High-Density Nuclear Symmetry Energy from Neutron Star Observations

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NSF

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U.S. DEPARTMENT OF ENERGY

Outline

(1) Why do we need to study nuclear symmetry energy $E_{sym}(\rho)$?
Why is the $E_{sym}(\rho)$ so uncertain especially at high densities?

(2) What have we learned about the symmetry energy from properties of canonical neutron stars ($\sim 1.4M_{\odot}$)?

(3) What can we learn about the high-dense symmetry energy from NICER+XMM-Newton's observation of the mass and radius of PSR J0740+6620 ($2.08 \pm 0.07M_{\odot}$) (previous talk by Cole Miller)?

Empirical parabolic law of the EOS of cold, neutron-rich nucleonic matter

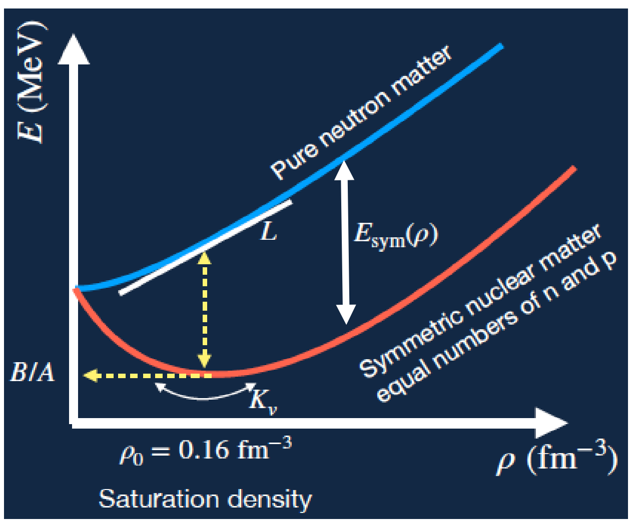
symmetry energy

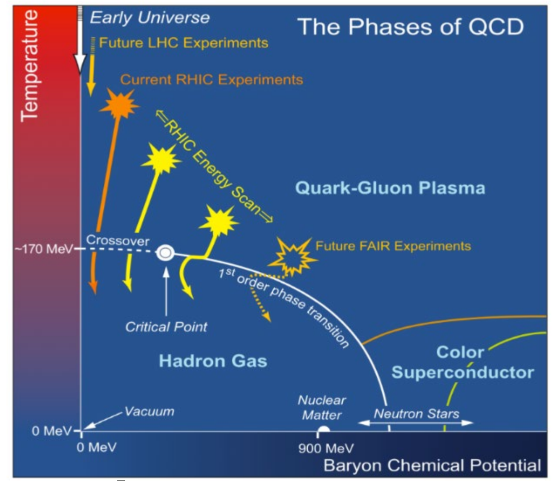
Isospin asymmetry δ

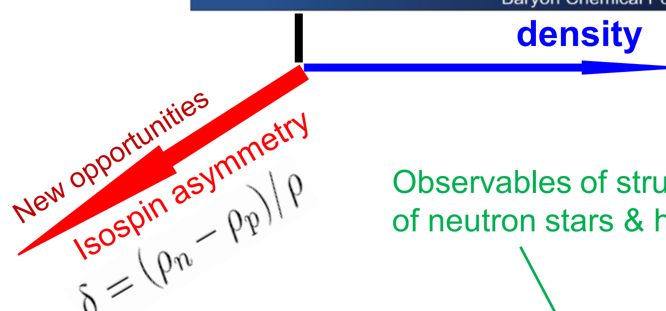
$$E(\rho_n,\rho_p) = E_0(\rho_n=\rho_p) + E_{sym}(\rho)\left(\frac{\rho_n-\rho_p}{\rho}\right)^2 + o(\delta^4)$$

Energy per nucleon in symmetric matter

Energy in asymmetric nucleonic matter







Observables of structures and collisions of neutron stars & heavy nuclei

$$E_{\mathrm{sym}}(\rho) = E_{\mathrm{sym}}(\rho_i) + \int_{\rho_i}^{\rho} \frac{P_{\mathrm{PNM}}(\rho_v) - P_{\mathrm{SNM}}(\rho_v)}{\rho_v^2}\, d\rho_v$$

Fundamental Microphysics Theories underlying each term in the EOS , what …, why …., where …how

Experimental and Observational Macrophysics underlying each observable and phenomenon, what …, why …., where …how

Empirical parameterizations

Transport model simulations of heavy-ion collisions, energy density functionals for nuclear structures, Bayesian inferences of EOS, properties of neutron stars, waveforms of gravitational waves, ….

$$E(\rho,\delta) = E_0(\rho) + E_{\rm sym}(\rho)\cdot\delta^2$$

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_0}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3, \quad (2.15)$$

$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{\rm sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_{\rm sym}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 \quad (2.16)$$

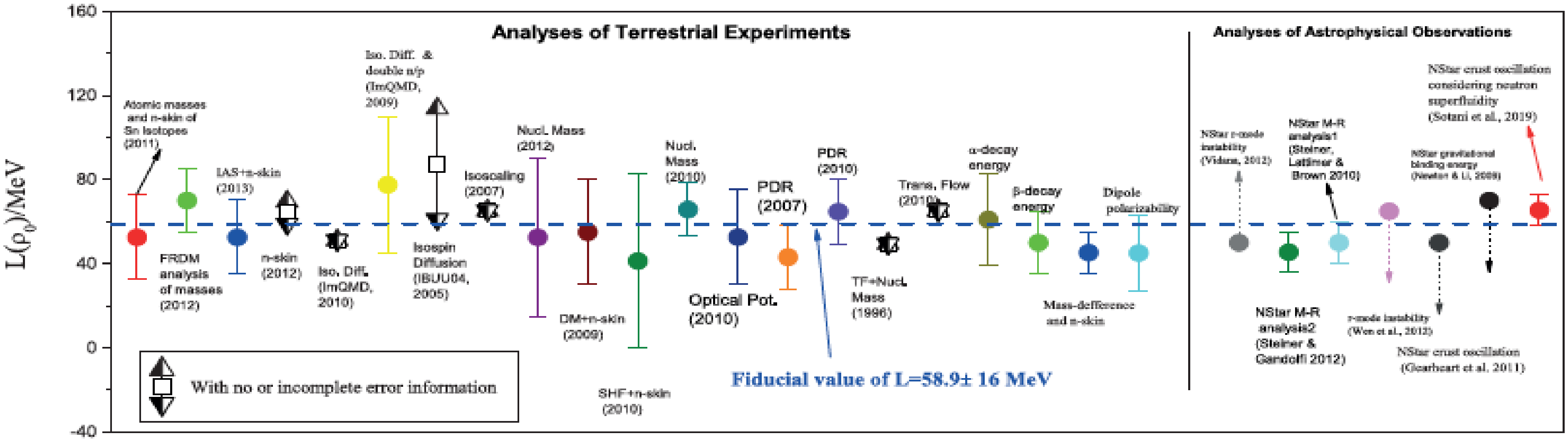
Near the saturation density $\boldsymbol{\rho}_0$, they are Taylor expansions, appropriate for structure studies. Just parameterizations when applied to heavy-ion collisions and the core of neutron stars

"Current" status of the restricted EOS parameter space:

Low density: $K_0 = 240 \pm 20$, $E_{\rm sym}(\rho_0) = 31.7 \pm 3.2$ and $L = 58.7 \pm 28.1$ MeV

High density: $-400 \leq K_{\rm sym} \leq 100$, $-200 \leq J_{\rm sym} \leq 800$, and $-800 \leq J_0 \leq 400$ MeV

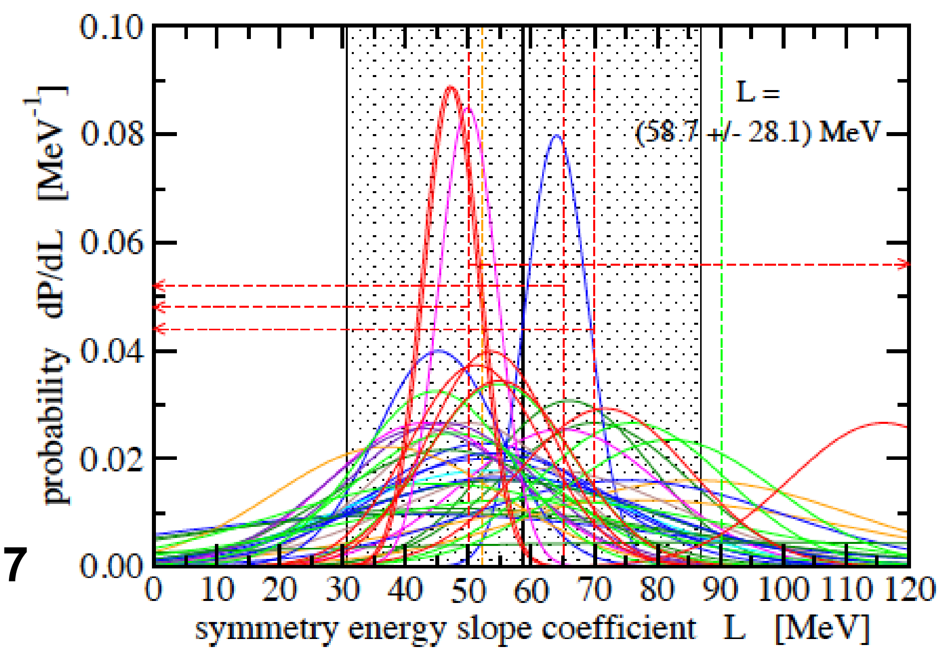
Constraints on L as of 2013 based on 29 analyses of data



Bao-An Li and Xiao Han, **Phys. Lett. B727 (2013) 276**

L=58.7±28.1 MeV

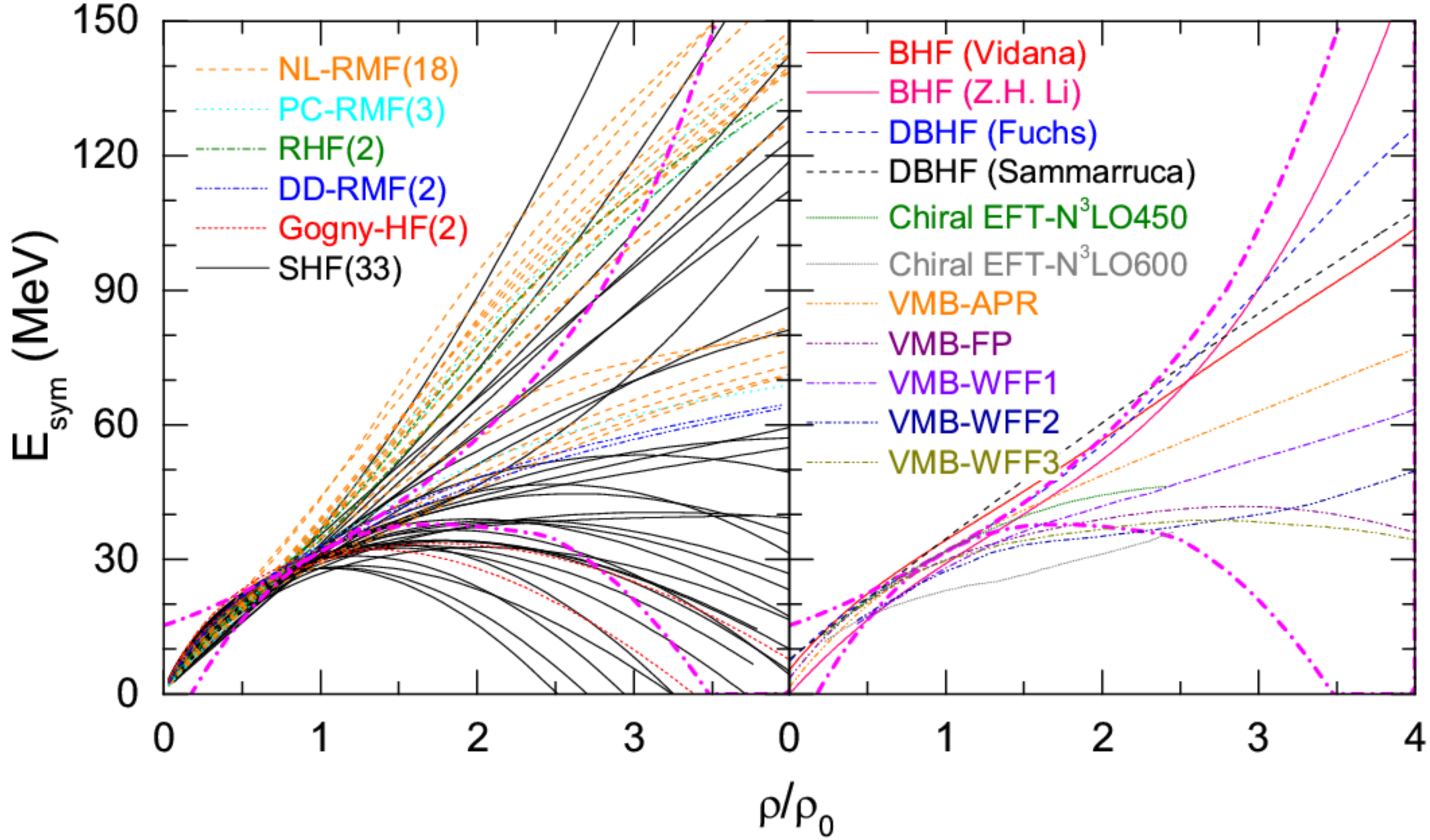
Fiducial value as of 2016 from surveying 53 analyses



M. Oertel, M. Hempel, T. Klähn, S. Typel

Review of Modern Physics 89 (2017) 015007

➢ Predicted high-density nuclear symmetry energy



N.B. Zhang and B.A. Li, EPJA 55, 39 (2019)

