Rip cosmology in Brans-Dicke Theory

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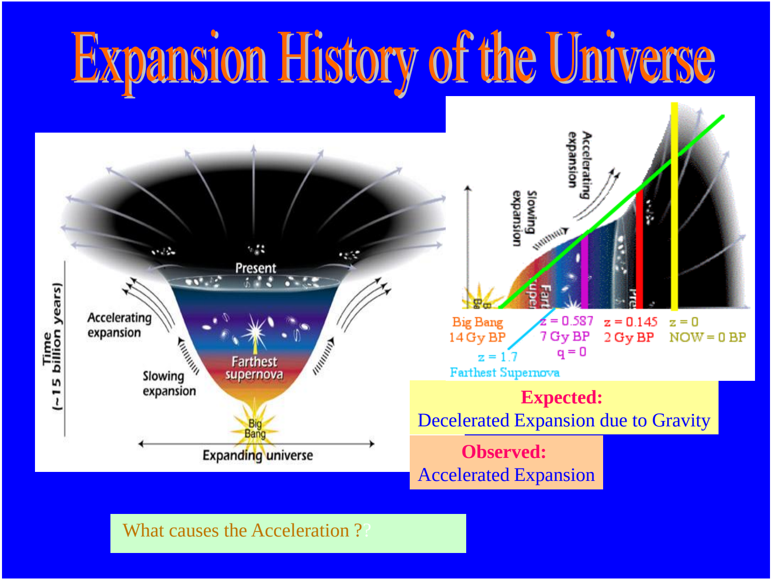
Plan of the Talk

- Late time cosmic acceleration and Phantom dark energy
- Dynamics in Generalized Brans-Dicke Theory
- GBD theory with a cosmological constant
- GBD dynamics with a Big Rip scenario
- Conclusion

Introduction: Late time cosmic acceleration



Introduction: Late time cosmic acceleration

The reason of cosmic acceleration is not exactly known.

It is attributed to an exotic dark energy with a negative pressure and low density.

Planck data estimates the largest contribution of dark energy with 68.3% in the energy budget of the Universe. Dark matter and baryonic matter comprises only 26.8% and 4.9% respectively.

Introduction: Late time cosmic acceleration

The Equation of state parameter (EOS) for dark energy is negative:

$$\omega_D = \frac{p_D}{\rho_D} < 0.$$

Uncertainty prevails in the determination of this EOS parameter. From different observations, ω_D may be constrained. We have some approximate ideas on this parameter as

- for ΛCDM model: $\omega_D = -1$
- for canonical scalar fields: $-\frac{2}{3} \leq \omega_D \leq -\frac{1}{3}$
- for phantom fields: $\omega_D < -1$.

Introduction: Phantom Dark Energy

Observational status on the EOS parameter include

- The 9 year WMAP survey of CMB measurements: $\omega_D = -1.073^{+0.090}_{-0.089}$
- Combined analysis of the data sets of SNLS3, BAO, Planck, WMAP9: $\omega_D = -1.06^{+0.11}_{-0.13}$ (Kumar and Xu, PLB737, 244, 2014)
- Planck 2018 results: $\omega_D = -1.03 \pm 0.03$ (N. Aghanim, et al., Planck Collaboration, arxiv:1807.06209.)

Observational results more or less favour a situation with $\omega_D < -1$ which may be explained by a phantom field.

A phantom field with negative kinetic energy may be unstable in quantum field theory but could be classically stable in classical cosmology. Also, the phantom field violates the energy conditions.

Introduction: Future Singularities

In the context of General Relativity, there occurs a singularity at the beginning of the Universe. We dub it as the Big Bang.

It may happen that, in future, the Universe may enter into a finite time singularity driven mostly by the phantom dark energy.

In the Big Rip future singularity, the scale factor of the Universe, the Hubble rate and its time derivative blow up at finite future cosmic time.

Introduction: Future singularities

Different possibilities of future singularities are

- Big Rip singularity (type-I singularity) where the scale factor and density become infinite in finite time (Caldwell, PRL91, 071301,2003)
- Sudden singularity (type-II singularity) where the pressure becomes infinite for finite scale factor and density (Barrow, CQG21, L79,2004)
- Type-III singularity, where the pressure and density blow up while keeping the scale factor finite (Nojiri et al., PRD 71, 063004, 2005)
- Type-IV singularity where the higher derivatives of the Hubble parameter diverge .(Nojiri et al., PRD 71, 063004, 2005)

Introduction: Proposal to cure finite time future singularities

Finite time future singularities lead to inconsistencies in the models. To avoid such inconsistencies, different scenarios have been proposed:

- To include quantum effects to delay the singularity
- To use special form of dark energy equation of state
- Dark matter-dark energy coupling
- Modification of gravity
- To consider a transient phantom acceleration where the EOS evolves asymptotically to -1.

