Relativistic density functional approach to quark matter in compact stars

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The Modern Physics of Compact Stars and Relativistic Gravity

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Outline

- **Relativistic density functional approach**
- $\mu = 0$ **and** $T \neq 0$
- $\mu \neq 0$ **and** $T = 0$
- **Modelling compact stars**

Relativistic density functional

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ - \hat{m})q + \mathcal{L}_V + \mathcal{L}_D - \mathcal{U}$$

- **Vector and diquark interaction**

$$\mathcal{L}_V = -G_V(\overline{q}\gamma_\mu q)^2, \quad \mathcal{L}_D = G_D \sum_{A=2,5,7} (\overline{q}i\gamma_5\tau_2\lambda_A q^c)(\overline{q}^c i\gamma_5\tau_2\lambda_A q)$$

- χ**-symmetric density functional**

$$(\overline{q}q)^2 + (\overline{q}i\vec{\tau}\gamma_5 q)^2 \ is\ \chi - invariant \quad \Rightarrow \quad \mathcal{U} = \mathcal{U}\left[(\overline{q}q)^2 + (\overline{q}i\vec{\tau}\gamma_5 q)^2\right]$$

$$\mathcal{U} = G_0\left[(1+\alpha)\langle\overline{q}q\rangle^2 - (\overline{q}q)^2 - (\overline{q}i\vec{\tau}\gamma_5 q)^2\right]^{\frac{1}{3}}$$

G_0 – coupling constant

$\alpha \geq 0$ – controls quark effective mass in the vacuum

Expansion of RDF around $\langle\bar{q}q\rangle$ and $\langle\bar{q}i\vec{\tau}\gamma_5 q\rangle=0$

$$\mathcal{U} = U + (\bar{q}q - \langle\bar{q}q\rangle)\,\Sigma_{MF} - G_S\,(\bar{q}q - \langle\bar{q}q\rangle)^2 - G_{PS}\,(\bar{q}i\vec{\tau}\gamma_5 q)^2 + \dots$$

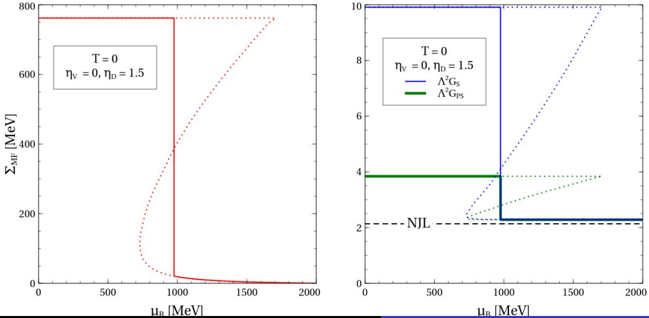
- **Expansion coeffisients**

  0^{th} order: $U = \mathcal{U}_{\langle\bar{q}q\rangle,\langle\bar{q}i\vec{\tau}\gamma_5 q\rangle}$

  1^{st} order: $\Sigma_{MF} = \frac{\partial U}{\partial\langle\bar{q}q\rangle}$ - mean field self-energy of quark

  2^{nd} order: $G_S = -\frac{1}{2}\frac{\partial^2 U}{\partial\langle\bar{q}q\rangle^2}$ - effective coupling in scalar channel

  2^{nd} order: $G_{PS} = -\frac{1}{6}\frac{\partial^2 U}{\partial\langle\bar{q}i\vec{\tau}\gamma_5 q\rangle^2}$ - effective coupling in pseudoscalar channel



$$\eta_i \equiv \frac{G_i}{G_S^\infty}$$

Comparison to NJL model

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ - \underbrace{(m + \Sigma_{MF})}_{\text{effective mass } m^*})q + G_S(\overline{q}q)^2 + G_{PS}\left(\overline{q}i\vec{\tau}\gamma_5 q\right)^2 + \ldots$$

- **Similarities:**
  1. current-current interaction
  2. scalar, pseudoscalar, vector, diquark, ... channels

- **Diferences:**
  1. high m^* at low T and/or μ - **phenomenological confinement**

     $$\langle\overline{q}q\rangle = \langle\overline{q}q\rangle_0 \;\Rightarrow\; m^* = m - \frac{2G_0}{3}\langle\overline{q}q\rangle_0^{1/3}\alpha^{-2/3} \;-\; \text{diverges at } \alpha \to 0$$

  2. medium dependent couplings

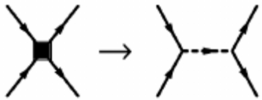
     low T and/or μ $\Rightarrow$ $G_S \neq G_{PS}$ $\Rightarrow$ broken $\chi-$symmetry

     high T and/or μ $\Rightarrow$ $G_S = G_{PS}$ $\Rightarrow$ restored $\chi-$symmetry

