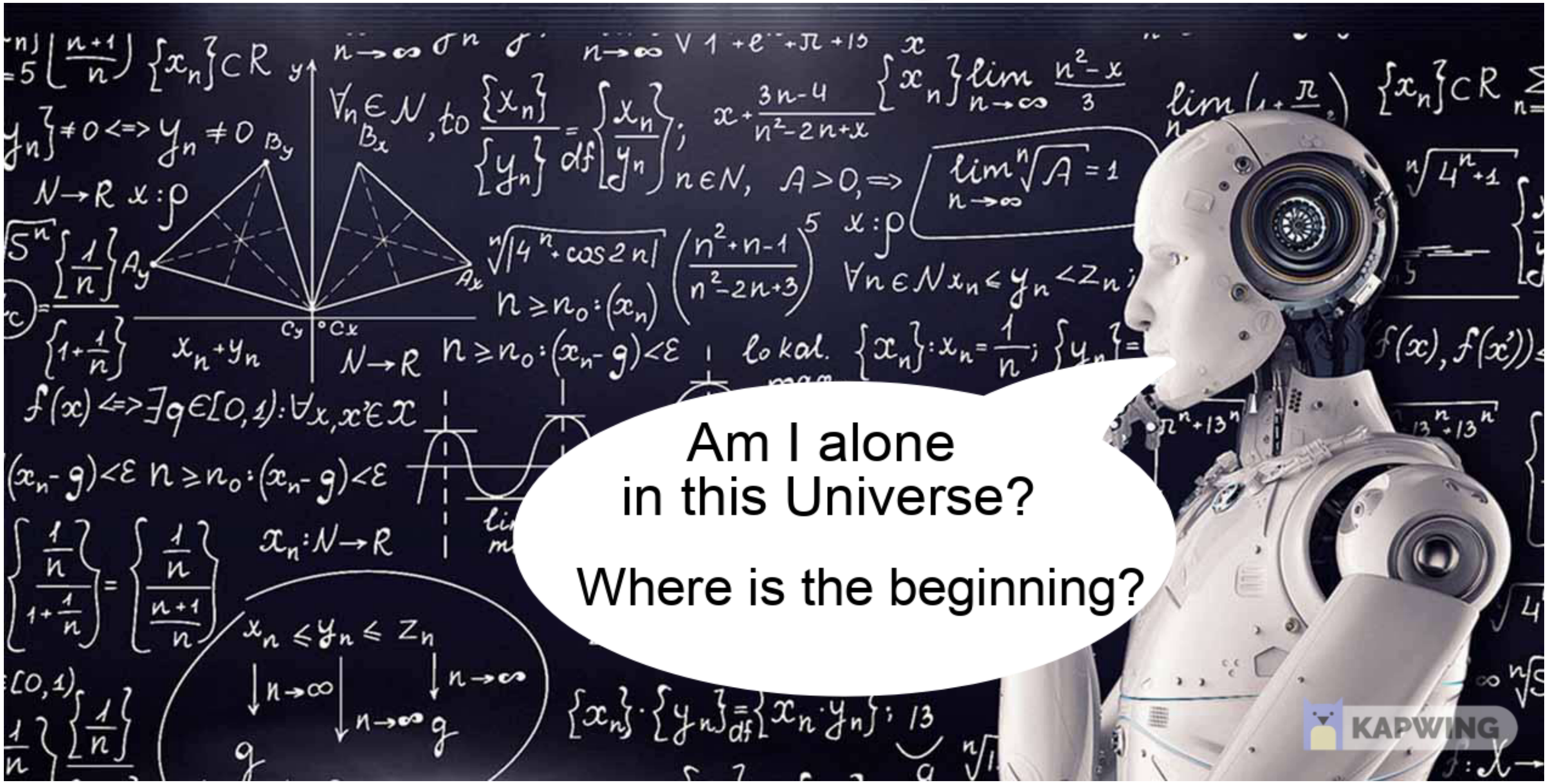
Cosmology ensuing from Machine Learning



Martiros Khurshudyan

Institute of Space Sciences (ICE-CSIC)
Barcelona, Spain

Agenda

A very brief review of Cosmology

What Machine Learning is?

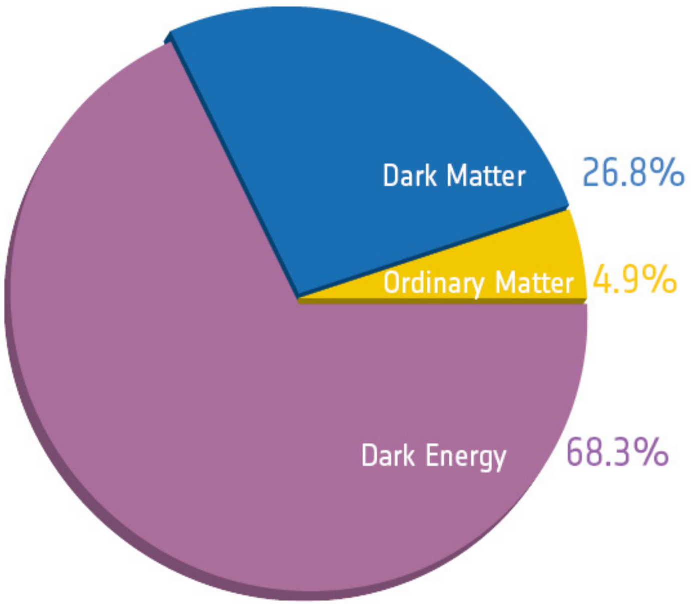
Machine Learning in Cosmology (Physics)

