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Optimization of Detector Modules for Measuring Gamma-ray Polarization in Positron Emission Tomography

Siddharth PARASHARI[*,‡], M. MAKEK[*], D. BOSNAR[*], T. BOKULIĆ[*], A. M. KOŽULJEVIĆ[*], Zdenka KUNCIC[†], and P. ŽUGEC[*]

[*]Department of Physics, Faculty of Science, University of Zagreb
[†]School of Physics, University of Sydney

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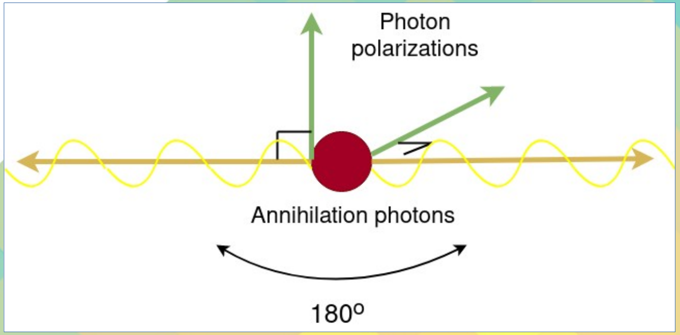
[‡]siddharth@phy.hr

Introduction

- Positron annihilation may result in two entangled and orthogonally polarized gamma photons.
- It is found that the polarization correlations can be utilized as an additional handle to improve Signal to Noise Ratio (SNR) which may improve medical imaging with Positron Emission Tomography (PET).
  [*Phys. Med. Bio. 59 (2014) 7587, Phys. Med. Bio. 61 (2016) 5803, Nat. Commun. 12 (2021) 2646*]



Photon polarization in positron annihilation event

Question ?

How to measure gamma-ray polarization in PET cost effectively ? → Compton scattering

The gamma polarization is related to the azimuthal angle in the Compton scattering process, so the initial correlation of polarization translates to the correlation of azimuthal angles in true coincidence events, which is not present in the background.

How to detect recoil electron and scattered photon efficiently ? → Single layer detectors

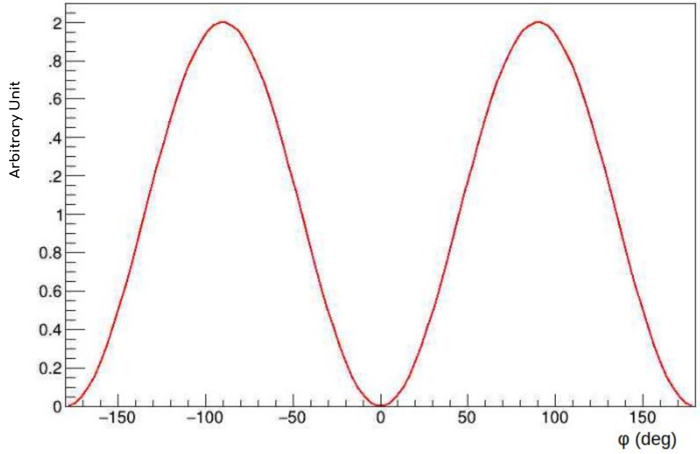
- Recoil e^- and scattered gamma in the same layer
- Scalability to large systems
- Proof of concept using 4x4 LFS crystals 3x3x20 mm^3 [*Nucl. Instr. Meth. A 958 (2020) 162835*]

Gamma polarization measurement via Compton scattering

Klein-Nishina differential cross-section for scattering of linearly polarized gamma photon

$$\frac{d\sigma}{d\Omega}=\frac{1}{2}r_0^2\left(\frac{k'}{k_0}\right)^2\left[\frac{k_0}{k'}+\frac{k'}{k_0}-2\sin^2\theta\cos^2\phi\right]$$

where θ is the scattering angle and φ is the angle between the scattering plane $(\vec{k}_0, \vec{k}')$ and the polarization vector



- Gamma is most likely to be scattered at azimuthal angle φ perpendicular to the polarization vector (cos φ = 0)
- The sensitivity to polarization is the largest for scattering at θ = 90°
- Polarization is correlated to the azimuthal scattering angle φ.

