Gravitational Wave Probes of
Physics Beyond Standard Model
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Primordial Black Holes and Cosmological Gravitational Waves

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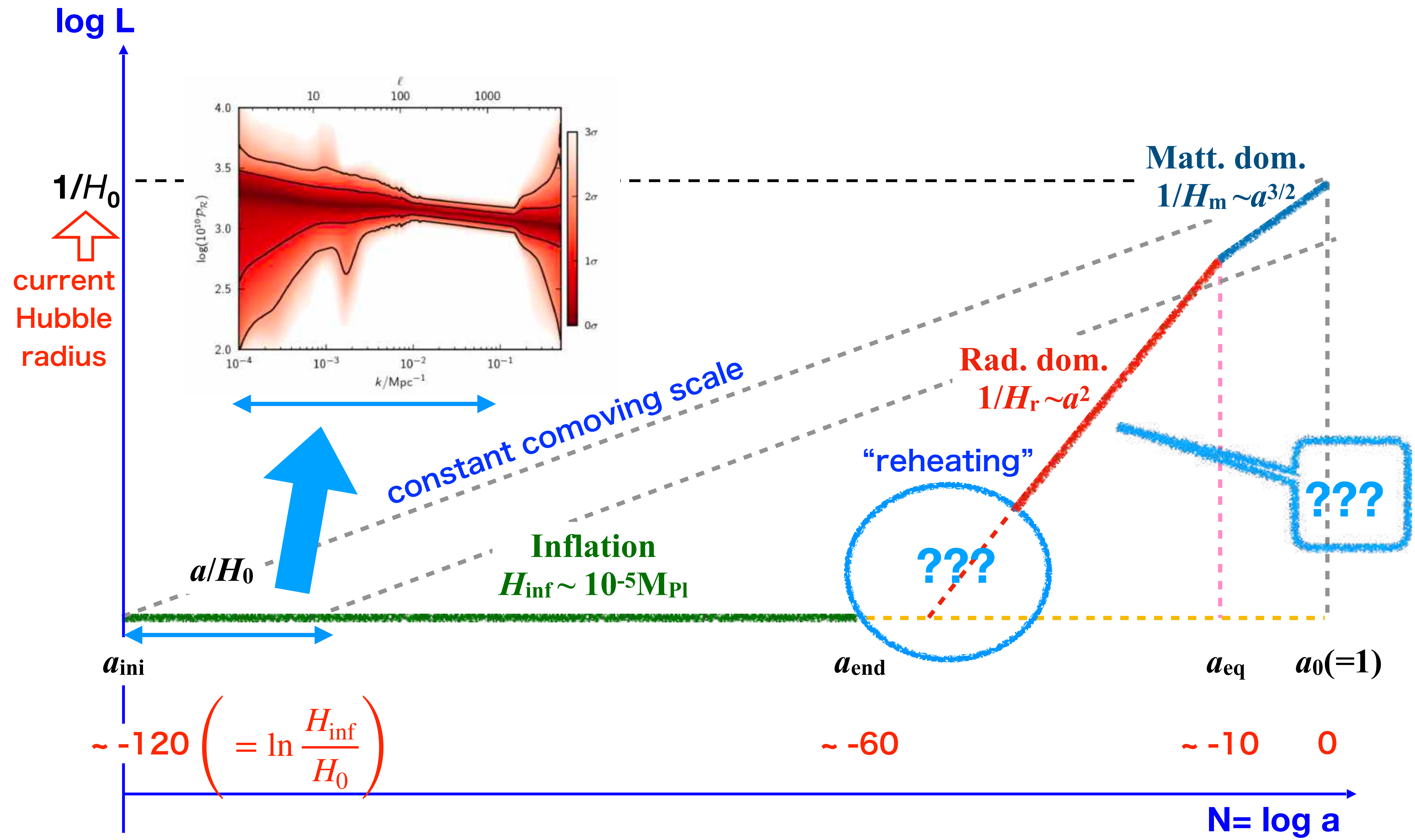
Introduction