Fundamental physics underlying nuclear symmetry energy

Single-nucleon (Lane) potential in isospin-asymmetric matter: A. M. Lane, Nucl. Phys. 35, 676 (1962).

$$U_{n/p}(k,\rho,\delta) = U_0(k,\rho) \pm U_{sym1}(k,\rho)\bullet\delta + U_{sym2}(k,\rho)\bullet\delta^2 + o(\delta^3)$$

Hugenholtz-Van Hove (HVH) theorem:

N.M. Hugenholtz, L. Van Hove, Physica 24 (1958) 363.

$$E_F = \frac{d\xi}{d\rho} = \frac{d(\rho E)}{d\rho} = E + \rho\frac{dE}{d\rho} = E + P/\rho$$

$$E_{sym}(\rho) = \frac{1}{3}\frac{k_F^2}{2M} + \frac{1}{2}U_{sym,1}(\rho,k_F) + \frac{k_F}{6}\left(\frac{\partial U_0}{\partial k}\right)_{k_F} - \frac{1}{6}\frac{k_F^4}{2M^3}$$

S. Fritsch, N. Kaiser, W. Weise, Nuclear Phys. A 750 (2005) 259.

Using the K-matrix theory:

K.A. Brueckner, J. Dabrowski, Phys. Rev. B 134 (1964) 722.
J. Dabrowski, P. Haensel, Phys. Lett. B 42 (1972) 163;

$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} + \frac{1}{2}U_{sym,1}(\rho,k_F), \quad m_0^*(\rho,k) = \frac{m}{1+\frac{m}{\hbar^2 k}\frac{\partial U_0(\rho,k)}{\partial k}},$$

$$L(\rho) = \frac{2}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} - \frac{1}{6}\left(\frac{\hbar^2 k^3}{m_0^{*2}}\frac{\partial m_0^*}{\partial k}\right)|_{k_F} + \frac{3}{2}U_{sym,1}(\rho,k_F) + \frac{\partial U_{sym,1}}{\partial k}|_{k_F}\cdot k_F + 3U_{sym,2}(\rho,k_F),$$

C. Xu, B.A. Li, L.W. Chen and C.M. Ko, NPA 865, 1 (2011)

$$m^*_{n-p} \approx 2\delta\frac{m}{\hbar^2 k_F}\left[-\frac{dU_{sym,1}}{dk} - \frac{k_F}{3}\frac{d^2U_0}{dk^2} + \frac{1}{3}\frac{dU_0}{dk}\right]_{k_F}\left(\frac{m_0^*}{m}\right)^2$$

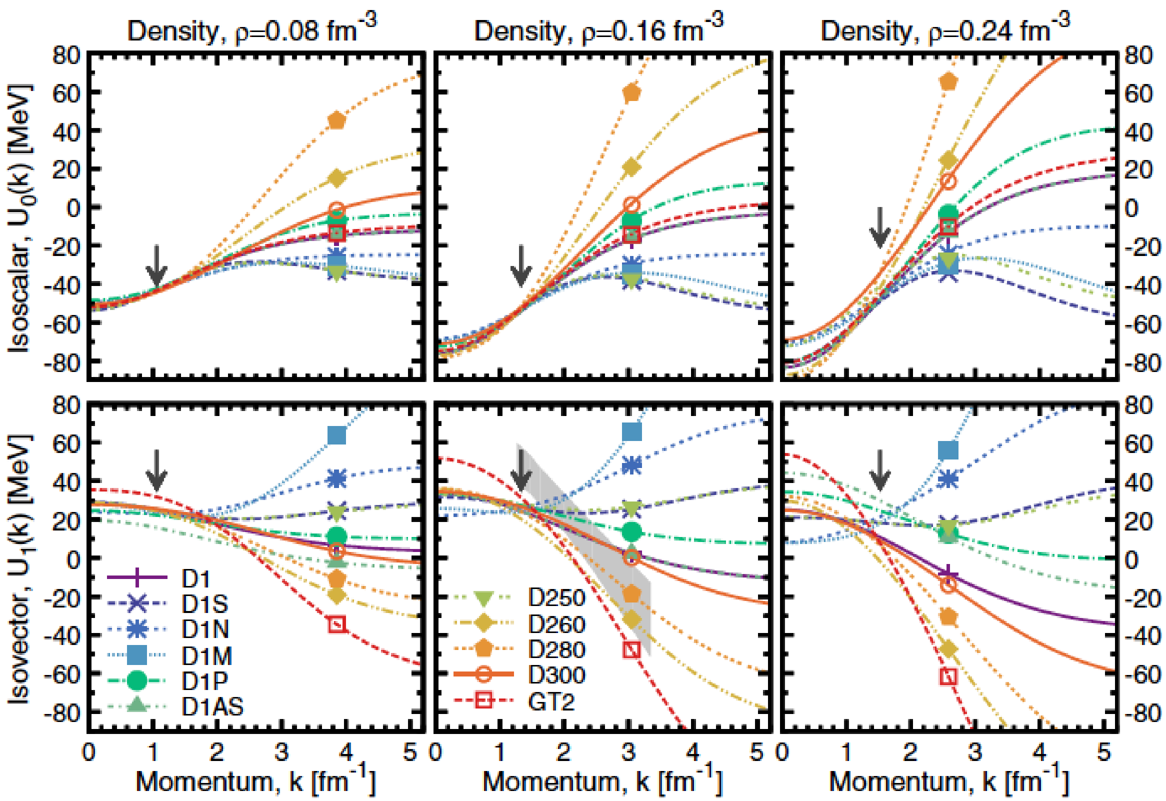
Bao-An Li, Bao-Jun Cai, Lie-Wen Chen and Jun Xu,
Progress in Particle and Nuclear Physics 99 (2018) 29–119

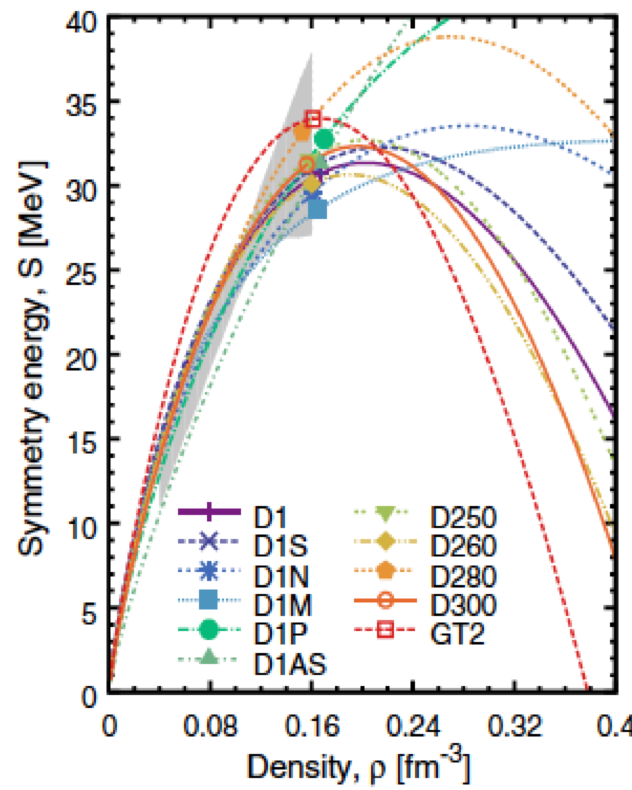
Density and momentum dependence of Isoscalar and Isovector potentials
Gogny Hartree-Fock predictions using 11 popular Gogny (finite-range) forces

PHYSICAL REVIEW C **90**, 054327 (2014)

Isovector properties of the Gogny interaction

Roshan Sellahewa and Arnau Rios





The most fundamental but lest known physics underlying the high-density symmetry energy

Spin-isospin dependence of strong interaction:

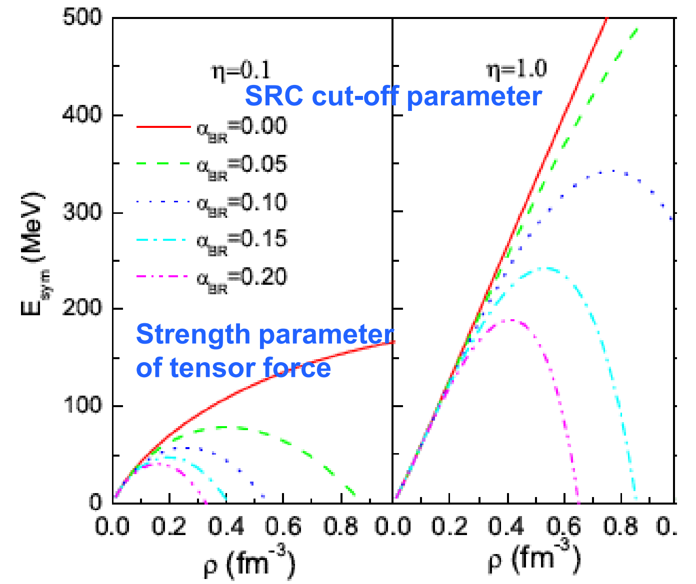
$V_{np}(T_0) \neq V_{np}(T_1)$

$$V_{T0} = V'_{np} \quad \text{(n-p pair in the T=0 state)}$$

$$V_{T1} = V_{nn} = V_{pp} = V_{np} \quad \text{(charge independence in the T=1 state)}$$

Tensor force due to pion and ρ meson exchange MAINLY in the T=0 channel

Effects of tensor force on E_{sym} (ρ) in a deuteron cluster model (i.e. n-p dominance) of nuclear matter



In a simple interacting Fermi gas model, the direct term:

Isospin-dependent correlation function

$$U_{sym}(k_F,\rho) = \frac{1}{4}\rho \int [V_{T1}(r_{ij}) f^{T1}(r_{ij}) - V_{T0}(r_{ij}) f^{T0}(r_{ij})] d^3 r_{ij}$$

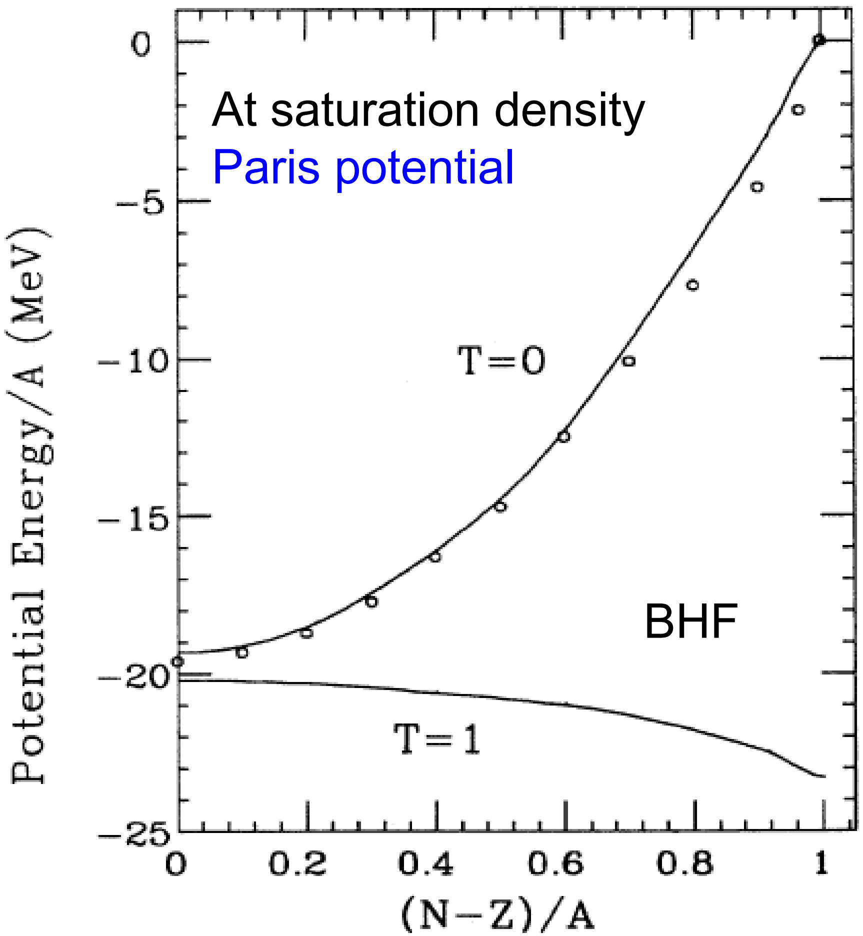
Isospin-dependent effective 2-body interaction

