

Nuclear Symmetry Energy from Experiment and Astrophysics

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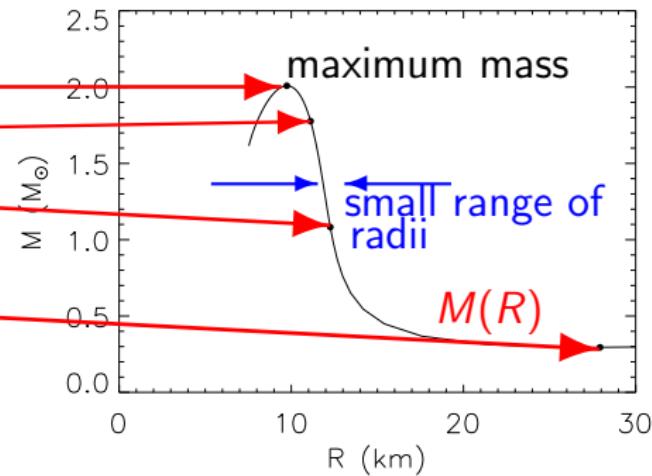
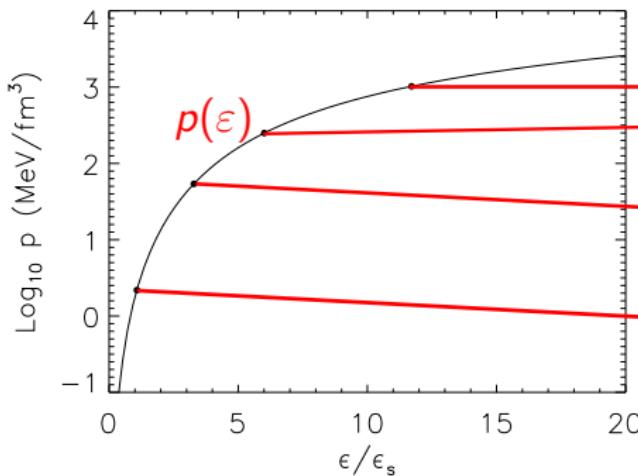
Main Topics

- ▶ Nuclear Symmetry Energy and its Connection to Neutron Stars
- ▶ Nuclear Masses
- ▶ Theoretical Neutron Matter Calculations
- ▶ Unitary Gas Considerations
- ▶ Neutron Skin Thicknesses
- ▶ Nuclear Dipole Properties
- ▶ Astrophysical Constraints

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^4} \frac{(mc^2 + 4\pi pr^3)(\varepsilon + p)}{r(r - 2Gm/c^2)}$$
$$\frac{dm}{dr} = 4\pi \frac{\varepsilon}{c^2} r^2$$

