Moving charges in particle physics and cosmic ray physics

A general signal theorem for all of electrodynamics

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Classical electrodynamics and reciprocity

Discusses the important preliminaries:
Maxwell's equations and Lorentz reciprocity

A general, electrodynamic signal theorem

Shows the (very quick) derivation of the theorem

Applications of the general theorem

Taken from particle physics and cosmic ray physics

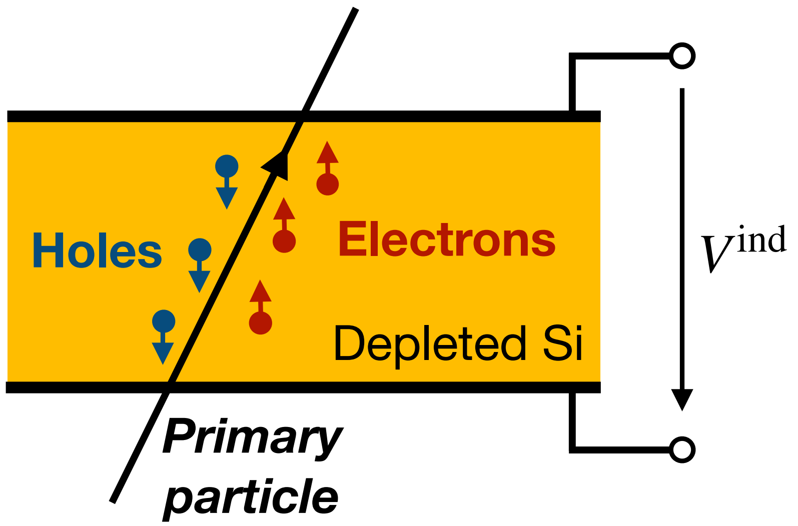
Drift tubes and transmission lines

Dipole antennas as particle detectors

The radio signature of neutrino-induced showers

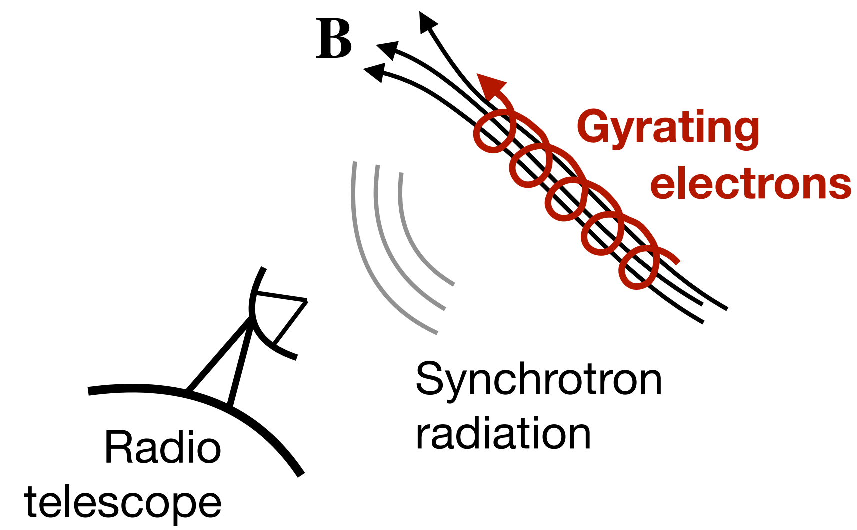
Introduction: signal formation in detectors

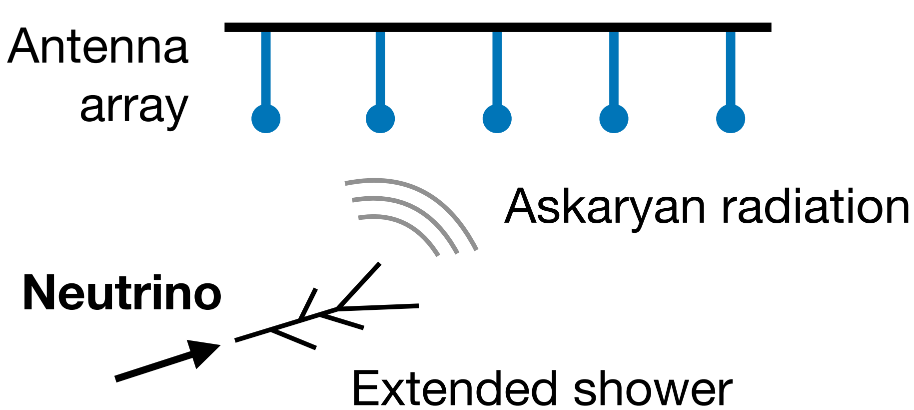
Detection of moving charged particles:
a fundamental problem across many branches of physics!