curvature perturbation,

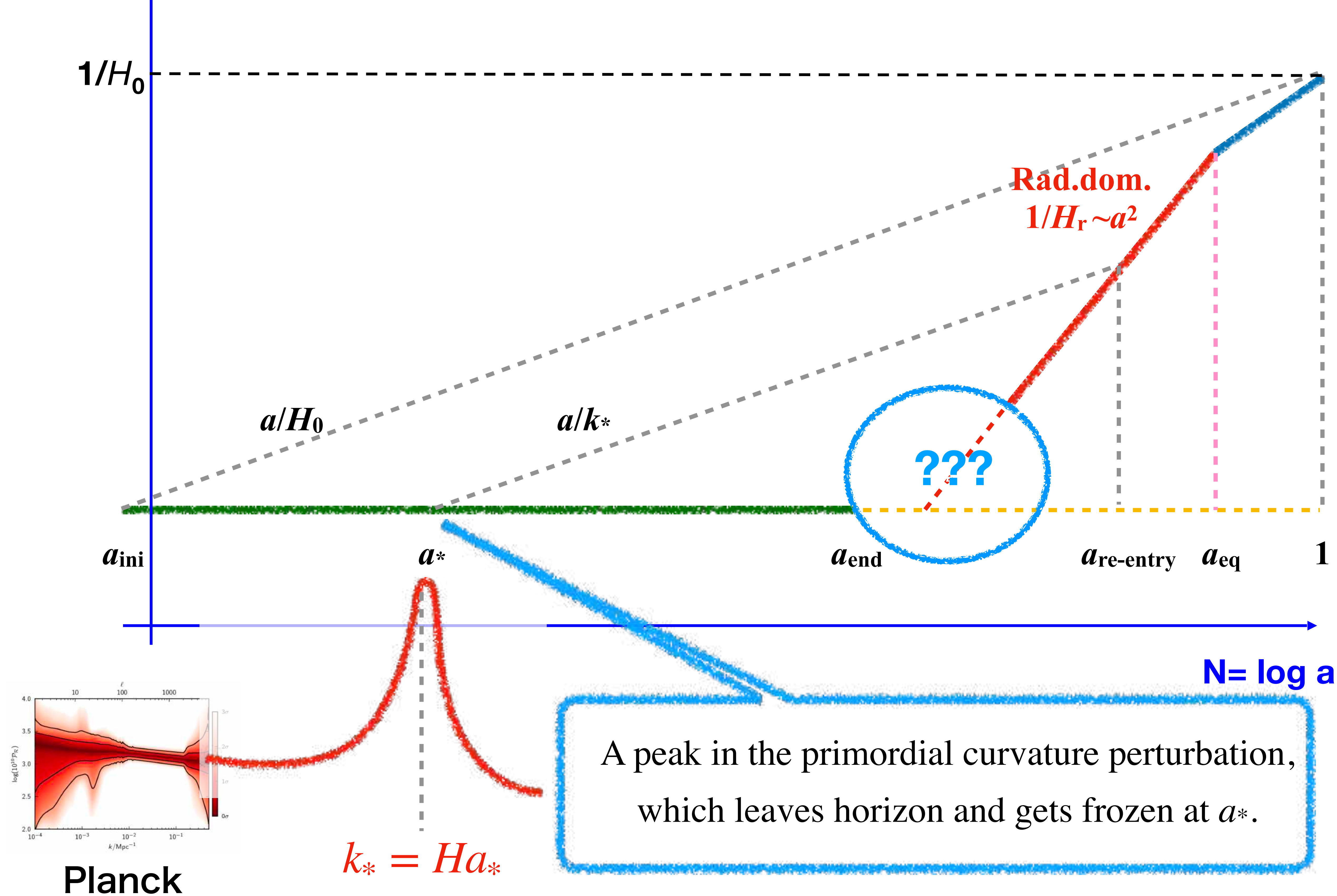
formation of PBHs,

and gravitational waves

cosmic spacetime diagram



PBH formation: conventional case



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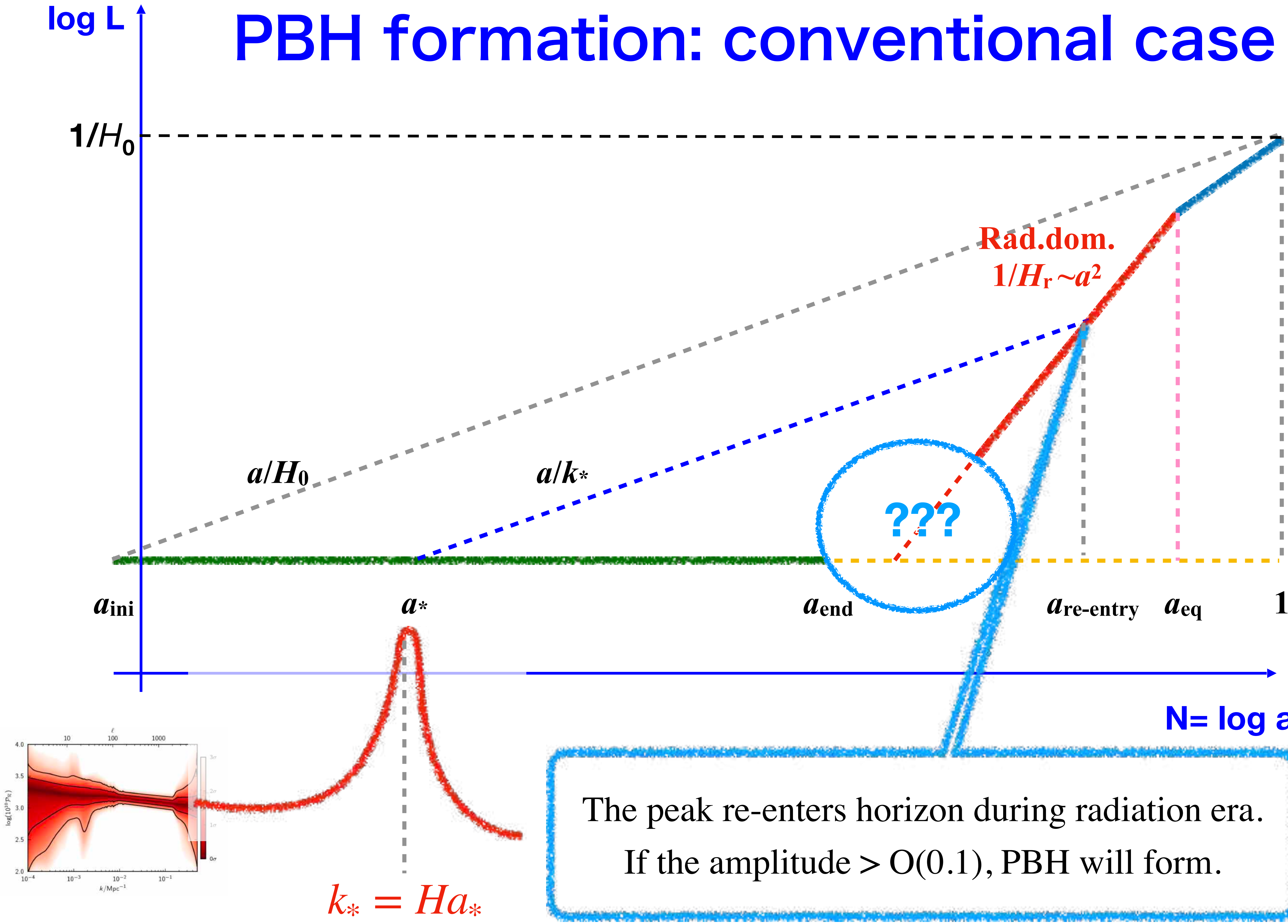
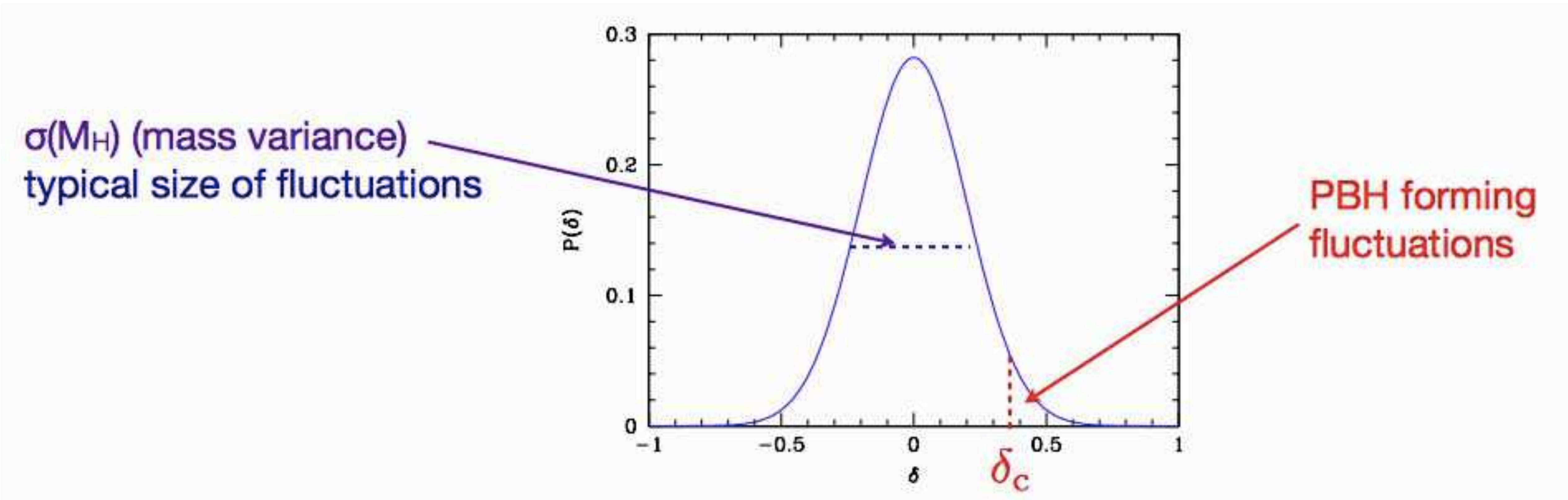


The peak re-enters horizon during radiation era.

If the amplitude > O(0.1), PBH will form.

