Scalar-connection gravity and spontaneous scalarization

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Scalar-tensor theories

- Scalar-tensor gravity (STG) is expected to reveal important and testable deviations from GR in strong regimes by studying the properties of high-density objects such as neutron stars!

- Although it is considered as an alternative theory of gravity, STG is however formulated through the strong foundations of GR as a geometric theory of spacetime!

- For instance, in Damour-Esposito-Farese model, gravity is mediated by a scalar field and metric tensor which are coupled as

$$S[\phi, g] = \frac{1}{16\pi}\int d^4x\sqrt{|g|}\left[g^{\mu\nu}R_{\mu\nu}(g) - 2g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right] + S_m\left[\psi_m, \Omega^2(\phi)g_{\mu\nu}\right]$$

Why a scalar-connection gravity (SCG)?

- In the geometric sector, STG is based on the metric tensor which is only of secondary importance: measurements of distances and angles...!
- Gravity is known to be a "manifestation" of the spacetime curvature, therefore, the essential concept is the connection!

"The essential achievement of GR, namely to overcome rigid space, is only indirectly connected with the introduction of a Riemannian metric. The directly relevant conceptual element is the displacement field Γ^k_{ij} which expresses the infinitesimal displacement of vectors" —- Albert Einstein.

Quantities in SCG

- In the absence of metric tensor, the number of quantities that one considers are less than in the metric case, recalling that scalars formed by contractions (using metric) are not allowed in the first place!

- In SCG one has (and forms) the following quantities without the need of "contraction"

$\Gamma^{\lambda}_{\mu\nu}(x)$	Spacetime connection (will be considered symmetric!)
$R_{\mu\nu}(\Gamma)$	Ricci tensor (at least the symmetric part!)
$u_{\mu}(x)u_{\nu}(x)$	Leading to fluid velocities!
$\nabla_{\mu}\phi\,\nabla_{\nu}\phi$	Leading to field "kinetic" terms!

Invariant action of SCG

- In the spirit of Eddington gravity, one constructs the invariant action

$$S[\phi, \Gamma] = \int d^4x \frac{\sqrt{|\kappa^{-1}[\mathrm{R}_{\mu\nu}(\Gamma) - \nabla_\mu \phi \nabla_\nu \phi] - \mathscr{A}(x, \phi)\mathrm{u}_\mu \mathrm{u}_\nu|}}{\mathscr{B}(x, \phi)}$$

Azri & Nasri PRD (2021)

- Matter will appear as a perfect fluid described by the parameters $\mathscr{A}(x)$ and $\mathscr{B}(x)$

- When compared with the purely affine models

Azri &Nasri PRD (2020), Azri &Demir (2018)

$$\mathscr{A} u_\mu u_\nu \longleftrightarrow \nabla_\mu \phi \nabla_\nu \phi \quad \text{“Kinetic!”}$$

$$\mathscr{B} \longleftrightarrow V(\phi) \quad \text{“Potential!”}$$

How to get the gravitational equations?

First vary w.r.t the connection, and remember that $\delta\Gamma^{\lambda}_{\mu\nu}$ is a tensor!

$$\delta_\Gamma S = 0 \Rightarrow \nabla_\lambda \left(\frac{1}{\mathscr{B}(x,\phi)} \sqrt{|K(\Gamma,\phi)|}\,(K^{-1})^{\mu\nu} \right) = 0$$

Covariant derivative w.r.t the connection!

$$K_{\mu\nu}(\Gamma,\phi) \equiv \kappa^{-1}[\mathrm{R}_{\mu\nu}(\Gamma) - \nabla_\mu\phi\nabla_\nu\phi] - \mathscr{A}(x,\phi)\mathrm{u}_\mu\mathrm{u}_\nu$$

This dynamical equation is at the heart of "generating" the metric in the SCG!

Generating the metric

- Notice the emergence of the inverse of $K_{\mu\nu}(\Gamma,\phi)$. Therefore, the latter must not vanish!
- In the vacuum case, this necessitates a nonzero cosmological term!

- One introduces the following (invertible) two-rank tensor such that

$$\frac{1}{\mathscr{B}(x,\phi)}\sqrt{|K(\Gamma,\phi)|}\,(K^{-1})^{\mu\nu} \equiv \sqrt{|g|}\,g^{\mu\nu}$$

It is from this identity that the field equations of the SCG will take the "familiar" form (Einstein equations-like!)