(Ray, Tarai, Mishra and Tripathy, Fortschrite der Physik, 2021, 2100086)

Dynamics in Brans-Dicke theory

The action for generalised Brans-Dicke theory in a Jordan frame

$$S = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}\phi^{,\alpha}\phi_{,\alpha} - V(\phi) + L_m\right],$$

where $\omega(\phi)$ is the modified Brans-dicke parameter, $V(\phi)$ is the self-interacting potential, R is the scalar curvature and L_m is the matter Lagrangian. The unit system we choose here is $8\pi G_0 = c = 1$.
The field equations

$$\begin{aligned}
G_{\mu\nu} &= \frac{T_{\mu\nu}}{\phi} + \frac{\omega(\phi)}{\phi^2}\left[\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}\right] \\
&+ \frac{1}{\phi}\left[\phi_{,\mu;\nu} - g_{\mu\nu}\Box\phi\right] - \frac{V(\phi)}{2\phi}g_{\mu\nu}, \\
\Box\phi &= \frac{T}{2\omega(\phi)+3} - \frac{2V(\phi) - \phi\frac{\partial V(\phi)}{\partial\phi}}{2\omega(\phi)+3} - \frac{\frac{\partial\omega(\phi)}{\partial\phi}\phi_{,\mu}\phi^{,\mu}}{2\omega(\phi)+3}.
\end{aligned}$$

(Tripathy et al., EPJC, 75, 149 (2015); Phys. of Dark Univ., 30, 100722 (2020); IJGMMP, 17, 2050056 (2020))

Dynamics in Brans-Dicke theory

A plane symmetric LRSBI model is considered through the metric,

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)(dy^2 + dz^2),$$

where A and B are the directional scale factors and are considered as functions of cosmic time t only. The metric corresponds to considering the YZ-plane as the symmetry plane and X as the axis of symmetry. The eccentricity of such a universe is given by $e = \sqrt{1 - \frac{A^2}{B^2}}$.

Dynamics in Brans-Dicke theory

$$\begin{aligned}
(2k+1)\xi^2 H^2 &= \frac{\rho}{\phi} + \frac{\omega(\phi)}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 - 3H\left(\frac{\dot{\phi}}{\phi}\right) + \frac{V(\phi)}{2\phi}, \\
2\xi\dot{H} + 3\xi^2 H^2 &= \frac{-p}{\phi} - \frac{\omega(\phi)}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 - 2\xi H\left(\frac{\dot{\phi}}{\phi}\right) - \frac{\ddot{\phi}}{\phi} + \frac{V(\phi)}{2\phi}, \\
(k+1)\xi\dot{H} + (k^2+k+1)\xi^2 H^2 &= -\frac{p}{\phi} - \frac{\omega(\phi)}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 - (k+1)\xi H\left(\frac{\dot{\phi}}{\phi}\right) - \frac{\ddot{\phi}}{\phi} + \frac{V(\phi)}{2\phi}.
\end{aligned}$$

where the anisotropic expansion rates along different spatial directions are considered as

$$H_x = \frac{\dot{A}}{A},\ H_y = H_z = \frac{\dot{B}}{B}$$

$H = \frac{1}{3}(k+2)H_y = \frac{1}{\xi}H_y$. Here we have defined an anisotropic parameter $\xi = \frac{3}{k+2}$.

Dynamics in Brans-Dicke theory

The evolution equation for the Brans-Dicke scalar field,

$$-\frac{\dot{H}}{H} - 3H = \frac{\dot{\phi}}{\phi}$$

which can also be expressed as

$$(q-2)H = \frac{\dot{\phi}}{\phi}$$

where $q = -1 - \frac{\dot{H}}{H^2}$ is the deceleration parameter. For a constant deceleration parameter the Brans-Dicke field evolves as $\phi \sim a^{q-2}$, or more specifically $\phi \sim (1+z)^{2-q}$.

