Equation of State table with exotic matter for supernova and neutron star merger

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Plan of the Talk

- Introduction
- Microphysics: strange equation of state (EoS)
- Results
- Summary and Outlook

Fate of Massive Remnant in GW170817

- Remnant formed in the merger has a mass $2.74^{+0.04}_{-0.01}$ M_{solar} assuming slow rotation of the components of the binary.
- It was not a prompt collapse because significant amount of ejecta was observed.
- Two possibilities - long lived neutron star or a delayed collapse to BH.
- Existence of a Neutron Star is not confirmed.
- If the remnant collapsed to a black hole the estimated upper bound of the NS mass is in the range $\sim 2.17 - 2.3 M_{solar}$.

Observation Results

- Observations of galactic massive pulsars set a lower limit on the maximum mass (M_{solar}) of neutron stars.

PSR J0348+0432	PSR J1810+1744
2.01$\pm$0.04	2.13 $\pm$ 0.04

- GW170827 calculation: Tidal deformability $70 \leq \widetilde{\Lambda} \leq 720$ estimates radius as 9-14 km for a 1.4 M_{solar} NS.
- NICER results of PSR J0030+0451 and PSR J0740+6620 for two different analyses are

Mass (M_{solar})	radius (km)	
$1.44^{+0.15}_{-0.15}$	$13^{+1.24}_{-1.06}$	Miller et. al 2019
$1.34^{+0.15}_{-0.16}$	$12.71^{+1.14}_{-1.19}$	Riley et. al 2019
$2.07^{+0.07}_{-0.07}$	$12.39^{+1.30}_{-0.98}$	Riley et.al 2021
$2.08^{+0.09}_{-0.09}$	$13.71^{+2.61}_{-1.50}$	Miller et. al 2021

Equation of State

- Equation of state (EoS) is an important microphyics input for simulations of stellar collapse and binary NS mergers.
- The theoretical modeling of EoS has undergone a sea of changes over the past several years.
- Traditional models of EoS based on two-body plus three-body interactions in non-relativistic approaches and the strong interaction Lagrangian in relativistic field theoretical approaches are widely used in numerical relativity simulations of compact astrophysical objects.
- Our motivation in this work is to compute an EoS including strange matter within the framework of relativistic field theoretical models.

Exotic Phases of Matter

Strangeness might appear in the post-bounce phase of a core-collapse supernova or in the massive remnant in NS merger in the form of

- Hyperons,
- Bose-Einstein condensates of Kaons,
- Quarks.

We construct new hyperon EoS BHB$\Lambda K^{-}\phi$

[T. Malik, S. Banik, and D. Bandyopadhyay,APJ, 910:96 , (2021)]

available at

i)https://universe.bits-pilani.ac.in/Hyderabad/sbanik/EoS

ii)CompStar Online Supernovae Equations of State (CompOSE)

Earlier version was hyperon EoS BHB$\Lambda\phi$ EoS

[S. Banik, M. Hempel, D.Bandyopadhyay ApJS214:22 (2014)]

- Hyperons produced at the cost of the nucleons.

$$\boxed{n + p \longrightarrow p + \Lambda + K^0, \quad n + n \longrightarrow n + \Sigma^- + K^+}$$

- Chemical equilibrium in compact star interior through weak processes,
- $p + e^- \longrightarrow \Lambda + \nu_e, \qquad n + e^- \longrightarrow \Xi^- + \nu_e$
- Condition for chemical equilibrium

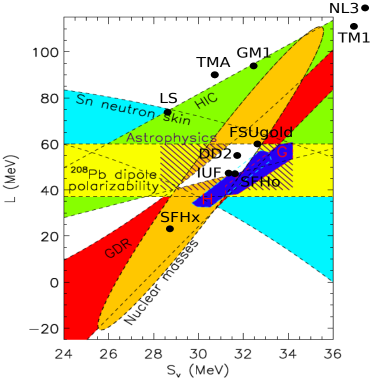
$$\mu_i = b_i \mu_n - q_i \mu_e$$

- Threshold Condition for Hyperons

$\mu_n - q_i \mu_e \geq m_B^* + g_{\omega B}\omega_0 + g_{\rho B}\rho_{03}\tau_3$