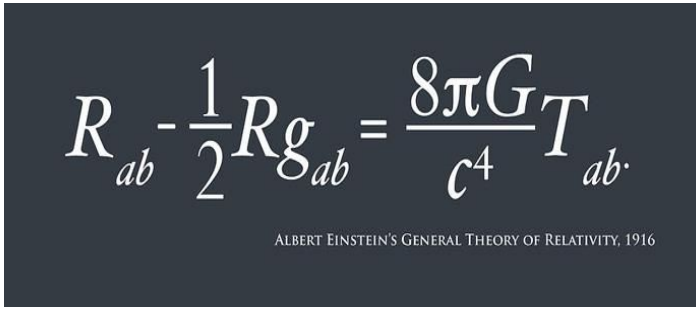
- Gaussian Processes in Cosmology
  - Model independent reconstruction of f(T) gravity
- Bayesian Machine Learning in Cosmology
  - Deviation from the cold dark matter paradigm

Conclusion

We and Universe
A very brief review of Cosmology

Large scale universe





Dark energy has enough negative pressure to work against gravity (accelerated expansion).

Models of dark energy – cosmological constant(the first dark energy model), varying cosmological constant models, quintessence, phantom, k-essence (scalar field representations), ghost dark energy, holographic dark energy (the energy density is parametrised), Chaplygin gas (dark energy and dark matter joint model with a non-linear EoS)…

