Effects of anisotropy on strongly magnetized neutron and strange quark stars in general relativity

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Introduction

- From the available observational evidence, it is difficult to confirm the strength of the magnetic field inside the compact stars.

- This urges researchers to come up with suitable theoretical models that help to investigate appropriately the effects of high magnetic fields on the physical parameters of the stellar objects.

- Magnetic flux conservation during stellar collapse leads to the presence of ultra-strong magnetic field inside of compact stars.

- Some researchers have found that at the center of inhomogeneous, ultradense and gravitationally bound compact stars, the magnetic field may be as high as $\sim 10^{19}$ G (Yuan & Zhang 1998; Tatsumi 2000; Ferrer et al. 2010).

Yuan, Y. F. & Zhang, J. L. 1998, A&A, 335, 969
Tatsumi, T. 2000, Physics Letters B, 489, 280.
Ferrer, E. J., de La Incera, V., Keith, J. P., et al. 2010, PhRvC, 82, 065802.

Presence of anisotropy in a compact stellar object

- Ruderman (1972), in his theoretical study, showed that the interactions become relativistic for the nuclear matter in the highly dense stellar interior and the presence of a type-P superfluid leads to a pressure anisotropy in the stars.

- Letelier (1980) and Bayin (1982) strongly argued that the presence of two (or more) fluids or a mixture thereof in the compact stars, may be the possible reason for pressure anisotropy, which Herrera & Santos (1997) confirmed later.

- Ferrer et al. (2010) showed that the presence of a strong magnetic field also induces anisotropy within the stars by breaking the spatial rotational [$\mathcal{O}(3)$] symmetry, which later Isayev & Yang (2011, 2012) confirmed in their articles.

Ruderman, M. 1972, ARA&A, 10, 427.
Herrera, L. & Santos, N. O. 1997, PhR, 286, 53.
Bayin, S. S. 1982, PhRvD, 26, 1262.
Isayev, A. A. & Yang, J. 2011, PhRvC, 84, 065802.
Letelier, P. S. 1980, PhRvD, 22, 807.
Isayev, A. A. & Yang, J. 2012, Physics Letters B, 707, 163.

Effect of magnetic field orientation on the stellar object

- Interestingly, researchers still do not agree unanimously on whether the maximum mass of a compact stellar object increases or decreases due to the presence of a strong magnetic field and it remains still an important open issue that needs to be resolved.

- Chu et al. (2014) tried to address this issue by introducing the idea of magnetic field orientation.

- When the local magnetic fields are directed towards the radial direction, they are termed radially oriented (RO), and when the magnetic fields are randomly oriented in the direction perpendicular to the radial direction (say along θ direction), they are referred to as transversely orientated (TO).

- Chu et al. (2014) showed in their study that not only the strength of the magnetic field but also the orientations of it has a significant effect on the maximum masses of the stellar object.

Chu, P.-C., Chen, L.-W., & Wang, X. 2014, PhRvD, 90, 063013.

Motivation of the present work

• It will be interesting to investigate the properties of anisotropic compact stars by considering the effects of magnetic field orientations and its spatial distribution in the TOV equation, which was ignored by Chu et al. (2014) in their study.

• In the present study, we consider the presence of the effective anisotropy that is arising due to (i) the local anisotropy of the fluid, and (ii) the presence of a strong magnetic field, which offers a more generalized and realistic situation to explore the properties of the compact objects.

Basic formalism and structural equations for magnetized compact stars

We consider the interior metric as follows:

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (1)$$

The energy-momentum tensor of the system is given by

$$T^{\mu\nu} = (\rho + p_t)u^\mu u^\nu - p_t g^{\mu\nu} + (p_r - p_t)\, v^\mu v^\nu$$
$$+\mathcal{M}B\left(g^{\mu\nu} - u^\mu u^\nu + \frac{B^\mu B^\nu}{B^2}\right) + \frac{B^2}{4\pi}\left(u^\mu u^\nu - \frac{1}{2}g^{\mu\nu}\right) - \frac{B^\mu B^\nu}{4\pi}, \quad (2)$$

where $u^\mu = \delta^\mu_0 e^{-\nu(r)/2}$ which is the time-like unit vector denoting the fluid 4-velocity of matter, whereas $v^\mu = \delta^\mu_1 e^{-\lambda(r)/2}$ represents the space-like unit vector in the radial direction. $\mathcal{M}$ is the magnetization per unit volume and $B^\mu B_\mu = -B^2$.