The Brans-Dicke parameter and the self-interacting potential are obtained from the field equations as

$$\omega(\phi) = \left(\frac{\dot{\phi}}{\phi}\right)^{-2}\left[-\frac{\rho+p}{\phi}-\frac{\ddot{\phi}}{\phi}+k\xi H\frac{\dot{\phi}}{\phi}-2\xi\dot{H}+2(k-1)\xi^2H^2\right],$$

$$V(\phi) = 2\phi\left[(2k+1)\xi^2H^2-\frac{\rho}{\phi}-\frac{\omega(\phi)}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+3H\frac{\dot{\phi}}{\phi}\right].$$

We wish to include a cosmological constant Λ, so that the total energy density may be expressed as $\rho_T = \rho + \Lambda$. The pressure corresponding to the cosmological constant is $-\Lambda$ and consequently the total pressure becomes $p_T = p - \Lambda$. For brevity, we will consider the field equations in the absence of any self interacting potential i.e $V(\phi) = 0$.
The GBD field equations in presence of the cosmological constant can be expressed as

$$\begin{aligned}
(2k+1)\xi^2 H^2 - \frac{\omega(\phi)}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + 3H\left(\frac{\dot{\phi}}{\phi}\right) &= \frac{\rho_T}{\phi}, \\
2\xi\dot{H} + 3\xi^2 H^2 + \frac{\omega(\phi)}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + 2\xi H\left(\frac{\dot{\phi}}{\phi}\right) + \frac{\ddot{\phi}}{\phi} &= \frac{-p_T}{\phi}, \\
(k+1)\xi\dot{H} + (k^2+k+1)\xi^2 H^2 + \frac{\omega(\phi)}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + (k+1)\xi H\left(\frac{\dot{\phi}}{\phi}\right) + \frac{\ddot{\phi}}{\phi} &= -\frac{p_T}{\phi}.
\end{aligned}$$

The Klein-Gordon wave equation for the scalar field becomes

$$\frac{\ddot{\phi}}{\phi} + 3H\frac{\dot{\phi}}{\phi} = \frac{\rho_T - 3p_T}{2\omega(\phi)+3} - \frac{\frac{\partial\omega(\phi)}{\partial\phi}\dot{\phi}^2}{2\omega(\phi)+3}.$$

GBD theory with a comsmological constant

The GBD field equations can be recast as equivalent Friedman equations:

$$\begin{aligned}(2k+1)\xi^2H^2 &= \rho_T+\rho_\phi,\\ \left(\frac{k+3}{2}\right)\xi\dot{H}+\left(\frac{k^2+k+4}{2}\right)\xi^2H^2 &= -\left(p_T+p_\phi\right),\end{aligned}$$

where

$$\begin{aligned}\rho_\phi &= (2k+1)\xi^2H^2\triangle\phi-3H\dot{\phi}+\frac{\omega(\phi)}{2}\left(\frac{\dot{\phi}^2}{\phi}\right),\\ p_\phi &= -\left(\frac{k^2+k+4}{2}\right)\xi^2H^2\triangle\phi-\left(\frac{k+3}{2}\right)\xi\dot{H}\triangle\phi+\ddot{\phi}+\left(\frac{k+3}{2}\right)\xi H\dot{\phi}+\frac{\omega(\phi)}{2}\left(\frac{\dot{\phi}^2}{\phi}\right).\end{aligned}$$

In the above we have introduced the quantity $\triangle\phi=1-\phi$ which tracks the departure of the scalar field from unity. In fact for $\triangle\phi=0$, we have $\rho_\phi=p_\phi=0$ and obviously the GR is recovered.

GBD theory with a comsmological constant

The EoS parameter for this effective DE along with the contribution from the vacuum energy density can be expressed as

$$\omega_{eff} = \frac{p_\phi - \Lambda}{\rho_\phi + \Lambda} = -1 + \frac{-\left(\frac{k+3}{2}\right)\xi \dot{H}\triangle\phi + \left[(2k+1) - \left(\frac{k^2+k+4}{2}\right)\right]\xi^2 H^2 \triangle\phi + f(\phi,\dot{\phi},\ddot{\phi})}{\Lambda + (2k+1)\xi^2 H^2 \triangle\phi + g(\phi,\dot{\phi})},$$

where

$$\begin{aligned} f(\phi,\dot{\phi},\ddot{\phi}) &= \ddot{\phi} + \left[\left(\frac{k+3}{2}\right)\xi - 3\right] H\dot{\phi} + \omega(\phi)\left(\frac{\dot{\phi}^2}{\phi}\right), \\ g(\phi,\dot{\phi}) &= -3H\dot{\phi} + \frac{\omega(\phi)}{2}\left(\frac{\dot{\phi}^2}{\phi}\right) \end{aligned}$$

