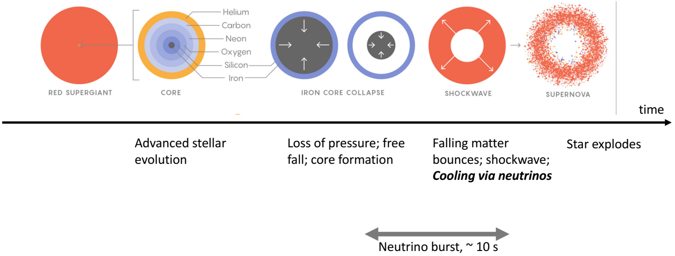
- In frequency domain: $h_c(f) \equiv 2f|\tilde{h}(f)|$ ($\tilde{h}$: Fourier transform).
- Zero frequency limit (ZFL):

$$
\lim_{f \to 0} h_c = \frac{|\Delta h|}{\pi} \lesssim 2.0\ 10^{-20} \left(\frac{|\alpha|_{max}}{0.04}\right) \left(\frac{E_{tot}}{3\ 10^{53}\ \mathrm{ergs}}\right) \left(\frac{r}{10\ \mathrm{kpc}}\right)^{-1}
$$

A phenomenological model of the supernova neutrino memory

Supernova neutrinos: a mini-review

Credit: Lucy Reading-Ikkanda/Quanta Magazine



Stellar death: core collapse

- neutrinos emitted thermally, $\langle E \rangle \simeq 10 - 18$ MeV, radius $R \simeq 100$ Km.
- $E_{tot} \sim 3\ 10^{53}$ ergs emitted in $\mathcal{O}(10)$ s burst.

Phases of neutrino emission: $L_\nu(t)$

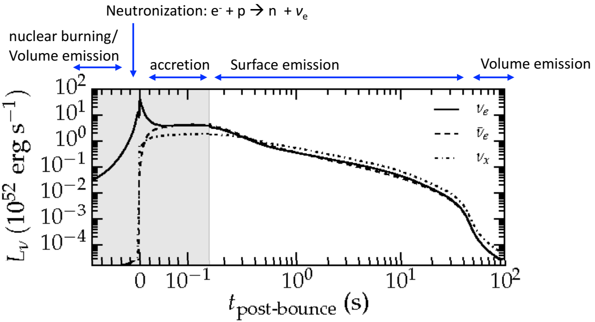


fig. from Roberts and Reddy, Handbook of Supernovae, Springer Intl., 2017

- accretion phase: $t \sim 0.003 - 0.5$ s: shockwave is stalled
- cooling phase: $t \sim 0.5 - 40$ s : shockwave re-energized by neutrino energy deposition, launches

near-core dynamics: $\alpha(t)$

- anisotropy develops during accretion, due to:
  - convection
  - large scale sloshing motion of shock front (Standing Accretion Shock Instability, SASI)
- anisotropy during cooling phase not simulated

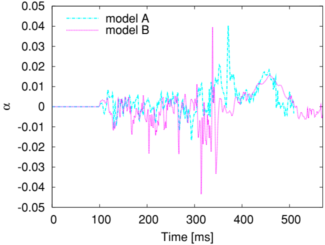


fig. from Kotake, Iwakami, Ohnishi and Yamada, Astrophys. J. 704 (2009) 951

Building a phenomenological model

- toy $L_\nu(t)$: global shape (only valid locally) :

$$L_\nu(t) = \lambda + \beta \ \exp\left(- \chi \, t \right) \ ,$$

- toy $\alpha(t)$: multi-Gaussian+constant:

$$\alpha(t) = \kappa + \sum_{j=1}^{N} \xi_j \ \exp\left(- \frac{(t - \gamma_j)^2}{2\sigma_j^2} \right) \ ,$$