Polarization correlations in paired Compton events

The cross-section for scattering of two linearly polarized γ-particles is given by

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_1\mathrm{d}\Omega_2}=\frac{r_0^4}{16}F(\theta_1)F(\theta_2)\left\{1-\frac{G(\theta_1)G(\theta_2)}{F(\theta_1)F(\theta_2)}\cos[2(\phi_1-\phi_2)]\right\}$$

The cross-section has maxima when $|\phi_1-\phi_2|=90^\circ$; keeping θ fixed.

The Polarimetric Modulation Factor μ is defined as,

$$\mu\equiv\frac{P(\phi_1-\phi_2=90^\circ)-P(\phi_1-\phi_2=0^\circ)}{P(\phi_1-\phi_2=90^\circ)+P(\phi_1-\phi_2=0^\circ)}$$

μ reaches maximum $\mu = 0.48$ for $\theta_1 = \theta_2 \approx 82^o$

Motivation

Modern PET detectors : → Highly segmented, large coverage

Most common setups to detect azimuthal polarization : → Detectors with 2 senisitive layers – 1st for measuring the recoil electron, 2nd for the scattered gamma

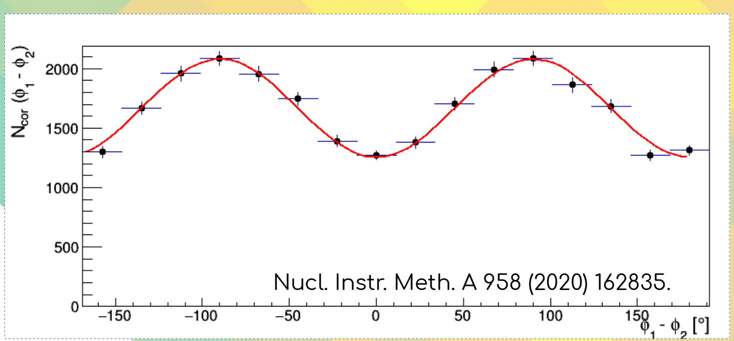
Drawbacks

A PET system based on 2 (or more)-layer detectors would dramaticaly increase the cost of the apparatus

- How to detect recoil electron and scattered photon efficiently ? → **Single layer detectors**

- Investigate the feasibility of the measurement of polarization correlations using single-layer detectors

Proof of concept using 4x4 LFS crystals 3x3x20 mm^3 →



Nucl. Instr. Meth. A 958 (2020) 162835.

→ **Energy resolution of detectors determines θ precision,**
Segmentation and material determine φ resolution and acceptance ←

Sensitivity of a detector system can be improved by :

1. Improving energy resolution — Choosing suitable detector material
2. Improving azimuthal resolution — Small pixel dimensions in a detector

Comparison of different setups for measurements of gamma polarization correlations

- The setup consists of a pair of modules, each containing 64 crystals in 8x8 configuration, polished on all sides and enclosed within a reflector. A schematic diagram of GaGG pixel detector of pitch 3.2 mm is shown in Fig 1 .
- The detector with different pixel sizes ranging from 1.9 mm to 3.0 mm were used. The details of respective detectors are provided in Table 1.
- The crystal matrices are read out by one-to-one matched silicon photo-multiplier (SiPM) arrays and processed by the TOFPET2 readout system.
- A Na-22 source was kept in between two modules of same type to measure the coincidence events.