fraction β that turns into PBHs

for Gaussian probability distribution



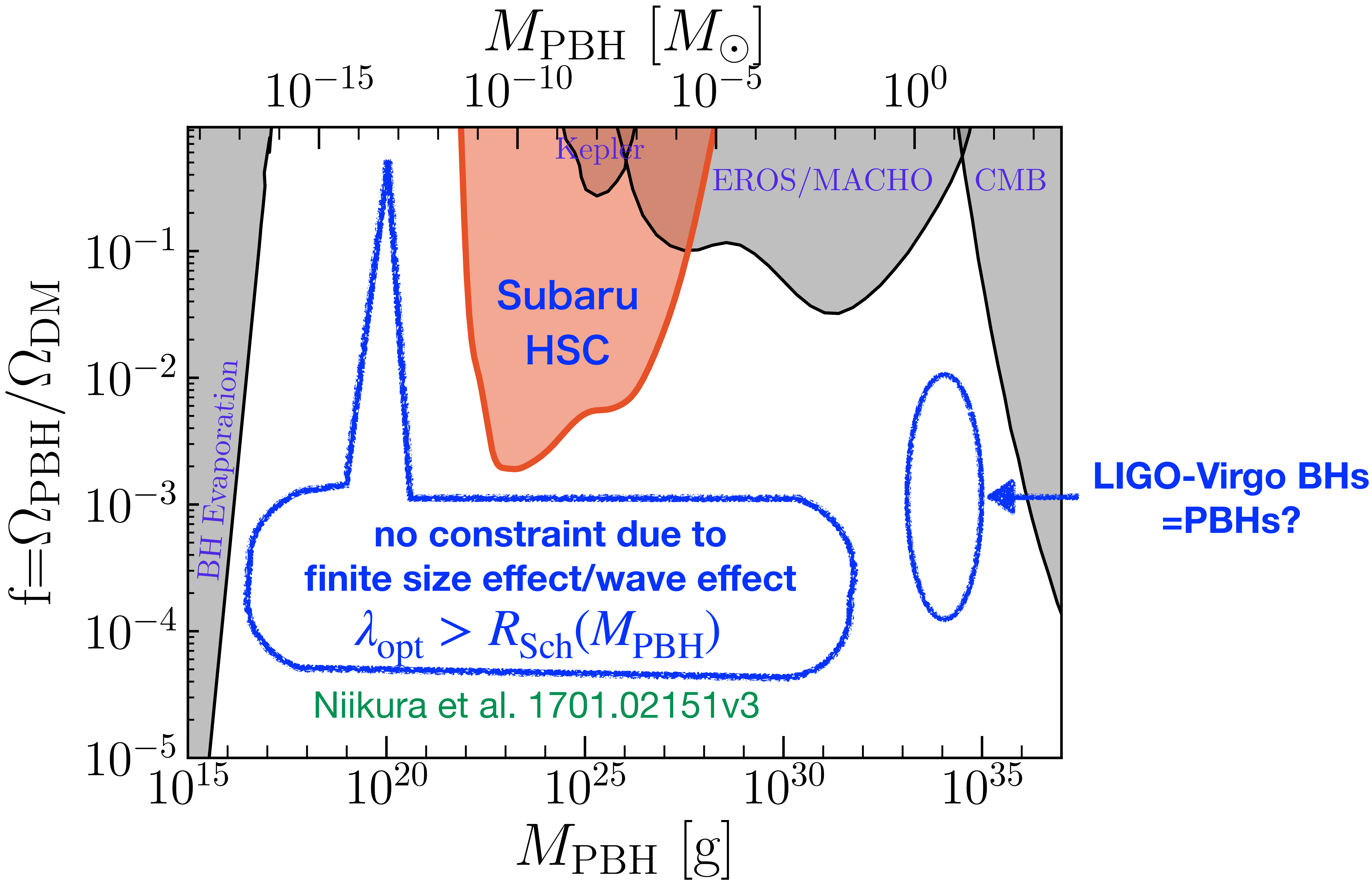
- When $\sigma_M << \delta_c$, β can be approximated by exponential:

$$\beta \approx \sqrt{\frac{2}{\pi}} \frac{\sigma_M}{\delta_c} \exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right)$$

$$\delta_c \equiv \left(\frac{\delta\rho_c}{\rho}\right)_{\rm crit} \sim 0.4$$

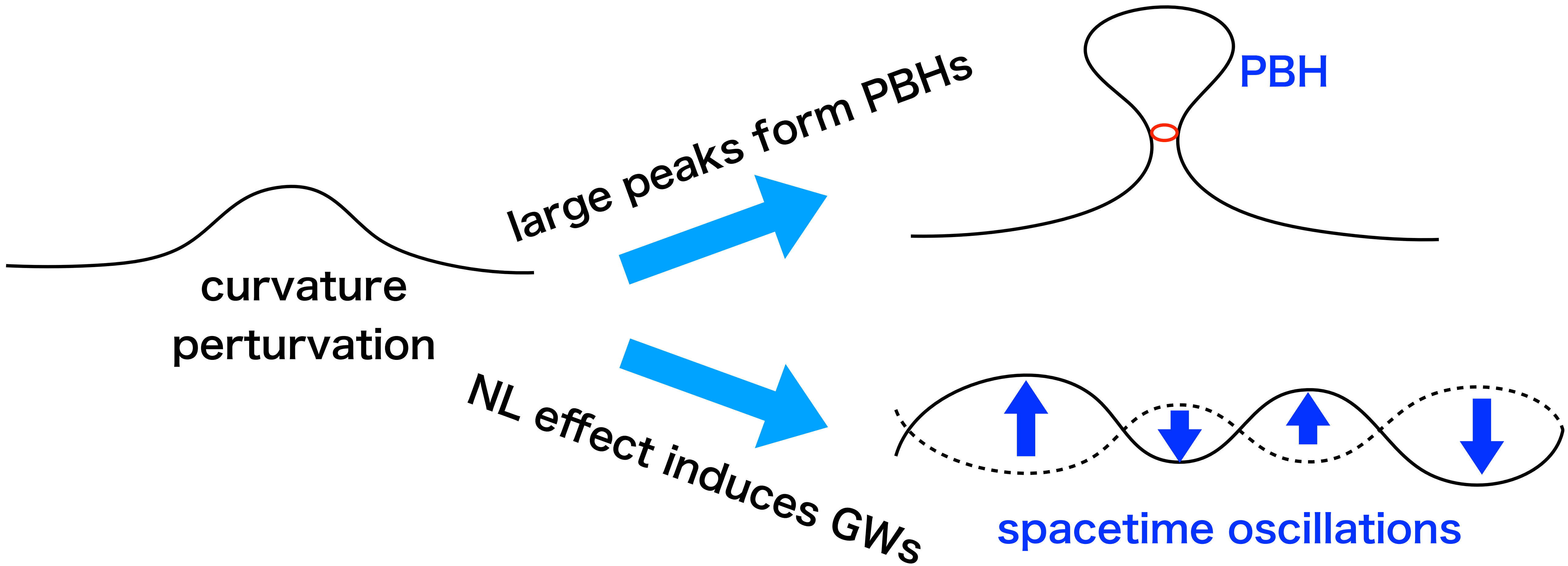
Carr ’75, …

PBH constraints



big window at $M_{\rm PBH} \approx 10^{17} - 10^{22}$ g ⟷ $T_{\text{re-entry}} \sim 10^4$ - 10^8 GeV

GWs can capture PBHs!