M.A. Preston and R.K. Bhaduri, Structure of the Nucleus, 1975

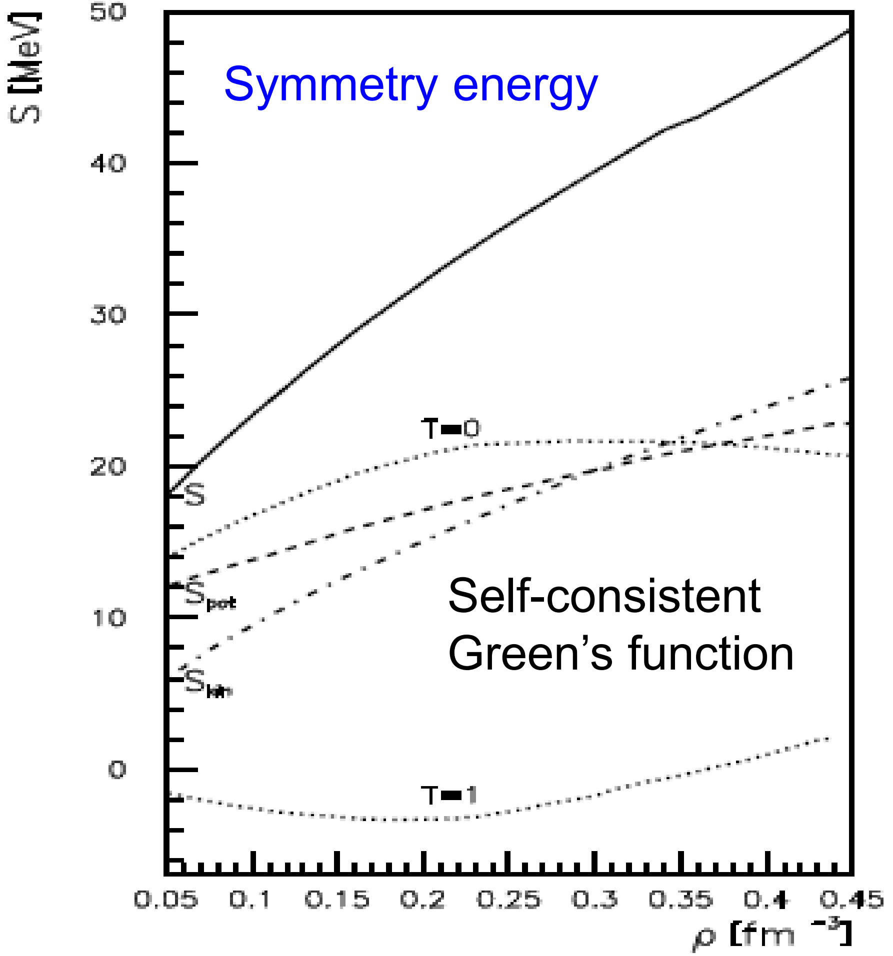
$$f(r) = 0, \text{ for } r < r_c \qquad r_c = \eta(3/4\pi\rho)^{1/3}$$

Chang Xu and Bao-An Li, PRC81, 064612 (2010)

Dominance of the isosinglet (T=0) interaction



I. Bombaci and U. Lombardo PRC 44, 1892 (1991)

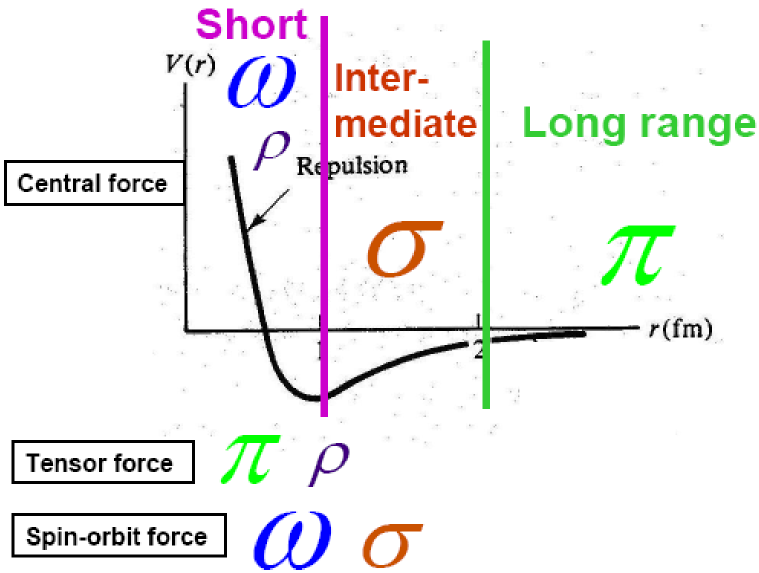


A.E.L. Dieperink,[1] Y. Dewulf,[2] D. Van Neck,[2] M. Waroquier,[2] and V. Rodin[3]

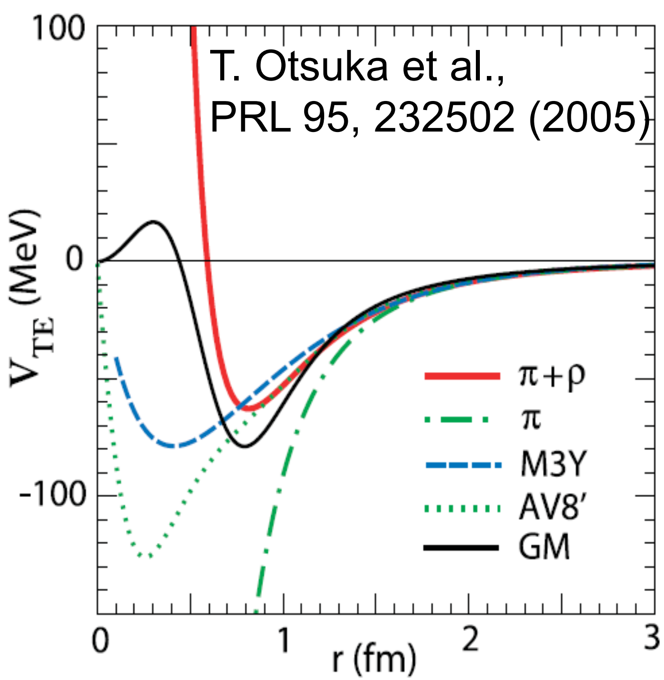
PRC68, 064307 (2003)

$$E_{sym}(\rho) = \frac{1}{2}\frac{\partial^2 E}{\partial \delta^2} \approx E(\rho)_{\text{pure neutron matter}} - E(\rho)_{\text{symmetric nuclear matter}}$$

The short and long range tensor force



Lecture notes of R. Machleidt
CNS summer school, Univ. of Tokyo
Aug. 18-23, 2005



$\pi(138)$ $$V_\pi = \frac{f_{\pi NN}^2}{8m_\pi^2}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\left[-\vec{\sigma}_1\cdot\vec{\sigma}_2 - S_{12}(\vec{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$$ Long-ranged tensor force

$\sigma(600)$ $$V_\sigma \approx \frac{g_\sigma^2}{\vec{q}^2+m_\sigma^2}\left[-1 - \frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$$ intermediate-ranged, attractive central force plus LS force

$\omega(782)$ $$V_\omega \approx \frac{g_\omega^2}{\vec{q}^2+m_\omega^2}\left[+1 - 3\frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$$ short-ranged, repulsive central force plus strong LS force

$\rho(770)$ $$V_\rho = \frac{f_\rho^2}{12M^2}\frac{\vec{q}^2}{\vec{q}^2+m_\rho^2}\left[-2\vec{\sigma}_1\cdot\vec{\sigma}_2 + S_{12}(\vec{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$$ short-ranged tensor force, opposite to pion

Tensor force contribution to the potential part of the symmetry energy within a simple model

G.E. Brown and R. Machleidt, Phys. Rev. C50, 1731 (1994).

S.-O. Bacnman, G.E. Brown and J.A. Niskanen, Phys. Rep. 124, 1 (1985).

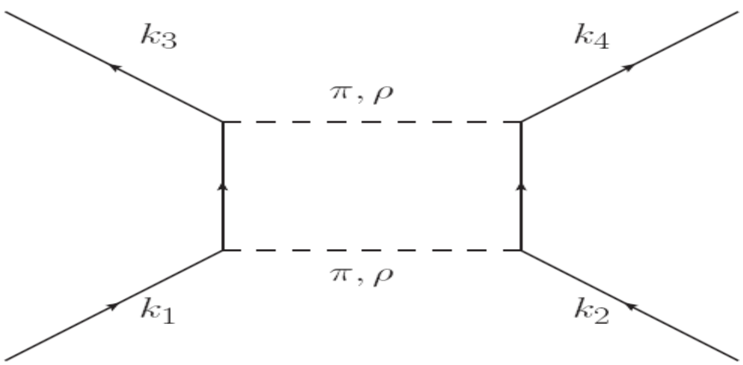
EFFECTIVE TENSOR INTERACTION IN NUCLEI *

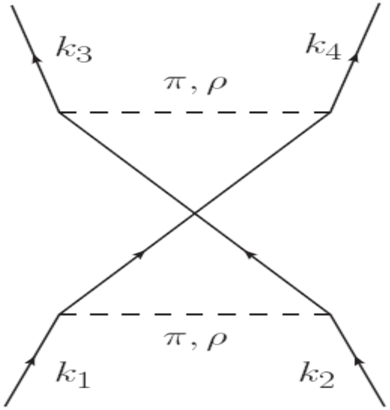
T. T. S. KUO and G. E. BROWN **PLB 18, 54 (1965)**

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

Received 25 June 1965

$$< V_{\rm sym} >= \frac{12}{e_{\rm eff}} < [V_t(\mathbf{r})]^2 >$$



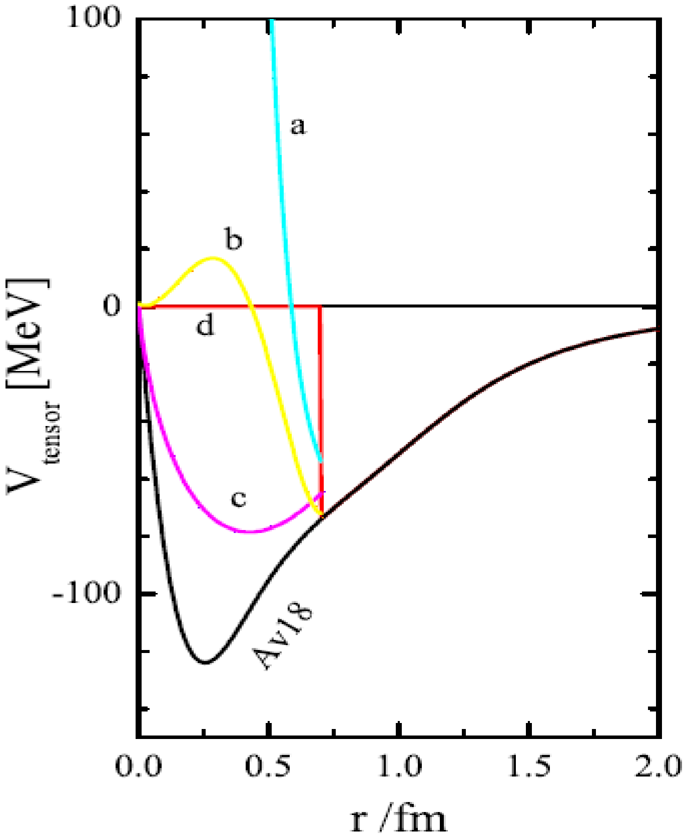


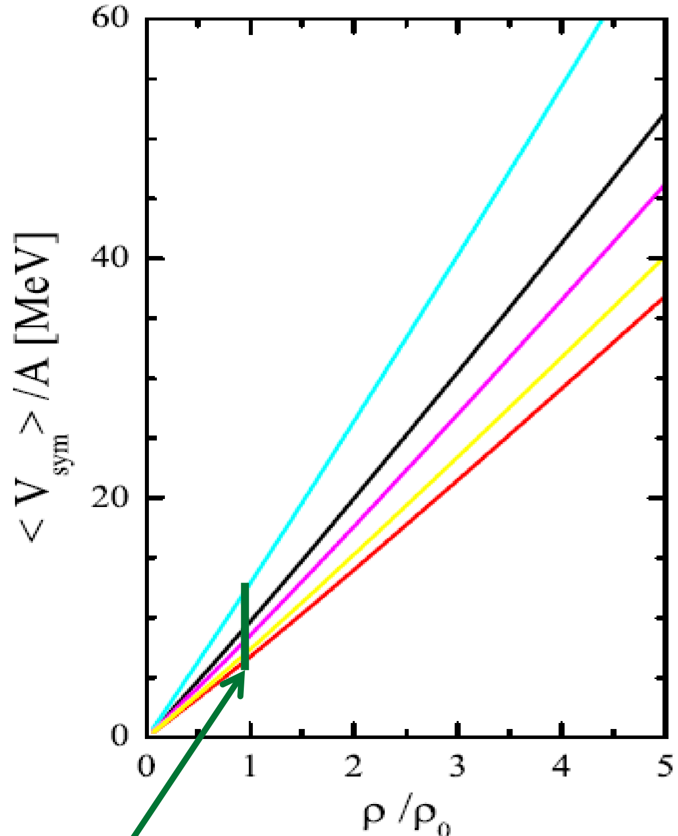
$$\frac{< V_{\rm sym} >}{A} = \frac{12}{e_{eff}} \cdot \frac{k_F^3}{12\pi^2}\{\frac{1}{4}\int V_t^2(r)d^3r - \frac{1}{16}\int [\frac{3j_1(k_F r)}{k_F r}]^2 V_t^2(r)d^3r\}$$

Ang Li and Bao-An Li, arXiv:1107.0496

C. Xu, A. Li and B.A. Li, Journal of Physics: Conference Series 420, 012190 (2013)

Short-range tensor forces affects the high-density symmetry energy





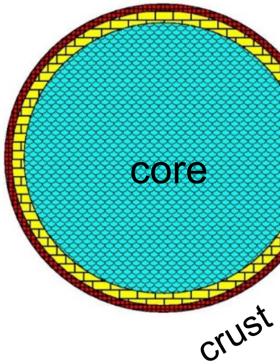
At saturation density, the 2nd order potential contribution
due to the tensor force is about 7-14 MeV, it is 9 MeV with Av18

How does the symmetry energy affect neutron star observables?