New Hyperon EoS

- should satisfy the experimental constraint on the value of parameter (*L*) corresponding to the density dependence of the symmetry energy
- should be consistent with $2M_\odot$ neutron star



J. M. Lattimer and Y. Lim, ApJ 771, 51 (2013)

- We construct the hyperon EoS tables for

	number density $n_B(g/cm^3)$	Temperature T(MeV)	+ve charge fraction $Y_q = n_p - n_K$
Range	$10^3 - 10^{15}$	$0.1 - 158.48$	$0.01 - 0.6$
Grid Spacing	$\Delta log_{10} n_B = 0.04$	$\Delta log_{10} T = 0.04$	$\Delta Y_q = 0.01$

- The EoS table consists of 301 n_B points, 81 T and 60 Y_q i.e. total one million data points.
- We adopt a Density Dependent Relativistic Mean Field (RMF) Model to describe uniform matter including hyperons
- At low temperature and sub-saturation density, matter is mainly composed of light and heavy nuclei coexisting with unbound nucleons. This is treated in the Nuclear Statistical Equilibrium model (Saha Equation) (Hempel and Schaffner, Nucl. Phys. A837, 210 (2010)).

Density Dependent Relativistic Model for EOS

- The interaction among baryons is mediated via scalar (σ) and vector (ω, ϕ, ρ) mesons.
- ϕ mesons account for the repulsive hyperon-hyperon interaction.
- The Lagrangian density for baryons is given by

$$\begin{aligned}
\mathcal{L}_B &= \sum_{B=N,\Lambda} \bar{\Psi}_B \left(i\gamma_\mu \partial^\mu - m_B^* - g_{\omega B}\gamma_\mu \omega^\mu - g_{\phi B}\gamma_\mu \phi^\mu \right. \\
& \left. - g_{\rho B}\gamma_\mu \boldsymbol{\tau}_B \cdot \boldsymbol{\rho}^\mu \right) \Psi_B + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) \\
& - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu \\
& - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu .
\end{aligned}$$

Ref: S. Banik, M. Hempel, D.Bandyopadhay, ApJS214:22(2014) ;

T, Malik,S.Banik, D.Bandyopadhyay, ApJ 910:96 , (2021)

EoM

In the mean field approximation, the meson field equations

$$m_\sigma^2\sigma = \sum_B g_{\sigma B} n_B^S, \quad m_\omega^2\omega_0 = \sum_B g_{\omega B} n_B,$$

$$m_\rho{}^2\rho_{03} = \frac{1}{2}\sum_B g_{\rho B}\tau_{3B} n_B, \quad m_\phi^2\phi_0 = \sum_B g_{\phi B} n_B.$$

The scalar density and baryon number density

$$n_B^S = 2\int \frac{d^3k}{(2\pi)^3}\frac{m_B^*}{E^*}\left(\frac{1}{e^{\beta(E^*-\nu_B)}+1} + \frac{1}{e^{\beta(E^*+\nu_B)}+1}\right),$$

$$n_B = 2\int \frac{d^3k}{(2\pi)^3}\left(\frac{1}{e^{\beta(E^*-\nu_B)}+1} - \frac{1}{e^{\beta(E^*+\nu_B)}+1}\right)$$

with $m_B^* = m_B - g_{\sigma B}\sigma$

The thermodynamic potential per unit volume for nucleons is given by

$$\frac{\Omega_B}{V} = \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{2}m_\omega^2\omega_0^2 - \frac{1}{2}m_\phi^2\phi_0^2 - \frac{1}{2}m_\rho^2\rho_{03}^2 - \Sigma^r \sum_{i=n,p,\Lambda} n_i$$
$$-2T\sum_B \int \frac{d^3k}{(2\pi)^3}[ln(1+e^{-\beta(E^*-\nu_B)}) + ln(1+e^{-\beta(E^*+\nu_B)})] .$$