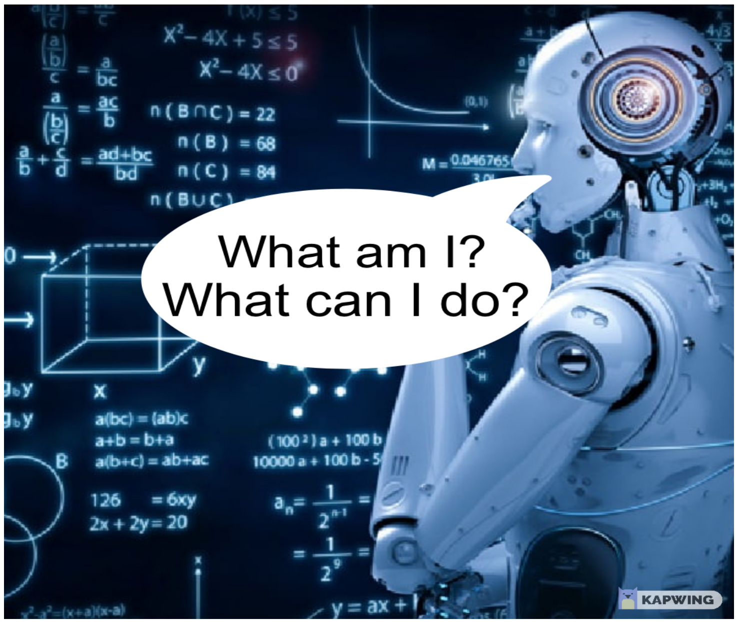
Alternative approach

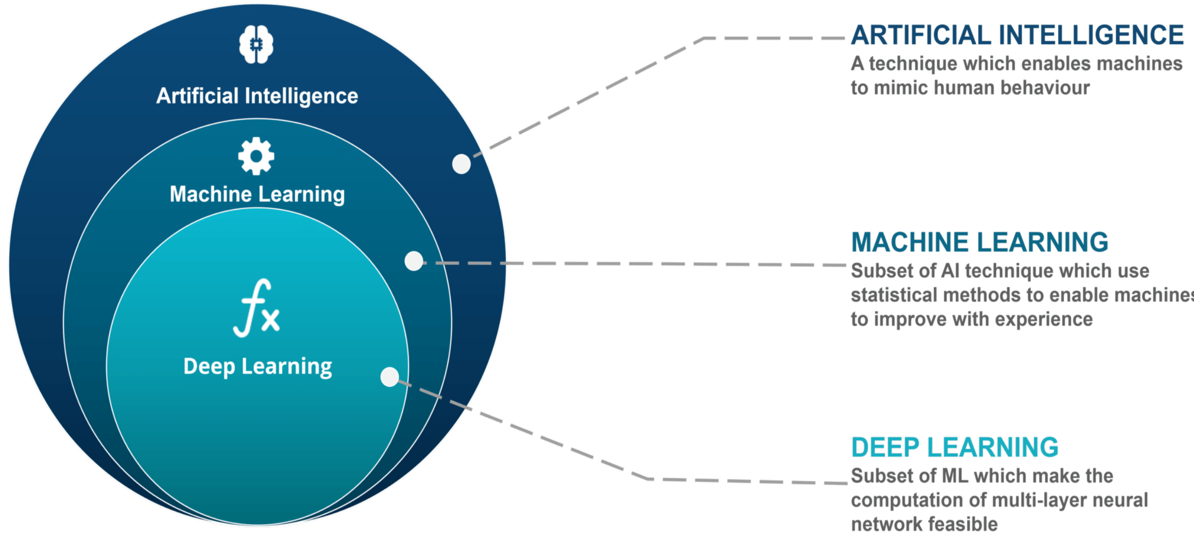
a modification of gravity

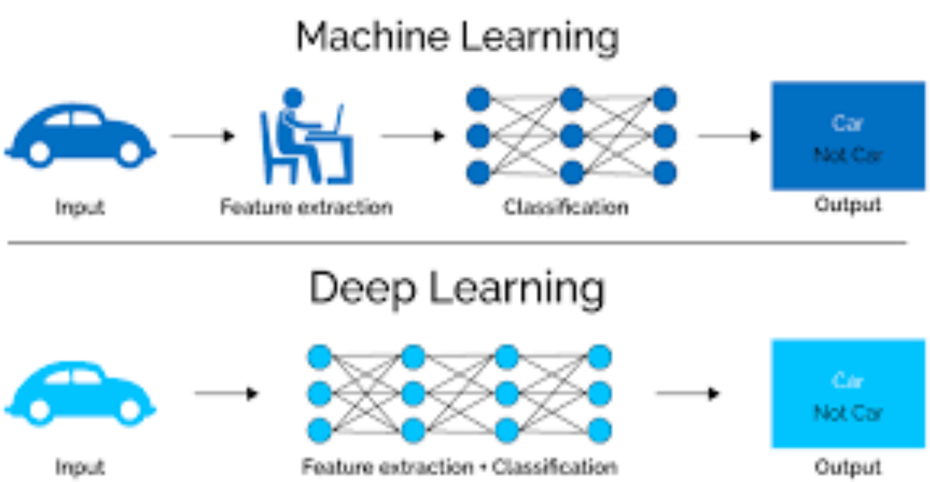
The accelerated expansion is model independent result.

It is an easy task to show this using H(z) data and Gaussian Processes

What Machine Learning is?







1. *A lot of data and computational resources are needed.*
2. *A bias in the data can cause a lot of problems (among other issues).*
3. *Physics can provide unique data for ML and can help better understand it.*
4. *ML can help better understand Physics*

Gaussian Processes in Cosmology

The Gaussian distribution presents a distribution of a random variable characterized by a mean and a covariance.

GP should be understood as a distribution over functions, characterized by a mean function and a covariance matrix.

z	$H(z)$	σ_H	z	$H(z)$	σ_H
0.070	69	19.6	0.4783	80.9	9
0.090	69	12	0.480	97	62
0.120	68.6	26.2	0.593	104	13
0.170	83	8	0.680	92	8
0.179	75	4	0.781	105	12
0.199	75	5	0.875	125	17
0.200	72.9	29.6	0.880	90	40
0.270	77	14	0.900	117	23
0.280	88.8	36.6	1.037	154	20
0.352	83	14	1.300	168	17
0.3802	83	13.5	1.363	160	33.6
0.400	95	17	1.4307	177	18
0.4004	77	10.2	1.530	140	14
0.4247	87.1	11.1	1.750	202	40
0.44497	92.8	12.9	1.965	186.5	50.4
0.24	79.69	2.65	0.60	87.9	6.1
0.35	84.4	7	0.73	97.3	7.0
0.43	86.45	3.68	2.30	224	8
0.44	82.6	7.8	2.34	222	7
0.57	92.4	4.5	2.36	226	8

TABLE I: The $H(z)$ and its uncertainty σ_H are in the unit of km s^{-1} Mpc^{-1}.

The covariance function (kernel) which correlates the function H(z) at different points

$$k(z,\hat{z}) = \sigma_f^2 \exp\left[-\frac{|z-\hat{z}|^2}{2l^2}\right]$$

where σ_f and l are the parameters know as hyperparameters. These parameters represent the length scales in the GP. l parameter corresponds to the correlation length along which the successive $f(x)$ values are correlated, while to control the variation in $f(x)$ relative to the mean of the process we need σ_f parameter.

Remember this tool is good for a model-independent analysis.

GaPP code by Marina Seikel et al...