Silicon detector: primary particle creates **charge clusters**; detector **signal** induced by **drifting electrons and holes**

Radio astronomy: **electrons gyrating** in interstellar magnetic field produce **synchrotron radiation**; leads to **signal in receiver of radio telescope**



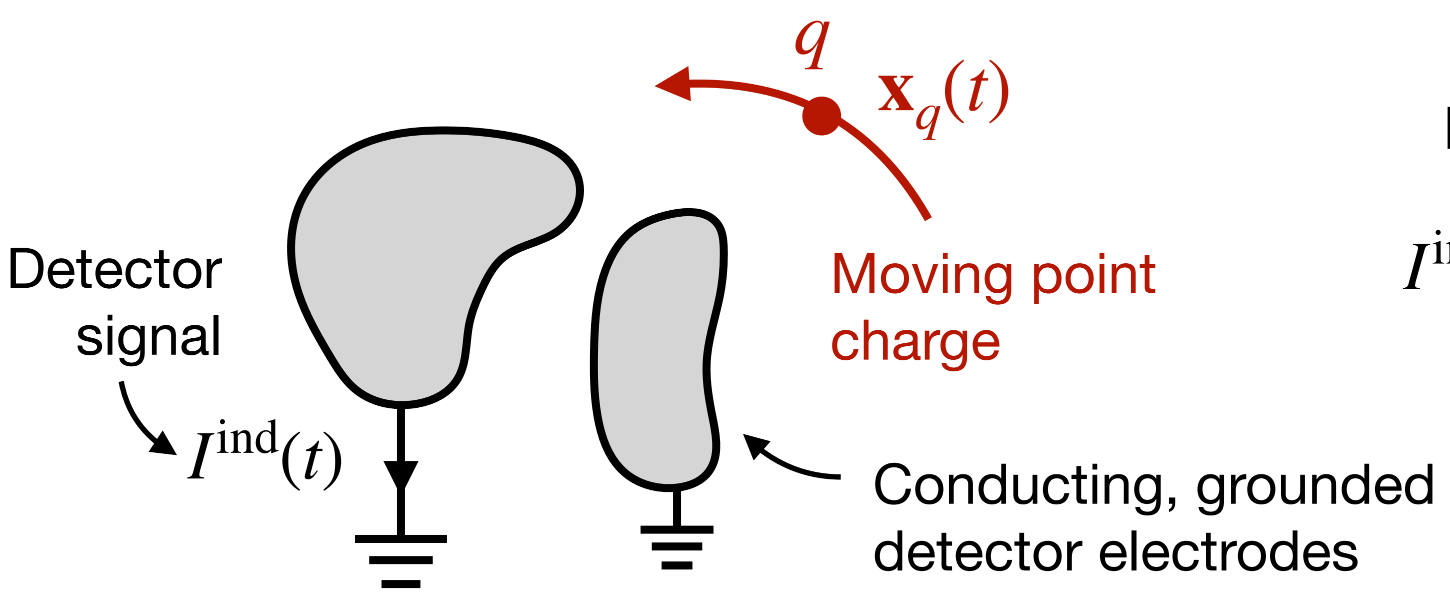


Radio neutrino observatories: ultra-high energy cosmic **neutrinos** induce **extended showers in arctic ice**; Askaryan radiation detected by **antenna array close to surface**

Introduction: signal formation in detectors

In all cases, signal formation is a well-defined problem in classical electrodynamics

For quasi-electro<u>static</u> situations *(where charges move nonrelativistically and radiation effects are negligible)*, **the Ramo-Shockley theorem may be used to compute the signal:**



Ramo-Shockley theorem:

$$I^{\mathrm{ind}}(t) = -\frac{q}{V_w}\mathbf{E}_w(\mathbf{x}_q(t)) \cdot \dot{\mathbf{x}}_q(t)$$

"Weighting field"

In the following: derive a similar signal theorem that holds for the full scope of electro<u>dynamics</u>

Maxwell's equations in geometry of detector + environment ⟵ **Core ingredients** ⟶ Lorentz reciprocity

Classical electrodynamics

Consider very general situation: **detector** and **environment** represented by a general, linear material

Such a material is characterised by permittivity (ε), permeability (μ) and conductivity (σ)

(For anisotropic materials these are 3 x 3 matrices)

$$\boxed{\begin{aligned} \mathbf{D}(\mathbf{x},\omega) &= \hat{\epsilon}\mathbf{E}(\mathbf{x},\omega) \\ \mathbf{J}(\mathbf{x},\omega) &= \hat{\boldsymbol{\sigma}}\mathbf{E}(\mathbf{x},\omega) \\ \mathbf{B}(\mathbf{x},\omega) &= \hat{\boldsymbol{\mu}}\mathbf{H}(\mathbf{x},\omega) \end{aligned}}$$

Any practical detector elements (wires, antennas, …) may be represented by a general such material distribution