- result: analytical $h(t)$

$$h(t) = \sum_{j=1}^{N} \left\{ \left[h_{1j} \left(\mathrm{erf}\left(\rho_j \ \tau_{1j}\right) + \mathrm{erf}\left(\rho_j (t - \tau_{1j})\right) \right) \right] + \left[h_{2j} \left(\mathrm{erf}\left(\rho_j \ \tau_{2j}\right) + \mathrm{erf}\left(\rho_j (t - \tau_{2j})\right) \right) \right] \right\}$$
$$+ \left[h_3 \left(\frac{\beta}{\chi} \left(1 - \exp\left(-t\chi\right) \right) + \lambda t \right) \right] \ ,$$

$$\tilde{h}(f) = \sum_{j=1}^{N} \left[\left(h_{1j}\, \frac{i}{\pi f} \exp\left(\frac{-\pi^2 f^2}{\rho_j^2} \right) \exp\left(i 2\pi f \tau_{1j} \right) \right) + \left(h_{2j}\, \frac{i}{\pi f} \exp\left(\frac{-\pi^2 f^2}{\rho_j^2} \right) \exp\left(i 2\pi f \tau_{2j} \right) \right) \right] + \left(\sqrt{2\pi}\; h_3\, \frac{\beta}{\chi} \left(\frac{1}{i 2\pi f} - \frac{1}{-\chi + i 2\pi f} \right) \right) ,$$

$$h_{1j} = \frac{2G}{rc^4} \sqrt{\frac{\pi}{2}} \beta \xi_j \sigma_j \exp\left(\frac{\chi}{2} (-2\gamma_j + \sigma_j^2 \chi) \right) ,$$

$$\rho_j = \frac{1}{\sqrt{2}\sigma_j} ,$$

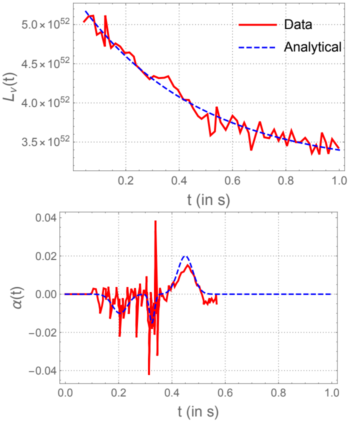
$$\tau_{1j} = \gamma_j - \sigma_j^2 \chi ,$$

$$h_{2j} = \frac{2G}{rc^4} \sqrt{\frac{\pi}{2}} \lambda \xi_j \sigma_j ,$$

$$\tau_{2j} = \gamma_j ,$$

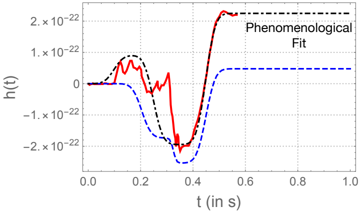
$$h_3 = \frac{2G}{rc^4} \kappa .$$

Comparison with numerical results



top data: Vartanyan and Burrows, Astrophys. J. 901 (2020) 108 ; **bottom** data: Kotake, Iwakami, Ohnishi and Yamada, Astrophys. J. 704 (2009) 951.

- toy model reproduces low frequency trends (relevant for Deci-Hz detectors)



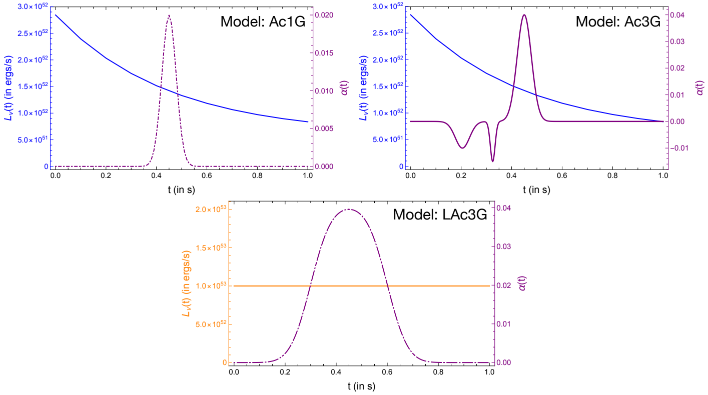
Data: Kotake, Iwakami, Ohnishi and Yamada, Astrophys. J. 704 (2009) 951.