(1) The proton fraction x is determined by the $E_{sym}(\rho)$ through charge neutrality and beta-equilibrium conditions:

$$x = 0.048[E_{sym}(\rho)/E_{sym}(\rho_0)]^3(\rho/\rho_0)(1-2x)^3$$

Critical for the cooling mechanism of protoneutron stars and associated neutrino emissions



(2) The pressure in the npe matter at beta equilibrium:

δ=1-2x

$$P(\rho,\delta) = \rho^2[\frac{dE_0(\rho)}{d\rho} + \frac{dE_{\rm sym}(\rho)}{d\rho}\delta^2] + \frac{1}{2}\delta(1-\delta)\rho E_{\rm sym}(\rho)$$

(3) The crust-core transition density and pressure is determined by setting the incompressibility of neutron star matter =0 (speed of sound becomes imaginary):

$$K_\mu = \rho^2\frac{d^2E_0}{d\rho^2} + 2\rho\frac{dE_0}{d\rho} + \delta^2\left[\rho^2\frac{d^2E_{sym}}{d\rho^2} + 2\rho\frac{dE_{sym}}{d\rho} - 2E_{sym}^{-1}(\rho\frac{dE_{sym}}{d\rho})^2\right] = 0$$

Lattimer & Prakash, Phys. Rep., 442, 109 (2007)

Microscopic diagnosis of n-skins in two Skyrme-Hartree-Fock models with similar EOSs for SNM and E_{sym} as the APR up to $1.5\rho_0$

$$S_1(\rho) = \frac{\hbar^2 k_F^2}{6m_0^*(\rho, k_F)}$$

$$S_2(\rho) = \frac{1}{2}U_{\mathrm{sym},1}(\rho, k_F) \ ,$$

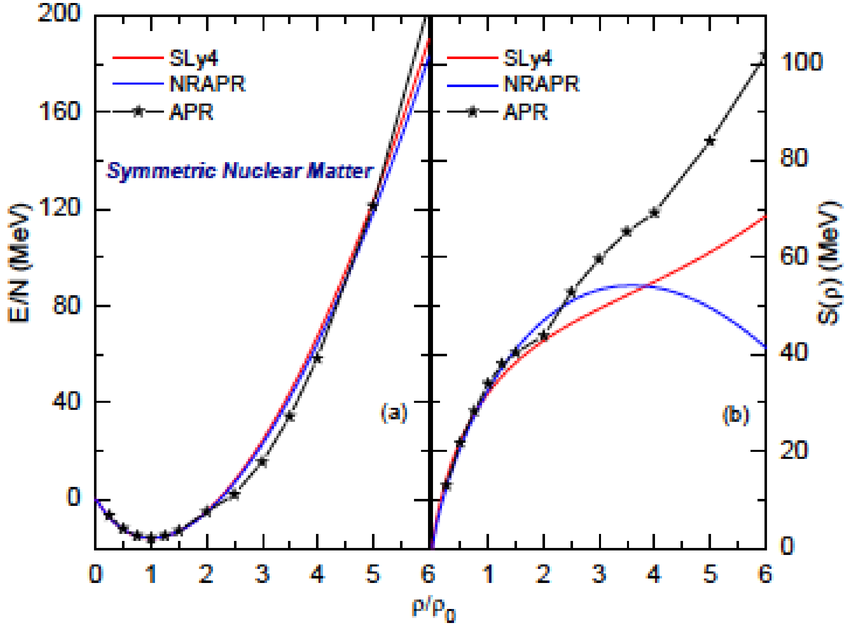
$$L_1(\rho) = \frac{2\hbar^2 k_F^2}{6m_0^*(\rho, k_F)} \equiv 2S_1(\rho)$$

$$L_2(\rho) = -\frac{\hbar^2 k_F^3}{6m_0^{*2}(\rho, k_F)} \frac{\partial m_0^*(\rho, k)}{\partial k}\bigg|_{k=k_F}$$

$$L_3(\rho) = \frac{3}{2}U_{\mathrm{sym},1}(\rho, k_F) \equiv 3S_2(\rho)$$

$$L_4(\rho) = \frac{\partial U_{\mathrm{sym},1}(\rho, k)}{\partial k}\bigg|_{k=k_F} \cdot k_F$$

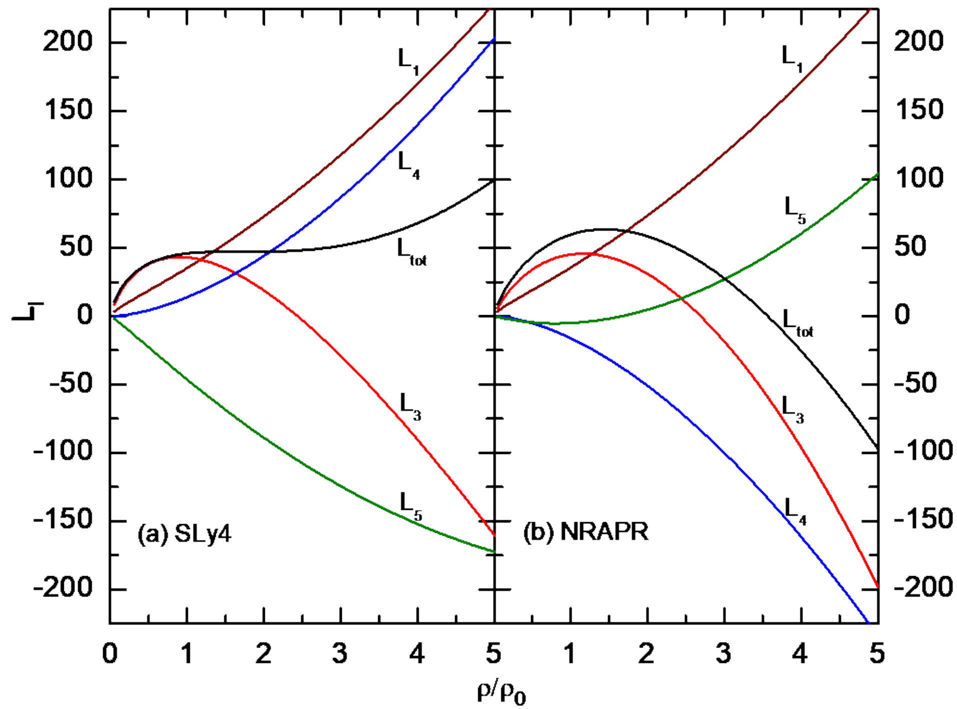
$$L_5(\rho) = 3U_{\mathrm{sym},2}(\rho, k_F) \ .$$



L_Sly4 = 45.9 MeV
L_NRAPR = 59.6 MeV

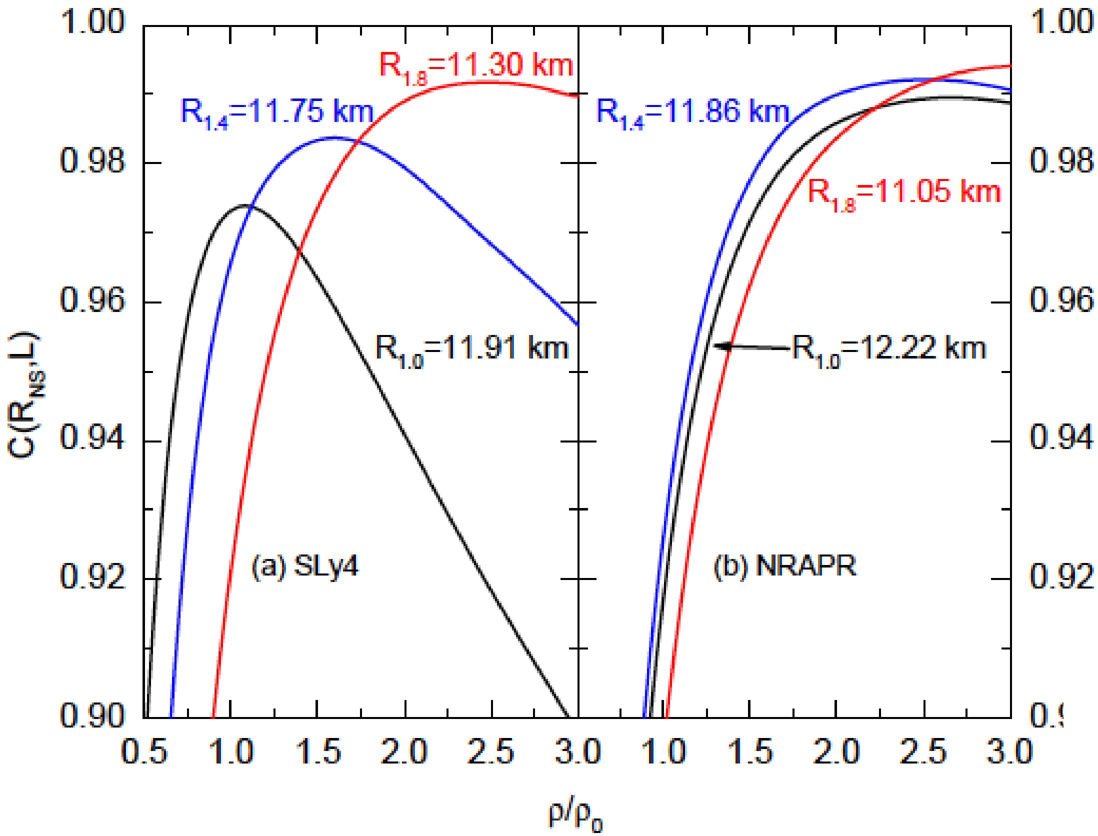
For ^{208}Pb

Rskin_Sly4 = 0.157 fm
Rskin_NRAPR = 0.184 fm



F. Fattoyev, W.G. Newton and Bao-An Li
PRC 90, 022801(R) (2014)

Correlations between radii of neutron stars of different masses and L(ρ)



$$S_1(\rho) = \frac{\hbar^2 k_F^2}{6m_0^*(\rho, k_F)}$$

$$S_2(\rho) = \frac{1}{2} U_{sym,1}(\rho, k_F) \ ,$$

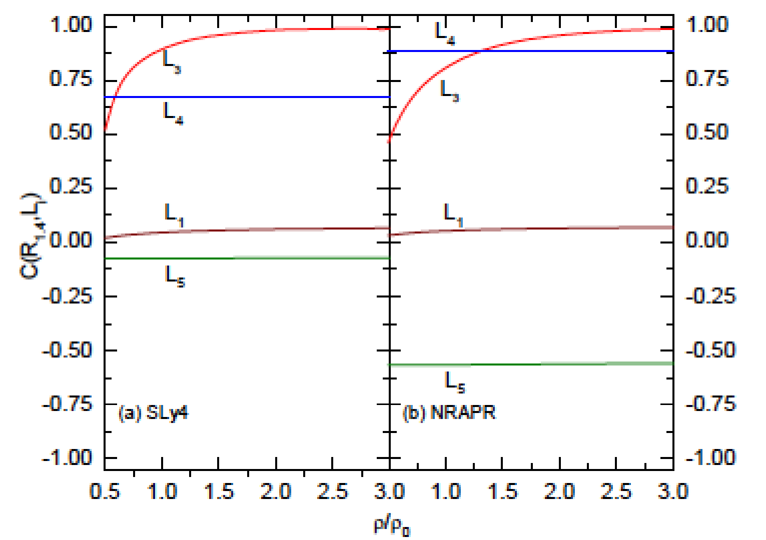
$$L_1(\rho) = \frac{2\hbar^2 k_F^2}{6m_0^*(\rho, k_F)} \equiv 2S_1(\rho)$$

$$L_2(\rho) = -\frac{\hbar^2 k_F^3}{6m_0^{*2}(\rho, k_F)} \left.\frac{\partial m_0^*(\rho, k)}{\partial k}\right|_{k=k_F}$$

$$L_3(\rho) = \frac{3}{2} U_{sym,1}(\rho, k_F) \equiv 3S_2(\rho)$$

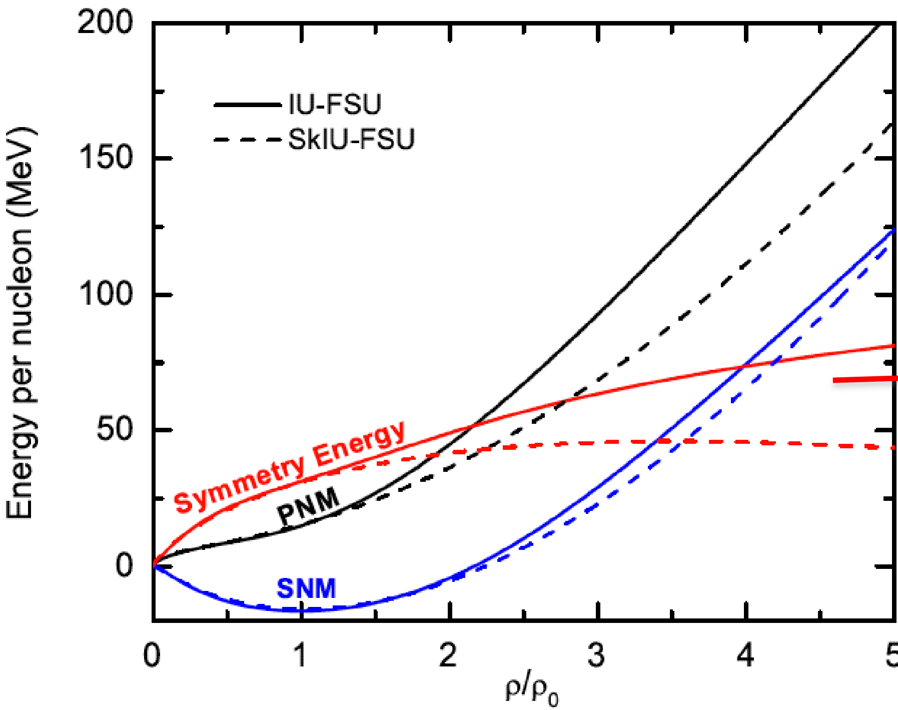
$$L_4(\rho) = \left.\frac{\partial U_{sym,1}(\rho, k)}{\partial k}\right|_{k=k_F} \cdot k_F$$