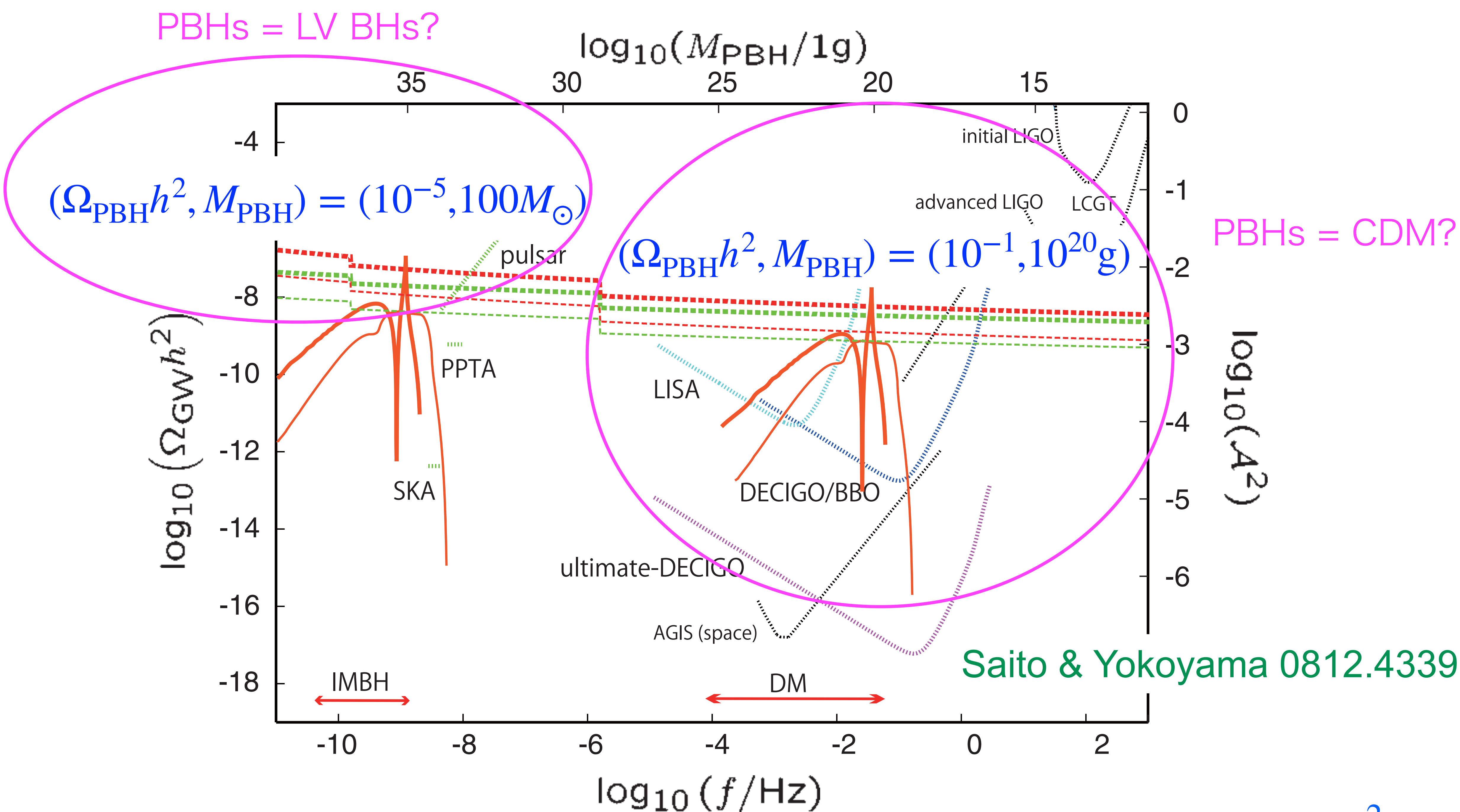
PBHs = CDM with $M_{PBH} \sim 10^{21}$g
generates GWs with f~10^{-3} Hz

⇒ Background GWs in LISA band

PBHs=LV BHs with M~10M_sol
generates GWs with f~10^{-8} Hz

⇒ PTA band

GWs can test PBH scenario!