• Ferrer et al. (2010) and Sinha et al. (2013) in their works obtained that magnetization is at least one order smaller compared to the magnetic pressure and magnetization has no effect on the physical properties of magnetized matter.

Sinha, M., Mukhopadhyay, B., & Sedrakian, A. 2013, NuPhA, 898, 43.

Basic formalism and structural equations for magnetized compact stars

• The system density $(\widetilde{\rho})$, which is the sum of the contribution from the matter and field, is given by

$$\widetilde{\rho} = \rho + \frac{B^2}{8\pi}. \tag{3}$$

• Depending on the magnetic field orientation, the system parallel pressure along the magnetic field reads

$$p_{\parallel} = \begin{cases} p_r - \frac{B^2}{8\pi}, & \text{for RO} \\ p_t - \frac{B^2}{8\pi}. & \text{for TO} \end{cases} \tag{4}$$

• Similarly, the system transverse pressure perpendicular to the magnetic field is given by

$$p_{\perp} = \begin{cases} p_t + \frac{B^2}{8\pi}, & \text{for RO} \\ p_r + \frac{B^2}{8\pi}. & \text{for TO} \end{cases} \tag{5}$$

Basic formalism and structural equations for magnetized compact stars

- The mass function of a star in the presence of a magnetic filed is defined as

$$m(r) = \int_0^r 4\pi r^2 \widetilde{\rho}\, \mathrm{d}r. \tag{6}$$

- Finally, the essential stellar structure equations needed to describe static, anisotropic, spherically symmetric compact objects in the presence of a strong magnetic field take the form

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi \left(\rho + \frac{B^2}{8\pi}\right) r^2, \tag{7}$$

$$\begin{cases} \dfrac{\mathrm{d}p_r}{\mathrm{d}r} = \dfrac{-(\rho+p_r)\dfrac{4\pi r^3\left(p_r-\frac{B^2}{8\pi}\right)+m}{r(r-2m)}+\frac{2}{r}\Delta}{\left[1-\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\frac{B^2}{8\pi}\right)\frac{\mathrm{d}\rho}{\mathrm{d}p_r}\right]}, & \text{for RO} \\ \dfrac{\mathrm{d}p_r}{\mathrm{d}r} = \dfrac{-\left(\rho+p_r+\frac{B^2}{4\pi}\right)\dfrac{4\pi r^3\left(p_r+\frac{B^2}{8\pi}\right)+m}{r(r-2m)}+\frac{2}{r}\Delta}{\left[1+\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\frac{B^2}{8\pi}\right)\frac{\mathrm{d}\rho}{\mathrm{d}p_r}\right]}, & \text{for TO} \end{cases} \tag{8}$$

where $\Delta = \left(p_t - p_r + \frac{B^2}{4\pi}\right)$ and $\left(p_t - p_r - \frac{B^2}{8\pi}\right)$, which denote the effective anisotropy of stellar structures for RO and TO, respectively.

Basic formalism and structural equations for magnetized compact stars

$$\text{Matter Isotropy} \rightarrow \begin{cases} \dfrac{\mathrm{d}p}{\mathrm{d}r} = \dfrac{-(\rho+p)\dfrac{4\pi r^3\left(p-\frac{B^2}{8\pi}\right)+m}{r(r-2m)}+\frac{B^2}{2\pi r}}{\left[1-\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\frac{B^2}{8\pi}\right)\frac{\mathrm{d}\rho}{\mathrm{d}p}\right]}, & \text{for RO} \\ \dfrac{\mathrm{d}p}{\mathrm{d}r} = \dfrac{-\left(\rho+p+\frac{B^2}{4\pi}\right)\dfrac{4\pi r^3\left(p+\frac{B^2}{8\pi}\right)+m}{r(r-2m)}-\frac{B^2}{4\pi r}}{\left[1+\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\frac{B^2}{8\pi}\right)\frac{\mathrm{d}\rho}{\mathrm{d}p}\right]}, & \text{for TO} \end{cases} \tag{9}$$

- If we consider isotropic matter distribution then $p_r = p_t = p$. As a result we have $\Delta \sim \mid B^2 \mid$ which has non-zero value at the stellar center. Hence, F_a also non-zero at the center.