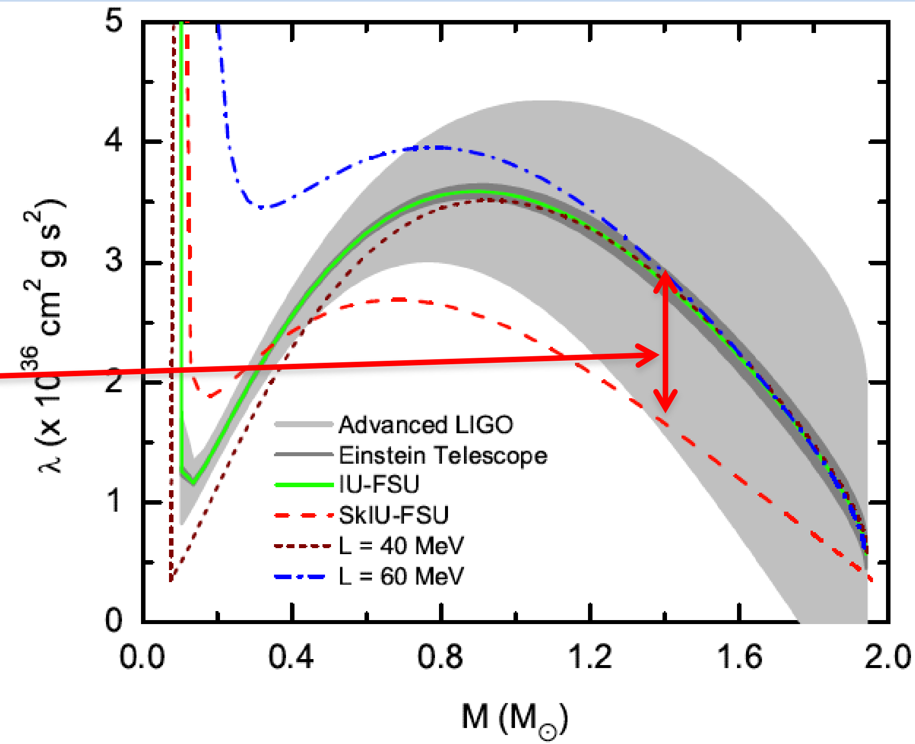
$$L_5(\rho) = 3U_{sym,2}(\rho, k_F) \ .$$



F. Fattoyev, W.G. Newton and Bao-An Li
PRC 90, 022801(R) (2014)

Imprint of high-density symmetry energy in GW signals: Tidal deformability and mergers





F. Fattoyev, J. Carvajal, W.G. Newton and B.A. Li, PRC87, 15806 (2013)

The tidal deformation:

$$\lambda = 2k_2R^5/(3G)$$

The quadrupole moment of m_1 due to m_2:

$Q_{m1}=k_2\ m_2\ R^5/d^3$

Given a EOS, the Love number k_2 and radius R for a given mass m can be solved from the Tolman-Oppenheimer-Volkoff Eq. coupled with a differential Eq. for the strength of the perturbed time-time component of the metric

Table 1. The radius $R_{1.4}$ data used in this work.

Radius $R_{1.4}$ (km) (90% confidence level)	Source	Reference
$11.9^{+1.4}_{-1.4}$	GW170817	(Abbott et al. 2018)
$10.8^{+2.1}_{-1.6}$	GW170817	(De et al. 2018)
$11.7^{+1.1}_{-1.1}$	QLMXBs	(Lattimer & Steiner 2014)
$11.9 \pm 0.8, 10.8 \pm 0.8, 11.7 \pm 0.8$	Imagined case-1	this work
11.9 ± 0.8	Imagined case-2	this work

Posterior probability distribution $$P(\mathcal{M}|D) = \frac{P(D|\mathcal{M})P(\mathcal{M})}{\int P(D|\mathcal{M})P(\mathcal{M})d\mathcal{M}},$$ **(Bayes' theorem)**

Likelihood: $$P[D(R_{1,2,3})|\mathcal{M}(p_{1,2,\cdots 6})] = \prod_{j=1}^{3} \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{obs},j}} \exp[-\frac{(R_{\mathrm{th},j} - R_{\mathrm{obs},j})^2}{2\sigma_{\mathrm{obs},j}^2}],$$

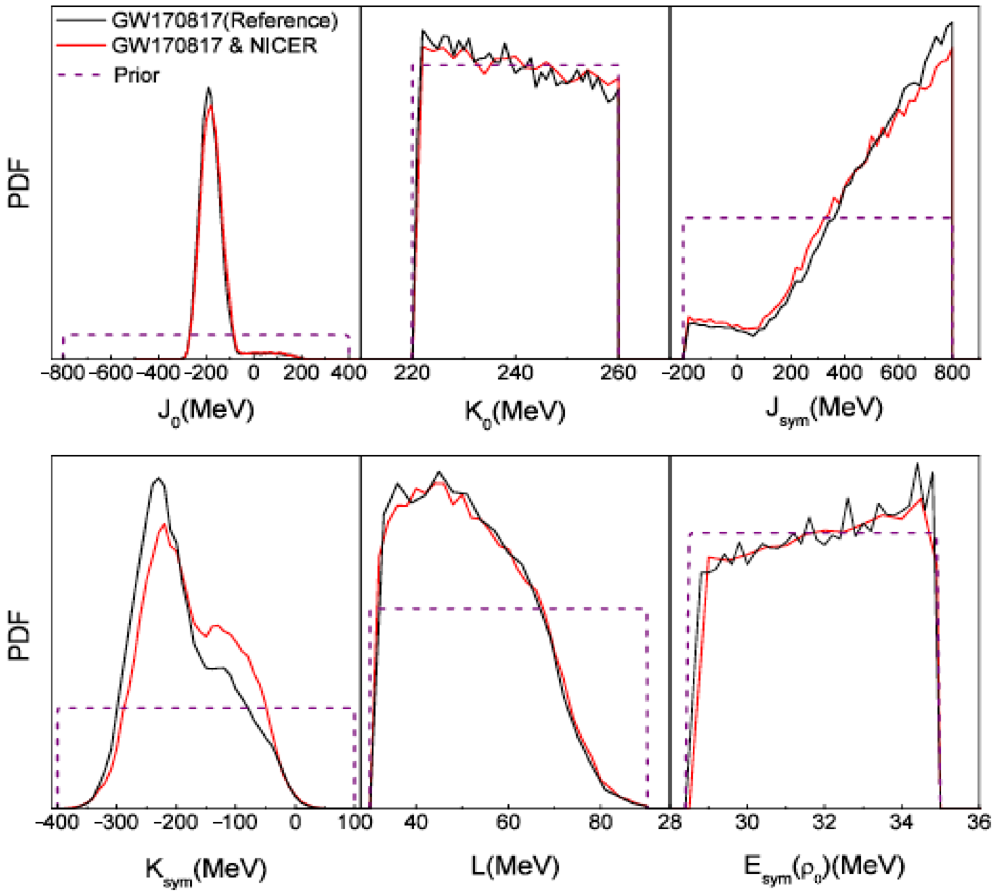
Uniform prior distribution P(M) in the ranges of

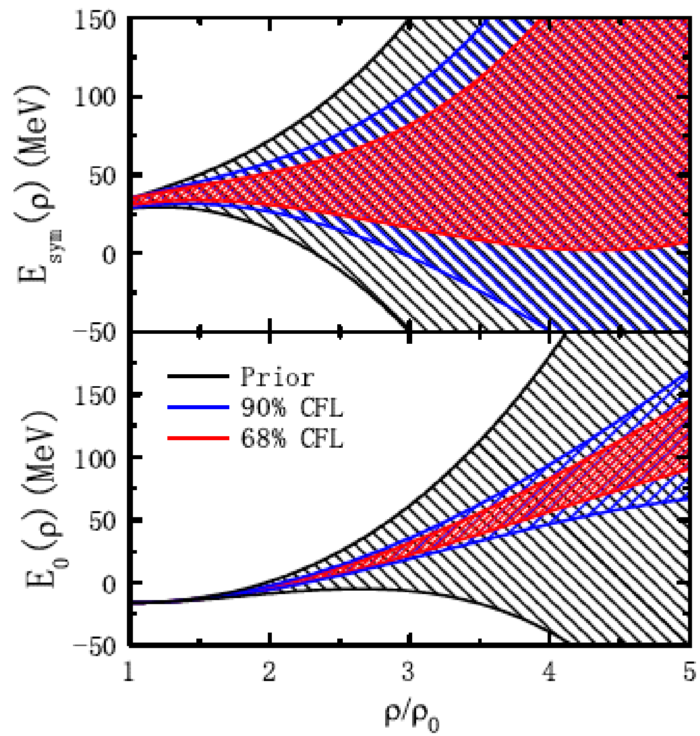
Table 2. Prior ranges of the six EOS parameters used

Parameters	Lower limit	Upper limit (MeV)
K_0	220	260
J_0	-800	400
K_{sym}	-400	100
J_{sym}	-200	800
L	30	90
$E_{\mathrm{sym}}(\rho_0)$	28.5	34.9

Bayesian inference of high-density E_{sym} from the radii $R_{1.4}$ of canonical neutron stars in 6D EOS parameter space

Posterior probability distribution function (PDF) of 6 EOS parameters from Bayesian analyses of GW170817 & NICER data for the canonical PSR J0030+0451