- toy $h(t)$ reproduces numerical result
  - dashed: computed from $L(t)$ and $\alpha(t)$
  - dot-dashed: toy formula for $h(t)$ with effective parameters

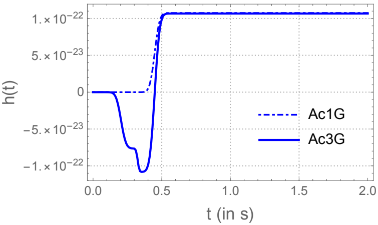
Case studies

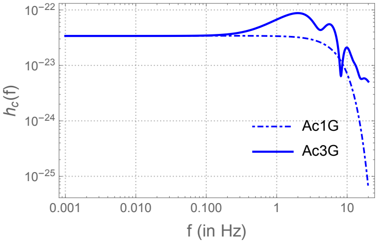
- *Accretion-only models*: zero anisotropy in cooling phase
  - $\delta t \sim 0.1 - 0.5 \text{ s} \rightarrow f \sim 2 - 10$ Hz
- *Long term evolution models*: anisotropy is non-zero throughout
  $\delta t \sim 1 - 10 \text{ s} \rightarrow f \sim 0.1 - 1$ Hz

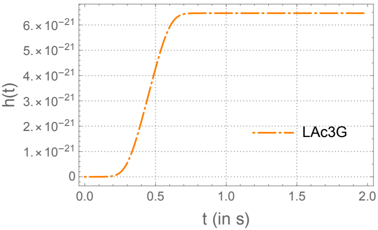
Accretion-only models: ingredients

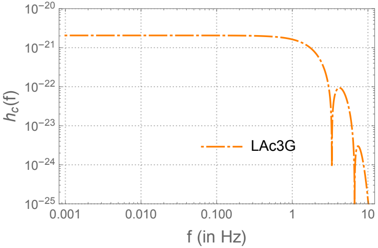


Accretion-only models: results (D=10 kpc)

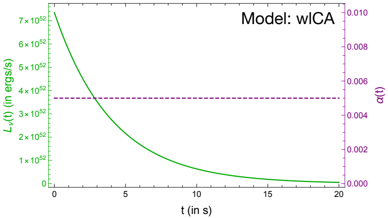


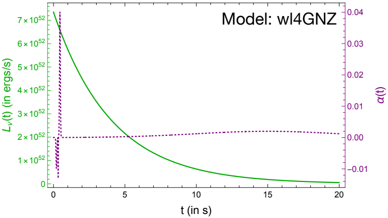




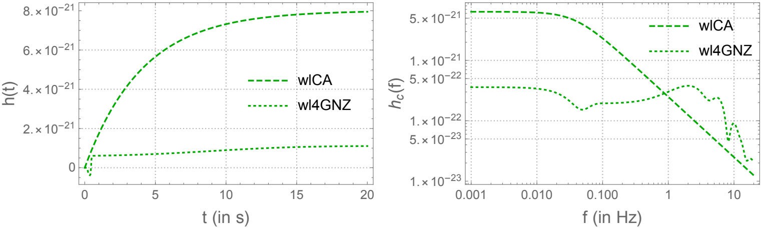


Long term evolution models: ingredients





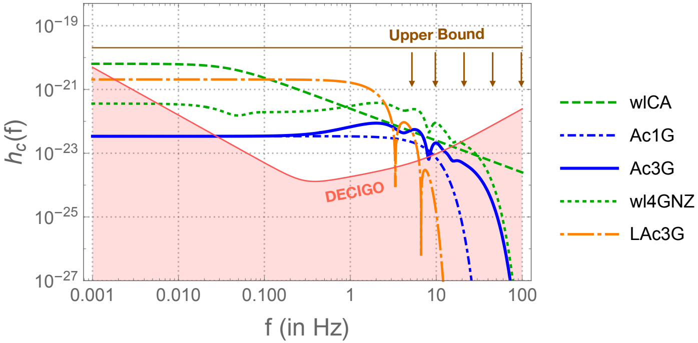
Long term evolution models: results (D=10 kpc)



Detectability and physics potential

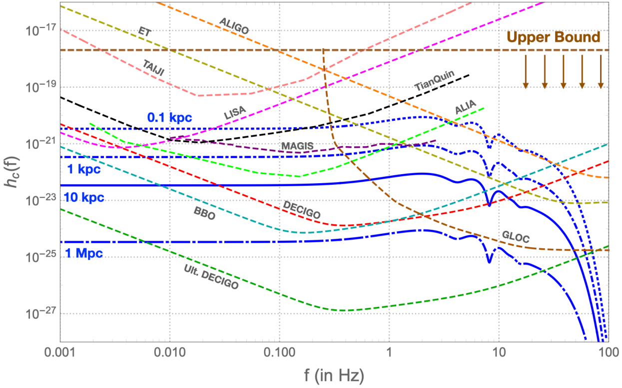
Memory at Deci-Hz detectors (D=10 kpc)

Detectable even in most pessimistic cases



Summary of detection prospects

Accretion only model, Ac3G. Note sensitivity up to Mpc distance and beyond!