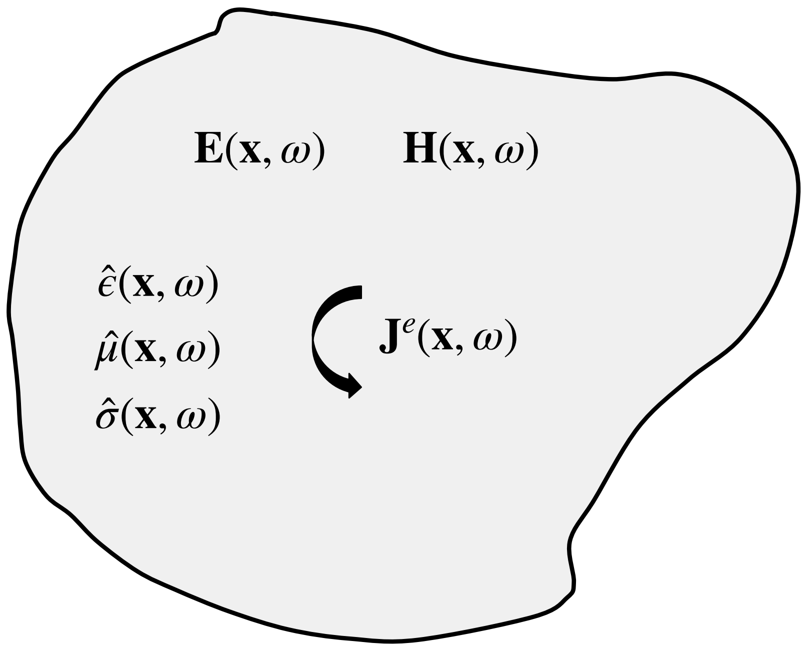
$$\boxed{\begin{aligned} \boldsymbol{\nabla} \cdot (\hat{\epsilon}\mathbf{E}) &= \rho, \\ \boldsymbol{\nabla} \cdot (\hat{\boldsymbol{\mu}}\mathbf{H}) &= 0, \\ \boldsymbol{\nabla} \times \mathbf{E} &= -i\omega\hat{\boldsymbol{\mu}}\mathbf{H}, \\ \boldsymbol{\nabla} \times \mathbf{H} &= \mathbf{J}^e + \hat{\boldsymbol{\sigma}}\mathbf{E} + i\omega\hat{\epsilon}\mathbf{E}. \end{aligned}}$$

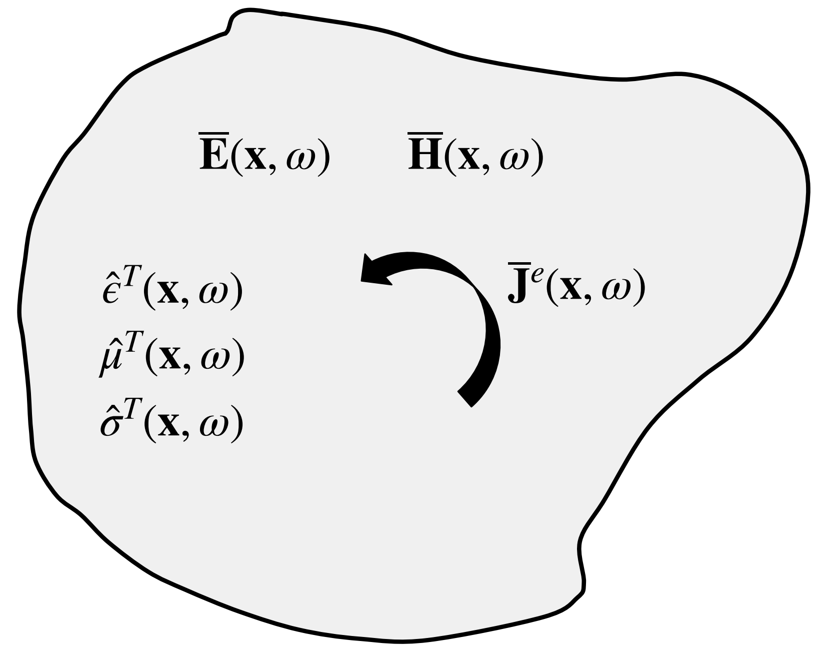
Dynamics of the fields determined by Maxwell's equations
(here written in the frequency domain)

Lorentz reciprocity

Compare two general electrodynamic situations:

Two arbitrary current densities $\mathbf{J}^e(\mathbf{x},\omega)$ and $\bar{\mathbf{J}}^e(\mathbf{x},\omega)$ lead to fields $\mathbf{E}$, $\mathbf{H}$ and $\bar{\mathbf{E}}$, $\bar{\mathbf{H}}$





The two situations have related material distributions

(Transposed material properties, i.e. identical material for <u>*symmetric*</u> *response matrices)*

$\rightarrow$

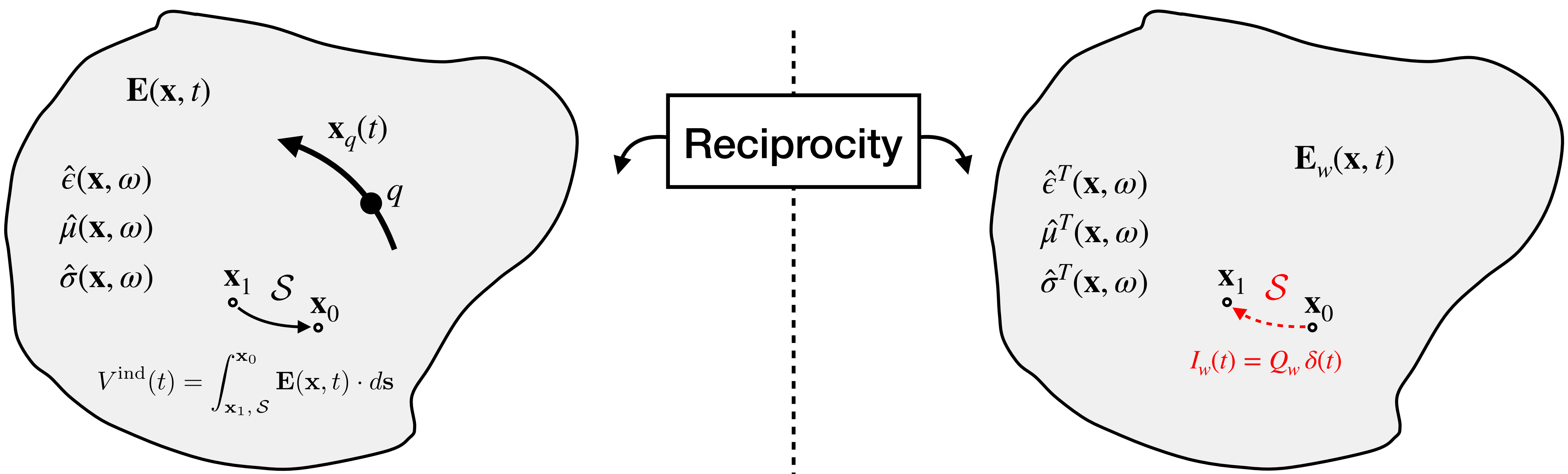
There exist relations also between the resulting fields