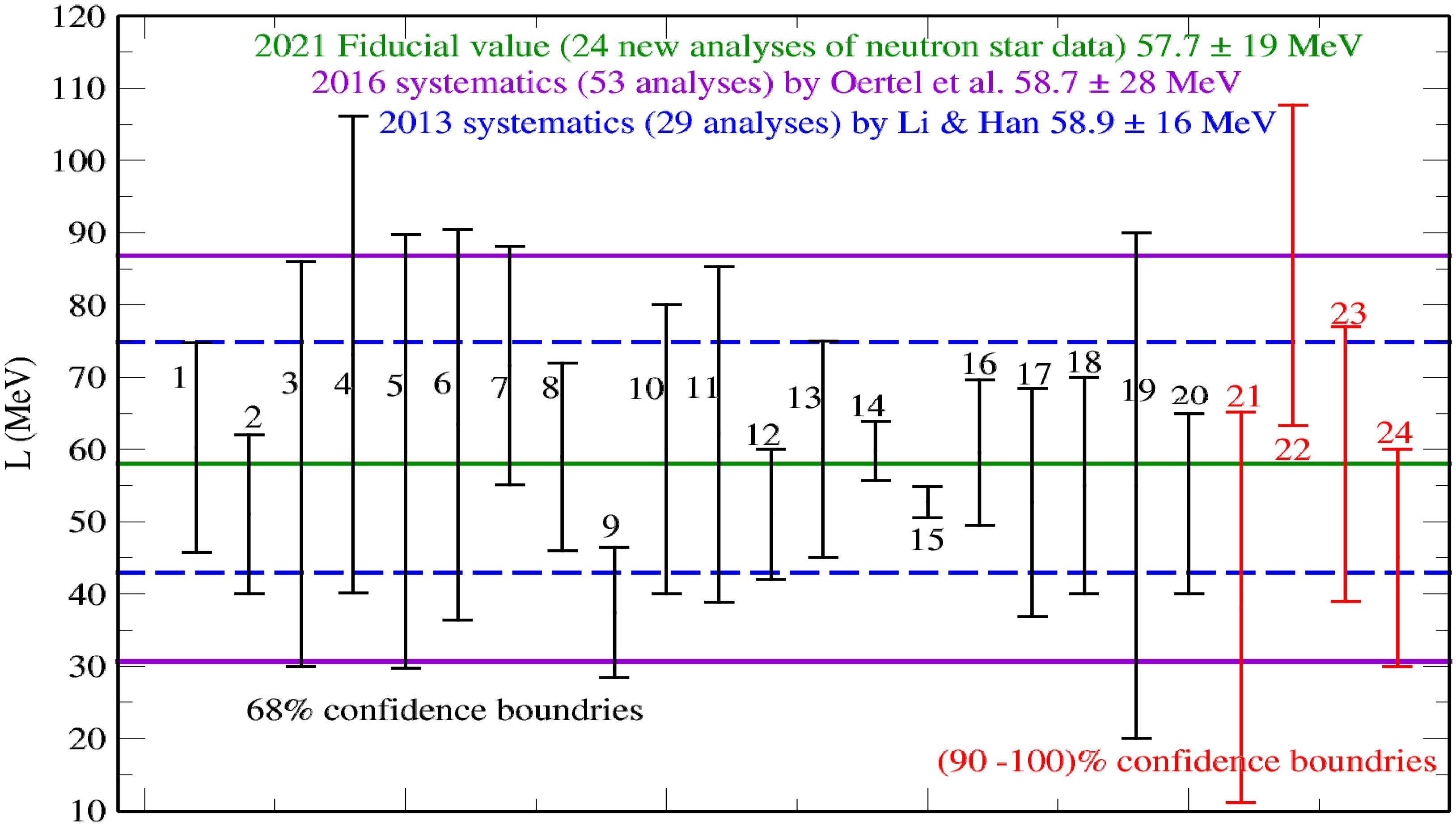




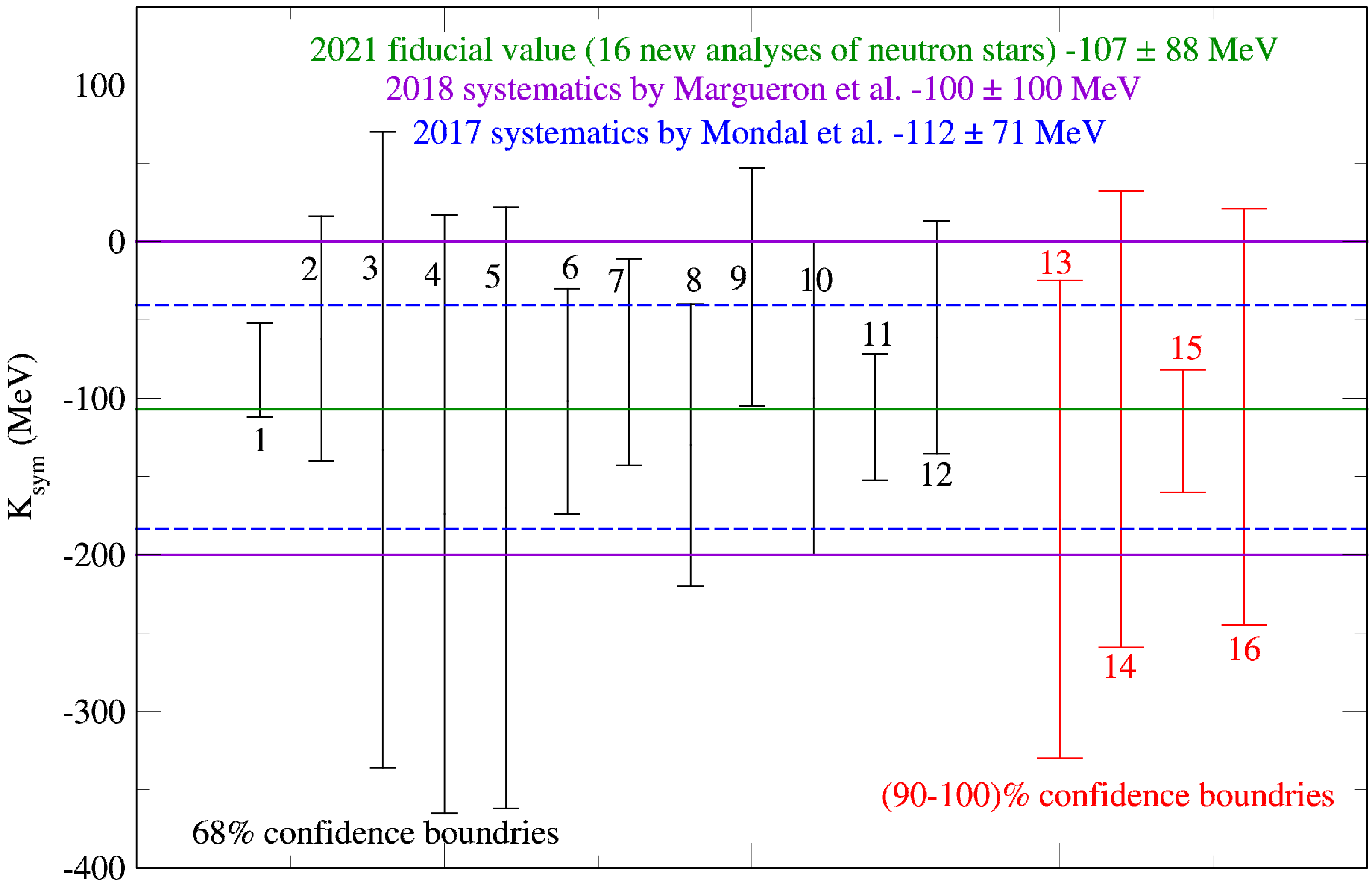
Wen-Jie Xie and Bao-An Li
APJ 883, 174 (2019)
APJ 899, 4 (2020)

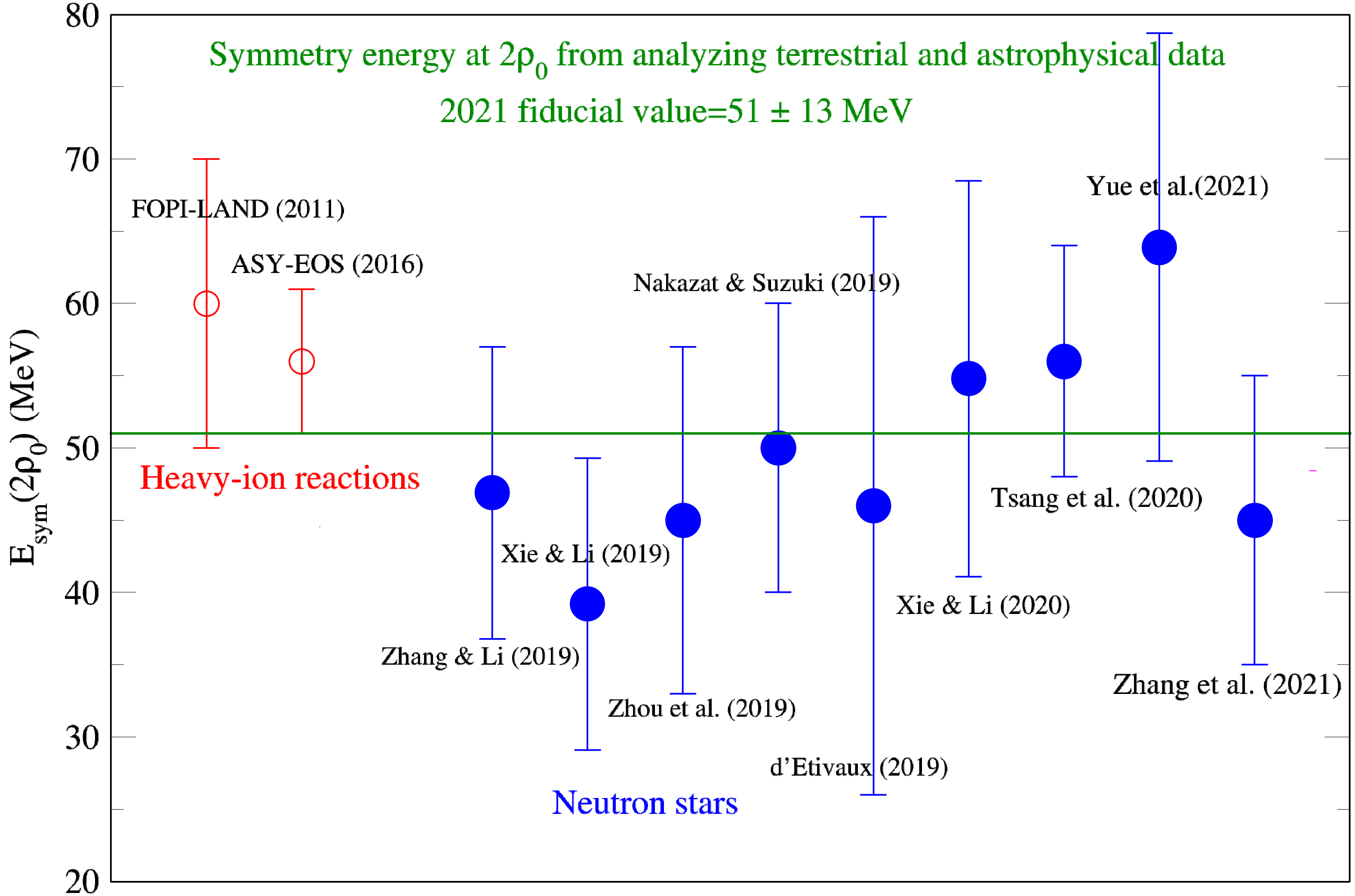
Progress in Constraining Nuclear Symmetry Energy Using Neutron Star Observables Since GW170817 by the community

Bao-An Li, Bao-Jun Cai, Wen-Jie Xie, Nai-Bo Zhang, **Universe 7, 182 (2021)**

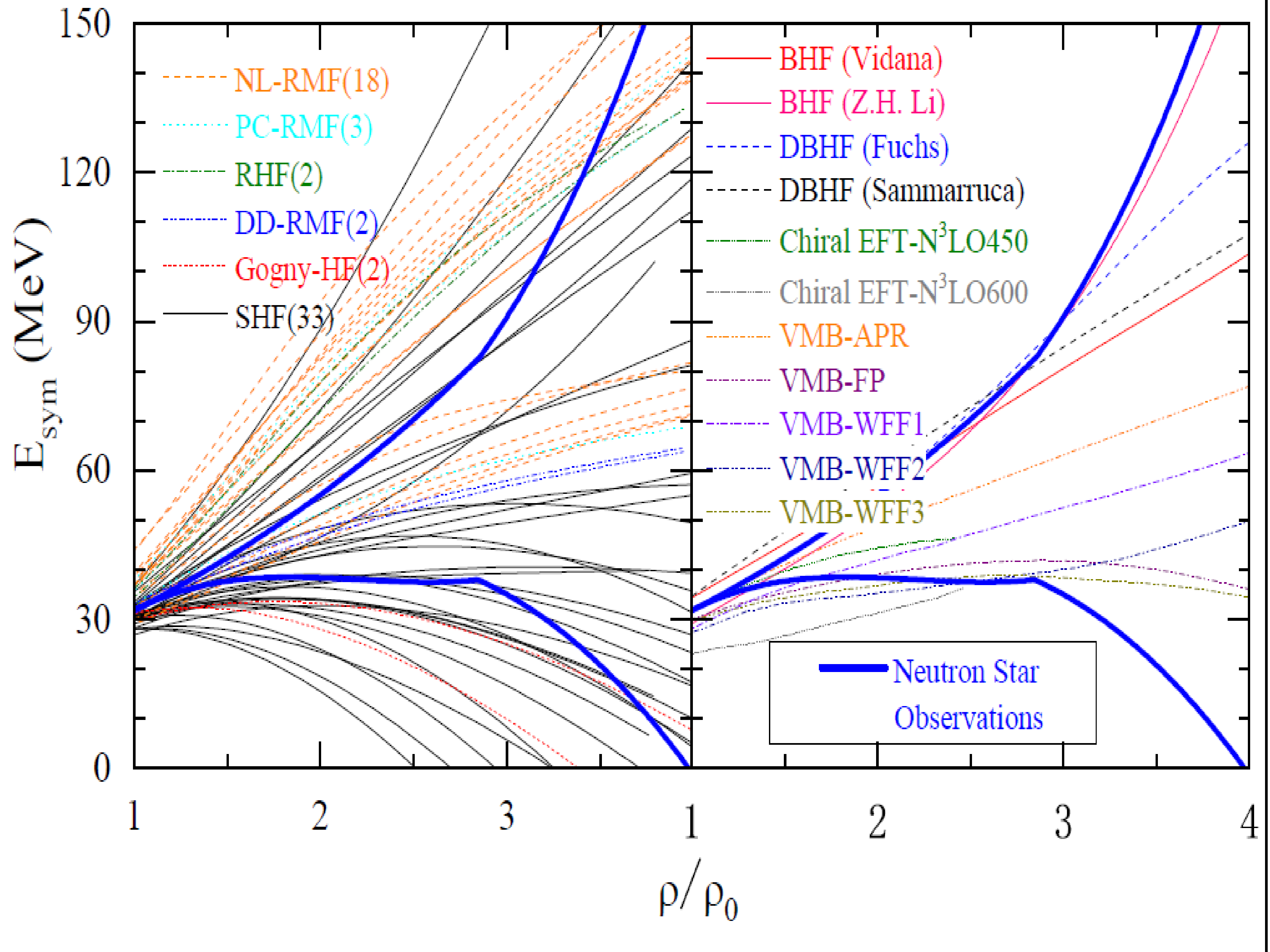


Curvature of the symmetry energy at saturation density





Symmetry energy at $2\rho_0$ from analyzing terrestrial and astrophysical data
2021 fiducial value=51 ± 13 MeV
$E_{sym}(2\rho_0)$ (MeV)
80
70
60
50
40
30
20
FOPI-LAND (2011)
ASY-EOS (2016)
Heavy-ion reactions
Nakazat & Suzuki (2019)
Yue et al.(2021)
Tsang et al. (2020)
Xie & Li (2019)
Zhang & Li (2019)
Zhou et al. (2019)
d'Etivaux (2019)
Xie & Li (2020)
Zhang et al. (2021)
Neutron stars



NL-RMF(18)
PC-RMF(3)
RHF(2)
DD-RMF(2)
Gogny-HF(2)
SHF(33)
BHF (Vidana)
BHF (Z.H. Li)
DBHF (Fuchs)
DBHF (Sammarruca)
Chiral EFT-N^{3}LO450
Chiral EFT-N^{3}LO600
VMB-APR
VMB-FP
VMB-WFF1
VMB-WFF2
VMB-WFF3
Neutron Star Observations
E_{sym} (MeV)
ρ/ρ_0
0
30
60
90
120
150
1
2
3
1
2
3
4

How can massive neutron stars help?