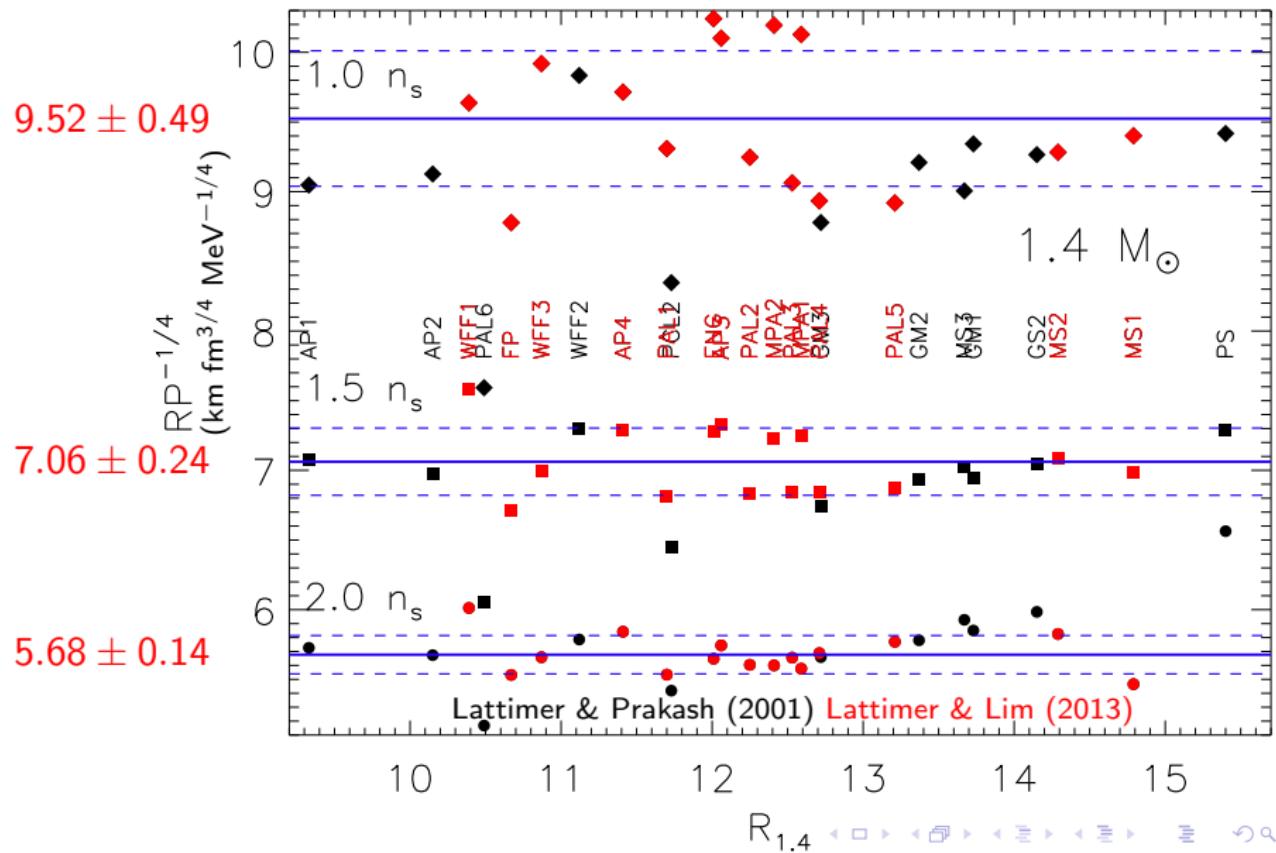


Equation of State



Observations

The Radius – Pressure Correlation



Nuclear Symmetry Energy and the Pressure

The symmetry energy is the difference between the energies of pure neutron matter ($x = 0$) and symmetric ($x = 1/2$) nuclear matter:

$$S(n) = E(n, x=0) - E(n, x=1/2)$$

Usually approximated as an expansion around the saturation density (n_s) and isospin symmetry ($x = 1/2$):

$$E(n, x) = E(n, 1/2) + (1-2x)^2 S_2(n) + \dots$$

$$S_2(n) = S_v + \frac{L}{3} \frac{n - n_s}{n_s} + \dots$$

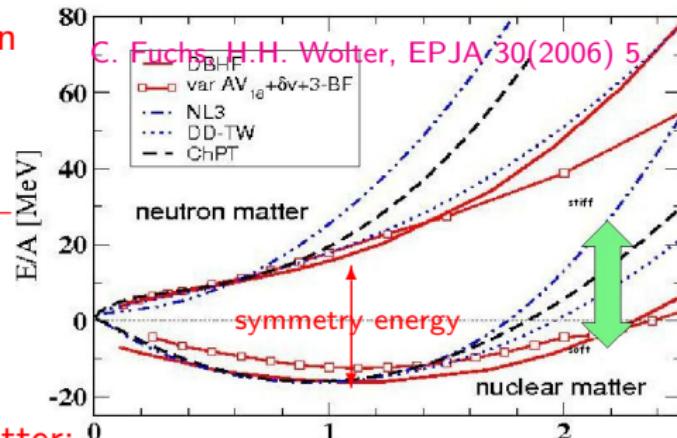
$$S_v \simeq 31 \text{ MeV}, \quad L \simeq 50 \text{ MeV}$$

