Ferromagnetic Neutron Stars in Scalar-Tensor Gravity

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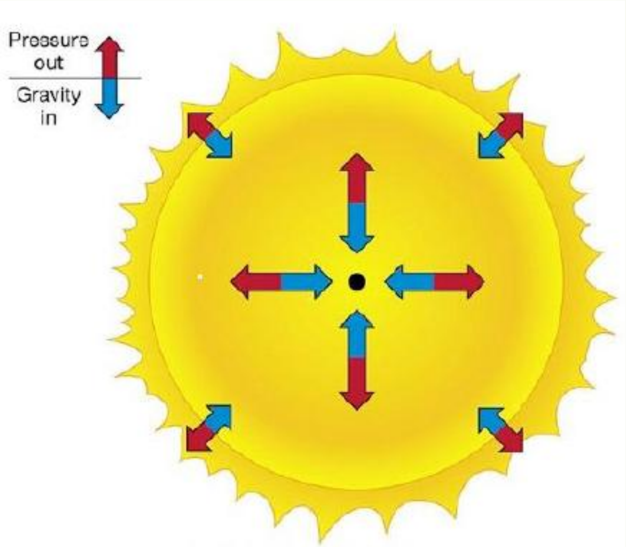
Neutron Star's Structure



Physics of Many-Body Systems

Gravity

Neutron Degeneracy Pressure

+

Repulsive Nuclear Forces

=

Gravitational Pressure

The Skyrme Interactions

- The Skyrme-like effective interactions,

$$V(\mathbf{r}_1,\mathbf{r}_2) = t_0(1+x_0P^\sigma)\delta(\mathbf{r}) + \frac{1}{2}t_1(1+x_1P^\sigma)(\mathtt{K}'^2\delta(\mathbf{r})+\delta(\mathbf{r})\mathtt{K}^2) + t_2(1+x_2P^\sigma)\mathtt{K}'.\delta(\mathbf{r})\mathtt{K} + \frac{1}{6}t_3(1+x_3P^\sigma)[\rho(\mathtt{R})]^\gamma\delta(\mathbf{r}) + iW_0(\sigma_1+\sigma_2).[\mathtt{K}'\times\delta(\mathbf{r})\mathtt{K}],$$

Eq. 1

with $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $R = (\mathbf{r}_1+\mathbf{r}_2)/2$, $\mathtt{K} = (\nabla_1 - \nabla_2)/2i$, and $P^\sigma = (1+\sigma_1.\sigma_2)/2$. Besides, $\mathtt{K}$, the relative momentum acts on the right and $\mathtt{K}'$ is its conjugate and acts on the left. Moreover, t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, γ, and W_0 are the Skyrme-force parameters.

Polarized Neutron Matter

- Consider a system of neutrons characterized by a total density ρ, which is the sum of the spin-up ($\rho\uparrow$) and spin-down ($\rho\downarrow$)

$$\rho = \rho\uparrow + \rho\downarrow \qquad \text{Eq. 2}$$

- At zero temperature, the total energy will be a function of $\rho\uparrow$ and $\rho\downarrow$ or,
- We define the spin polarization parameter Δ,

$$\Delta = \frac{\rho\uparrow - \rho\downarrow}{\rho} \qquad \text{Eq. 3}$$

Polarized Neutron Matter

- If $\Delta = 0$,

The system is totally unpolariazed,

- If $\Delta = -1 \; or \; 1$

The system is totally polarized,

$$\Delta = \frac{\rho \uparrow - \rho \downarrow}{\rho}$$

- If $-1 < \Delta < 1$

The system is partially polarized

Polarized Neutron Matter

- The energy per particle for non ferromagnetic neutron matter,

$$E_{\mathrm{NFM}}/N = \frac{3}{5}\frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} + \frac{1}{4}t_0(1-x_0)n + \frac{1}{24}t_3(1-x_3)n^{\gamma+1} + \frac{3}{40}(3\pi^2)^{2/3}\Theta n^{5/3},$$