- For isotropic matter distribution in the presence magnetic field at the center F_h and F_g are zero, whereas $F_a \neq 0$. It leads to instability at the stellar centre due to non-equilibrium of the forces.

- This situation can be taken care by considering effective anisotropy due to combined effect from local matter anisotropy and magnetic field.

Ansatz for Anisotropy

We require a functional form for the anisotropy (Δ) to close the system of equation, which follows the essential assumptions given bellow:

- To maintain the stability of the system via equilibrium of the forces (non-diverging nature), the anisotropic force essentially should be zero at the center.

- The anisotropy should vary with position inside the system and also depends non-linearly on p_r. (Bowers & Liang 1974; Silva et al. 2015).

- Based on the present study, the functional form of the anisotropy should include the anisotropic effects due to both the local anisotropy of the fluid and the presence of a strong magnetic field. It is also important to include the effects due to magnetic field orientation.

Bowers, R. L. & Liang, E. P. T. 1974, ApJ, 188, 657.
Silva, H. O., Macedo, C. F. B., Berti, E., et al. 2015, Classical and Quantum Gravity, 32, 145008.

Ansatz for Anisotropy

To include the effects of the magnetic field and its orientation, here we modify the Bowers-Liang anisotropic form, which reads

$$\Delta = \begin{cases} \kappa \dfrac{(\rho + p_r)\left(\rho + 3\,p_r - \frac{B^2}{4\pi}\right)}{\left(1 - \frac{2m}{r}\right)} r^2, & \text{for RO} \\ \kappa \dfrac{\left(\rho + p_r + \frac{B^2}{4\pi}\right)\left(\rho + 3\,p_r + \frac{B^2}{2\pi}\right)}{\left(1 - \frac{2m}{r}\right)} r^2, & \text{for TO} \end{cases} \tag{10}$$

where the dimensionless constant κ controls the strength of anisotropy in the system. Note that we consider the parametric values of κ well within its limiting values given by $\left[-\frac{2}{3}, \frac{2}{3}\right]$ (Silva et al. 2015).

Equation of state

• In order to study the matter of NSs, We consider SLy EOS, which is moderately stiff in classification. Based on Skyrme-type energy density functional, Douchin and Haensel (2001) proposed SLy EOS, which is widely used in the literature to discuss NSs. Note that SLy EOS is equally consistent in the NS core and the crust.

• We consider the MIT bag model EOS to study the SQM distribution for SQSs reads

$$p_r = \frac{1}{3}\left(\rho - 4\,\mathcal{B}\right). \tag{11}$$

Our numerical results are computed for a bag constant of $\mathcal{B} = 60\ \mathrm{MeV/fm^3}$, which corresponds to strongly bound SQM (Weber 2005).

Douchin, F. & Haensel, P. 2001, A& A, 380, 151.

Profile for density-dependent magnetic field magnitude

• For NSs following Bandyopadhyay et al. (1997, 1998), we choose the profile for the density-dependent magnetic field strength in such a way that the magnetic field at the stellar core, B_c, complies with the virial theorem and the surface magnetic field, B_s, fits observed values is given by

$$B(\rho) = B_s + B_0 \left[1 - \exp\left\{-\eta \left(\frac{\rho}{\rho_0}\right)^{\gamma}\right\}\right], \quad (12)$$

• For SQSs, where the surface density is $\rho_s \neq 0$, a modification for the profile of magnetic field strength is required to ensure that the asymptotic value B_s is obtained at the surface, given by