Here, $\beta = 1/T$, $E^* = \sqrt{(k^2 + m_B^{*2})}$ and Σ^r is the rearrangement term.

$$P_B = -\Omega_B/V.$$

The energy density is given by,

$$\epsilon_B = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2$$
$$+2\sum_B \int \frac{d^3k}{(2\pi)^3}E^*\left(\frac{1}{e^{\beta(E^*-\nu_B)}+1} + \frac{1}{e^{\beta(E^*+\nu_B)}+1}\right) .$$

Parameters of the Model

- The density dependent couplings (DD2 parameter set):

$$g_{\alpha N} = g_{\alpha N}(n_0) f_\alpha(x)$$

$$f_\alpha(n_b/n_0) = a_\alpha \frac{1 + b_\alpha (x + d_\alpha)^2}{1 + c_\alpha (x + d_\alpha)^2}$$

  Here n_0 is the saturation density, $\alpha = \sigma, \omega$ and $x = n_b/n_0$.
- For ρ mesons, $g_{\rho N} = g_{\rho N}(n_0) exp[-a_\rho (x - 1)]$.
- The scaling factors for vector and isovector mesons from the SU(6) symmetry relations of the quark model $\frac{1}{2} g_{\omega\Lambda} = \frac{1}{3} g_{\omega N}$; $g_{\rho\Lambda} = 0$; $2g_{\phi\Lambda} = -\frac{2\sqrt{2}}{3} g_{\omega N}$
- Scalar-Λ hyperon is obtained from the potential depth of Λ hyperon in saturated nuclear matter: $U_\Lambda^N(n_0) = \Sigma_\Lambda^v - \Sigma_\Lambda^s$
- The potential depth $U_\Lambda^N(n_0) = -30$ MeV from Λ hypernuclei data.

Extended NSE model

Internal excitations, Coulomb screening and excluded volume effects are included.
The total canonical partition function is given by,

$$Z(T, V, \{N_i\}) = Z_{nuc} \prod_{A,Z} Z_{A,Z} \, Z_{Coul}.$$

The free energy density is
$f = \sum_{A,Z} f^0_{A,Z}(T, n_{A,Z}) + f_{Coul}(n_e, n_{A,Z}) + \xi f^0_{nuc}(T, n'_n, n'_p) - T \sum_{A,Z} n_{A,Z} \ln\kappa$
The last term goes to infinity when available volume fraction of nuclei (κ) is zero near n_0.

Mergeing the EoS

For the merging of the two tables, we follow

- the minimum free energy per baryon at fixed T, n_B, and $Y_q(n_B) = n_p - n_K$
- hyperon fraction is small i.e. 10^{-5}.

Kaon Condensation

- Bose-Einstein condensate of negatively charged kaons might appear in dense matter .
- This idea was extended to understand the non-observation of a neutron star in SN1987A.

Antikaon condensation

The original problem dates back to 1986-87 when Kaplan and Nelson first demonstrated within a chiral $SU(3)_\times SU(3)_R$ model that K^- mesons may undergo Bose-Einstein condensation in dense matter formed in heavy ion collisions as well as in neutron stars.

- Processes responsible for p-wave π condensates or s-wave K condensates (attractive interaction)

$$n \to p + \pi^-, \quad n \to p + K^-$$

$$e^- \to \pi^- + \nu_e, \quad e^- \to K^- + \nu_e$$

Kaons being bosons are able to condensate in the lowest momentum state, hence is energetically favourable.

- The Lagrangian density for (anti)kaons in minimal coupling scheme:

$$\mathcal{L}_K = D^*_\mu \bar{K} D^\mu K - m_K^{*2} \bar{K} K \ ,$$

where $D_\mu = \partial_\mu + i g_{\omega K} \omega_\mu + i g_{\rho K} \tau_K \cdot \boldsymbol{\rho}_\mu$ and $m_K^* = m_K - g_{\sigma K} \sigma$.