Lorentz reciprocity relation

$$\boxed{\int_{\mathbb{R}^3} dV\, \bar{\mathbf{E}}(\mathbf{x},\omega)\cdot\mathbf{J}^e(\mathbf{x},\omega) = \int_{\mathbb{R}^3} dV\, \mathbf{E}(\mathbf{x},\omega)\cdot\bar{\mathbf{J}}^e(\mathbf{x},\omega)}$$

A general signal theorem

To derive signal theorem, apply Lorentz reciprocity theorem to two specific situations:



Point charge moves along arbitrary trajectory, induces detector signal

Signal is voltage measured between detector **terminals** at $\mathbf{x}_1$ and $\mathbf{x}_0$
(Path dependent! Path is part of signal definition.)

Point charge is removed and a delta-like current source connected to terminals

Detector acts as transmitting antenna, resulting field distribution $\mathbf{E}_w$ is weighting field

Lorentz reciprocity relation resolves to

$$\boxed{V^{\mathrm{ind}}(t) = -\frac{q}{Q_w} \int_{-\infty}^{\infty} dt' \, \mathbf{E}_w(\mathbf{x}_q(t'), t - t') \cdot \dot{\mathbf{x}}_q(t')}$$

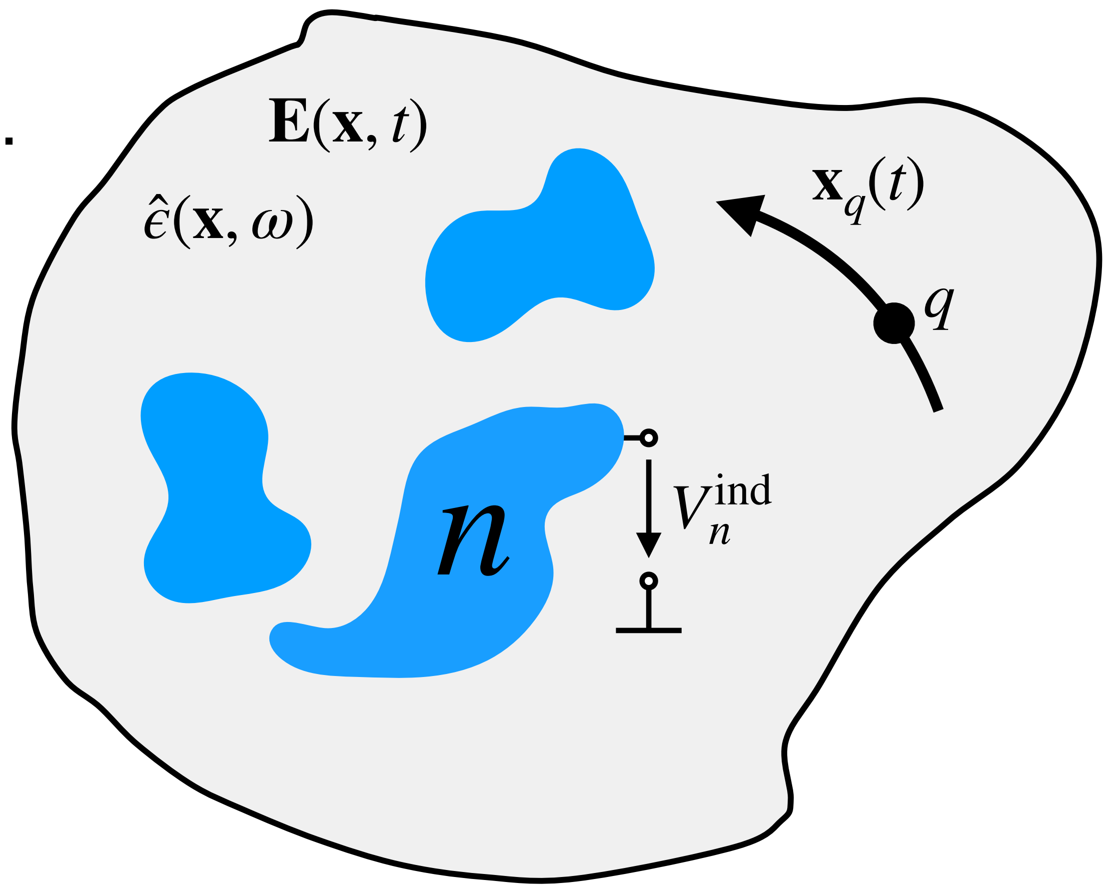
Nonrelativistic limit

This is a fully general, electrodynamic, signal theorem …

… it contains the Ramo-Shockley theorem
(and its electrostatic extensions) as **special cases**