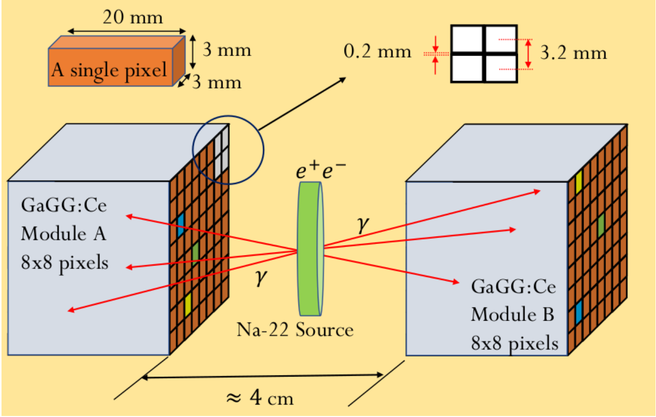


Fig. 1. Schematic diagram of experimental setup – example for GaGG:Ce 8x8 matrix

Table. 1. List of detector modules used with their respective properties

Setup	Array No.	Crystal Material	Pixel Dimensions †	Array Type	Gluing Agent	Pitch (mm)	Mean Resolution at 511 keV (%)
GaGG 2.9	1	GaGG:Ce	2.9 x 2.9 x 20	64 pixels (8x8 Matrix)	Optical Glue	3.2	8.3 ± 0.4
	2						8.4 ± 0.5
GaGG 2.9	3						8.4 ± 0.5
	4						8.7 ± 0.6
GaGG 2.9	5						9.0 ± 0.8
	6						8.6 ± 0.6
LYSO 2.0	7	LYSO:Ce	2 x 2 x 20		Optical Glue	2.2	13.8 ± 1.0
	8						13.7 ± 1.1
GaGG 3.0	9	GaGG:Ce	3 x 3 x 20		Optical Glue	3.2	9.7 ± 0.8
	10				Silicon Pad		11.2 ± 0.9
LYSO 1.9	11	LYSO:Ce	1.9 x 1.9 x 20		Optical Glue	2.2	14.7 ± 1.1
	12						15 ± 1.3
GaGG 1.9	13	GaGG:Ce	1.9 x 1.9 x 20		Optical Glue		8 ± 0.4
	14						8.1 ± 0.7

† **All pixel dimesions are in mm.**

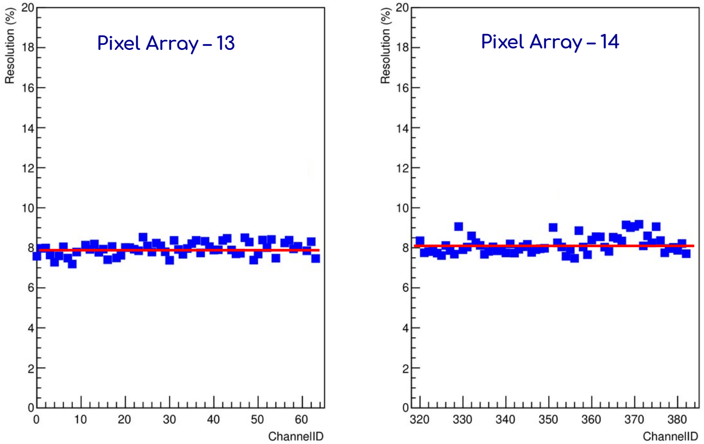


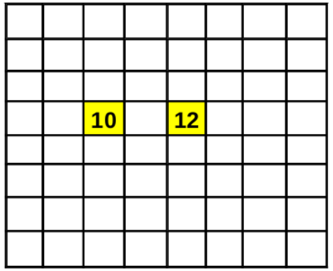
Fig. 2. Individual pixel resolution (%) at 511 keV in the GaGG 1.9 mm detector modules.

Data analysis

Selection of Compton Events –

The Compton events in each module are selected requiring that exactly two pixels fire [Fig. 3(a)], that the energy deposited in the module is within 511 keV±3σ [Fig. 3(b)] and that pixel energies correspond to Compton kinematics, [Fig. (c), (d)].