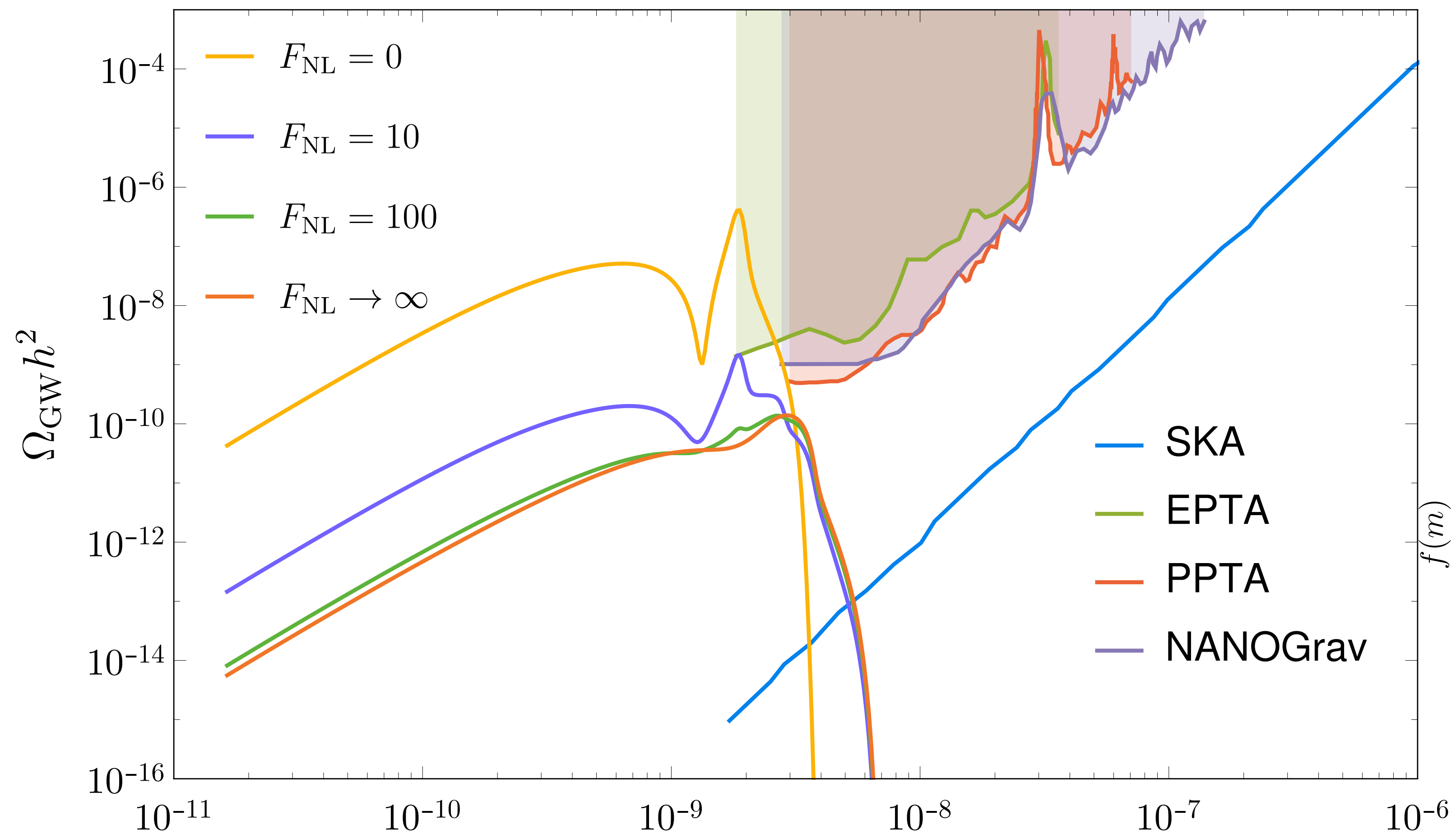


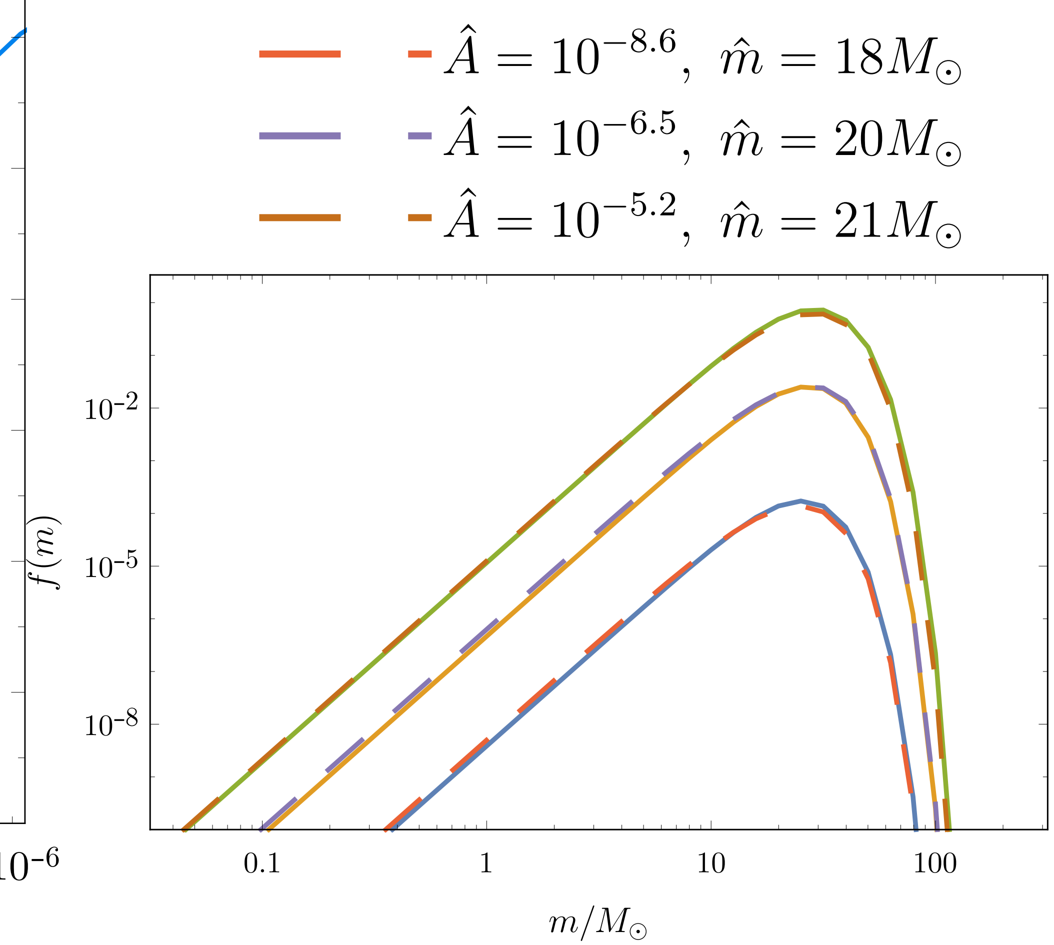
$$M_{PBH} \sim 0.1 M_\odot \left(\frac{1\mathrm{GeV}}{T}\right)^2 \sim 10 M_\odot \left(\frac{1\mathrm{pc}^{-1}}{k}\right)^2$$

Testing LV BH=PBH scenario

"Pulsar Timing Array Constraints on …" Cai, Pi, Wang & Yang '19

$$f_{\rm peak} \sim 6.7 \times 10^{-9} \left(\frac{M_{\rm PBH}}{M_\odot} \right)^{-1/2} \ {\rm Hz}$$





for $f(m)$ peaked at $m \sim 17 M_\odot$

Gaussian case seems on the verge of exclusion/or detection!

➡ lots of speculations after recent NANOGrav 12.5 years result…

NANOGrav collaboration '20

Isocurvature Perturbation due to inhomogeneous PBH distribution

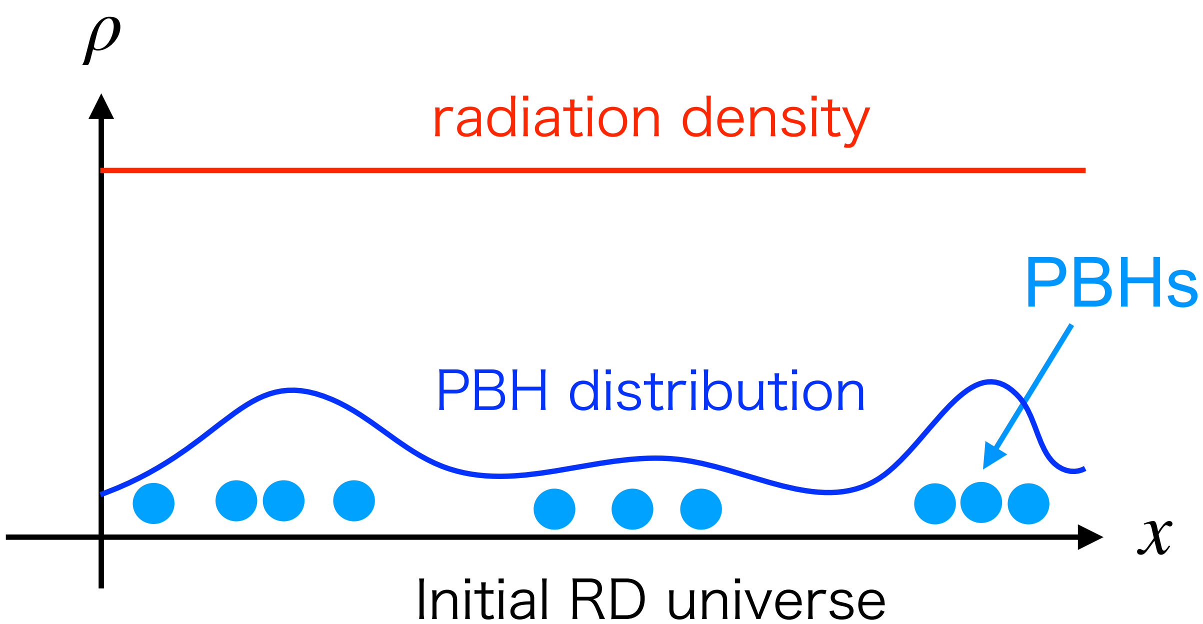
What if PBHs have completely evaporated?

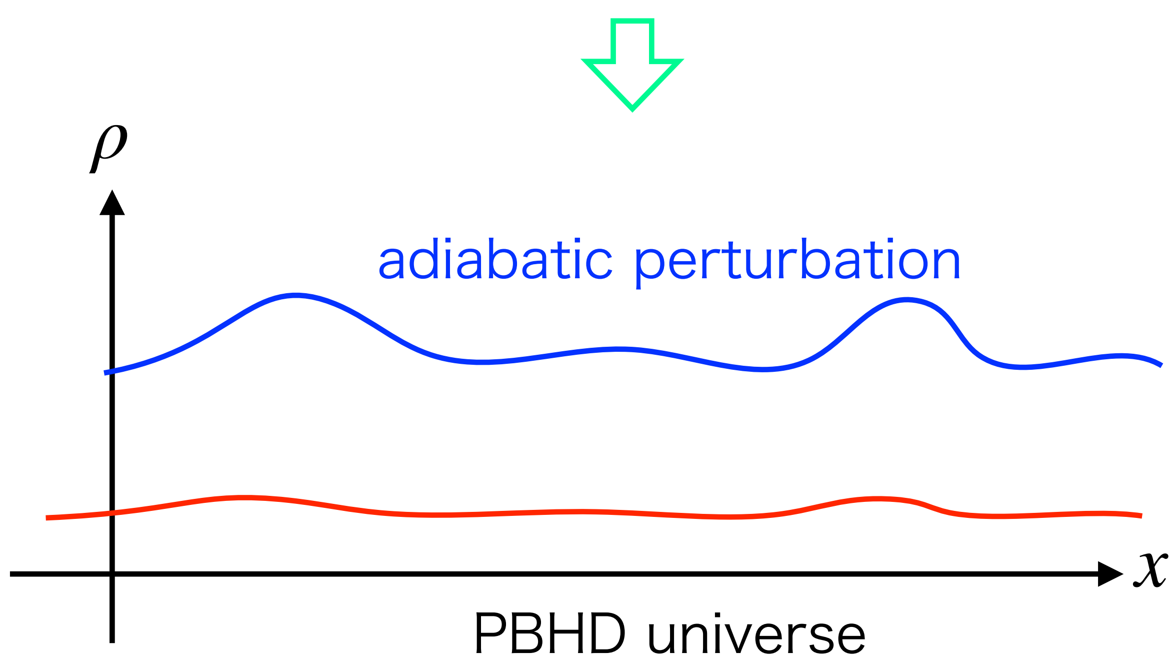
For $M_{\rm PBH} \lesssim 10^8\,{\rm g}, \quad t_{\rm evap} \lesssim 1\,{\rm s}$