Generating the metric

- The previous equations can be reverted to

$$R_{\mu\nu}(g) = 8\pi \left[\mathscr{A}(x,\phi)\mathrm{u}_\mu \mathrm{u}_\nu + \mathscr{B}(x,\phi) g_{\mu\nu} \right] + \nabla_\mu \phi \nabla_\nu \phi$$

$$\nabla_\lambda \left(\sqrt{|g|}\, g^{\mu\nu} \right) = 0$$ ✓

Recall that the connection is symmetric! therefore, the last identity implies that the connection $\Gamma^\lambda_{\mu\nu}(x)$ is reduced to the Levi-Civita connection of $g_{\mu\nu}(x)$

Gravitational field equations

- The "generated" tensor field will play the role of the metric tensor $g_{\mu\nu}(x)$

Since the connection is symmetric!

$$\nabla_\lambda(\sqrt{|g|}g^{\mu\nu}) = 0 \quad \Rightarrow \quad \Gamma^\mu_{\alpha\beta}(x) = \frac{1}{2}g^{\mu\nu}\left(\partial_\beta g_{\alpha\nu} + \partial_\alpha g_{\beta\nu} - \partial_\nu g_{\alpha\beta}\right)$$

Recall that

$$\frac{1}{\mathscr{B}(x,\phi)}\sqrt{|K(\Gamma,\phi)|}\,(K^{-1})^{\mu\nu} \equiv \sqrt{|g|}\,g^{\mu\nu} \qquad \otimes$$

Signature?!

Only those field configurations where the tensor $K_{\mu\nu}(\phi,\Gamma)$ has the signature (-,+,+,+) are considered!

Gravitational field equations

- One constructs the Einstein tensor

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi\left[\mathscr{A}\,u_\mu u_\nu + \left(\frac{1}{2}\mathscr{A} - \mathscr{B}\right)g_{\mu\nu}\right] + \nabla_\mu\phi\,\nabla_\nu\phi - \frac{1}{2}\,g_{\mu\nu}\,\nabla^\lambda\phi\,\nabla_\lambda\phi$$

In addition to the scalar fields, perfect fluid source appears in terms of its energy-momentum tensor

$$T^m_{\mu\nu} = \mathscr{A}\,\mathrm{u}_\mu \mathrm{u}_\nu + \left(\frac{1}{2}\mathscr{A} - \mathscr{B}\right)g_{\mu\nu}$$

- Notice the GR-limit in which

$$\mathscr{A}(x,0) = \rho + P$$

$$\mathscr{B}(x,0) = (\rho - P)/2$$

The equation of motion for the scalar field

- Variation w.r.t to the scalar field leads to

$$\Box \phi = 4\pi \left(\frac{\partial \mathscr{A}(x,\phi)}{\partial \phi} \underbrace{g^{\mu\nu} \mathrm{u}_\mu \mathrm{u}_\nu}_{-1} + 2 \frac{\partial \mathscr{B}(x,\phi)}{\partial \phi} \right)$$

Here we have used the expression of the "generated" metric!

The system will then develop potential-like terms. Needless to say, in the absence of matter, this equations will simply describe a propagating massless scalar field.

Damour-Esposito-Farese model from SCG

- Although the scalar-connection gravity can be considered as an alternative theory of gravity different from scalar-tensor theory, it might be relevant to produce the latter for certain cases!
- Based on the previous remarks on the limit of the theory (GR), one may choose

$$\mathscr{A}(x,\phi) = (\rho + P)e^{\beta\phi^2/2}$$

$$\mathscr{B}(x,\phi) = \frac{1}{2}(\rho - P)e^{\beta\phi^2}$$

$\longrightarrow$ i.e, with a sole parameter β

To linear order in perturbation (about the GR solution!)

✓ $$\Box\,\delta\phi \simeq -4\pi\beta\left(-\rho + 3P\right)\delta\phi$$ $\longrightarrow$ Scalarization!

Non-minimal to minimal coupling

- The SCG action can be written as

$$S[\varphi,\Gamma] = \int d^4x \frac{\sqrt{|\kappa^{-1}[\omega(\varphi)\mathrm{R}_{\mu\nu}(\Gamma) - \nabla_\mu\varphi\nabla_\nu\varphi] - \tilde{\mathscr{A}}(x)\mathrm{u}_\mu\mathrm{u}_\nu|}}{\tilde{\mathscr{B}}(x)}$$

This describes the non-minimal coupling dynamics where the Scalar fields interacts "explicitly" with the curvature!