For ex. Pixels 10 & 12 fired in a Compton Event



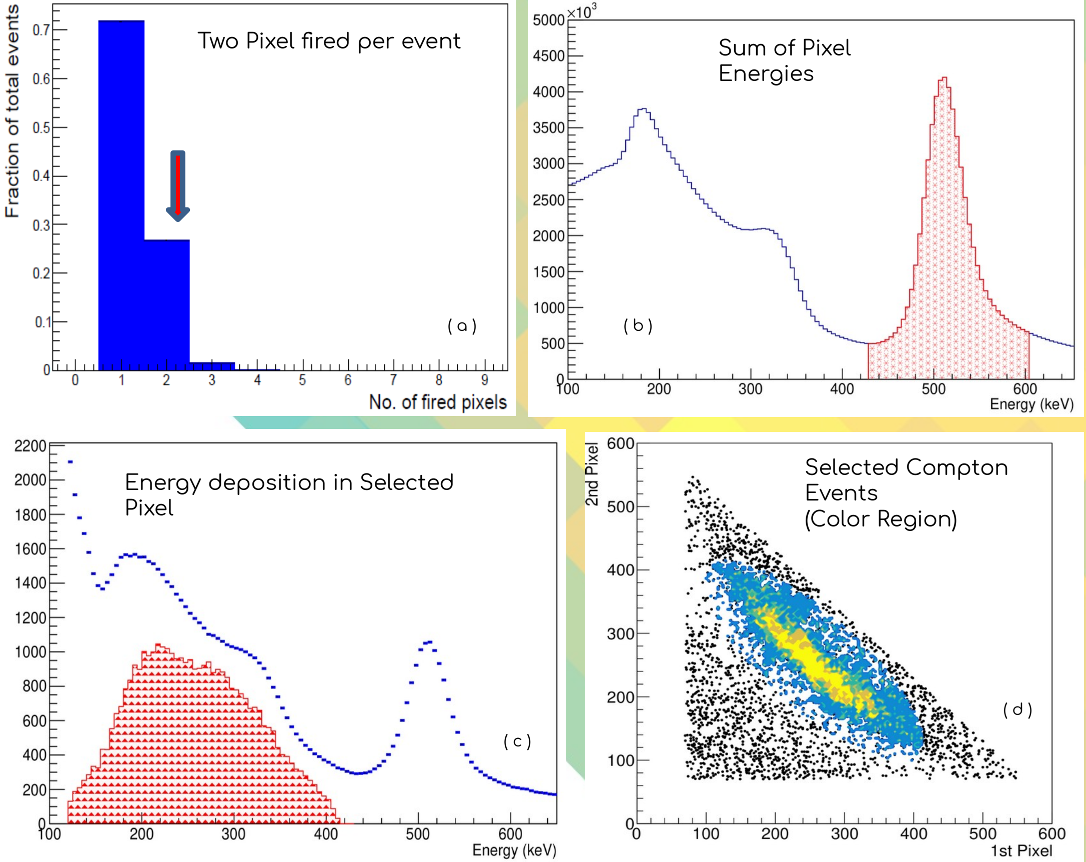


Fig. 3. (a) Number of pixel fired in all events; (b) sum of pixel energies in Compton events; (c) energy deposited in an individual pixel fired in Compton event; (d) selected Compton events

Data analysis

Reconstruction of Compton Scattering Angles –

The Compton scattering angle $\theta = \operatorname{acos}\left[m_e c^2\left(\frac{1}{E_e+E_{\gamma'}} - \frac{1}{E_{\gamma'}}\right) - 1\right]$

We always assume forward scattering, $E_e' < E_\gamma'$ so that the pixel with lower energy deposit corresponds to recoil electron detection – simulation shows this is correct in 55-60% of the cases, depending on inter-pixel distance.