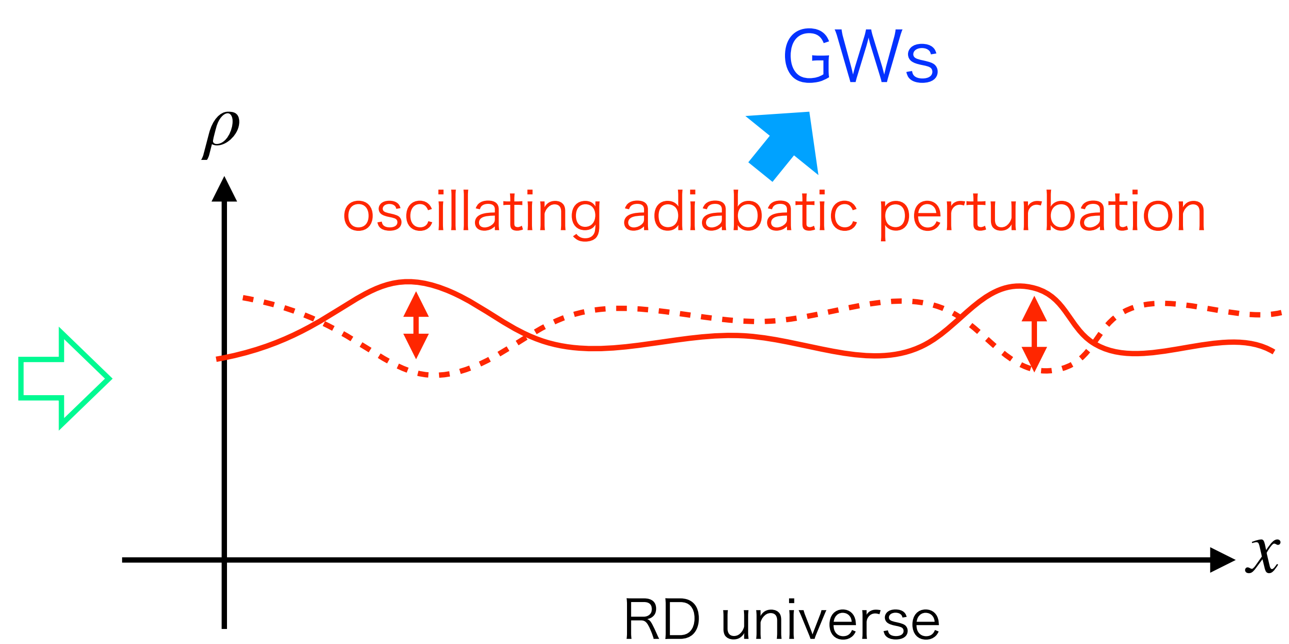
Even if PBHs are unclustered, inhomogeneities due to random distribution may induce GWs when the universe is reheated by PBH evaporation

Papanikolaou et al., arXiv:2010.11573

Domenech, Lin & MS, arXiv:2012.08151







Induced GWs from PBH evaporation

Domenech, Lin & MS, arXiv:2012.0851

- If the transition from PBHD to RD is slow ($\Delta t \sim H^{-1}$) as in the case of decaying particles, there will be no significant production of induced GWs.

Inomata et al., arXiv:1904.12878

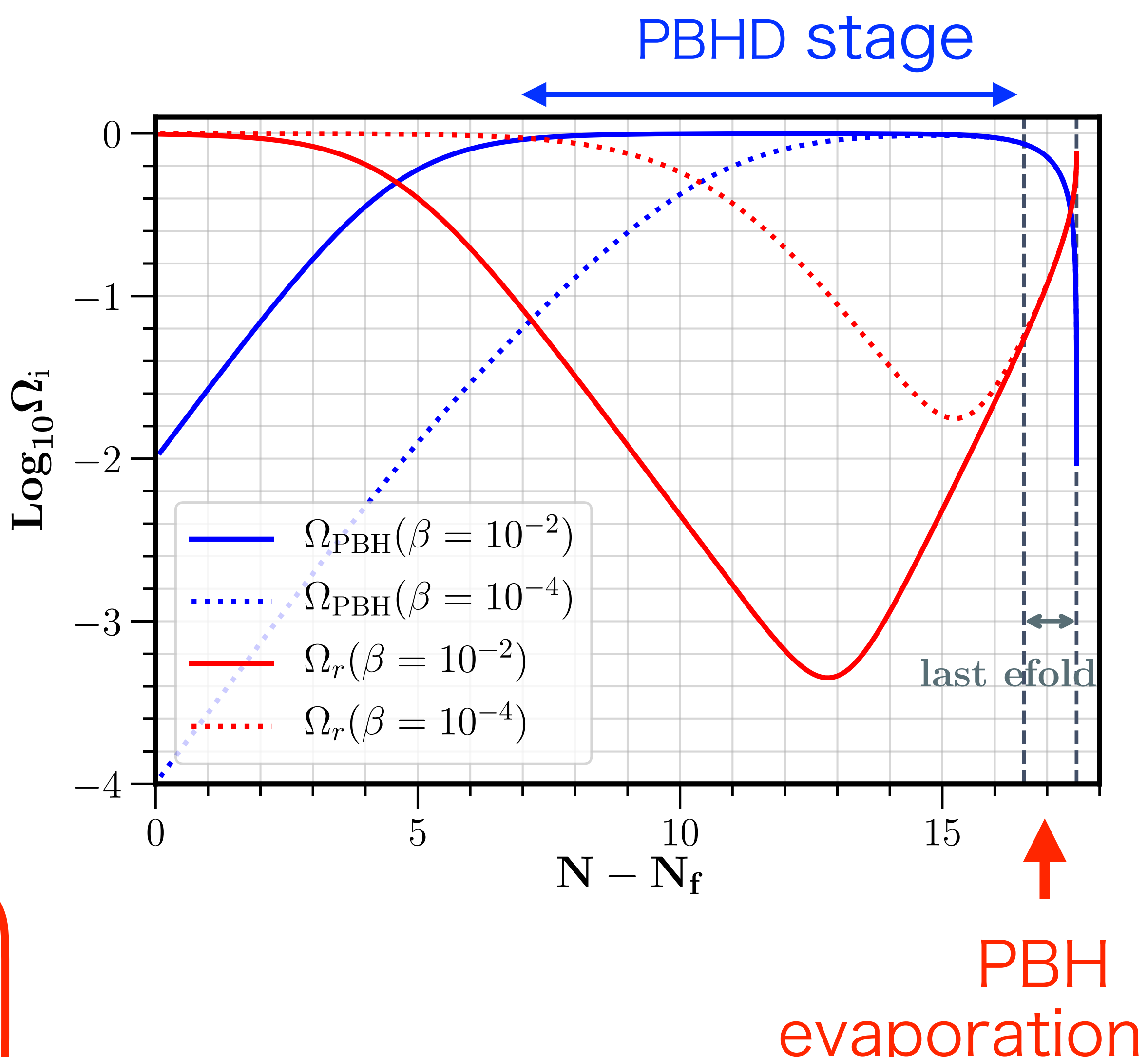
$$Q = Q_0 e^{-\Gamma t} \quad \rightarrow \quad \frac{1}{\Delta t} = \frac{1}{Q}\frac{dQ}{dt} = -\Gamma = const\,.$$

- A fast transition leads to strong enhancement of induced GWs on sub-horizon scales, which is the case for PBH evaporation.