Detector consisting of N insulated electrodes

Electrostatic limit: charge moves nonrelativistically, no wave propagation



Weighting field for electrode *n* is a gradient field …

$$\mathbf{E}_w(\mathbf{x}, t) = -\boldsymbol{\nabla}\phi_{w,n}(\mathbf{x}; Q_w)\,\Theta(t)$$

… and the general theorem reduces to

$$V_n^{\text{ind}}(t) = \frac{q}{Q_w}\phi_n(\mathbf{x}_q(t); Q_w)$$

Signal determined by weighting potential

Nonrelativistic limit

This is a fully general, electrodynamic, signal theorem …

… it contains the Ramo-Shockley theorem
(and its electrostatic extensions) as **special cases**

Detector consisting of N grounded electrodes

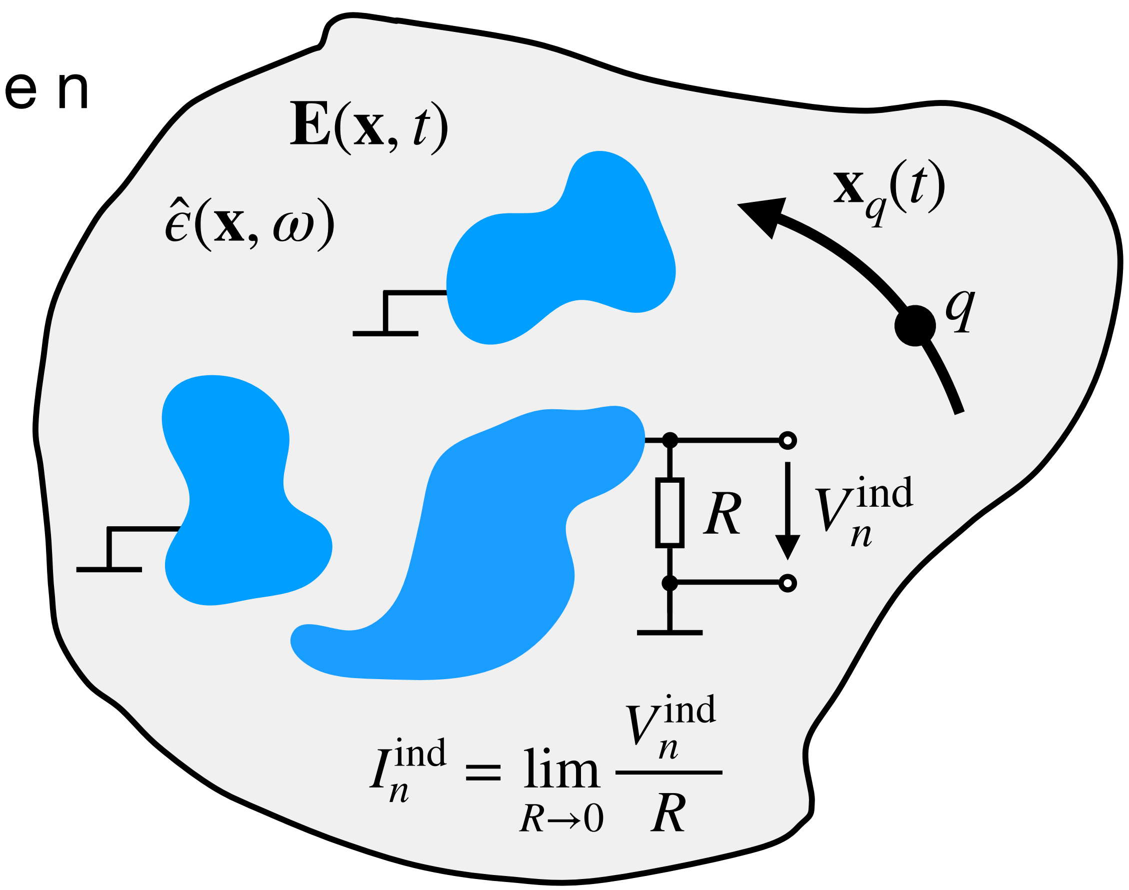
Electrostatic limit: charge moves nonrelativistically, no wave propagation

Small resistance R inserted in path from electrode n to ground to "measure" induced current

Recover the original Ramo-Shockley theorem:

$$I_n^{\mathrm{ind}}(t) = -\frac{q}{V_w}\mathbf{E}_n(\mathbf{x}_q(t); V_w) \cdot \dot{\mathbf{x}}_q(t)$$

(Known) non-relativistic theorems for other scenarios *(space charge, conductive media, …)* **equally emerge as special cases**