- The thermodynamic potential for (anti)kaons is,

$$\frac{\Omega_K}{V} = T \int \frac{d^3k}{(2\pi)^3} [ln(1 - e^{-\beta(\omega_{K^-} - \mu)}) + ln(1 - e^{-\beta(\omega_{K^+} + \mu)})] \ ,$$

- The in-medium energies of $K^\pm$ mesons are given by

$$\omega_{K^\pm} = \sqrt{(k^2 + m_K^{*2})} \pm \left(g_{\omega K} \omega_0 + g_{\rho K} \rho_{03} + g_{\phi K} \phi_0 \right) \ .$$

- Threshold condition for (anti) kaon condensation
$\mu = \omega_{K^-} = m_K^* - g_{\omega K} \omega_0 - g_{\rho K} \rho_{03} - g_{\phi K} \phi_0$.

(Ref:N.K. Glendenning, J. Schaffner-Bielich,Phys. Rev. **C60**, 025803 (1999) ,
S.B., D. Bandyopadhyay, Phys. Rev. **C64** (2001) 055805.)

EoM:in the presence of antikaons

$$
\begin{aligned}
m_\sigma^2 \sigma &= \sum_B g_{\sigma B} n_B^S + \sum_K g_{\sigma K} n_K, \\
m_\omega^2 \omega_0 &= \sum_B g_{\omega B} n_B - \sum_K g_{\sigma K} n_B, \\
{m_\rho}^2 \rho_{03} &= \frac{1}{2} \sum_B g_{\rho B} \tau_{3B} n_B + \sum_K g_{\sigma K} n_K \tau_{3K}, \\
m_\phi^2 \phi_0 &= \sum_B g_{\phi B} n_B - \sum_K g_{\sigma K} n_K .
\end{aligned}
$$

The net (anti)kaon number density is given by $n_K = n_K^C + n_K^T$.

S. Banik, W. Greiner, D. Bandyopadhyay Phys.Rev.C78:065804,2008

T. Malik, S. Banik, and D. Bandyopadhyay,APJ910:96 , (2021)

Kaon-Meson couplings

- Kaon-meson couplings are not density-dependent.
- The kaon-vector meson coupling constants are also estimated exploiting the quark model and isospin counting rule i.e. $g_{\omega K} = \frac{1}{3} g_{\omega N}$ and $g_{\rho K} = g_{\rho N}$
- The scalar coupling constant is determined from the real part of K^- optical potential $U_{K^-} = -g_{\sigma K}\sigma_0 - g_{\omega K}\omega_0 + \Sigma^r$ at n_0.
- $U_{K^-} = -120$ MeV is considered so that the cold beta-equilibrated EoS is compatible with 2 M_{solar} neutron stars.
- -60MeV $< U_{K^-} < -200$ MeV as indicated by the unitary chiral model calculations and phenomenological fit to kaonic atom data.

Accuracy and Consistency Checks of the EoS table

Thermodynamic consistency is achieved by the condition:

$$f = \mu_n n_n + \mu_p n_p + \mu_\Lambda n_\Lambda + \mu n_K - P,$$

where $f = \epsilon - T\mathcal{S}$.
The modulus of the relative thermodynamic accuracy is given by

$$\Delta = \frac{T\mathcal{S} - P + \mu_n n_n + \mu_p n_p + \mu_\Lambda n_\Lambda + \mu n_K}{\epsilon} - 1 \sim 10^{-7}.$$