Inomata et al., arXiv: 2003.10455

$$\frac{1}{\Delta t} = \left|\frac{1}{M}\frac{dM}{dt}\right| = \frac{1}{3(t_{\rm ev}-t)} \gg H \text{ as } t \rightarrow t_{\rm ev}$$

may lead to strong constraints on early PBH dominance model



Constraints on early PBH dominated universe

Domenech, Lin & MS, arXiv:2012.08151
Domenech, Takhistov & MS, arXiv:2105.06816

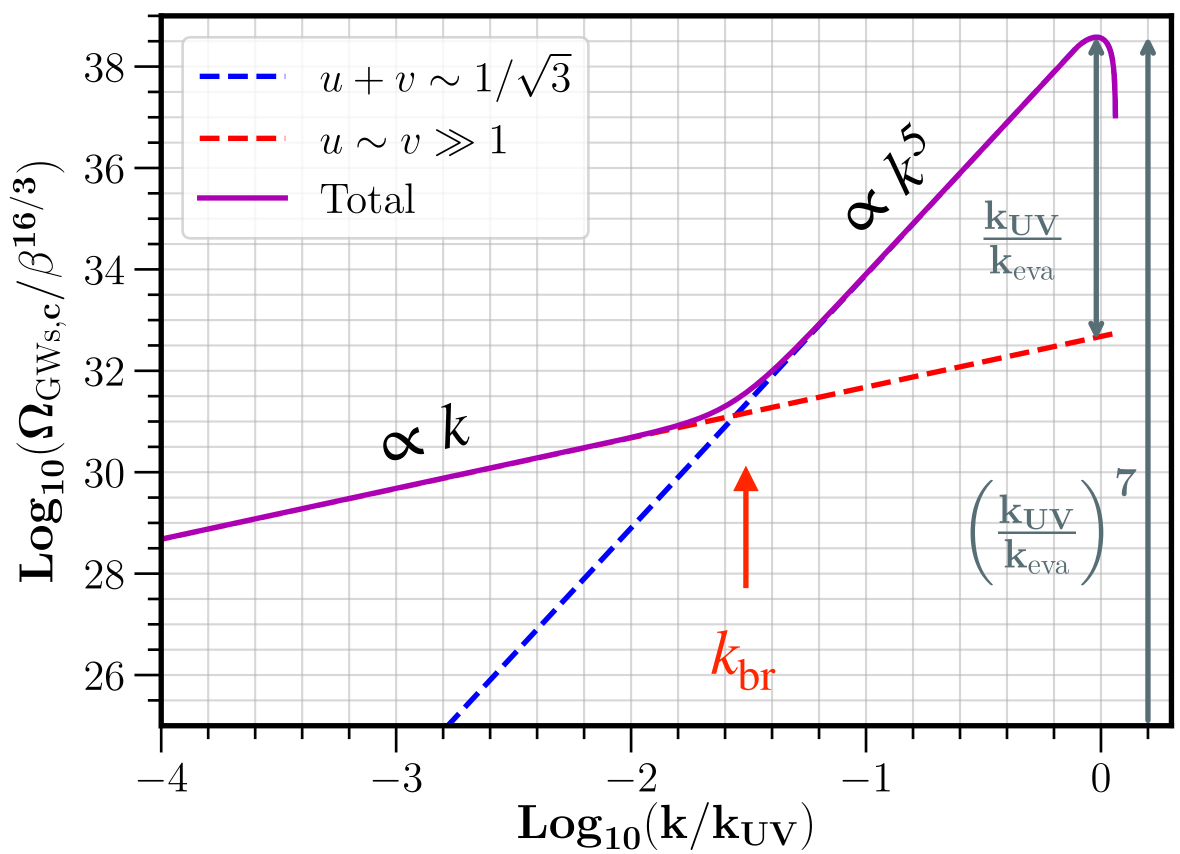
- Assumptions
  - Monochromatic mass function for PBHs.
  - Poisson distribution for $\delta n_{\rm PBH}/n_{\rm PBH}$: $\mathscr{P}_S(k) = \frac{2}{3\pi}\left(k/k_{\rm UV}\right)^3; \quad k < k_{\rm UV} = n_{\rm PBH}^{-1/3}$

- Resulting spectrum
  - sharp rise ~ k^5 near the peak.
  - Peak value:

$$\left(\frac{\Omega_{GW,max}}{\Omega_{r,0}}\right) \approx 5\times10^{34}\,\beta^{16/3}\left(\frac{M}{10^4\,{\rm g}}\right)^{14/3}$$