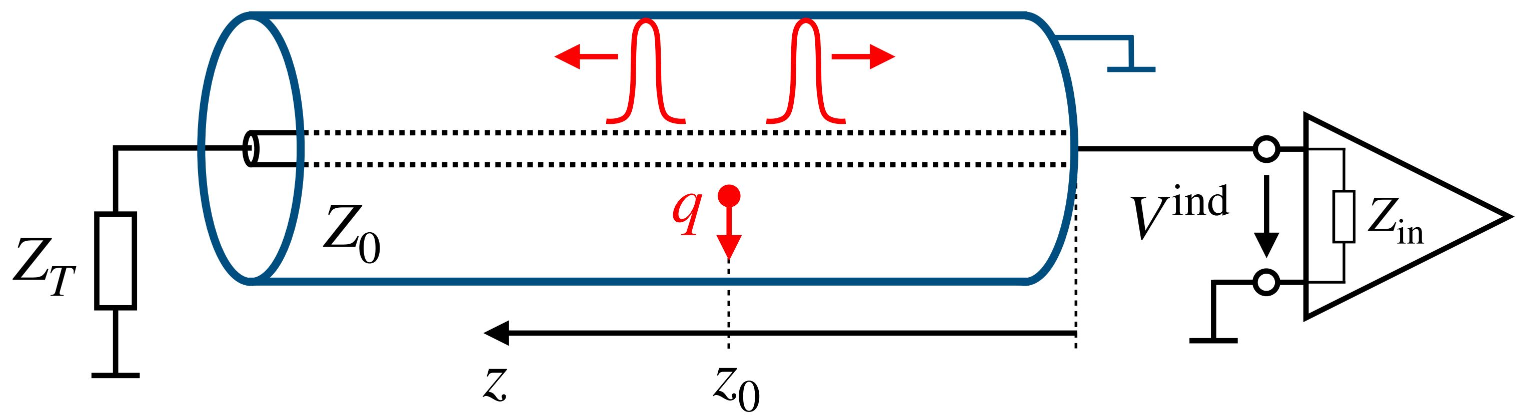
Drift tubes and transmission lines

General theorem allows coherent modelling of detectors where the signal includes electro<u>static</u> and electro<u>dynamic</u> components

Example: long drift tube with transmission line character

Slowly (radially) drifting charge induces signal at z = z_0

Signal propagates along transmission line



Electrodynamic weighting field can be computed analytically
(easy for the TEM approximation)

Calculation shows that induced signal is:

$$\boxed{V^{\text{ind}}(t) = \frac{1}{Q_w} \int dt'\, V(z_0, t - t') I^{\text{ind}}(z_0, t')}$$

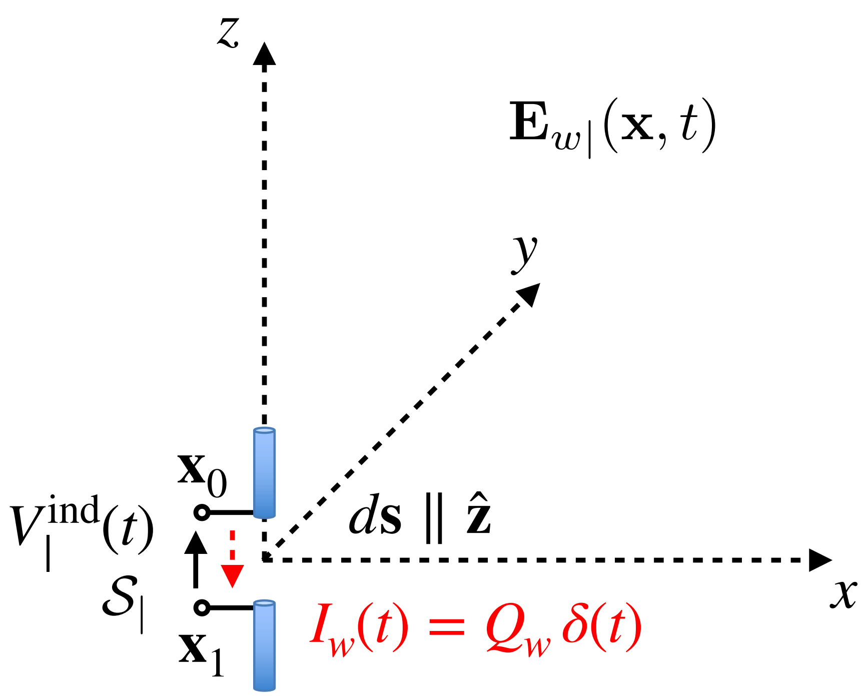
Propagating waves along drift tube
(Green's function of transmission line eq.)

Current induced locally at z_0 (computed with original Ramo-Shockly theorem)

Proves that signal induction and signal transmission <u>factorise</u>!