Marina Seikel, Chris Clarkson, Mathew Smith, JCAP 06 (2012), 036

Gaussian Processes in Cosmology

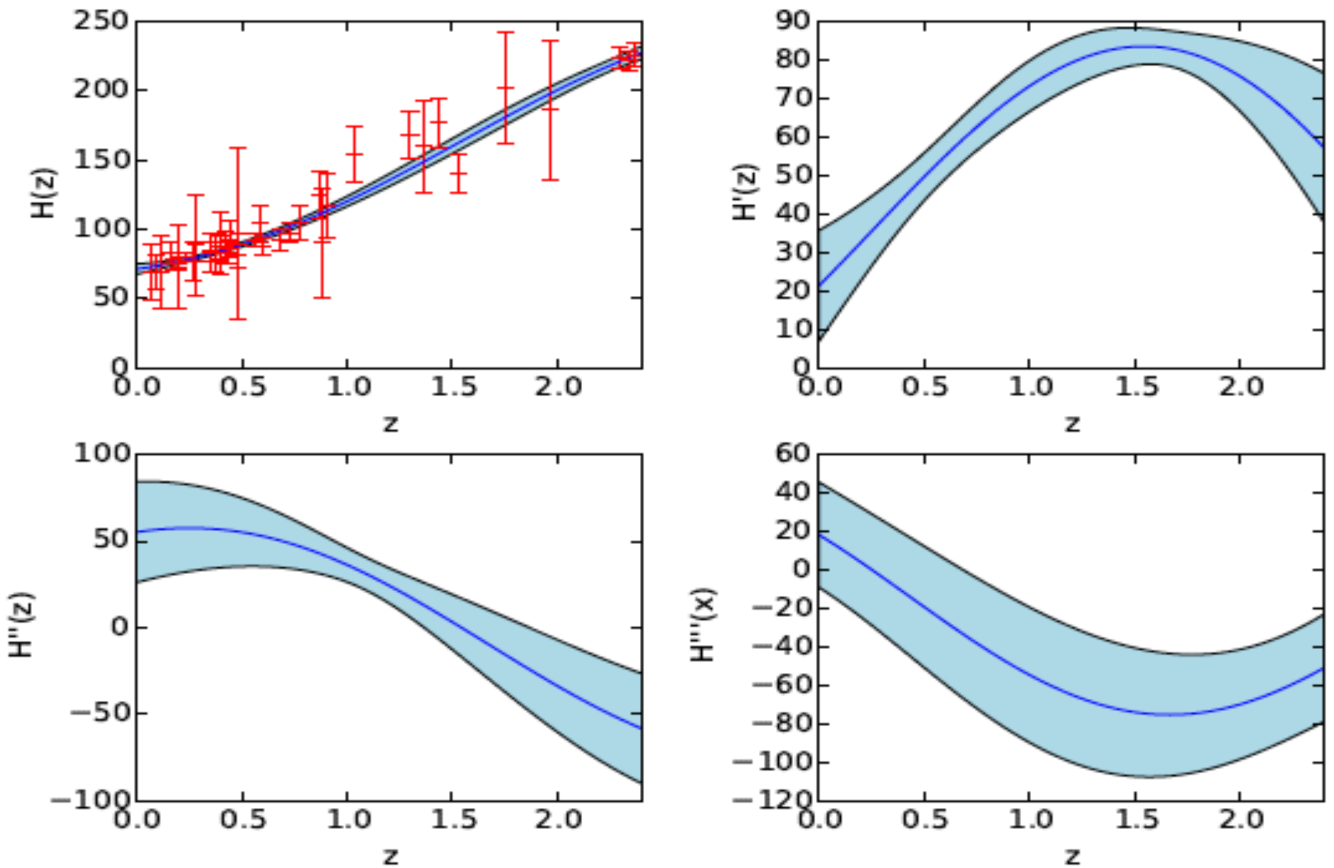


FIG. 2: GP reconstruction of $H(z)$, $H'(z)$, $H(z)''$, and $H(z)'''$ for the 30-point sample deduced from the differential age method, with the additional 10-point sample obtained from the radial BAO method. The $'$ means derivative with respect to the redshift z.

1. When we use GP we do not need to consider parameterized functions to represent data.

2. The GP allows to learn the form of the function which is in the best way represents given data

Gaussian Processes in Cosmology

f(T) Cosmology Reconstruction (torsional gravity)

$$S = \frac{1}{16\pi G}\int d^4x\, e[T + f(T) + L_m]$$

$$H^2 = \frac{8\pi G}{3}\rho_m - \frac{f}{6} + \frac{Tf_T}{3}$$

$$\dot{H} = -\frac{4\pi G(\rho_m + P_m)}{1 + f_T + 2Tf_{TT}}$$

with $H \equiv \dot{a}/a$ the Hubble function and where dots denoting derivatives with respect to t. Additionally, in the above equations ρ_m and P_m are respectively the energy density and pressure of the matter fluid.

The torsion scalar

$$T = -6H^2$$

As a next step we define an effective dark energy sector with energy density and pressure respectively given by

$$\rho_{\rm DE} \equiv \frac{3}{8\pi G}\left[-\frac{f}{6} + \frac{Tf_T}{3}\right]$$

$$P_{\rm DE} \equiv \frac{1}{16\pi G}\left[\frac{f - f_T T + 2T^2 f_{TT}}{1 + f_T + 2Tf_{TT}}\right]$$

$$w \equiv \frac{P_{\rm DE}}{\rho_{\rm DE}} = -\frac{f/T - f_T + 2Tf_{TT}}{[1 + f_T + 2Tf_{TT}][f/T - 2f_T]}$$

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_{\rm DE})$$

$$\dot{\rho}_m + 3H(\rho_m + P_m) = 0$$

$$\dot{\rho}_{\rm DE} + 3H(\rho_{\rm DE} + P_{\rm DE}) = 0$$

Yi-Fu Cai, M. Khurshudyan, E. N. Saridakis, Astrophys. J. 888 (2020) 62

Gaussian Processes in Cosmology

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$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_{\rm DE})$$

$$\rho_m = \frac{3}{8\pi G}H_0^2\Omega_{m0}(1+z)^3$$

$$\dot{\rho}_{\rm DE} + 3H(\rho_{\rm DE} + P_{\rm DE}) = 0$$

$$f_T \equiv \frac{df(T)}{dT} = \frac{df/dz}{dT/dz} = \frac{f'}{T'}$$

$$f'(z) \approx \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

$$f(z_{i+1}) - f(z_i) = 6(z_{i+1} - z_i)\frac{H'(z_i)}{H(z_i)}\left[H^2(z_i) - H_0^2\Omega_{m0}(1+z_i)^3 + \frac{f(z_i)}{6}\right]$$

The final result

(due to the reconstruction mean)

$$f(T) \approx -2\Lambda + \xi T^2$$

with $-2\Lambda = -6H_0^2(1 - \Omega_{m0})$

Note: We always can use GP to constrain cosmological models

Yi-Fu Cai, M. Khurshudyan, E. N. Saridakis, Astrophys. J. 888 (2020) 62

Bayesian Machine Learning in Cosmology

Bayesian Machine Learning:

• We need to define the model to be used to provide a so-called generative process. In our case, it will be the cosmological model. We use the model itself to constrain its parameters.