For an isotropic model with $k = 1$, the effective DE EoS parameter reduces to

$$\omega_{eff} = -1 + \frac{-2\dot{H}\triangle\phi + f(\phi,\dot{\phi},\ddot{\phi})}{\Lambda + 3H^2\triangle\phi + g(\phi,\dot{\phi})}.$$

GBD theory with a comsmological constant

At high redshift zone, the EoS parameter reduces to

$$\omega_{eff}(z \gg 1) = -1 + \frac{-\left(\frac{k+3}{2}\right)\xi\dot{H} + \left[(2k+1) - \left(\frac{k^2+k+4}{2}\right)\right]\xi^2 H^2}{(2k+1)\xi^2 H^2}.$$

For small values of $|\triangle\phi|$, in the low redshift zone, we may neglect all the terms in the denominator in ω_{eff} compared to Λ. Similarly, we may neglect the function $f(\phi, \dot{\phi}, \ddot{\phi})$ in the numerator, so that the effective DE EoS parameter becomes

$$\omega_{eff} \simeq -1 - \frac{\triangle\phi}{\Lambda}\left[-\left(\frac{k+3}{2}\right)\xi\dot{H} + \left[(2k+1) - \left(\frac{k^2+k+4}{2}\right)\right]\xi^2 H^2\right].$$

GBD theory with a comsmological constant

Considering a small departure from cosmic isotropy i.e $\triangle k = k - 1 \simeq 0$, we get a quintessence-like behaviour for $\triangle\phi > 0$ and a phantom-like behaviour for $\triangle\phi < 0$.

For an anisotropic Universe, we get

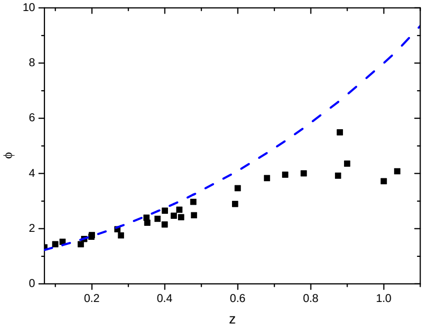
$$\phi \simeq \frac{H_0}{H}(1+z)^3,$$

so that

$$\triangle\phi \simeq 1 - \frac{H_0}{H}(1+z)^3.$$

In the low redshift region, the BD scalar field behaves like $\sim (1+z)^3$.

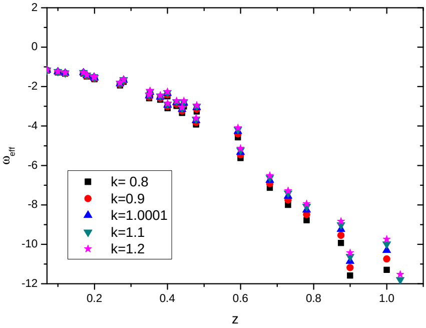
GBD theory with a comsmological constant



The BD scalar field constructed for low redshift region from the available observational $H(z)$ data.

The figure shows that the scalar field is always greater than 1 and consequently, for the low redshift region, we have $\triangle\phi < 0$ and we should expect a phantom-like behaviour for our model.

GBD theory with a comsmological constant



The effective equation of state ω_{eff} as constructed from the observational $H(z)$ data for five representative values of the anisotropy parameter k.

The effective equation of state parameter evolves from a region $\omega_{eff} < -1$ to $\omega_{eff} = -1$. For all the values of k chosen in the work, we get phantom-like behaviour.

A Big Rip Scenario

We consider a Big Rip scenario where the scale factor evolves as

$$a(t) \simeq (t_{BR} - t)^{\alpha},$$

where t_{BR} is the epoch where the scale factor blows up. α is a constant parameter related to the EOS parameter as

$$\alpha = \frac{2}{3(1+\omega_D)}.$$

Since, in phantom field dominated Universe, the EOS parameter is less than unity i.e. $\omega_D < -1$, we have $\alpha < 0$.

We have the Hubble rate

$$H(t) = -\frac{\alpha}{t_{BR} - t} = -\frac{H^2}{\alpha},$$

and the deceleration parameter as

$$q = -1 + \frac{1}{\alpha}.$$

A Big Rip Scenario

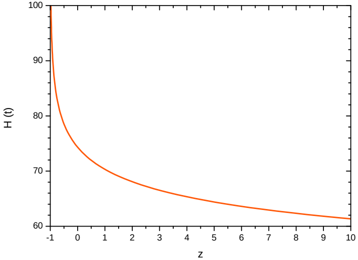
While the Hubble rate contains two adjustable parameters, the deceleration parameter contains only one parameter.