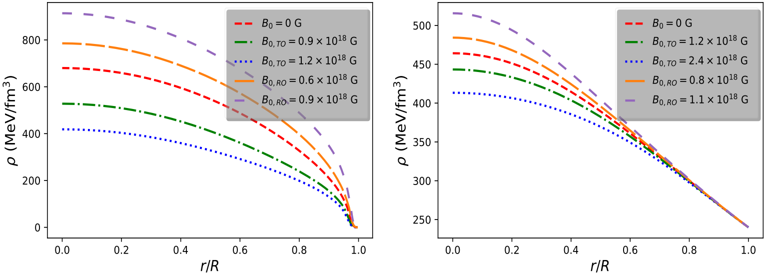
$$B(\rho) = B_s + B_0 \left[1 - \exp\left\{-\eta \left(\frac{\rho - \rho_s}{\rho_0}\right)^{\gamma}\right\}\right]. \quad (13)$$

In the present study, we shall consider values of B_s given by 10^{13} and 10^{15} G for NSs and SQSs, respectively. However, we found that our results are not sensitive to the particular choice of the value of B_s.

Bandyopadhyay, D., Chakrabarty, S., & Pal, S. 1997, PhRvL, 79, 2176.
Bandyopadhyay, D., Pal, S., & Chakrabarty, S. 1998, Journal of Physics G Nuclear Physics, 24, 1647.

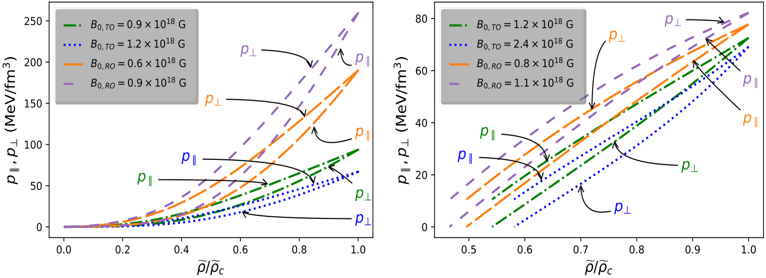
Matter density

Variation of (i) matter density (ρ), (ii) radial pressure (p_r) and (iii) tangential pressure (p_t) with radial coordinate r/R for $2.01 \pm 0.04\ M_\odot$ NS candidate PSR J0348+0432 and $1.97 \pm 0.04\ M_\odot$ SQS candidate PSR J1614−2230. All the left panels represent variation of the physical parameters for NS matter, whereas right panels represent SQS matter. Here and in what follows $\kappa = 0.5$, $\eta = 0.2$, $\gamma = 2$ and $\mathcal{B} = 60\ \mathrm{MeV/fm^3}$.



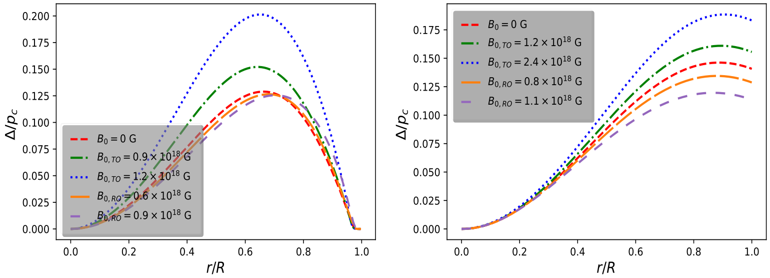
System pressures

Variation of parallel pressure ($p_{\parallel}$) and transverse pressure ($p_{\perp}$) with the system density ($\widetilde{\rho}$) normalized to the central system density ($\widetilde{\rho}_c$) for $2.01 \pm 0.04\ M_{\odot}$ NS candidate PSR J0348+0432 and $1.97 \pm 0.04\ M_{\odot}$ SQS candidate PSR J1614−2230. The left panel features NS pressure profiles, whereas the right panel presents SQS pressure profiles. The dotted, dash-dotted, long-dashed and space-dashed curves correspond to $B_0 = 2.4 \times 10^{18}$ G (TO), $B_0 = 1.2 \times 10^{18}$ G (TO), $B_0 = 0.8 \times 10^{18}$ G (RO) and $B_0 = 1.1 \times 10^{18}$ G (RO), respectively.