• We then should update our prior belief. Moreover, we need to update them each time to get the posteriors.

Why (Bayesian) Machine Learning?

1. We want to learn instead of doing a simple fit.

2. When we have learned we can do predictions and extend the results to the unseen regimes (no data yet).

3. This can be useful for designing new experiments and observations

4. The real data can be used at the end to validate learned results.

Bayes theorem

Likelihood of the Evidence given that the Hypothesis is True

Prior Probability of the Hypothesis

$$P(H|E) = \frac{P(E|H) * P(H)}{P(E)}$$

Posterior Probability of the Hypothesis given that the Evidence is True

Prior Probability that the evidence is True

Hard to calculate due to complicated integrals or summations.

We need an approximation which over time will become closer to the real answer.

Bayesian Machine Learning in Cosmology

Let me discuss what means Interacting Dark Energy

1. Constraints from observational data shows that interaction can exist
2. Gaussian Process allows model independent reconstruction of Q

$$\dot{\rho}_{de} + 3H(\rho_{de} + P_{de}) = -Q$$

$$\dot{\rho}_{dm} + 3H(\rho_{dm} + P_{dm}) = Q$$

$$H^2 = \frac{1}{3}\rho \quad \dot{H} = -\frac{1}{2}(\rho + 3P)$$

It is obvious that in this way we introduce effective dark energy and effective dark matter models.

Why do interacting dark energy models work better than non-interacting models (in most cases)?
Do we need to revise our understanding of dark matter on the cosmological scales?
Do we need to revise our understanding of dark energy on the cosmological scales?

$$H(z) = H_0 E(z, \Omega_{dm}, \Omega_b, \omega_{dm}, \omega_{de})$$

$$E(z, \Omega_{dm}, \Omega_b, \omega_{dm}, \omega_{de}) = \left[\Omega_b(1+z)^3 + \Omega_{dm}(1+z)^{3(1+\omega_{dm})} + (1 - \Omega_{dm} - \Omega_b)(1+z)^{3(1+\omega_{de})}\right]^{1/2}$$

E. Elizalde, J. Gluza, **M. Khurshudyan**, arXiv:2104.01077

Bayesian Machine Learning in Cosmology

Model with $\omega_{dm} \neq 0$ dark matter and cosmological constant

$$E(z, \Omega_{dm}, \Omega_b, \omega_{dm}) = \left[\Omega_b(1+z)^3 + \Omega_{dm}(1+z)^{3(1+\omega_{dm})} + \Omega_{de}\right]^{1/2} \quad \text{where } \Omega_{de} = 1 - \Omega_{dm} - \Omega_b$$

Model 1	H_0	Ω_{dm}	Ω_b	ω_{dm}
when $z \in [0,2]$	68.42 ± 0.24 km/s/Mpc	0.289 ± 0.007	$0.0037^{+0.0032}_{+0.0025}$	-0.035 ± 0.005
when $z \in [0,2.5]$	68.54 ± 0.24 km/s/Mpc	0.291 ± 0.007	$0.0035^{+0.0036}_{-0.0024}$	-0.022 ± 0.005
when $z \in [0,5]$	68.95 ± 0.25 km/s/Mpc	0.271 ± 0.003	$0.00035^{+0.00038}_{-0.00022}$	-0.028 ± 0.001

TABLE II: Best fit values and 1σ errors estimated for the Λ + Baryonic Matter + Dark Matter (with $\omega_{dm} \neq 0$) model, when $z \in [0,2]$, $z \in [0,2.5]$ and $z \in [0,5]$, respectively. The results have been obtained from a Bayesian (Probabilistic) Machine Learning approach, where the generative process is based on Eq. (1) and Eq. (9) using $H_0 \in [64.0, 80.0]$, $\Omega_{dm} \in [0.23, 0.4]$, $\Omega_b \in [0.0, 0.1]$ and $\omega_{dm} \in [-0.1, 0.1]$ flat priors. The analysis is based on 10 chains and in each chain, 10,000 "observational" data-sets from the model have been simulated or generated.

Problem with Physics? Problem with observations?

Recently, an interesting issue which goes under the name of the H_0 tension problem has appeared.

the Planck-CMB data analysis provides a value $H_0 = 67.4 \pm 0.5$ km/s/Mpc when the ΛCDM scenario is assumed

a local measurement coming from the Hubble Space Telescope yields $H_0 = 73.52 \pm 1.62$ km/s/Mpc

We learned that on cosmological scales there is a deviation from the cold dark matter paradigm.