The non-minimal coupling function can be absorbed easily without geometric transformation as in STG (by using metric transformation.) Here, this is realized by performing a simple field-redefinition $\nabla_\mu\phi\nabla_\nu\phi \to \omega^{-1}(\varphi)\nabla_\mu\varphi\nabla_\nu\varphi$

Non-minimal to minimal coupling

- The SCG action reads

$$S[\phi,\Gamma] = \int d^4x \frac{\sqrt{|\kappa^{-1}[R_{\mu\nu}(\Gamma) - \nabla_\mu\phi\nabla_\nu\phi] - \omega^{-1}(\phi)\tilde{\mathscr{A}}(x)u_\mu u_\nu|}}{\omega^{-2}(\phi)\tilde{\mathscr{B}}(x)}$$

- Notice that the parameter (kinetic-like term) of matter is rescaled by $\omega^{-1}(\phi)$,whilst the potential-like term is rescaled by $\omega^{-2}(\phi)$

- The previous study can be realized now as

$$\omega^{-1}\tilde{\mathscr{A}}(x) \to \mathscr{A}(x,\phi)$$
$$\omega^{-2}\tilde{\mathscr{B}}(x) \to \mathscr{B}(x,\phi)$$

General setup

- Here, one may not follow the previous mechanism based on the transition from minimal to nonminimal coupling which leads to specific rescaling of the fluid parameters!

- Rather, the general setup is based on the fact that the matter parameters can be coupled to the scalar in different ways. One generic case is to take

$$\mathscr{A}(x,\phi) = (\rho + P)e^{\beta_1\phi^2/2}$$
$$\mathscr{B}(x,\phi) = \frac{1}{2}(\rho - P)e^{\beta_2\phi^2}$$

i.e, with different parameters β_1, β_2

To linear order in perturbation (about the GR solution!)

$$\Box\,\delta\phi \simeq 4\pi\left[\left(2\beta_2 - \beta_1\right)\rho - \left(2\beta_2 + \beta_1\right)P\right]\delta\phi$$

General setup: Spontaneous scalarization

- In terms of the equation of state

$$\Box\,\delta\phi \simeq -4\pi(\beta_1 - 2\beta_2)\rho\left[1 + \frac{\beta_1 + 2\beta_2}{\beta_1 - 2\beta_2}\omega(\rho)\right]\delta\phi$$

- Thus, the spontaneous scalarization mechanism in this class of SCG is to be determined in line with specific bounds of the two parameters β_1, β_2

- One distinguishes different scenarios:
    - For $\beta_1 = -2\beta_2$ the effective potential that drives spontaneous scalarization arises only through the energy density of matter. However, in this case the complete suppression of the instability will not be quite similar through both parameters $\mathscr{A}$ and $\mathscr{B}$!

General setup: Spontaneous scalarization

- The second scenario is when the dynamics is driven only by pressure, this arises here when $\beta_1 = 2\beta_2$, thus

$$\Box\,\delta\phi \simeq -\,8\pi\beta_1 P\delta\phi$$

If $\beta_1 < 0$, which is a trivial choice to eventually suppress the unstable modes, then a negative effective mass square $-8\pi\beta_1 P$ requires $P < 0$.

This cannot describe a physical high density object such as a neutron star unless the negative sign is conventional, i.e, the inward gravitational pull!

General setup: Spontaneous scalarization

- a neutron star can be approximated to a nonrelativistic matter where the energy density term dominates, hence

$$\Box\, \delta\phi \simeq 4\pi \left(2\beta_2 - \beta_1\right) \rho\, \delta\phi$$

- If $\beta_2 < \beta_1/2$ spontaneous growth occurs when the GR solution $\phi = 0$ is unstable due to the negative (effective) mass squared. This has an effective wavelength

$$\lambda_{eff} = \sqrt{\frac{\pi}{|2\beta_2 - \beta_1|\rho}}$$

In terms of the compactness C

$$\lambda_{eff} \simeq \frac{R}{\sqrt{|2\beta_2 - \beta_1|C}}$$

R is the radius of the star!

General setup: Spontaneous scalarization

- In order to fit various observational constraints such as those coming from the pulsar-white dwarf binary, it might be necessary even for SCG to embody a massive scalar field. The system, then, would gain an effective mass as

$$m_{eff}^2 = 4\pi \left(2\beta_2 - \beta_1\right) \rho + m_\phi^2$$

- To accurately probe the SCG, one mainly has to examine various classes of the theory by providing a generic phenomenological study of other relevant forms for $\mathscr{A}$ and $\mathscr{B}$ and adopting specific equations of state of neutron stars Azri & Nasri (in progress.)

Summary

- The aim is to provide a scalar-connection gravity (SCG) where unlike the famed scalar-tensor theories, gravity is not mediated by the metric tensor a priori !
- In the SCG, gravity is mediated by a scalar and a connection. It is the latter which is necessary for the curvature, thus, gravity !
- In SCG, small perturbations around the GR (unstable) solution grows up due to an induced effective potential proportional to both energy density and pressure of matter $\longrightarrow$ scalarization !
- SCG leads to an equivalent prediction of scalar-tensor theory, namely the Damour-Esposito-Farese model of spontaneous growth, for particular cases! The general setup leads to different predictions!

Thank you