Wen-Jie Xie and Bao-An Li
APJ 899, 4 (2020)

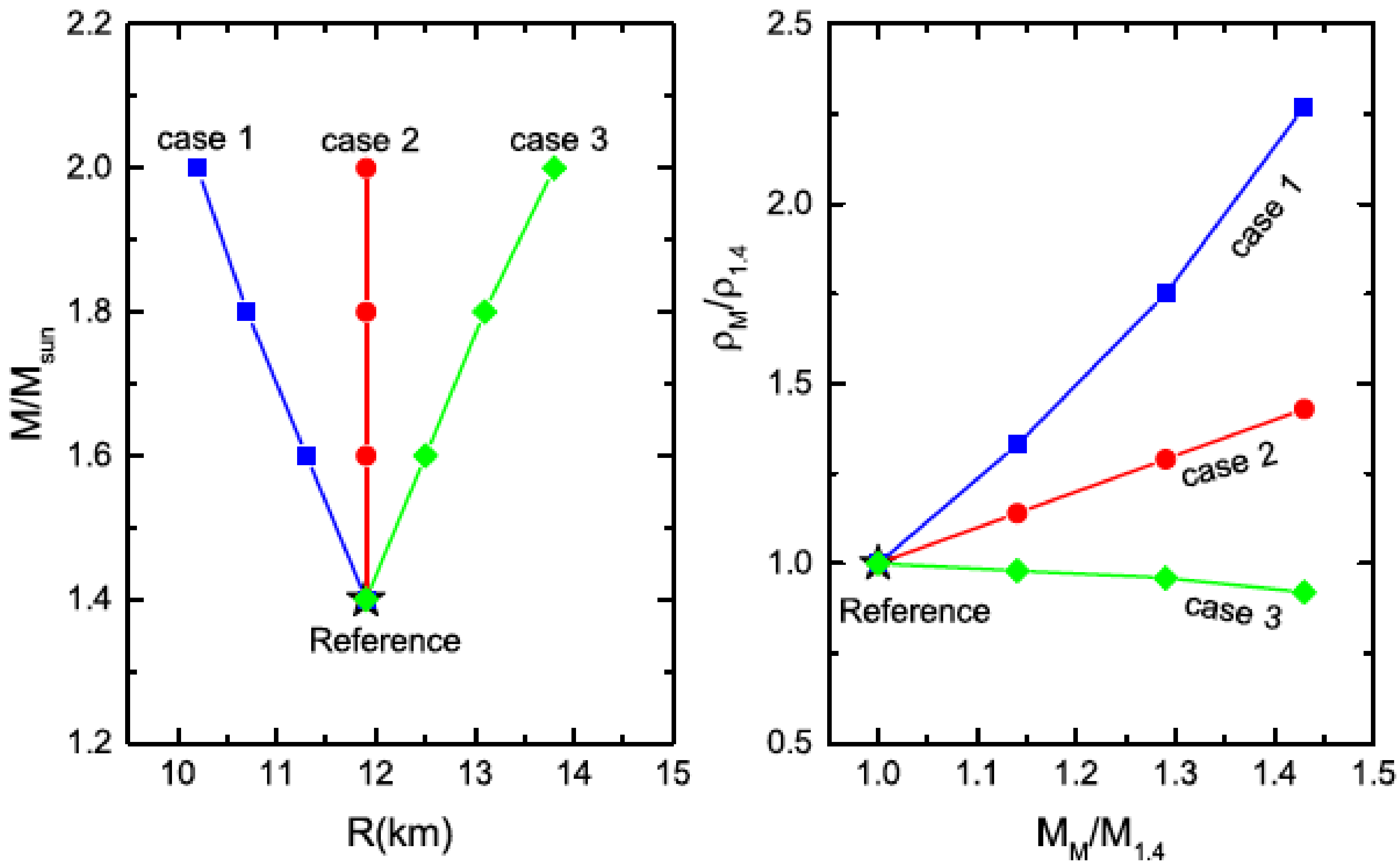
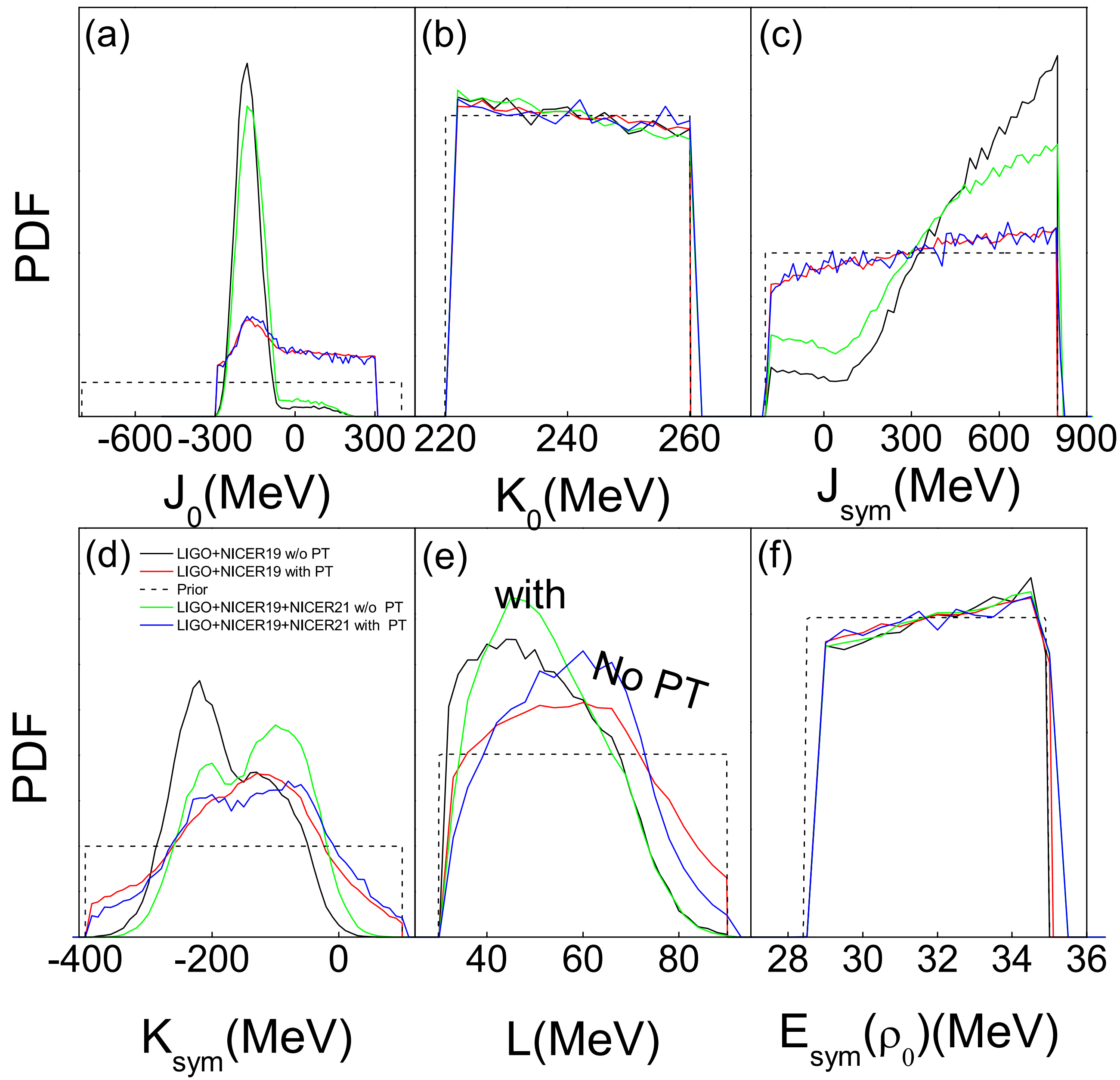


Figure 9. (**Left**) Representative mocked mass–radius correlations considered for massive NSs with respect to the reference of $R_{1.4} = 11.9 \pm 1.4$ km at a 90% confidence level for canonical NSs from GW170817. (**Right**) The corresponding average density in NSs of mass M scaled by that of canonical NSs as a function of the mass ratio $M/M_{1.4}$. Taken from [67].

Bayesian analysis using GW170817+NICER's radius data for both PSR J0740+6620 and J0030+0451



Radius of J0740

$13.7^{+2.6}_{-1.5}$ km

Miller et al

$12.39^{+1.30}_{-0.98}$ km

Riley et al.

Solving the NS inverse-structure problems by calling the TOV solver within 3 Do-Loops: Given an observable→ Find ALL necessary EOSs

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{\boxed{J_0}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3, \quad (2.15)$$

$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{\boxed{K_{\rm sym}}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{\boxed{J_{\rm sym}}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 \quad (2.16)$$

$$E(\rho,\delta) = E_0(\rho) + E_{\rm sym}(\rho)\cdot\delta^2$$

Fix the saturation parameters $E_0(\rho_0)$, $E_{sym}(\rho_0)$ and L at their most probable values currently known

Example:

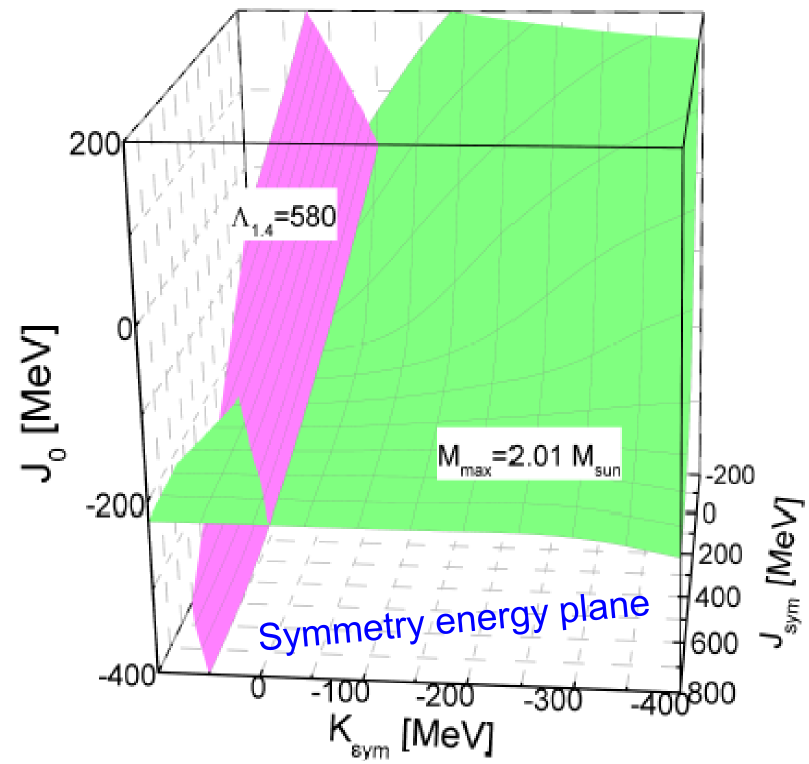
J_0=-189 is found given M_{max}=2.01M_{sun}

J_0 loop

Inversion by brute force

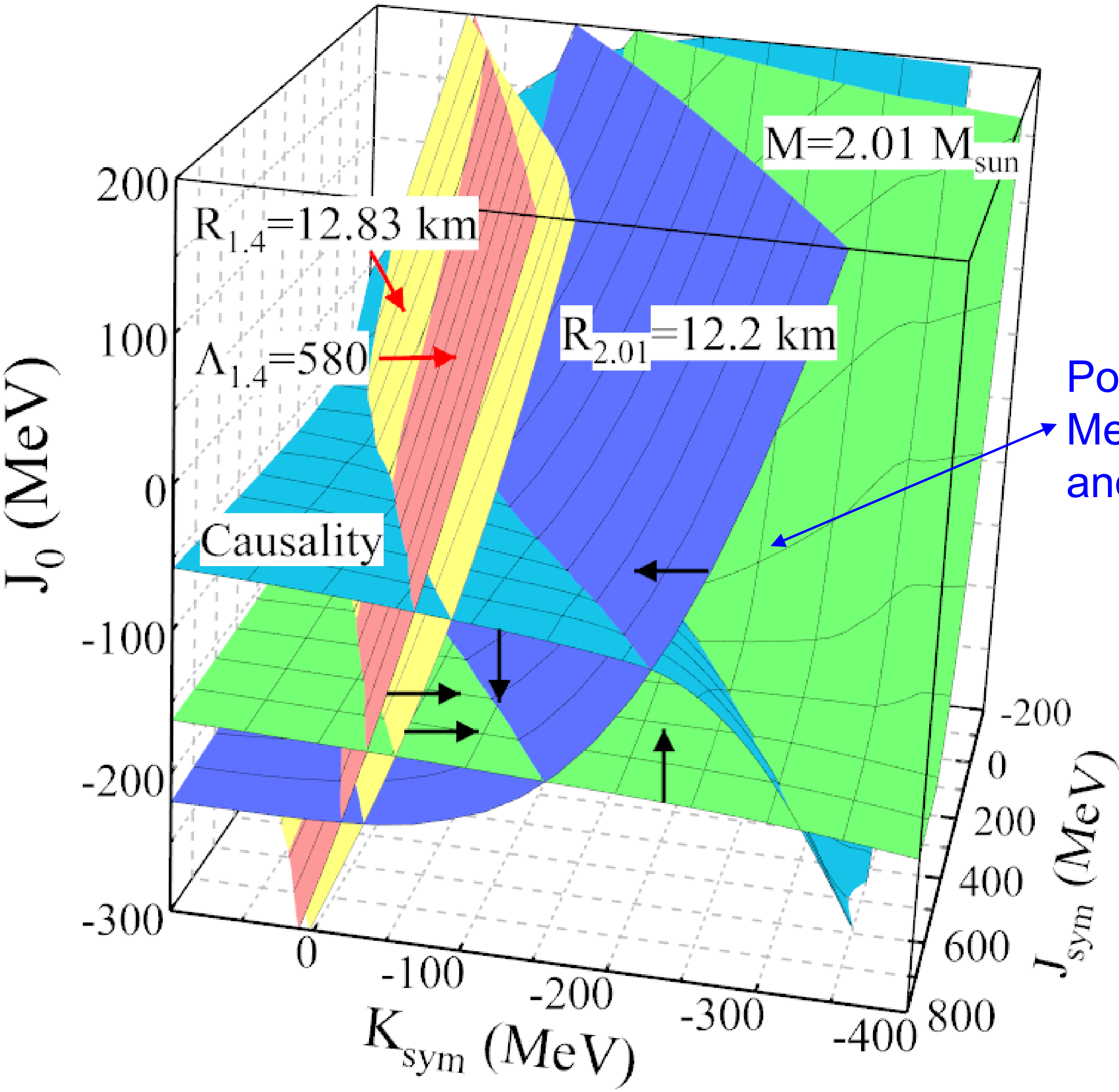
TOV

at K_{sym}=-200 & J_{sym}=400
inside the K_{sym} and J_{sym} loops



N.B. Zhang, B.A. Li and J. Xu, APJ 859, 90 (2018)