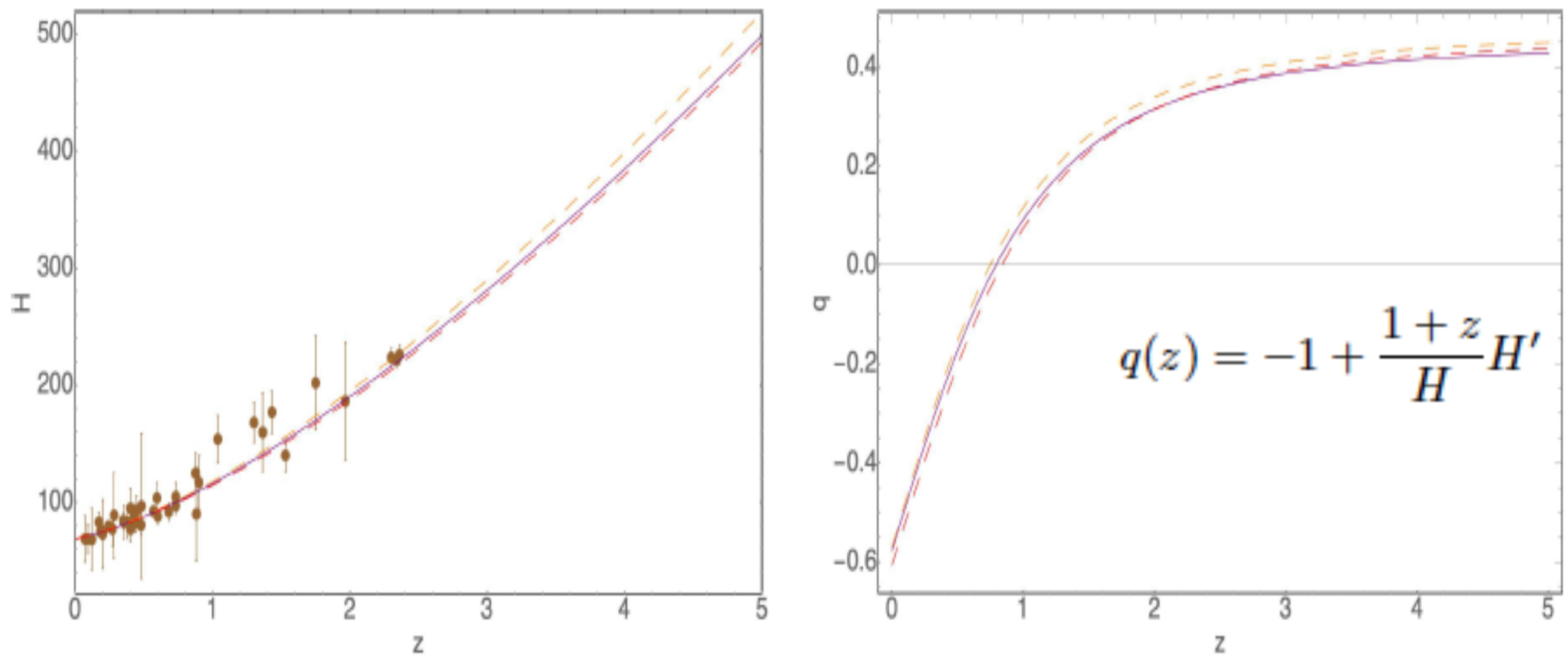
But this is not enough to solve the H0 tension problem. But is this due to the fact that we forced the dark energy to be the cosmological constant?

E. Elizalde, J. Gluza, **M. Khurshudyan**, arXiv:2104.01077

Bayesian Machine Learning in Cosmology

Model with $\omega_{dm} \neq 0$ dark matter and cosmological constant

$$E(z, \Omega_{dm}, \Omega_b, \omega_{dm}) = \left[\Omega_b(1+z)^3 + \Omega_{dm}(1+z)^{3(1+\omega_{dm})} + \Omega_{de}\right]^{1/2} \quad \text{where } \Omega_{de} = 1 - \Omega_{dm} - \Omega_b$$



Comparing learned results with existing expansion rate data (red dots) shows that the learning was quite successful.

According to learned results, we foresaw that future expansion rate data up to z = 5, when becomes available, will support the considered model.

E. Elizalde, J. Gluza, **M. Khurshudyan**, arXiv:2104.01077

Bayesian Machine Learning in Cosmology

Model with $\omega_{dm} \neq 0$ dark matter and $\omega_{de} \neq -1$ dark energy

$$E(z, \Omega_{dm}, \Omega_b, \omega_{dm}, \omega_{de}) = \left[\Omega_b(1+z)^3 + \Omega_{dm}(1+z)^{3(1+\omega_{dm})} + (1-\Omega_{dm}-\Omega_b)(1+z)^{3(1+\omega_{de})}\right]^{1/2}$$

Model 2	H_0	Ω_{dm}	Ω_b	ω_{dm}	ω_{de}
when $z \in [0,2]$	73.52 ± 0.24 km/s/Mpc	0.271 ± 0.012	$0.0036^{+0.0038}_{-0.0025}$	-0.0505 ± 0.0097	-1.049 ± 0.044
when $z \in [0,2.5]$	73.47 ± 0.23 km/s/Mpc	0.268 ± 0.011	$0.0036^{+0.0041}_{-0.0022}$	-0.0354 ± 0.0093	-1.091 ± 0.039
when $z \in [0,5]$	73.65 ± 0.15 km/s/Mpc	0.254 ± 0.008	0.0069 ± 0.0045	-0.0752 ± 0.0047	-1.064 ± 0.009