Eq. 4

Chabanat et al, NPA, 1997

- The energy per particle for ferromagnetic (FM) neutron matter

$$\begin{aligned}\frac{E(\rho\uparrow,\rho\downarrow)}{N} =& \frac{\hbar^2}{2m}\frac{1}{\rho}(\tau\uparrow+\tau\downarrow) + \frac{1}{4\rho}\left(2t_2\left(x_2+1\right)\right)\left(\tau\uparrow\rho\uparrow+\tau\downarrow\rho\downarrow\right)\\ &+ \frac{1}{4\rho}\left(t_1\left(1-x_1\right)+t_2(x_2+1)\right)\left(\tau\uparrow\rho\downarrow+\tau\downarrow\rho\uparrow\right)\\ &+ \frac{1}{\rho}\left(\frac{1}{6}t_3\left(1-x_3\right)\rho^{\alpha}+t_0\left(1-x_0\right)\right)\rho\uparrow\rho\downarrow\,.\end{aligned}$$

Eq. 5

Rios and Polls, PRC, 2005

$$\tau_\sigma = \tfrac{3}{5}(6\pi^2\rho_\sigma)^{2/3}\rho_\sigma = \tfrac{3}{10}(3\pi^2\rho)^{2/3}\rho(1\pm\Delta)^{5/3},$$

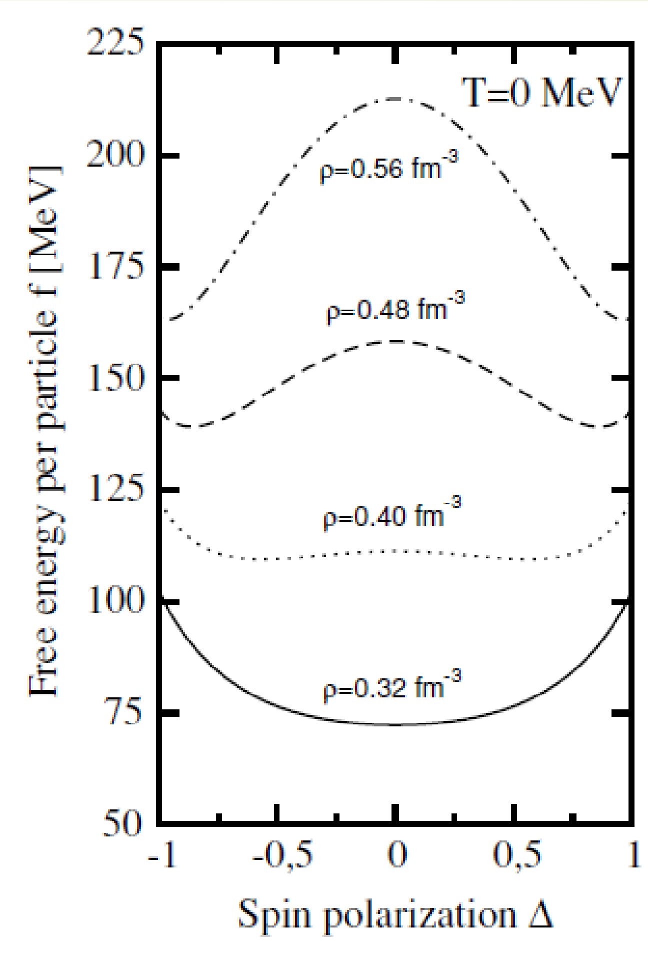
- Considering $\rho\uparrow = \frac{\rho}{2}(1+\Delta)$ and $\rho\downarrow = \frac{\rho}{2}(1-\Delta)$,

$$\begin{aligned}
\frac{E(\Delta)}{N} =& \frac{3\hbar^2}{20m}\left(3\pi^2\right)^{2/3}\rho^{2/3}\left((1-\Delta)^{5/3}+(\Delta+1)^{5/3}\right) \\
&+\frac{3}{40}t_2\left(x_2+1\right)\left(3\pi^2\right)^{2/3}\rho^{5/3}\left((1-\Delta)^{8/3}+(\Delta+1)^{8/3}\right) \\
&+\frac{3}{80}\left(t_1\left(1-x_1\right)+t_2\left(x_2+1\right)\right)\left(3\pi^2\right)^{2/3}\rho^{5/3}\Big((\Delta+1)(1-\Delta)^{5/3} \\
&+(\Delta+1)^{5/3}(1-\Delta)\Big)+\frac{1}{4}t_0\left(1-x_0\right)\left(1-\Delta^2\right)\rho^2 \\
&+\frac{1}{24}t_3\left(1-x_3\right)\rho^{\alpha+1}\left(1-\Delta^2\right).
\end{aligned} \qquad \text{Eq. 6}$$