Summary and discussion

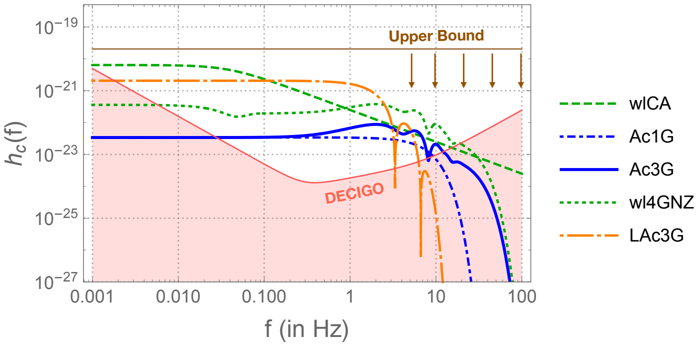
Summary and caveats

- the SN neutrino memory is detectable at (the most powerful) Deci-Hz interferometers
- A new phenomenological model is available
  - consistent with numerical simulations
  - fully analytical, useful for phenomenological studies, detector response studies, data fits, etc.
- Uncertainties:
  - $\mathcal{O}(10)$ uncertainty on $\alpha(t)$ (3D simulations result pessimistic)
  - anisotropy in cooling phase unknown
  - matter contribution to memory (sub-dominant at $f \lesssim 0.1$ Hz?)

Physics potential

- Another General Relativity prediction will be confirmed
- A new *multimessenger* component: neutrinos + GW (100 Hz scale) + GW memory (0.1-10 Hz) + astro
  - potential for supernova alerts!
  - test anisotropy $\rightarrow$ probe fluid dynamics in accretion phase
- memory + neutrinos: probe invisible cooling channels
  - sterile neutrinos, light scalars, invisible neutrino decay, etc.
- tests of gravity, room for theoretical developments
  - non-linear memory, quantum effects, etc.

Thank you!


Upper Bound
DECIGO
$h_c(f)$
f (in Hz)
10^{-19}
10^{-21}
10^{-23}
10^{-25}
10^{-27}
0.001
0.010
0.100
1
10
100
wICA
Ac1G
Ac3G
wl4GNZ
LAc3G

Backup

Alternate form of the analytical formulae

Using the approximation (accurate within 1%):

$$\mathrm{erf}\,(x) \simeq \tanh\,(mx), \;\; \text{with } m = \sqrt{\pi}\,\log(2)\,,$$

one can rewrite the results as:

$$h(t) = \sum_{j=1}^{N} \left[\left\{ h_{1j} \left(\tanh\,(m\rho_j\; \tau_{1j}) + \tanh\,\left(m\rho_j (t - \tau_{1j}) \right) \right) \right\} + \left\{ h_{2j} \left(\tanh\,(m\rho_j\; \tau_{2j}) + \tanh\,\left(m\rho_j (t - \tau_{2j}) \right) \right) \right\} \right] + h_3 \left(\frac{\beta}{\chi} \left(1 - \exp\,(-t\chi) \right) + \lambda t \right)\,,$$

$$\tilde{h}(f) = \sum_{j=1}^{N} \left[\left(h_{1j} \frac{i\pi}{m\rho_j} \,\mathrm{csch}\, \left(\frac{\pi^2 f}{m\rho_j} \right) \exp\,\left(i2\pi f \tau_{1j} \right) \right) + \left(h_{2j} \frac{i\pi}{m\rho_j} \,\mathrm{csch}\, \left(\frac{\pi^2 f}{m\rho_j} \right) \exp\,\left(i2\pi f \tau_{2j} \right) \right) \right] + \left(\sqrt{2\pi}\; h_3 \frac{\beta}{\chi} \left(\frac{1}{i2\pi f} - \frac{1}{-\chi + i2\pi f} \right) \right).$$

Longer accretion model, LAc3G.