TABLE III: Best fit values and 1σ errors estimated for the X (Dark Energy with $\omega_{de} \neq -1$) + Baryonic Matter + Dark Matter (with $\omega_{dm} \neq 0$) model, when $z \in [0,2]$, $z \in [0,2.5]$ and $z \in [0,5]$, respectively. The results have been obtained from the Bayesian (Probabilistic) Machine Learning approach, where the generative based process is based on Eqs. (1) and (2), using $H_0 \in [64.0, 80.0]$, $\Omega_{dm} \in [0.23, 0.4]$, $\Omega_b \in [0.0, 0.1]$, $\omega_{dm} \in [-0.1, 0.1]$, and $\omega_{de} \in [-1.5, -0.4]$ as flat priors. The analysis is based on 10 chains and in each chain, 10,000 "observational" data-sets from the model have been simulated or generated.

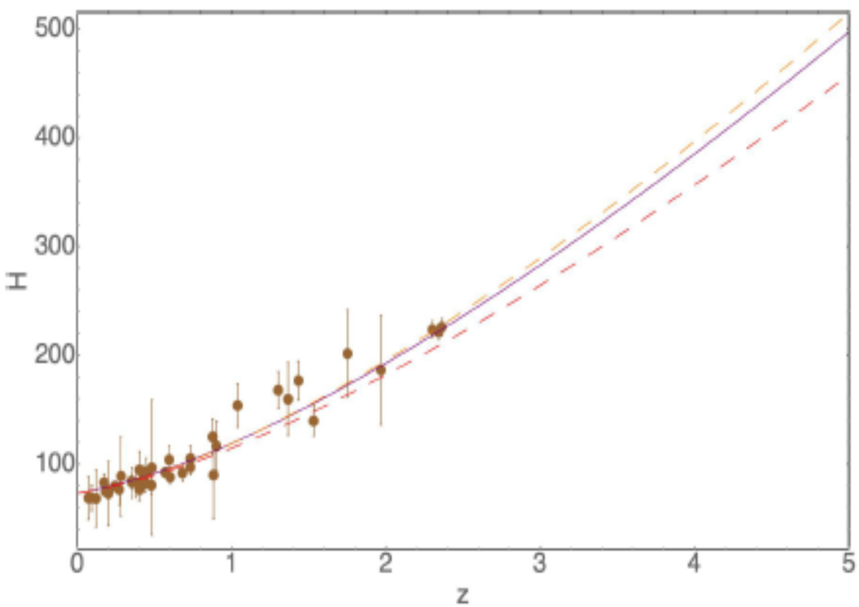
1. We learned that on cosmological scales there is a deviation from the cold dark matter paradigm.
2. The H0 tension problem can be solved without IDE, but just deviating from the cold dark matter .
3. This explains why interacting dark energy models work.
4. We see the constraints on dark energy are consistent with known results.
5. Eventually, we learned that understanding both dark energy and dark matter is very important to solve the H0 tension problem. We need to be very careful about this point.

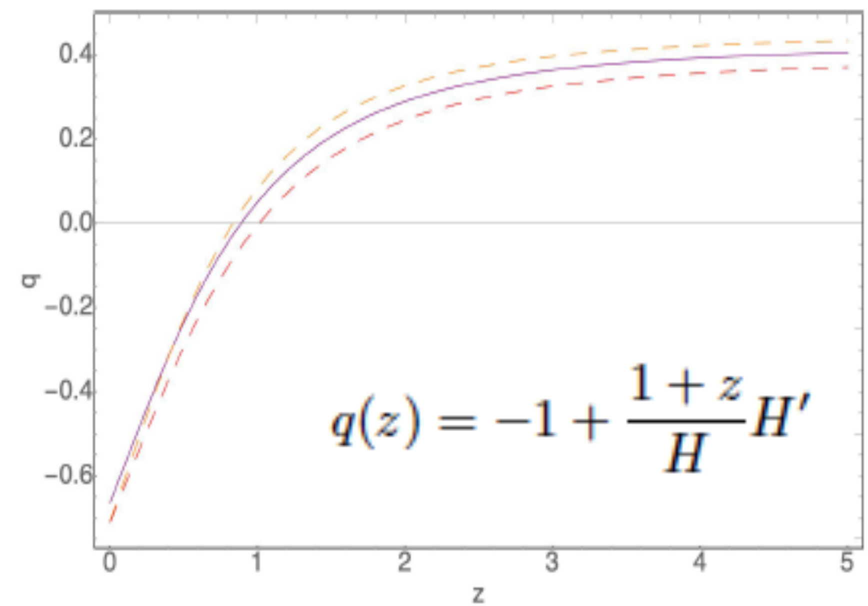
E. Elizalde, J. Gluza, **M. Khurshudyan**, arXiv:2104.01077

Bayesian Machine Learning in Cosmology

Model with $\omega_{dm} \neq 0$ dark matter and $\omega_{de} \neq -1$ dark energy

$$E(z, \Omega_{dm}, \Omega_b, \omega_{dm}, \omega_{de}) = \left[\Omega_b(1+z)^3 + \Omega_{dm}(1+z)^{3(1+\omega_{dm})} + (1 - \Omega_{dm} - \Omega_b)(1+z)^{3(1+\omega_{de})}\right]^{1/2}$$





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Conclusion

Machine Learning is a set of algorithms cooperating together in a very clever way.