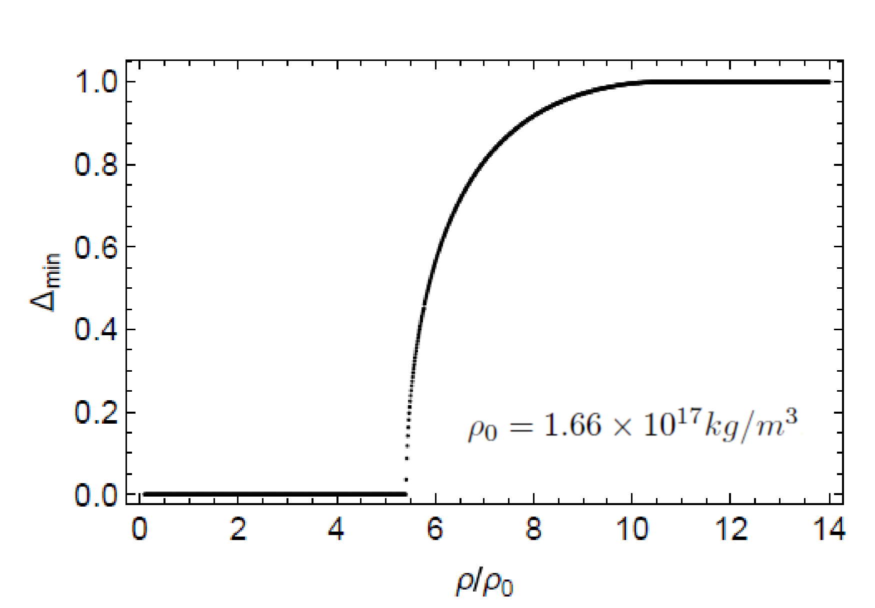
- We need to find a polarization parameter Δ that minimizes the energy for a specific density,

- Energy is symmetry as a function of the polarization,
- The ground state of the system is not necessarily in the polarized or unpolarized phase.

Rios and Polls, PRC, 2005



- **Fig. 1.** Neutron matter free energy per particle atzero temperature as a function of polarization for several densities.



- **Fig. 2.** Polarization associated to the minimum of the energy for neutron matter as a function of density

- The EOS of FM neutron matter is softer than NFM one.