Extrapolated to pure neutron matter:

$$E_N = E(n_s, 0) \approx S_v + E(n_s, 1/2) \equiv S_v - B, \quad P_N = P(n_s, 0) = L n_s / 3$$

Neutron star matter (beta equilibrium) is nearly neutron matter:

$$\frac{\partial(E + E_e)}{\partial x} = 0, \quad P(n_s, x_\beta) \simeq \frac{L n_s}{3} \left[1 - \left(\frac{4 S_v}{\hbar c} \right)^3 \frac{4 - 3 S_v / L}{3 \pi^2 n_s} \right]$$



Energy Expansions

$$S(u) = E_N(u) - E_{1/2}(u), \quad u = n/n_s$$

$$E_{1/2}(u) = -B + \frac{K_{1/2}}{18}(u-1)^2 + \frac{Q_{1/2}}{162}(u-1)^3 + \dots$$

$$E_N(u) = S_V - B + \frac{L}{3}(u-1) + \frac{K_N}{18}(u-1)^2 + \frac{Q_N}{162}(u-1)^3 + \dots$$

$$S(u) = S_V + \frac{L}{3}(u-1) + \frac{K_{\text{sym}}}{18}(u-1)^2 + \frac{Q_{\text{sym}}}{162}(u-1)^3 + \dots$$

Empirical saturation properties:

$$n_s = 0.155 \pm 0.005 \text{ fm}^{-3}, \quad B = 16 \pm 1 \text{ MeV}, \quad K_{1/2} = 230 \pm 20 \text{ MeV}$$

261 nuclear interactions fit to nuclei yield these correlations:

$$K_{\text{sym}} = 3.501L - 305.67 \pm 24.26 \text{ MeV}$$

$$Q_{\text{sym}} = -6.443L + 708.74 \pm 118.14 \text{ MeV}$$

$$Q_{1/2} = -0.870L - 354.71 \pm 178.04 \text{ MeV}$$

Symmetry Parameter Correlation from Masses

Liquid drop model approximately valid

$$E_{\text{sym}}(N, Z) = (S_V A - S_s A^{2/3}) I^2$$

$$\chi^2 = \frac{1}{N\sigma_D^2} \sum_{i=1}^N (E_{\text{ex},i} - E_{\text{sym},i})^2$$

$$\chi_{vv} = \frac{2}{N\sigma_D^2} \sum_{i=1}^N I_i^4 A_i^2 \simeq 61.6 \sigma_D^{-2}$$

$$\chi_{ss} = \frac{2}{N\sigma_D^2} \sum_{i=1}^N I_i^4 A_i^{4/3} \simeq 1.87 \sigma_D^{-2}$$

$$\chi_{vs} = -\frac{2}{N\sigma_D^2} \sum_{i=1}^N I_i^4 A_i^{5/3} \simeq -10.7 \sigma_D^{-2}$$

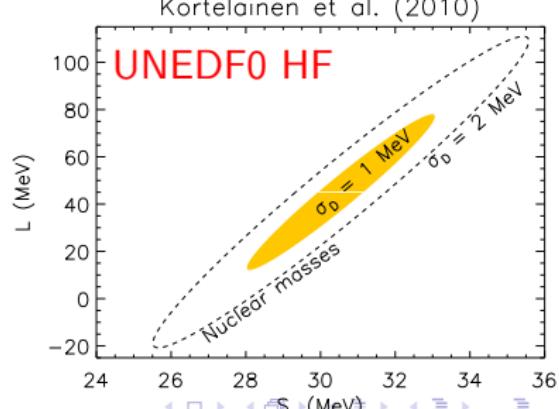
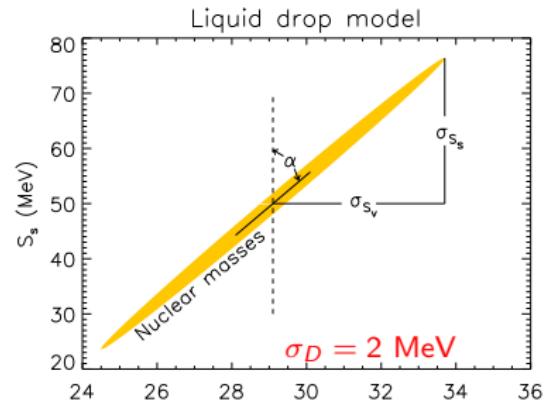
$$\sigma_{S_V} = \sqrt{\frac{2\chi_{ss}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}} \simeq 2.3 \sigma_D$$

$$\sigma_{S_s} = \sqrt{\frac{2\chi_{vv}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}} \simeq 13.2 \sigma_D$$