Inversion of n-star observables in high-density EOS parameter space



Power of simultaneous Measurements of mass and radius

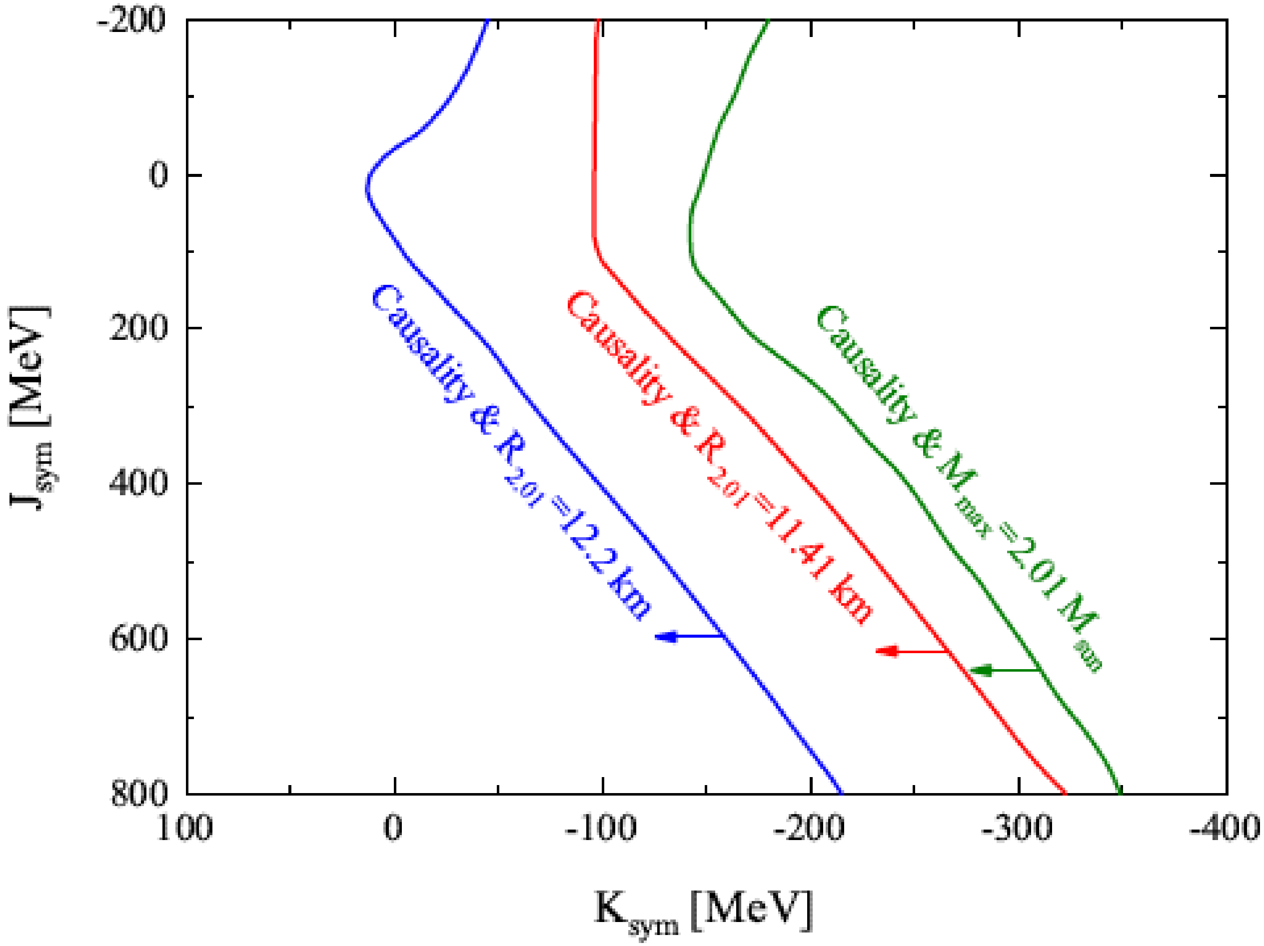
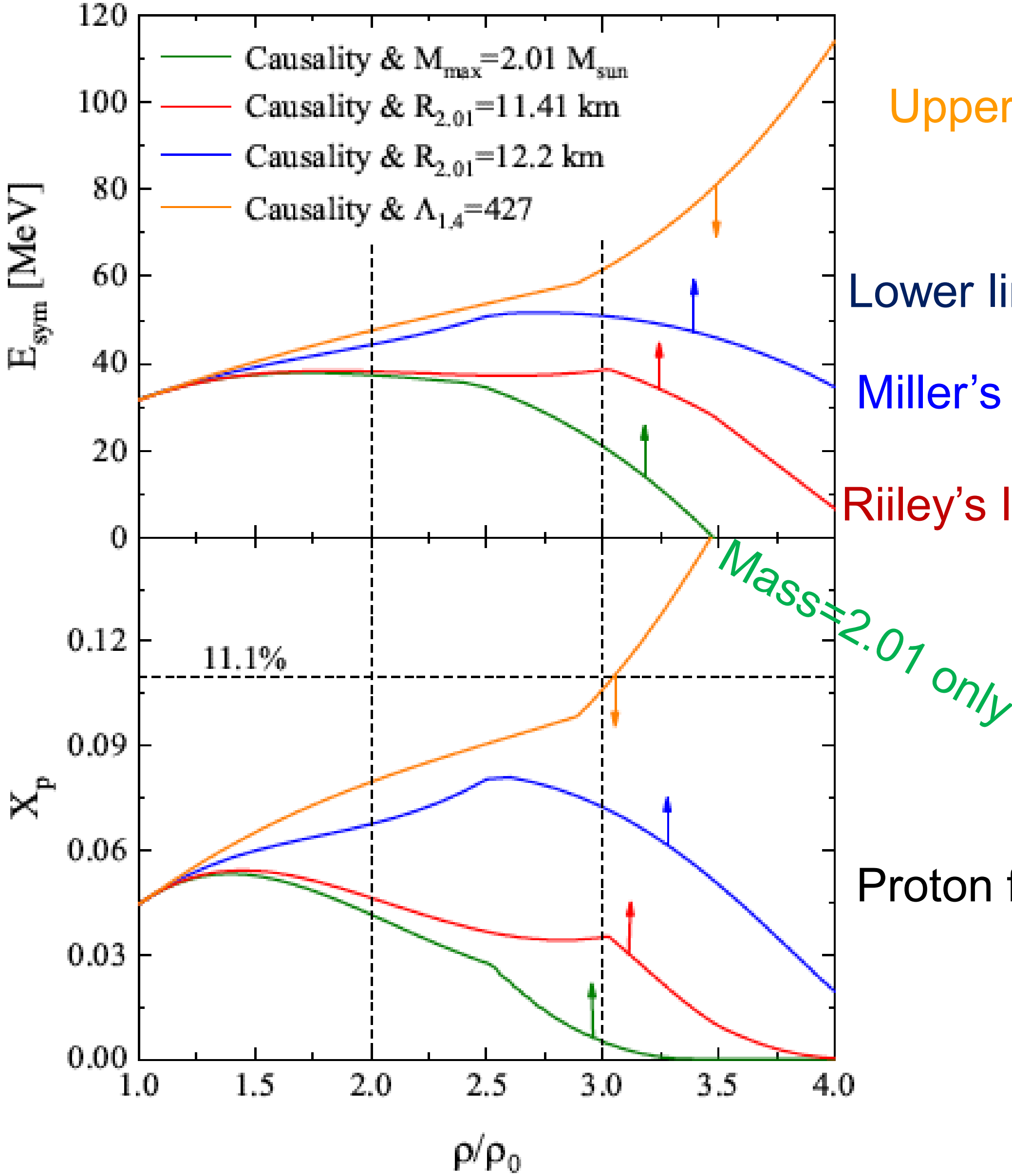


Figure 5. The projection of the crossline between surfaces of causality condition and $M_{\rm max} = 2.01\ {\rm M}_{\odot}$ (green line), $R_{2.01} = 11.41$ km (red line) and $R_{2.01} = 12.2$ km (blue line), respectively, on the $K_{\rm sym}-J_{\rm sym}$ plane for $L = 58.7$ MeV. The arrows point to the directions satisfying the corresponding constraints.

Impact of NICER's Radius Measurement of PSR J0740+6620 on Nuclear Symmetry Energy at Suprasaturation Densities, arXiv:2105.11031

Nai-Bo Zhang and Bao-An Li, APJ (2021) in press.



Upper limit on E_{sym} from GW170817

Lower limit on E_{sym} from PSR J0740+6620

Miller's lower radius

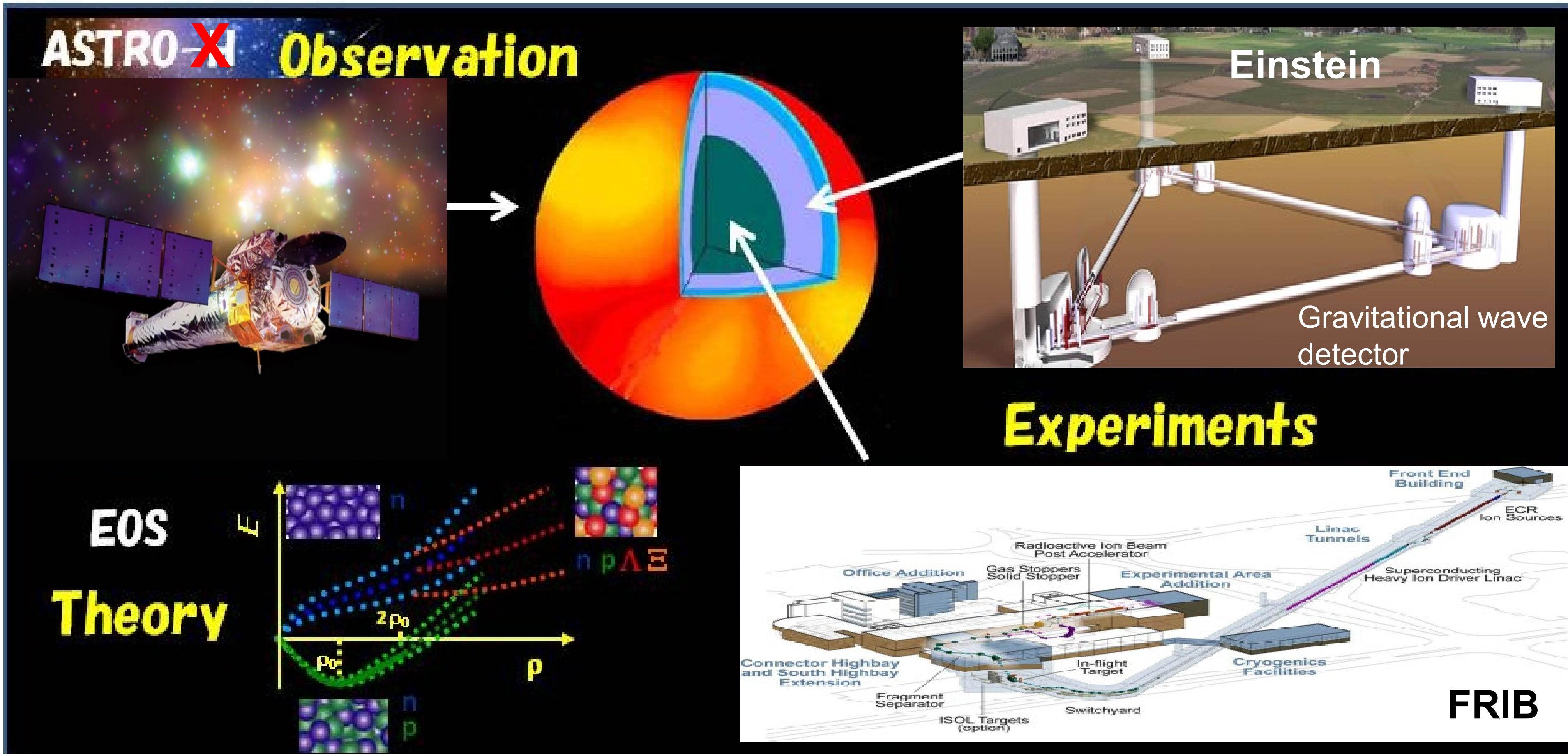
Riiley's lower radius

Proton fraction in PSR J0704+6620

From Earth to Heaven: multi-messengers of nuclear EOS

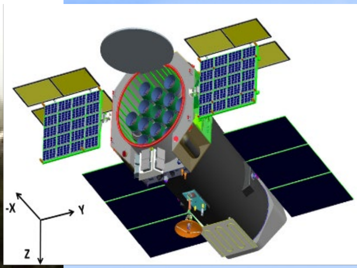
(1) Significant progress has been made in fixing nuclear symmetry energy below twice the saturation density

(2) Truly multi-messenger approach to probe the EOS of dense neutron-rich matter = astrophysical observations + terrestrial experiments + theories + …



A road map towards determining the nature and EOS of ... tter

Collaboration is the key for success!



Reaction Theory

Shell Model
Continuum Shell Model

ab initio
GFMC, NCSM, CCM

NN and
many-nucleon
forces
EFT

Neutron Stars
EOS

Collective Models
Algebraic Models

MANY BODY

ASTRO

Under Construction

FAIR/Germany
RIKEN/Japan,
FRIB/USA
HIAF/China
ROAN/Korea
NICA/Russia

Transport
of neutrons,protons,pions

THEORY

EXPERIMENT

Heavy-ion Collisions

Ground-based
gravitational wave detectors