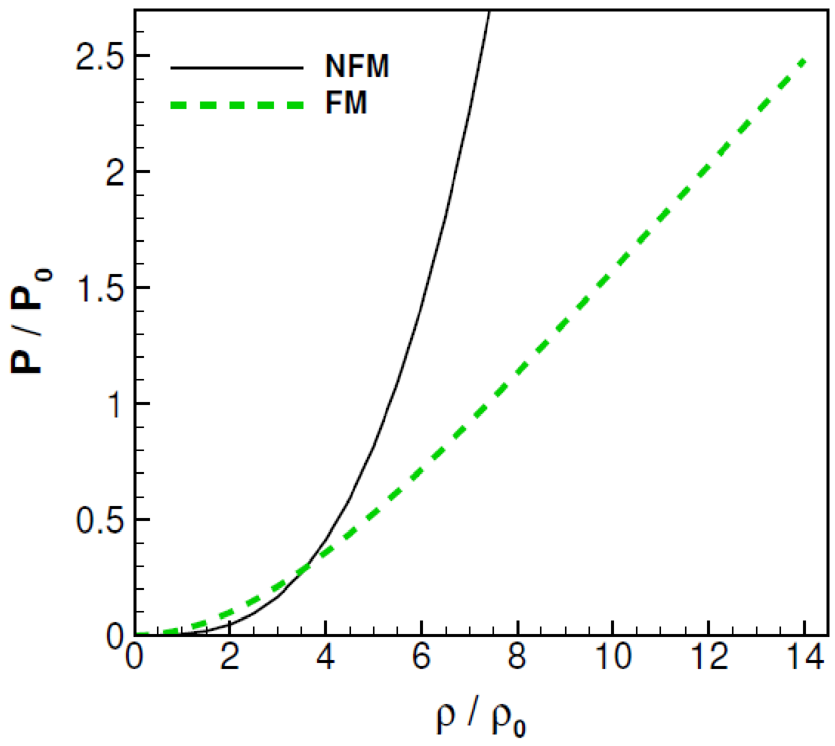
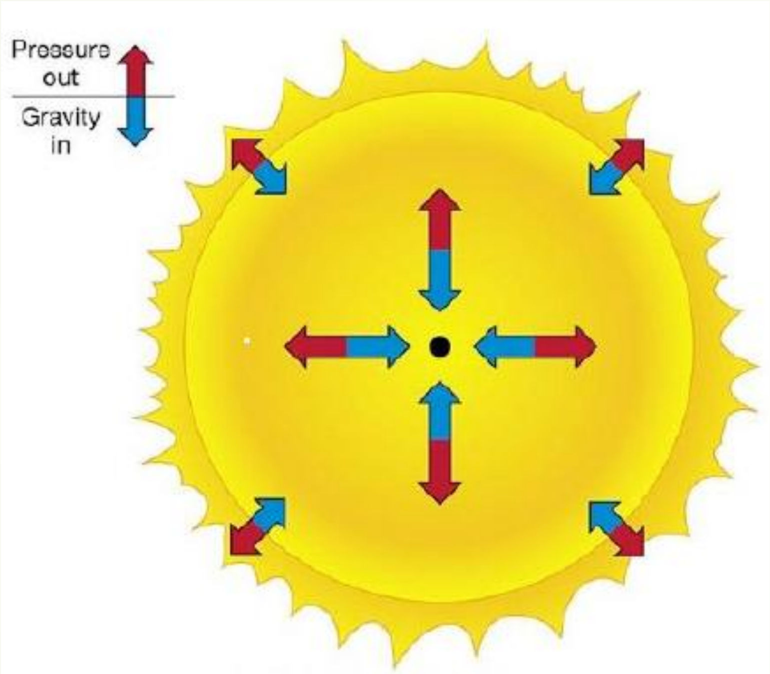


Figure 3. Pressure, P, of the non-ferromagnetic (NFM) and ferromagnetic (FM) neutron matter in SLy230a model versus the density, ρ, with $P_0 = 5 \times 10^{34} kg/ms^2$ and $\rho_0 = 1.66 \times 10^{17} kg/m^3$.

Neutron Star's Structure

Physics of Many-Body Systems



Gravity Theories

Gravity

General Relativity (GR)

Scalar-Tensor Theory of Gravity (STT)

Scalar-Tensor Theories of Gravity (STTs)

Motivations

- The standard GR framework do not explain the acceleration expansion of the Universe very well
- This problem can be solved by considering STTs
- STTs have fruitful applications in cosmology, both in the models of cosmic inflation and dark sector

How we can study them

- Solar system experiments (the weak-field regime) do not give more information about STTs,
- Some of important feature of it (scalar effects) arise in the strong-field regime,

The Jordan Frame

- The spacetime line element for a spherical symmetry,

$$ds^2 = -N(r)^2 dt^2 + A(r)^2 dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

- We denote the physical metric or Jordan metric by

$$\tilde{g}_{\mu\nu} := a(\phi)^2 g_{\mu\nu}$$

And a(ϕ) is the coupling function,

$$\text{Model 1 (M1)}: \qquad a(\phi) = \left[\cosh\left(\sqrt{3}\beta(\phi - \phi_0)\right)\right]^{\frac{1}{3\beta}}$$

$$\text{Model 2 (M2)}: \qquad a(\phi) = e^{\frac{1}{2}\beta(\phi-\phi_0)^2},$$

The Spontaneous Scalarization

- When coupling constant is sufficiently negative i.e. $\beta \lesssim -4.35$, new configurations with a nonzero scalar field can arise
- The phenomenon of spontaneous scalarization is happened
- The phenomenon of spontaneous scalarization can lead to significant deviations of the basic neutron star properties from GR