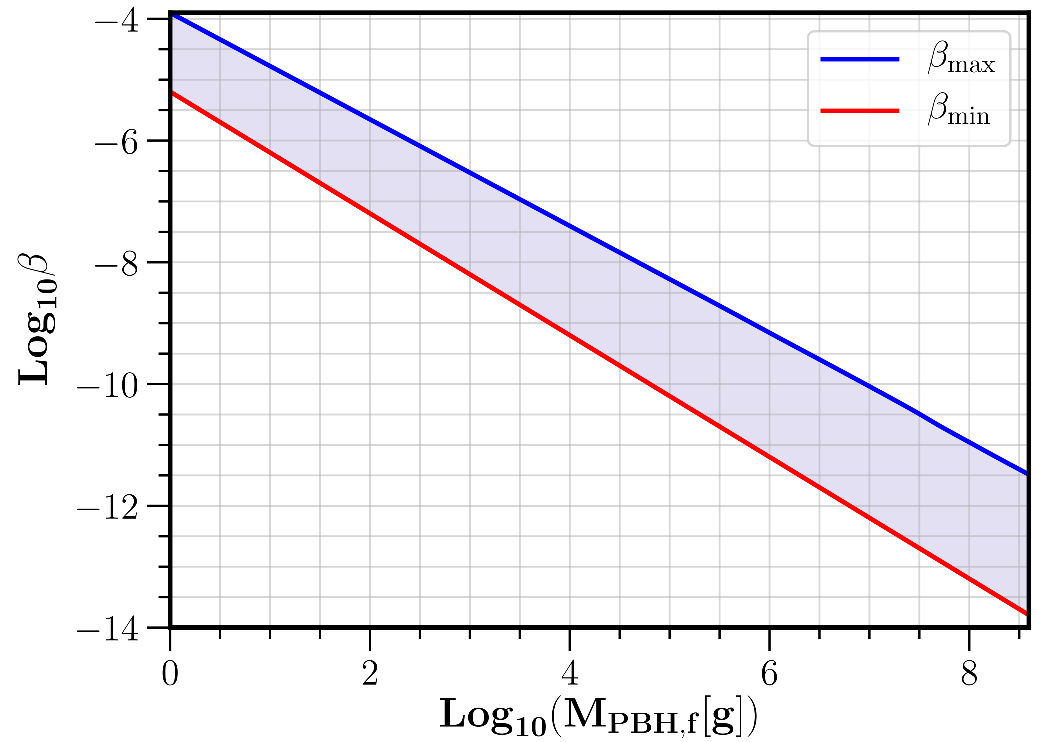
β : PBH fraction at formation

constraints on β



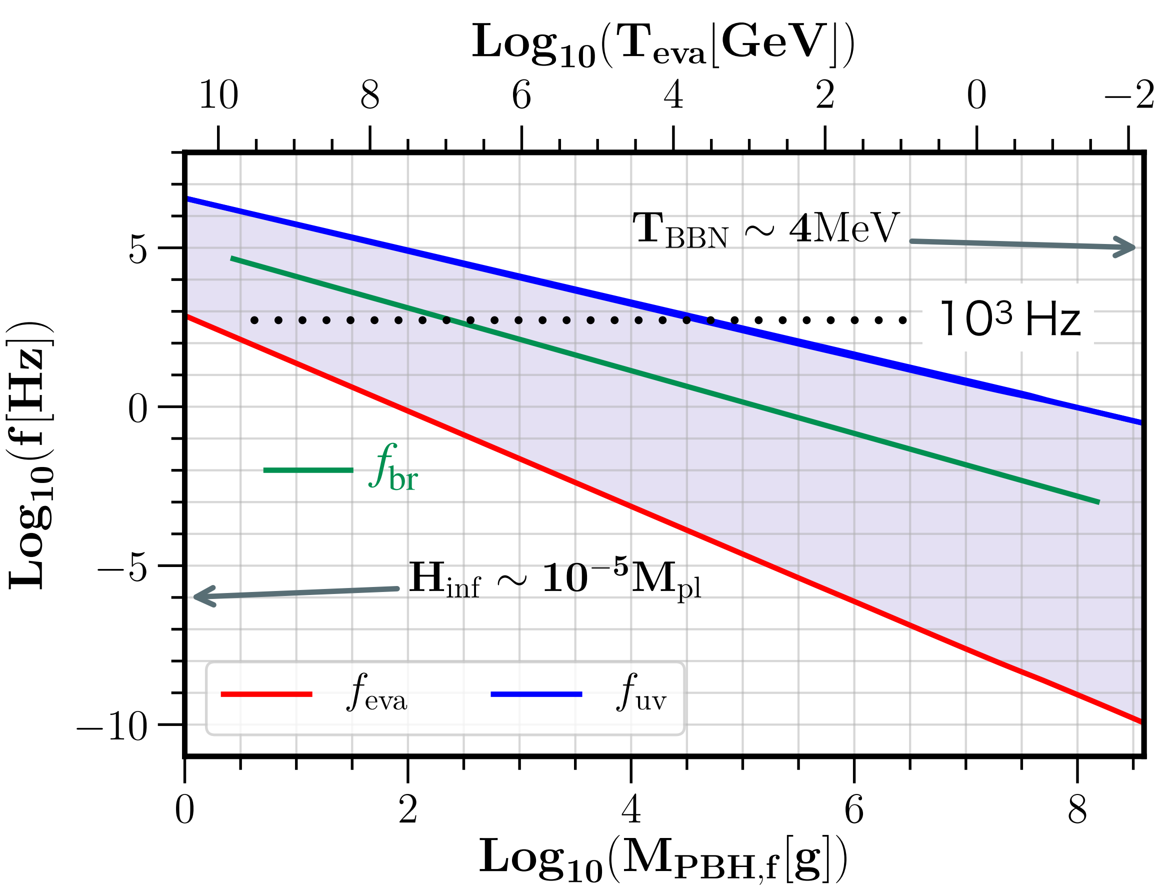
$$k_{\rm br} \approx 0.04\,k_{\rm UV}\left(M_{\rm PBH}/10^4\,{\rm g}\right)^{-1/6}$$

Constraints on β and frequencies

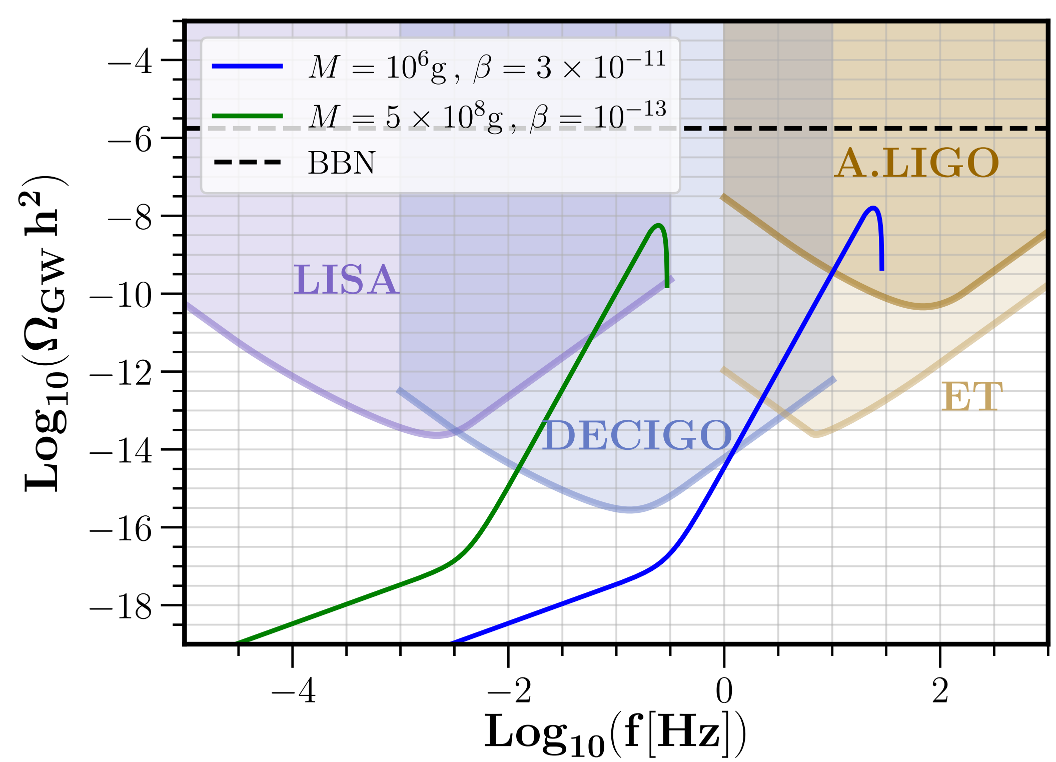


$$\beta_{\rm max} \approx 3\times 10^{-8}\left(\frac{M_{\rm PBH}}{10^5\,{\rm g}}\right)^{-7/8}$$

$$\beta_{\rm min} \approx 6\times 10^{-10}\left(\frac{M_{\rm PBH}}{10^5\,{\rm g}}\right)^{-1}$$



frequency range vs M_{PBH}



GW detectors sensitivity curves

Caviat . . .

For the primordial isocurvature perturbation,

$$\mathcal{P}_S(k) = \frac{2}{3\pi}\left(k/k_{\rm UV}\right)^3; \quad k < k_{\rm UV} = n_{\rm PBH}^{-1/3}$$

the resulting curvature perturbation at PBH dominated Universe is

$$\Phi = \frac{3}{4}\left(\frac{k_{\rm eq}}{k}\right)^2 S \sim 0.3\left(\frac{k_{\rm eq}}{k_{\rm UV}}\right)^2\left(\frac{k}{k_{\rm UV}}\right)^{-1/2} \qquad \text{for} \quad k_{\rm eq} < k < k_{\rm UV}$$

➡ The density perturbation becomes nonlinear for $k > k_{\rm NL}$:

$$\frac{\delta\rho}{\rho} = \frac{2}{3}\left(\frac{k}{aH}\right)^2 \Phi \sim 0.1\left(\frac{a_{\rm evap}}{a_{\rm eq}}\right)\left(\frac{k}{k_{\rm UV}}\right)^{3/2} \gtrsim 1$$

$$\text{for} \quad k_{\rm UV} > k > k_{\rm NL} \sim 5\left(\frac{a_{\rm eq}}{a_{\rm evap}}\right)^{2/3} k_{\rm UV}$$

$$\left(\frac{a_{\rm eq}}{a_{\rm evap}}\right)^{2/3} \approx \exp\left[-\frac{8}{9}\left(\log\frac{\beta}{10^{-7}} + \log\frac{M}{10^4\,{\rm g}}\right)\right]$$

Caviat . . .

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need more studies!

take-home messages:

- PBHs may play central roles in GW cosmology

➡ **PBH-GW Cosmology!**

- (nonlinear) isocurvature perturbations may play important roles in PBH-GW cosmology