The angle ϕ is determined by the positions of the fired pixels as, $\tan\phi = \frac{y_{\gamma'} - y_e}{x_{\gamma'} - x_e}$;

$(x_e, y_e) \longrightarrow$ pixel coordinates where recoil electron is detected

$(x_{\gamma'}, y_{\gamma'}) \longrightarrow$ pixel coordinates where scattered gamma is detected

Detector Acceptance –

- The detector azimuthal acceptance is not uniform.
- The scattered gamma photons are more attenuated for the φ angles covered by pixel pairs with a large inter-pixel distance, d.
- The acceptance-corrected φ distribution is obtained as $\varphi = \varphi_{measured}/\varphi_{norm}$, where the φ_{norm} is the distribution of all triggered Compton events obtained in a high-statistics run.
- The acceptance-corrected φ distribution for coincident Compton events is shown in Fig. 4(b).

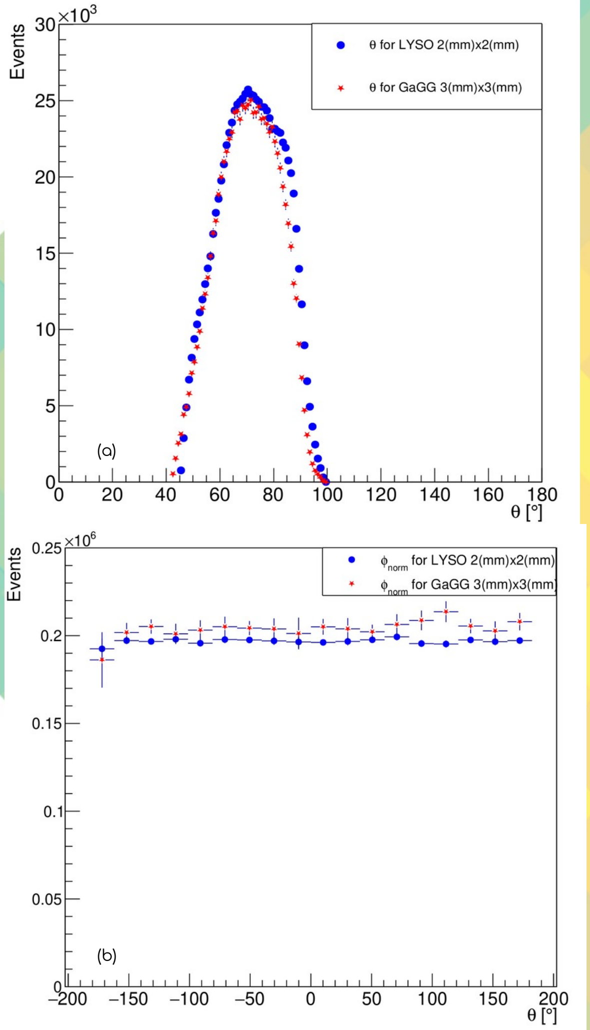


Fig. 4. Reconstructed (a) θ and (b) ϕ (acceptance corrected), for both GaGG 3.0 mm and LYSO 2.0 mm detectors.

Results – polarization correlations of gamma-ray from positron annihilation

- $\phi_1 - \phi_2$ **Distributions :** For various inter-pixel distances d
  For different angular range in $\theta_{1,2}$

- **Modulation factor μ extracted by fit :** $N_{cor}(\phi_1 - \phi_2) = M\left[1 - \mu\cos 2(\phi_1 - \phi_2)\right]$