Bosonization of (pseudo)scalar interaction

- **Hubbard-Stratonovich transformation**

$$\exp\left[\int dx\ G_\phi(\overline{q}\hat{\Gamma}q)^2\right] = \int [D\phi]\exp\left[-\int dx\left(\frac{\phi^2}{4G_\phi} + \overline{q}\phi\hat{\Gamma}q\right)\right]$$



- **Bosonized Lagrangian** ($m^* = m + \Sigma_{MF}$ **- effective quark mass)**

$$\begin{aligned}\mathcal{L} &= \overline{q}(i\partial\!\!\!/ - m^*)q + \mathcal{L}_V + \mathcal{L}_D - U + \langle\overline{q}q\rangle\Sigma_{MF} \\ &- \frac{\sigma^2}{4G_S} - \frac{\vec{\pi}^2}{4G_{PS}} - \overline{q}(\sigma + i\vec{\pi}\vec{\tau}\gamma_5)q + \sigma\langle\overline{q}q\rangle\end{aligned}$$

- **Field equations for** σ **and** $\vec{\pi}$

$$\begin{cases}\sigma = 2G_S(\langle\overline{q}q\rangle - \overline{q}q) \\ \vec{\pi} = 2G_{PS}(\langle\overline{q}i\vec{\tau}\gamma_5 q\rangle - \overline{q}i\vec{\tau}\gamma_5 q)\end{cases} \Rightarrow \langle\sigma\rangle = \langle\vec{\pi}\rangle = 0 \Rightarrow \sigma,\vec{\pi} - \text{beyond MF}$$

comment: $\langle\sigma\rangle = 0$ does not assume χ-symmetry since $\langle\overline{q}q\rangle \neq 0$

Thermodynamic potential

- **Fully bosonized Lagrangian**

$$\mathcal{L} = \overline{Q}\hat{S}_{NG}^{-1}Q + \underbrace{\frac{\omega_\mu\omega^\mu}{4G_V} - \frac{\Delta_A\Delta_A^*}{4G_D} - \frac{\sigma^2}{4G_S} - \frac{\vec{\pi}^2}{4G_{PS}}}_{\text{meson part } \mathcal{L}_{\text{mes}}} - \underbrace{U + \langle\overline{q}q\rangle\Sigma_{MF}}_{\text{MF part } \mathcal{L}_{\text{MF}}}$$

$Q = \frac{1}{\sqrt{2}}\begin{pmatrix} q \\ q^c \end{pmatrix}$ - Nambu-Gorkov quark bispinor

$\hat{S}_{NG}^{-1} = \begin{pmatrix} i\partial\!\!\!/ + \psi\!\!\!/ - m^* - \sigma - i\vec{\tau}\vec{\pi}\gamma_5 & i\Delta_A\gamma_5\tau_2\lambda_A \\ i\Delta_A\gamma_5\tau_2\lambda_A & i\partial\!\!\!/ + \psi\!\!\!/ - m^* - \sigma - i\vec{\tau}\vec{\pi}\gamma_5 \end{pmatrix}$ - $\mathrm{Nambu-Gorkov}$ quark propagator

- **Statistical partition (quark fields are integrated out)**

$$\mathcal{Z} = \int [D\omega_\mu][D\Delta_A][D\Delta_A^*][D\sigma][D\vec{\pi}] \exp\left[\int dx\,(\mathcal{L}_{mes} + \mathcal{L}_{MF}) + \frac{1}{2}\ln\det(\beta S_{NG}^{-1})\right]$$

$$\Omega = -\frac{1}{\beta V}\ln\mathcal{Z}$$

Zero chemical potential

- **Quark propagator**

$$\mu = 0 \quad \Rightarrow \quad \omega_\mu = 0,\ \Delta_A = 0 \quad \Rightarrow \quad \hat{S}_{NG}^{-1} = \begin{pmatrix} \hat{S}_{MF}^{-1} + \Sigma_{mes} & 0 \\ 0 & \hat{S}_{MF}^{-1} + \Sigma_{mes} \end{pmatrix}$$

$\hat{S}_{MF}^{-1} = i\not{\partial} - m^* + \mu\gamma_0$ - MF propagator of quarks

$\Sigma_{mes} = -\sigma - i\vec{\tau}\vec{\pi}\gamma_5$ - beyond MF quark self-energy due to meson correlations

- **Gaussian aproximation**

$$\begin{aligned} \frac{1}{2}\ln\det(\beta\hat{S}_{NG}^{-1}) &= \mathrm{tr}\ln(\beta\hat{S}_{MF}^{-1}) + \mathrm{tr}\ln(1 + \hat{S}_{MF}\Sigma_{mes}) \\ &\simeq \mathrm{tr}(\beta\hat{S}_{MF}^{-1}) + \mathrm{tr}(\hat{S}_{MF}\Sigma_{mes}) + \frac{1}{2}\mathrm{tr}(\hat{S}_{MF}\Sigma_{mes})^2 \end{aligned}$$