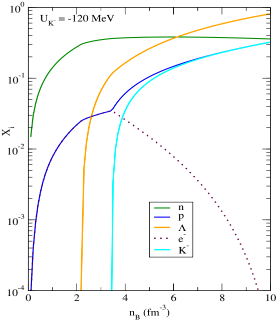
Sum rule of particle fractions (X_i) is satisfied by the EoS table given by

$$X_n + X_p + X_s + X_A + X_\Lambda = 1.$$

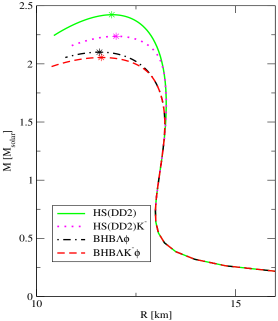
Finally, the EoS table fulfills the thermodynamic stability criteria:

$$\frac{d\mathcal{S}}{dT} > 0, \quad \frac{dP}{dn_B} > 0.$$

β-equilibrated matter



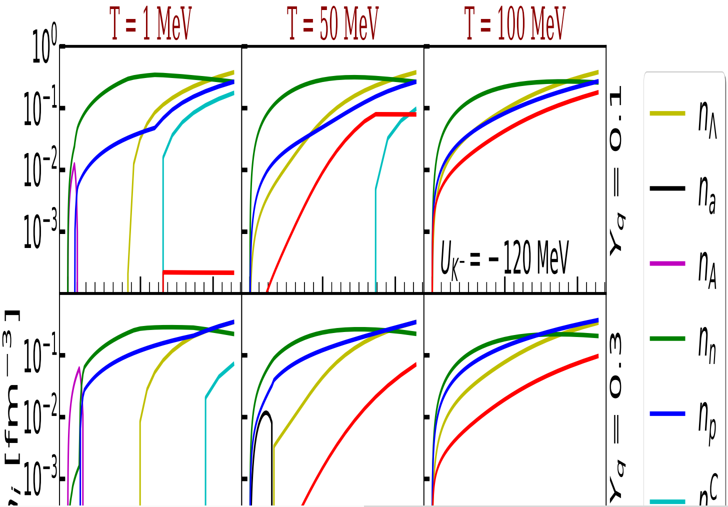
Particle fractions in a β-equilibrated cold NS as a function of n_B for BHB$\Lambda K^-\phi$ EoS



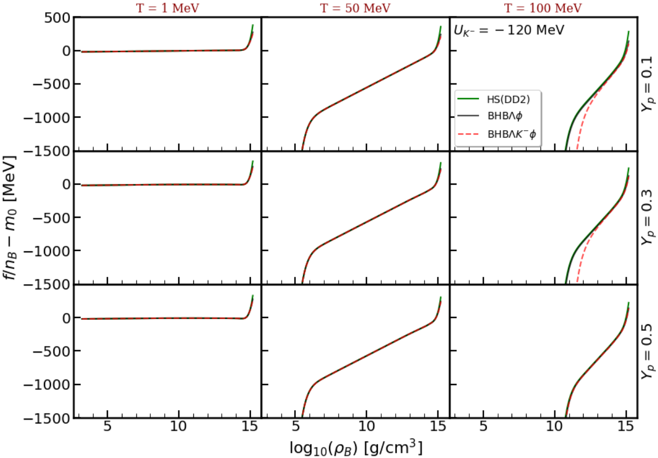
Masses of the NS sequence for nucleonic and strange EoSs * marks maximum mass.

The maximum masses for the four EoSs are 2.42, 2.24, 2.1 and 2.05 M_{solar}, their corresponding radii being 11.89, 12.0, 11.58, 11.62 km respectively.