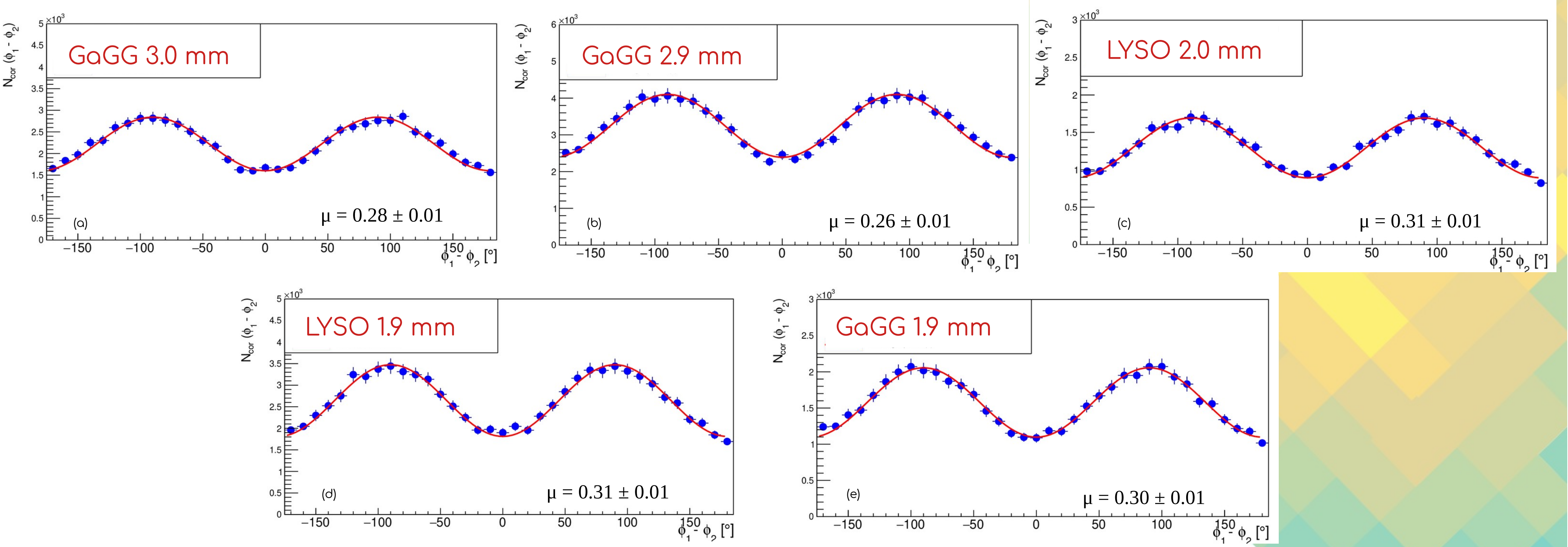


Fig 5. Observed azimuthal difference distributions for (a) GaGG_3.0mm (b) GaGG_2.9mm (c) LYSO_2.0mm (d) LYSo_1.9mm & (e) GAGG_1.9mm for 72° < $\theta_{1,2}$ < 90°

Comparison of modulation factors in detector modules at different scattering angles (θ±dθ)

- The dependence of the modulation factor, μ, at different $\theta_{1,2}$ ranges is explored in each detector configuration.
- The obtained modulation factors, μ for **72° < $\theta_{1,2}$ < 92°** and **60° < $\theta_{1,2}$ < 80°** at inter-pixel distance **d > 4.5 (mm)** is shown for all modules.
- Larger modulation are observed for **72° < $\theta_{1,2}$ < 92°** as compared to 60° < $\theta_{1,2}$ **< 80°** angular range, which is expected from the theory

  [*Nature, vol. 160, Sep. 1947*]

Table. 1. Modulation factor μ from measurements in Detector Modules for 72° < $\theta_{1,2}$ < 92° at inter-pixel distance d > 4.5 (mm).

Detector Module	Azimuthal Resolution <Δφ>°	Modulation (μ)
GaGG 3.0 mm	18.8	0.28 ± 0.01
GaGG 2.9 mm	18.2	0.26 ± 0.01
LYSO 2.0 mm	16.7	0.31 ± 0.01
LYSO 1.9 mm	15.8	0.31 ± 0.01
GaGG 1.9 mm	15.3	0.30 ± 0.01

Table. 2. Modulation factor μ from measurements in Detector Modules for 60° < $\theta_{1,2}$ < 80° at inter-pixel distance d > 4.5 (mm).