We may consider the deceleration parameter at the present epoch as $q_0 = -1.08 \pm 0.29$ (Camarena et al. Phys.Rev. Res., 2020, 2, 013028) to constrain the parameter as $\alpha = -12.49$.

The corresponding EOS parameter becomes $\omega_D = -1.0533$ which is in close agreement with some recent measurements as mentioned earlier.

A Big Rip Scenario

The Hubble rate for such scenario predicting a finite time future singularity behaves as



The evolutionary behaviour of the Hubble parameter.

The Hubble parameter increases with the cosmic expansion and blows up at some finite future time. Assuming a Hubble parameter at the present epoch as $H_0 = 74.3\ km\ s^{-1}\ Mpc^{-1}$, the big rip may occur at a cosmic time $t_{BR} \simeq 16.14\ Gyr$.

Behaviour of the BD scalar field

Since

$$-\frac{\dot{H}}{H} - 3H = \frac{\dot{\phi}}{\phi},$$

we obtain

$$\frac{\dot{\phi}}{\phi} = (1 - 3\alpha) H$$

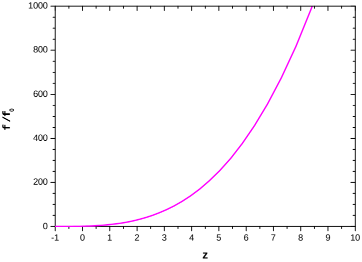
which on integration yields

$$\frac{\phi}{\phi_0} = \left(\frac{H_0}{H}\right)^{(1-3\alpha)},$$

where ϕ_0 is the value of the BD scalar field at the present epoch.
Also we have

$$\frac{\ddot{\phi}}{\phi} = -3\left(\frac{1}{\alpha} - 3\right) H^2.$$

Behaviour of the BD scalar field



The evolutionary behaviour of the BD scalar field.

The BD Scalar field decreases from a large value at an initial epoch to vanishingly small values at large cosmic time. As has been mentioned earlier, in the low redshift region, the BD scalar field behaves as $\sim (1+z)^3$.

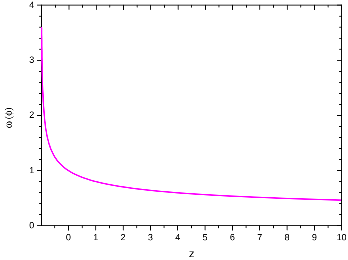
Behaviour of the BD parameter

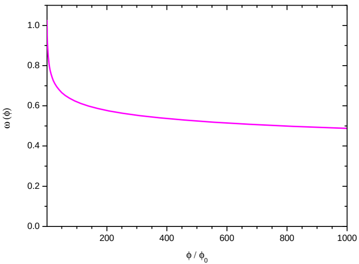
In the GBD theory, the BD parameter is dynamical quantity and its behaviour depends on the behaviour of the BD scalar field. The BD parameter is expressed as

$$\omega(\phi) = \left(\frac{\dot{\phi}}{\phi}\right)^{-2}\left[-\frac{\rho+p}{\phi} - \frac{\ddot{\phi}}{\phi} + k\xi H\frac{\dot{\phi}}{\phi} - 2\xi\dot{H} + 2(k-1)\xi^2 H^2\right].$$

The energy density may be obtained from the conservation equation as $\rho = \rho_0\left(\frac{H}{H_0}\right)^2$ and consequently the pressure as $p = \left(\frac{2}{3\alpha} - 1\right)\rho_0\left(\frac{H}{H_0}\right)^2$.

Behaviour of the BD parameter





The evolutionary behaviour of the BD parameter.

The BD parameter increases with cosmic expansion and also blows up at $t \simeq t_{BR}$. Also, within the purview of the present formalism, in order to get a rip scenario at a finite future, we require a large negative value for the BD parameter at the present epoch i.e $\omega(\phi_0) \simeq -3.62 \times 10^9$.

Summary and Conclusion

- We discussed the possibility of a Big Rip scenario within the formalism of a generalized Brans-Dicke theory.
- The finite time future singularity or the Big Rip may occur for phantom dominated dark energy models where the EOS parameter is $\omega_D < -1$.
- We have shown that, a GBD theory with a cosmological constant favours a phantom field like behaviour and a Big Rip scenario may occur within this formalism.
- A Big Rip scenario is discussed where the scale factor, Hubble rate blow up at a finite rip time predicted from the model to be $t_{BR} \simeq 16.14\ Gyr$.
- For a finite time doomsday to occur, we require a large negative value of the BD parameter.

Thank You