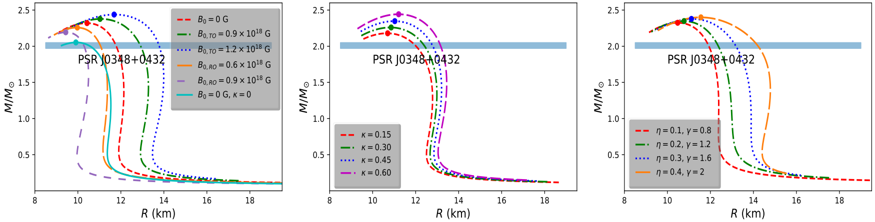
Anisotropy

Variation of anisotropy Δ, normalized to central pressure (p_c), with radial coordinate r/R for $2.01 \pm 0.04\ M_\odot$ NS candidate PSR J0348+0432 and $1.97 \pm 0.04\ M_\odot$ SQS candidate PSR J1614−2230. The left and right panels present NS and SQS anisotropy stress profiles, repectively.



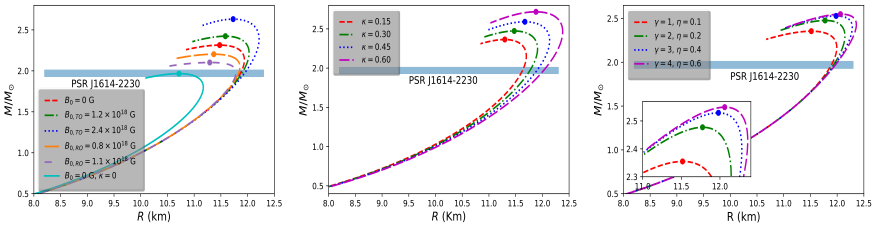
Mass-radius curve for NSs

Variation of stellar mass $M/M_\odot$ with stellar radius R. Solid circles represent the maximum-mass star of each stellar sequence. Here, we plot $M/M_\odot$ vs. R for NS due to (i) varying B_0 and $\kappa = 0.5$ (left), (ii) varying κ, where $B_0 = 6 \times 10^{17}$ G (middle) and (iii) varying η and γ, where $B_0 = 6 \times 10^{17}$ G and $\kappa = 0.5$ (right).



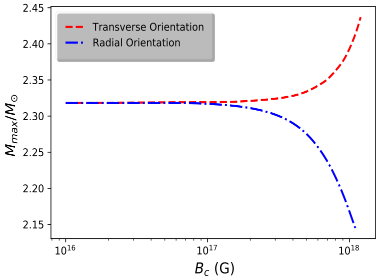
Mass-radius curve for SQSs

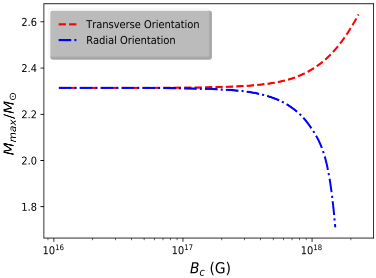
Here, we plot $M/M_\odot$ vs. R for SQS due to (i) varying B_0, where $\kappa = 0.5$ (left), (ii) varying κ, where $B_0 = 2.4 \times 10^{18}$ G (middle) and (iii) varying η and γ, where $B_0 = 1.2 \times 10^{18}$ G and $\kappa = 0.5$ (right).



Mass asymmetry due to orientations of Magnetic field

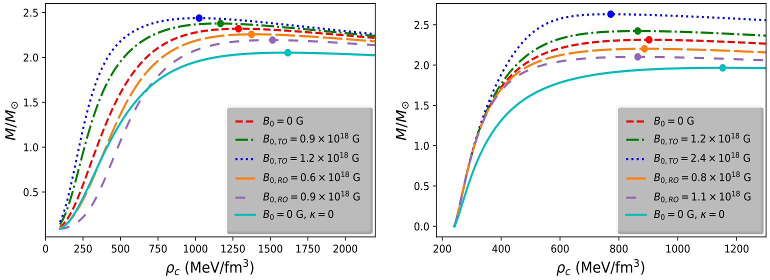
Variation of maximum mass M_{max} with central magnetic field strength B_c for NSs and SQSs in the left and right panels, respectively.