Dipole antennas as particle detectors

Very simple electrodynamic particle detectors:
electric and magnetic dipole antennas



Infinitesimal electric dipole
Induced signal measures local electric field

Weighting field: *outward propagating shock front with dipolar characteristic*

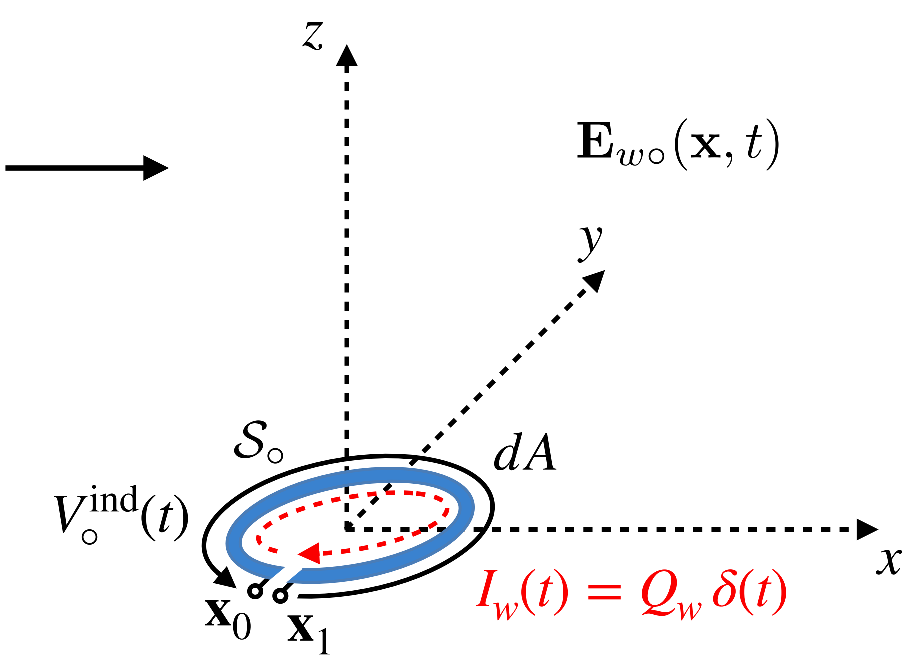
$$E^\theta_{w|}(r,\theta) = -\frac{Q_w ds}{4\pi\epsilon}\frac{\sin\theta}{r^3}\left[\Theta\left(t - \frac{r\,n}{c}\right) + \frac{r\,n}{c}\delta\left(t - \frac{r\,n}{c}\right) + \left(\frac{r\,n}{c}\right)^2\delta'\left(t - \frac{r\,n}{c}\right)\right]$$
$$E^r_{w|}(r,\theta) = -2\frac{Q_w ds}{4\pi\epsilon}\frac{\cos\theta}{r^3}\left[\Theta\left(t - \frac{r\,n}{c}\right) + \frac{r\,n}{c}\delta\left(t - \frac{r\,n}{c}\right)\right],$$

(in spherical coordinates)

Infinitesimal magnetic dipole
Induced signal measures local flux change

Weighting field also known analytically, similar character:

$$E^\phi_{w\circ}(r,\theta) = \frac{Q_w\,dA\,\mu}{4\pi}\frac{\sin\theta}{r^2}\left[\delta'\left(t - \frac{r\,n}{c}\right) + \frac{r\,n}{c}\delta''\left(t - \frac{r\,n}{c}\right)\right]$$



The radio signature of cosmic ray showers

Large-scale antenna arrays important in cosmic ray physics:

Ultra high energy cosmic rays (charged particles, neutrinos) induce extended showers in the atmosphere, or in ice

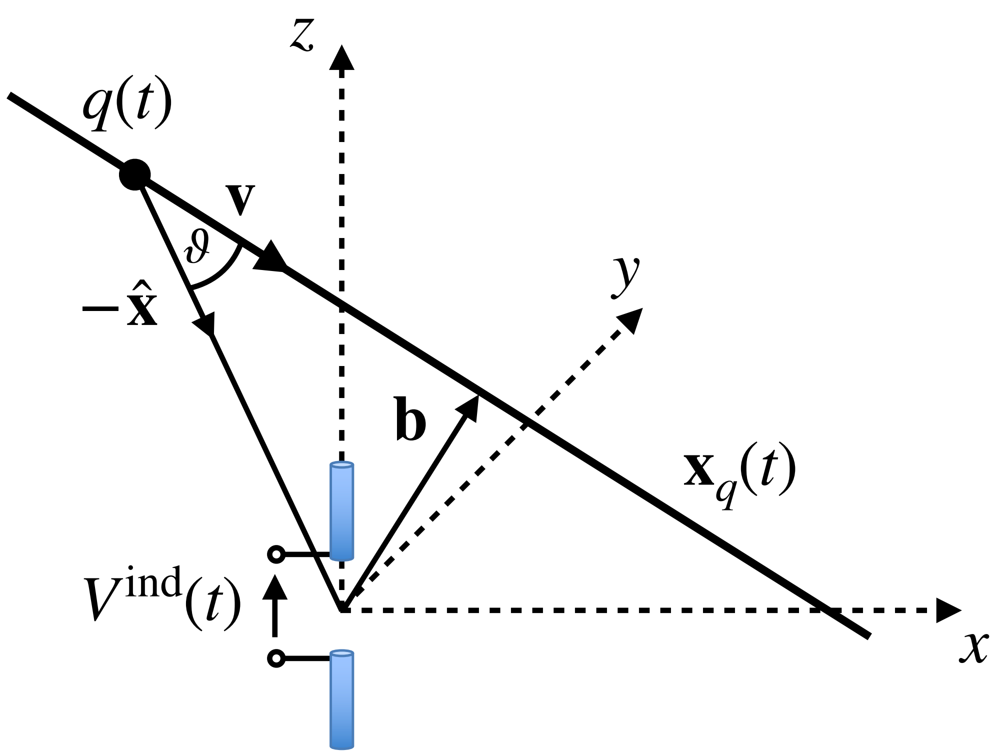
Radio emissions from shower detectable at surface level