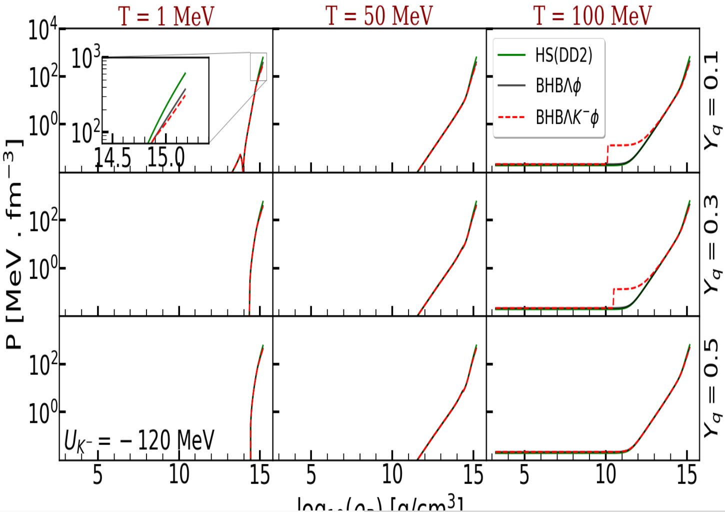
Number density



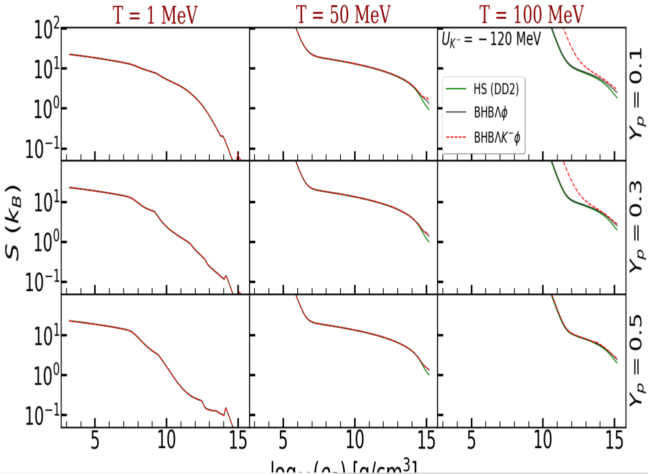
Free energy


T = 1 MeV
T = 50 MeV
T = 100 MeV
$U_{K^-} = -120$ MeV
HS(DD2)
BHBΛϕ
BHBΛK^-ϕ
$Y_p = 0.1$
$Y_p = 0.3$
$Y_p = 0.5$
$f/n_B - m_0$ [MeV]
$\log_{10}(\rho_B)$ [g/cm^3]

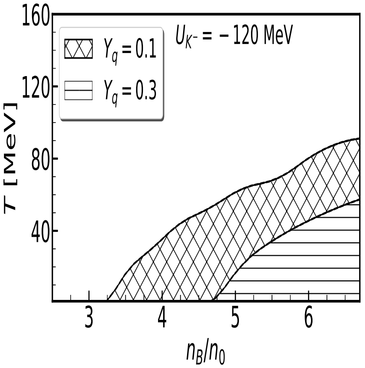
Pressure

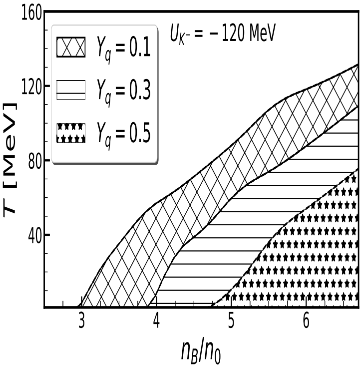

T = 1 MeV
T = 50 MeV
T = 100 MeV
HS(DD2)
BHBΛφ
BHBΛK⁻φ
Yq = 0.1
Yq = 0.3
Yq = 0.5
P [MeV . fm⁻³]
U_K⁻ = −120 MeV
14.5
15.0
5
10
15

Entropy per baryon



Phase Diagram





Shaded regions are K^- condensed phases, demarcated from the hadron phases by the corresponding solid lines of Y_q.

Summary

- A new EoS table including (anti)kaons and hyperons for core collapse supernova and binary NS merger simulations s constructed.
- The low density, non-uniform matter of this EoS table is generated within an extended version of the NSE model with excluded volume.
- The uniform matter is described in a finite temperature DDRMF model.
- At low T and low Y_q, the system is populated with K^- condensate and a very small amount of thermal (anti)kaons.
  A high fraction of Λs at low density does not favour the onset of K^- condensate.
  At high temperature, only the thermal (anti)kaons populate the matter.
- The mass values for cold, β-equilibrated matter is within observational constraints.
- Our results for radius corresponding to 1.4M$_{solar}$ neutron star is slightly higher than that of GW170817 whereas it is consistent with the NICER observation.