Detector Module	Azimuthal Resolution <Δφ>°	Modulation (μ)
GaGG 3.0 mm	19.2	0.17 ± 0.01
GaGG 2.9 mm	18.9	0.13 ± 0.01
LYSO 2.0 mm	17.3	0.23 ± 0.01
LYSO 1.9 mm	16.2	0.25 ± 0.01
GaGG 1.9 mm	15.8	0.17 ± 0.01

**Note :- The measured modulation factors are sensitive to the statistics and may change within ± 2%.

Continued :

- The obtained modulation factors, μ are compared for angular ranges **$72° < \theta_{1,2} < 92°$**, **$77° < \theta_{1,2} < 87°$** and **$80° < \theta_{1,2} < 84°$** at inter-pixel distance **d > 4.5 (mm)** for all modules.

- Better Modulations are achieved within a narrower angular range **$80° < \theta_{1,2} < 84°$** i.e. more closer to the maxima condition $|\phi_1 - \phi_2| = 90°$.

Table. 3. Modulation factor μ from measurements in Detector Modules for $77° < \theta_{1,2} < 87°$ at inter-pixel distance d > 4.5 (mm).

Detector Module	Azimuthal Resolution <Δφ>°	Modulation (μ)
GaGG 3.0 mm	18.9	0.29 ± 0.01
GaGG 2.9 mm	18.2	0.29 ± 0.01
LYSO 2.0 mm	16.9	0.33 ± 0.01
LYSO 1.9 mm	15.8	0.32 ± 0.01
GaGG 1.9 mm	15.3	0.34 ± 0.01

Table. 4. Modulation factor μ from measurements in Detector Modules for $80° < \theta_{1,2} < 84°$ at inter-pixel distance d > 4.5 (mm).

Detector Module	Azimuthal Resolution <Δφ>°	Modulation (μ)
GaGG 3.0 mm	18.9	0.29 ± 0.02
GaGG 2.9 mm	18.3	0.29 ± 0.02
LYSO 2.0 mm	16.9	0.34 ± 0.02
LYSO 1.9 mm	15.8	0.33 ± 0.01
GaGG 1.9 mm	15.3	0.34 ± 0.02

**Note :- The measured modulation factors are sensitive to the statistics and may change within ± 2%.

Modulation factors in LYSO 1.9 mm at different scattering angles

The dependence of the modulation factor, μ, on the azimuthal resolution **<Δφ>°** at different $\theta_{1,2}$ is explored in each detector configuration. An examples of obtained Modulation factors, μ with azimuthal resolution **<Δφ>°** for **LYSO 1.9 mm detector** is shown in figure.

Observations –

- Larger modulation factors are observed for angular ranges closer to 82° in comparison to 70°.
- Rising modulation amplitude is observed with lower Azimuthal Resolution <Δφ>° or larger inter-pixel distances d(mm).

- **We have achieved a well pronounced polarimetric performance of finer segmented detector modules than observed previously** [*Nucl. Instrum. Methods Phys. Res. A, vol. 958, Apr. 2020*].