Shower front develops negative excess charge:

(Annihilation of positive charges + scattering of electrons from the surrounding medium)

→ **Askaryan radiation**

Simple model for situation: time-dependent point charge, observed by dipole antenna →

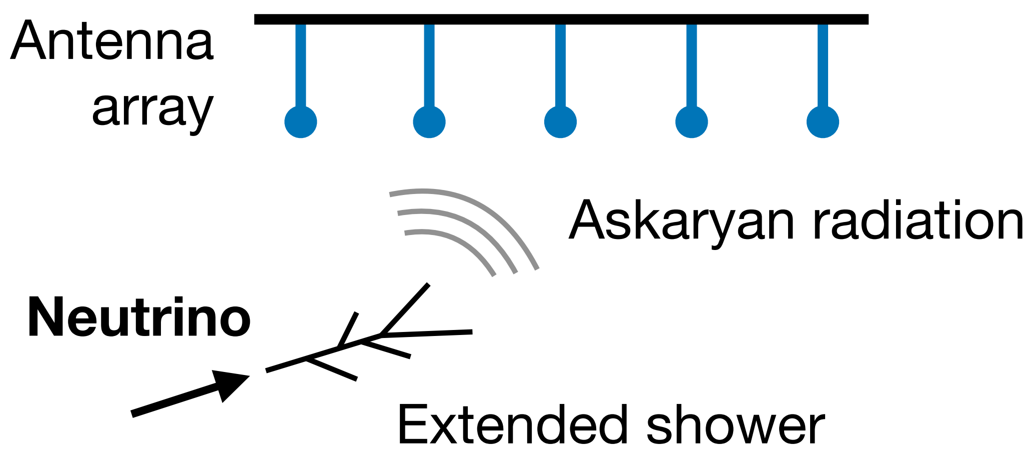


$$
\begin{aligned}
V_{|}^{\text{ind}}(t) &= V_{\text{Front}}^{\text{ind}}(t) + V_{\text{Ions}}^{\text{ind}}(t) + V_{\text{Askaryan}}^{\text{ind}}(t) = \\
&= -\frac{ds}{4\pi\epsilon}\left[\frac{q(t)}{|1-n\beta\cos\vartheta|^3}\frac{1}{r_q^2}\left(1-n^2\beta^2\right)\left(\frac{z_q}{r_q}+n\beta_z\right)\right]_{t_{\text{ret}}} \\
&\quad + \frac{ds}{4\pi\epsilon}\int_{-\infty}^{t_{\text{ret}}} dt'\,\dot{q}(t')\frac{z_q}{r_q^3} \\
&\quad - \frac{ds}{4\pi\epsilon}\left[\frac{\dot{q}(t)}{|1-n\beta\cos\vartheta|(1-n\beta\cos\vartheta)}\frac{n}{r_q c}\left(n\beta_z + n\beta\cos\vartheta\frac{z_q}{r_q}\right)\right]_{t_{\text{ret}}}
\end{aligned}
$$

Analytic solution for induced signal contains **Coulomb field of shower front**, **shower ion tail**, and **Askaryan radiation**

Radio neutrino observatories

Antenna arrays generate huge instrumented volumes in the arctic / antarctic ice
(several km^3, much larger than IceCube)



Complicated modelling of detector signal:

Radiation propagation through (inhomogeneous) ice, reflections off the ice surface, …

Important physics background: downgoing muons impact ice from above

Directly computing the signal by numerically solving Maxwell's equations for the shower is intractable!

Showers can arrive in any orientation w.r.t. array, no symmetries to exploit; need cm^3 grid over km^3 volume

But: weighting field of the antenna / array is approximately cylindrically symmetric!

Realistic weighting fields are tractable numerically

Currently being implemented in NuRadioMC [ref], enabling practical first-principles radio modelling for the first time!

Summary

Lorentz reciprocity gives rise to a fully general signal theorem, applicable to all devices detecting fields and radiation from charged particles

$$\boxed{V^{\mathrm{ind}}(t) = -\frac{q}{Q_w}\int_{-\infty}^{\infty} dt'\, \mathbf{E}_w(\mathbf{x}_q(t'), t - t') \cdot \dot{\mathbf{x}}_q(t')}$$

The Ramo-Shockley theorem (and related extensions) emerge in various quasi-electrostatic limits of the general result

Coherent modelling of devices where both electrostatic induction and electrodynamic effects determine the signal
(e.g. drift tubes and transmission lines)

Important applications in cosmic ray physics:
enables high-fidelity simulations of radio emissions of cosmic-ray showers, not possible with present techniques

References

Publication with more details, derivations, and examples:

Contents lists available at ScienceDirect

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Signals induced on electrodes by moving charges, a general theorem for Maxwell's equations based on Lorentz-reciprocity

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