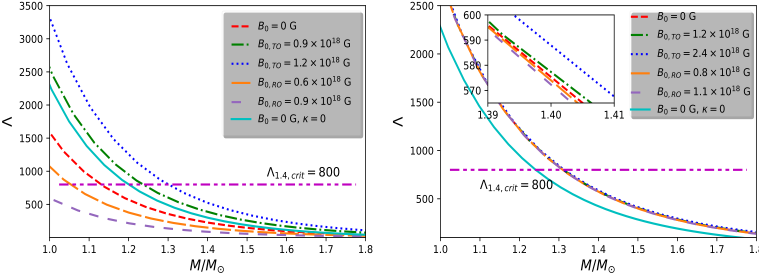
Mass-central density curve

$M/M_{\odot}$ as a function of the matter central matter density ρ_c for NSs and SQSs in the left and right panels, respectively. Solid circles represent the maximum mass stars of each stellar sequence.



Tidal deformability

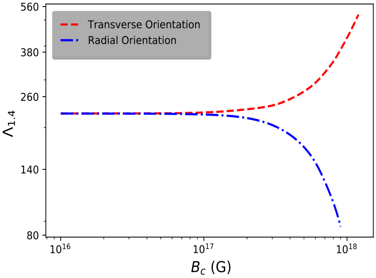
We present the compatibility of our model with respect to the tidal deformability (Λ) for both NSs and SQSs, constrained from the observation of GW emission related to GW170817 event, detected by the LIGO/Virgo Collaboration (LVC) (Abbott et al. 2017,2018,2019). The investigation by LVC sets an upper limit of Λ associated with $1.4M_{\odot}$ pulsars ($\Lambda_{1.4}$) by Abbott et al. (2017) which is given as $\Lambda_{1.4} < 800$ for the low-spin cases.

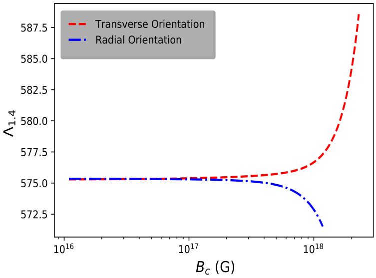


Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2017, PhRvL, 119, 161101.
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2018, PhRvL, 121, 161101.
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2019, Physical Review X, 9, 011001.

Tidal deformability

We also show the effect of magnetic field orientation on $\Lambda_{1.4}$, which increases with increasing B_c for the TO case and decreases with increasing B_c for the RO case. Chu et al. (2021) found the same dependency of $\Lambda_{1.4}$ on the magnetic field orientations which confirms our results in the case of anisotropic magnetized compact stars. Hence, through this work, we explore that for anisotropic magnetized stars, anisotropy, magnetic field strength and orientations of the magnetic field have a significant effect on $\Lambda_{1.4}$.

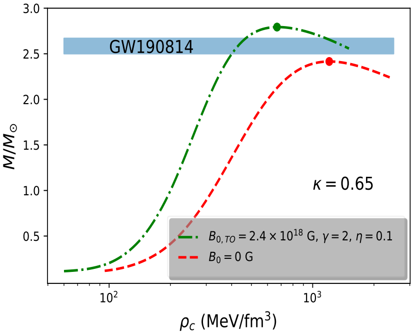




Chu, P.-C., Zhou, Y., Jiang, Y.-Y., et al. 2021, European Physical 891 Journal C, 81, 93.

Our model can explain the mass range $2.5 - 2.67\ M_\odot$ observed in GW190814

With the appropriate choice of the physical parameters, such as $B_0 = 2.5 \times 10^{18}$ G, $\eta = 0.1$, $\gamma = 2$ and $\kappa = 0.65$, we find that the maximum possible mass of NSs is $\sim 3.06\ M_\odot$, comfortably accommodates the anomalously high mass of the lighter object associated with GW190814.



The highlight of the major results

- Our study reveals that in a magnetized compact stellar object the magnetic field strength, the magnetic field orientations and the anisotropy significantly influence physical properties of the stars.

- Through our work, we are able to demonstrate that to study magnetised compact stars, it is essential to consider the effective anisotropies of both the fluid and the magnetic field.

- Based on the orientation of the magnetic field, the maximum mass of static magnetized compact stars may be enhanced or reduced which resolves the long-standing issue.

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Thank you!