- **Polarization operators of bosons**

$$\Pi_\sigma = -\mathrm{tr}(\hat{S}_{MF})^2, \quad \Pi_\pi = -\mathrm{tr}(\hat{S}_{MF}i\gamma_5)^2$$

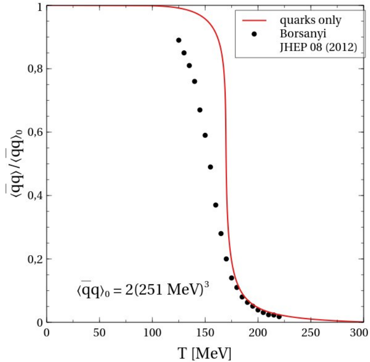
Zero chemical potential

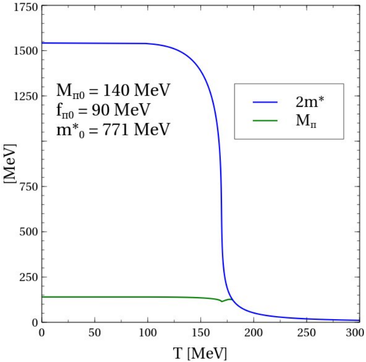
$$\Omega = \underbrace{\Omega_{MF}}_{\text{mean field}} + \underbrace{\frac{1}{2\beta V}\mathrm{tr}\left(\frac{1}{2G_S} - \Pi_\sigma\right)}_{\sigma-\text{mesons}} + \underbrace{\frac{3}{2\beta V}\mathrm{tr}\left(\frac{1}{2G_{PS}} - \Pi_\pi\right)}_{\pi-\text{mesons}}, \quad \langle \bar{q}q \rangle = \frac{\partial \Omega}{\partial m}$$

- **Meson propagators and masses**

$$D_\sigma^{-1} = \frac{1}{2G_S} - \Pi_\sigma, \quad D_\pi^{-1} = \frac{1}{2G_{PS}} - \Pi_\pi$$

$D_i^{-1} = 0 \Rightarrow$ dispersion relation $E_i(k)$ with mass $M_i = E_i(0)$ for $i = \sigma, \pi$





Finite chemical potential (mean field level)

$$\Omega = -\frac{\omega^2}{4G_V} + \frac{\Delta^2}{4G_D} + U - \langle\bar{q}q\rangle\Sigma_s$$

$$- 2\sum_f \int \frac{dk}{(2\pi)^3}\Bigg[\overbrace{\frac{\xi_f^+ + \xi_f^-}{2} + T\ln\left(1+e^{-\beta\xi_f^+}\right) + T\ln\left(1+e^{-\beta\xi_f^-}\right)}^{1\ \text{unpaired color state}}$$

$$\underbrace{+E_f^+ + E_f^- + 2T\ln\left(1+e^{-\beta E_f^+}\right) + 2T\ln\left(1+e^{-\beta E_f^-}\right)}_{2\ \text{paired color states}}\Bigg]$$

- **Single quark energies**

$$\xi_f^\pm = \sqrt{k^2 + m_f^{*2}} \mp \mu_f^*, \quad E_f^\pm = \sqrt{\xi_f^{\pm\,2} + \Delta^2}, \quad \mu_f^* = \mu_f - \omega$$

- **Vector field, diquark gap, χ-condensate**

$$\frac{\partial\Omega}{\partial\omega} = 0, \qquad \frac{\partial\Omega}{\partial\Delta} = 0, \quad \langle\bar{q}q\rangle = \sum_f \frac{\partial\Omega}{\partial m_f}$$

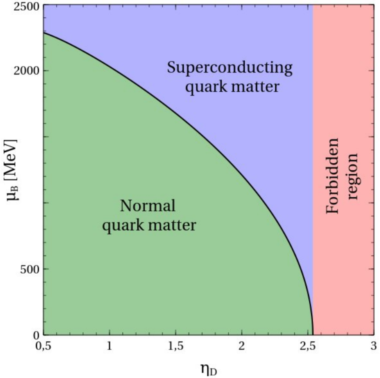
Superconductivity onset ($T = 0$)

- **Diquark gap equation**

$$\frac{\partial \Omega}{\partial \Delta} = 0 \quad \Rightarrow \quad \Delta = 0 \quad \text{or} \quad 1 = 4G_D \sum_f \int \frac{dk}{(2\pi)^3} \left[\frac{1}{E_f^+} + \frac{1}{E_f^-} \right]$$