Field equations in STT

- This class of theories is defined by the following action written in geometric units (c=G=1)

$$S[g_{\mu\nu};\phi;\Psi_m] = \frac{1}{16\pi}\int d^4x\sqrt{-g}\,(R - 2\nabla_\mu\phi\nabla^\mu\phi) + S_m[\Psi_m; a(\phi)^2 g_{\mu\nu}],$$

- The field equations obtained from varying the action in Eq. (3)with respect to the metric $g_{\mu\nu}$and the scalar field ϕ,

$$\begin{aligned} G_{\mu\nu} - 2\nabla_\mu\phi\nabla_\nu\phi + g_{\mu\nu}\nabla_\rho\phi\nabla^\rho\phi &= 8\pi a^2 \tilde{T}_{\mu\nu}, \\ \nabla^\mu\nabla_\mu\phi &= -4\pi a^4 \alpha \tilde{T}, \end{aligned}$$

- considering the neutron star as a perfect fluid with stress-energy tensor, $\tilde{T}^{\mu\nu} = \tilde{\epsilon}\tilde{u}^\mu\tilde{u}^\nu + \tilde{p}(\tilde{g}^{\mu\nu} + \tilde{u}^\mu\tilde{u}^\nu),$

- stress-energy tensor is conserved, $\tilde{\nabla}_\nu \tilde{T}^{\mu\nu} = 0,$

TOV equations in STT

- The neutron star's structure equations (TOV equations) in STT,

1 $$\frac{dm}{dr} = 4\pi r^2 a^4 \tilde{\epsilon} + \frac{r}{2}(r-2m)\left(\frac{d\phi}{dr}\right)^2 \quad (8)$$

2 $$\frac{d\ln N}{dr} = \frac{4\pi r^2 a^4 \tilde{p}}{r-2m} + \frac{r}{2}\left(\frac{d\phi}{dr}\right)^2 + \frac{m}{r(r-2m)} \quad (9)$$

3 $$\frac{d^2\phi}{dr^2} = \frac{4\pi r a^4}{r-2m}\left[\alpha(\tilde{\epsilon}-3\tilde{p}) + r(\tilde{\epsilon}-\tilde{p})\frac{d\phi}{dr}\right] - \frac{2(r-m)}{r(r-2m)}\frac{d\phi}{dr} \quad (10)$$

4 $$\frac{d\tilde{p}}{dr} = -(\tilde{\epsilon}+\tilde{p})\left[\frac{4\pi r^2 a^4 \tilde{p}}{r-2m} + \frac{r}{2}\left(\frac{d\phi}{dr}\right)^2 + \frac{m}{r(r-2m)} + \alpha\frac{d\phi}{dr}\right], \quad (11)$$

The fifth eq.
EOS

Mendes and Ortiz, PRD, 2016

Numerical methods

- The boundary conditions

$$m(0) = 0, \quad \lim_{r\to\infty} N(r) = 1, \quad \lim_{r\to\infty} \phi(r) = \phi_0,$$

$$\frac{d\phi}{dr}(0) = 0, \quad \tilde{p}(0) = p_c, \quad \tilde{p}(R_s) = 0,$$

- and

$$\phi_s - \phi_0 + \frac{2\psi_s}{\sqrt{\dot{\nu}_s^2 + 4\psi_s^2}} \operatorname{arctanh}\left[\frac{\sqrt{\dot{\nu}_s^2 + 4\psi_s^2}}{\dot{\nu}_s + 2/R_s}\right] = 0$$

$\psi_s := (d\phi/dr)_s$ and $\dot{\nu}_s := 2(d\ln N/dr)|_s = R_s\psi_s^2 + 2m_s/[R_s(R_s - 2m_s)]$