Gaussian Process is one of the ML algorithms working directly with data.

It reconstructs the possible form of the function representing data. This can be used to perform model-independent analysis.

It is useful for dark energy study, for constraining cosmological models, for the reconstruction of modified theories of gravity, etc... Even more, it appears very useful for the Swampland criteria study.

E. Elizalde, M. Khurshudyan, Phys. Rev. D 99 (2019) 10, 103533

Bayesian Machine Learning is another ML algorithm based on data generation.

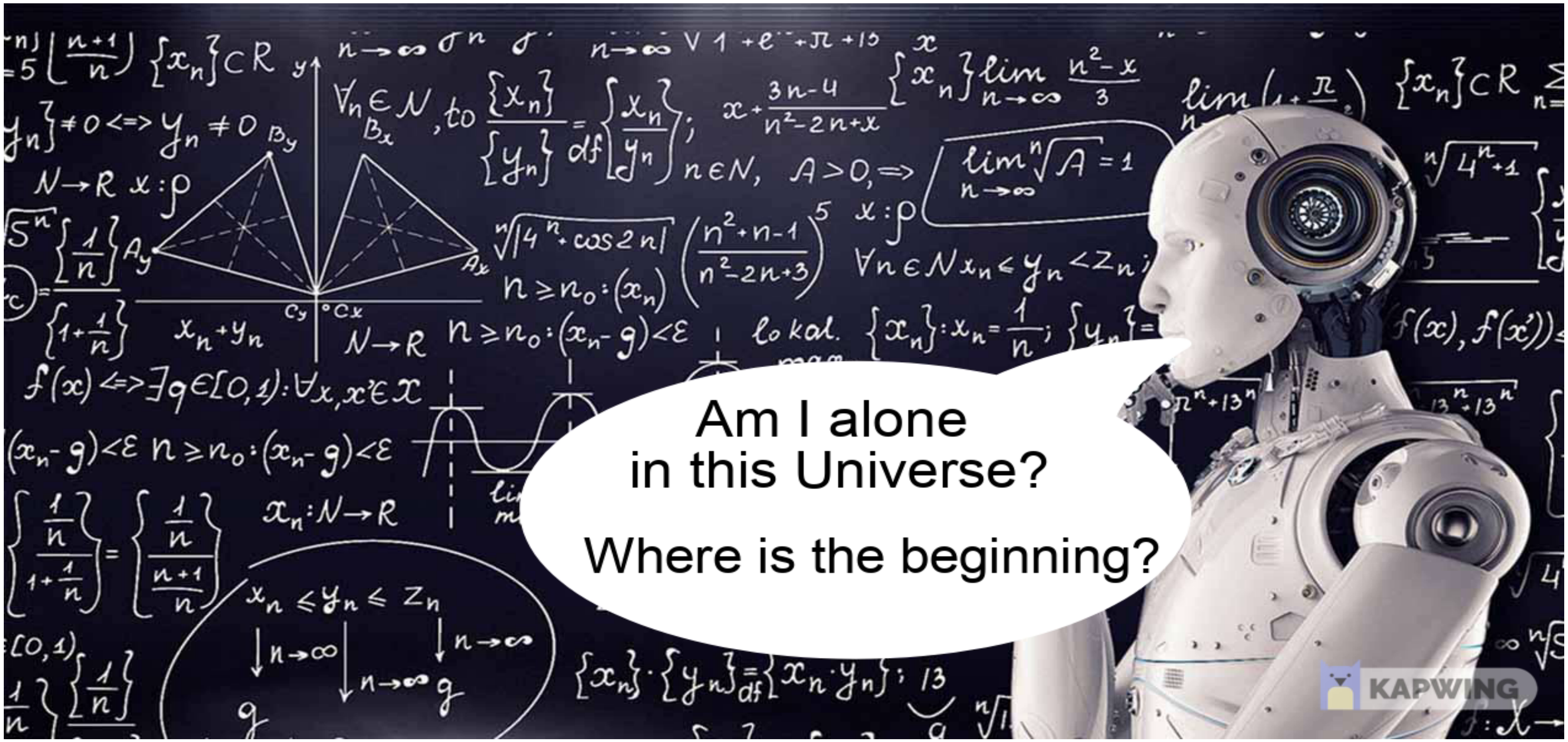
Its particular application allows seeing a deviation from the cold dark matter paradigm giving a solution to the H0 tension problem and explains why interacting dark energy models work.

E. Elizalde, M. Khurshudyan, Eur. Phys. J. C 81 (2021) 4, 335

E. Elizalde, M. Khurshudyan, arXiv:2006.12913

E. Elizalde et al., Phys. Rev. D 102 (2020) 12, 123501

Thank You for Your attention



Have more specific questions or ideas and eventually wish to live a message?
contact me at email - [*khurshudyan@ice.csic.es*](mailto:khurshudyan@ice.csic.es)