- **Superconductivity onsets when two solutions coincide**

$$\left.\frac{\partial^2 \Omega}{\partial \Delta^2}\right|_{\Delta=0} = \frac{1}{2G_D} - 2\sum_f \int \frac{dk}{(2\pi)^3} \left[\frac{1}{E_f^+} + \frac{1}{E_f^-} \right]_{\Delta=0} = 0 \quad \Rightarrow \quad \mu_B = \mu_B(G_D)$$



No vacuum superconductivity

$$\Downarrow$$

$$\eta_D \lesssim 2.5$$

Effective mass and diquark gap

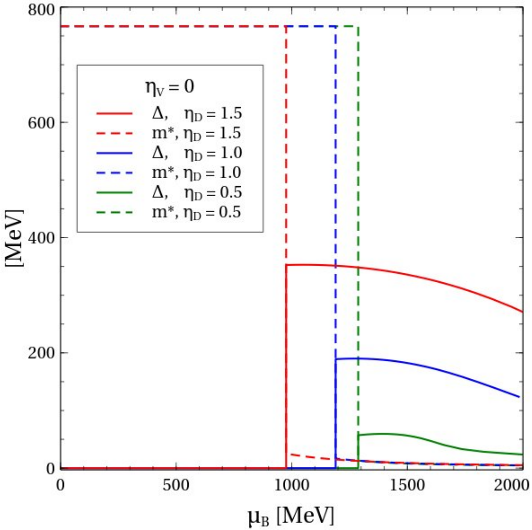
- **Effective mass**

$$m^* = m + 4G_D \sum\!\!\!\!\!\!\int \frac{m_f}{\epsilon_f}$$

$$\times \left[\frac{\xi_f^+}{E_f^+} + \frac{\xi_f^-}{E_f^-} + 1 - \theta(-\xi_f^+) \right]$$

- **Diquark gap**

$$1 = 4G_D \sum\!\!\!\!\!\!\int \left[\frac{1}{E_f^+} + \frac{1}{E_f^-} \right]$$



Hybrid EoS

- **Charge neutrality:** electrons
- **β-equilibrium:**

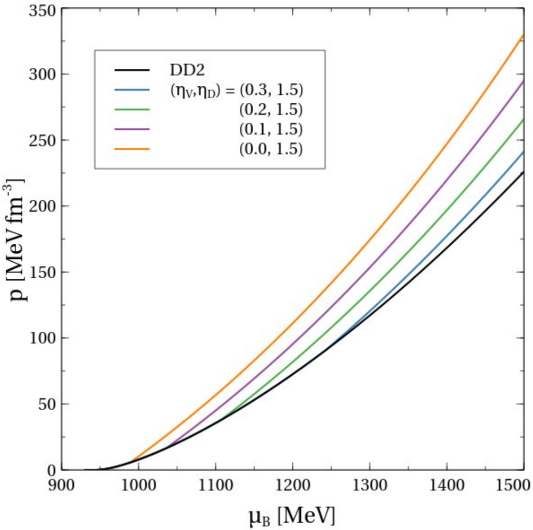
$$\mu_d = \mu_u + \mu_e$$

- **Hadron EoS:** DD2

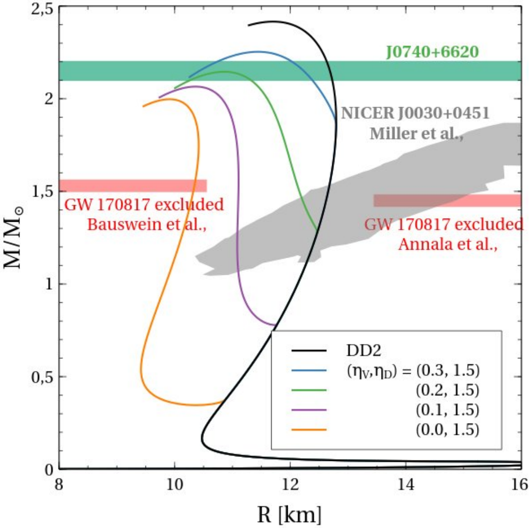
S. Typel et al., PRC 81, 015803 (2010)

- **Maxwell construction:**

$$p_q(\mu_B^c) = p_h(\mu_B^c)$$



Compact stars with quark core



Conclusions

- χ-symmetric formulation of the String-Flip Model, derivation of the effective "confining" NJL model
- Medium dependent quark-meson couplings, derived consistently with model and symmetries
- Straightforward addition of vector and diquark channels
- Good agreement with the observational data even at weak repulsion
- Next steps:
  1. strangeness
  2. other mesonic states and baryons beyond mean-field
  3. unified quark-hadron EoS (hadron dissociation via Mott effect, hadronic correlations via Beth-Uhlenbeck approach)
  4. ...