RESULTS

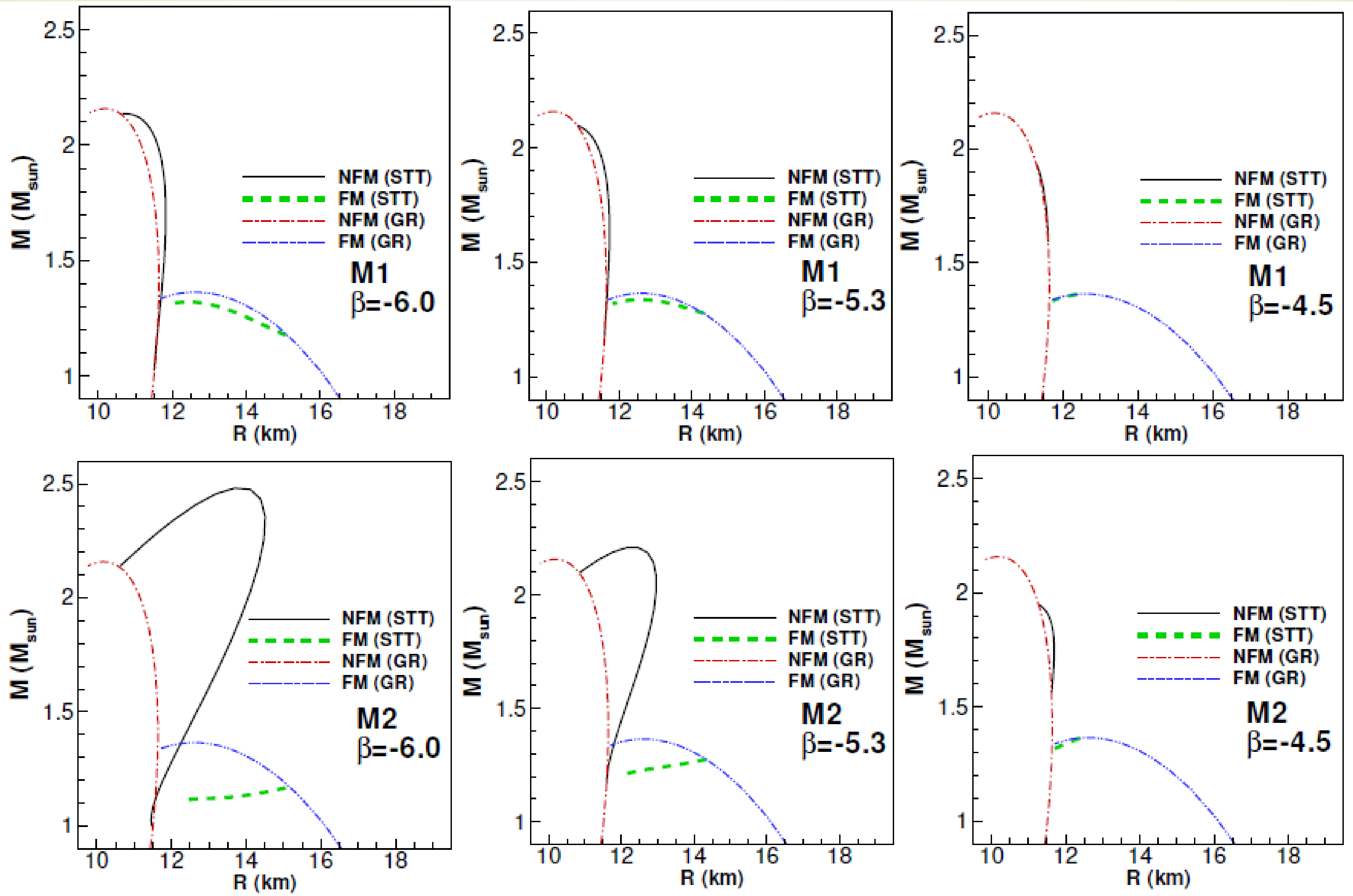


Figure 4. Mass versus the radius for NFM and FM neutron stars in STT and GR applying M1 and M2 with different values of the coupling constant, β. The lines which show the allowed region for neutron stars are also presented.

Conclusion

- The EOS of FM neutron matter is softer than NFM neutron matter
- The scalarized NSs have different configurations from NSs in GR
- The maximum mass of NFM of neutron stars is higher than FM of neutron matter
- The maximum mass in STT (2.48 $M_\odot$) is higher than maximum mass in GR.