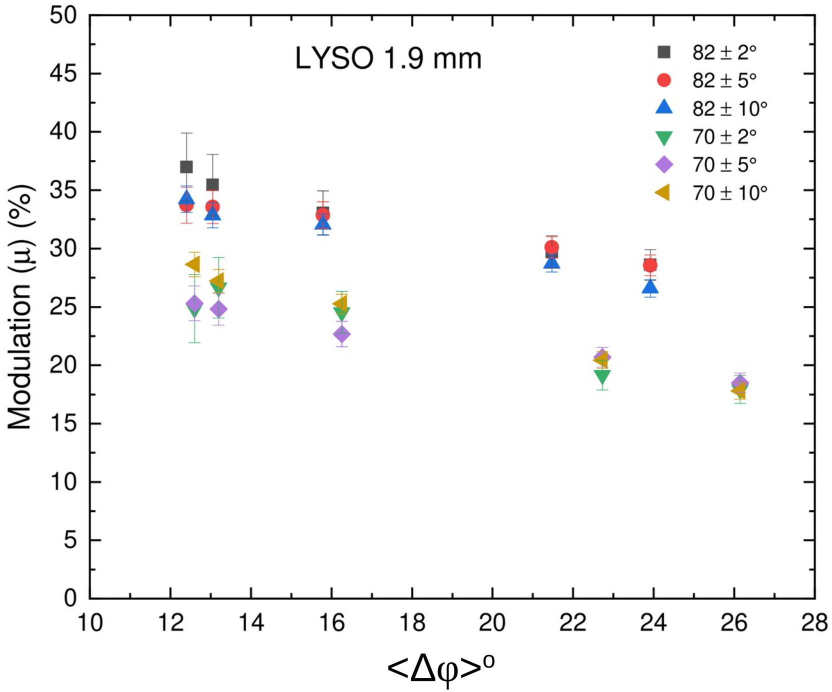


Fig. 6. Modulation (%) vs azimuthal resolution <Δφ>° at different angular ranges for LYSO 1.9 mm detector.

Comparison of μ vs $\Delta\varphi$ among modules at different scattering angles

We compared the polarimetric performance **μ** of individual detectors with azimuthal resolution <**Δφ>°** at different angular ranges.

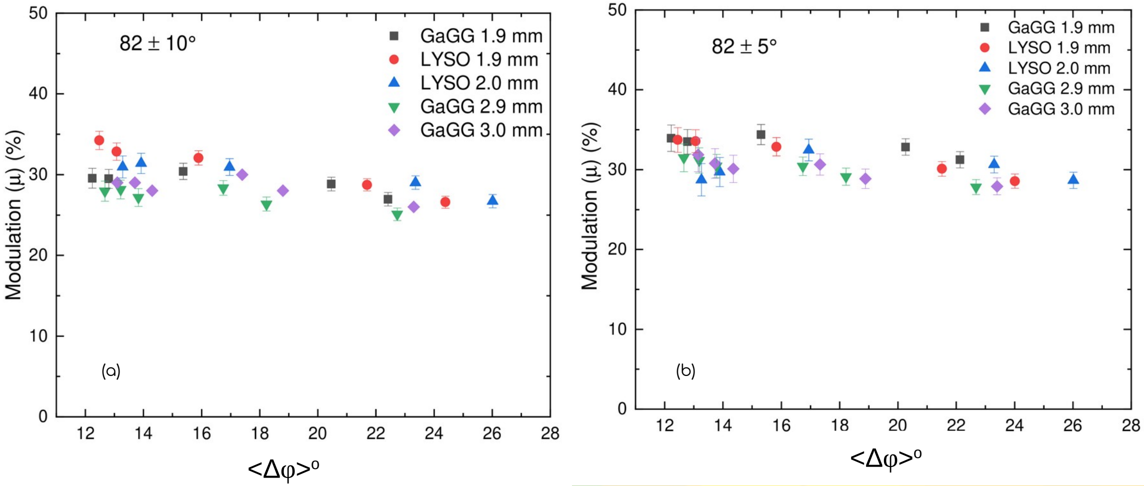


Fig. 7. Comparison of modulation (%) vs azimuthal resolution <Δφ>° for all detectors at angular ranges (a) **72° < $\theta_{1,2}$ < 92° and (b) 77° < $\theta_{1,2}$ < 87°**

Observations –

- Modulation factors increases further for 77° < $\theta_{1,2}$ < 87° as we select events closer to scattering angle 82°.

Summary

- Angular correlation of annihilation quanta were successfully measured with the single-layer pixelated scintillation detectors.
- Polarimetric performance of different detector modules from 1.9 – 3.0 mm pixel sizes was studied successfully.
- All detector setups exhibit good performance, however, higher modulation was obtained with finely segmented pixel detectors.
- The single-layer concept offers cost-efficient scalability to larger systems.

Future goal –

It has been successfully demonstrated that finer segmented pixel modules can be used to measure polarization correlations in annihilation quanta. With this motivation, the detector modules can now be tested with phantoms for image reconstructions to obtain realistic estimates of SNR taking advantage of azimuthal correlations.

Acknowledgments

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CONTACT:

makek@phy.hr, **siddharth@phy.hr**