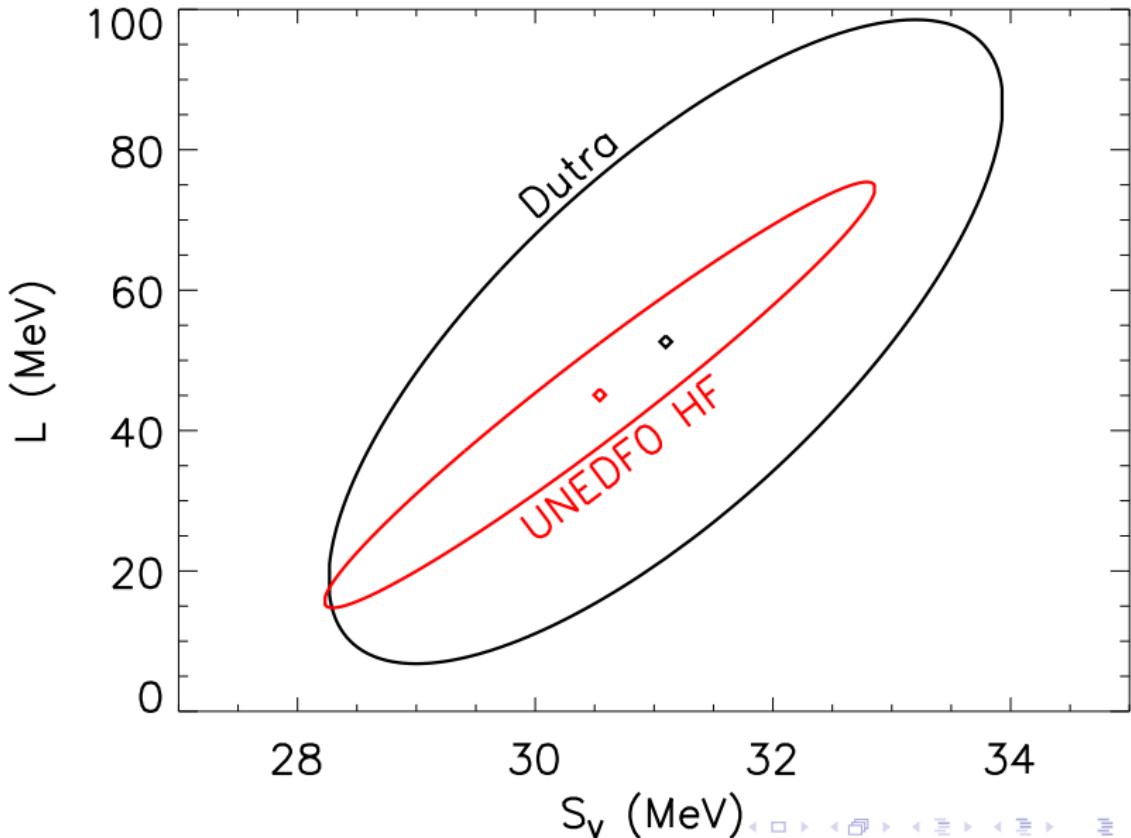
$$\alpha = \frac{1}{2} \tan^{-1} \frac{2\chi_{vs}}{\chi_{vv} - \chi_{ss}} \simeq 9^\circ.8$$

$$r_{vs} = -\frac{\chi_{vs}}{\sqrt{\chi_{vv}\chi_{ss}}} \simeq 0.997$$

S_s is highly correlated with L and S_V .

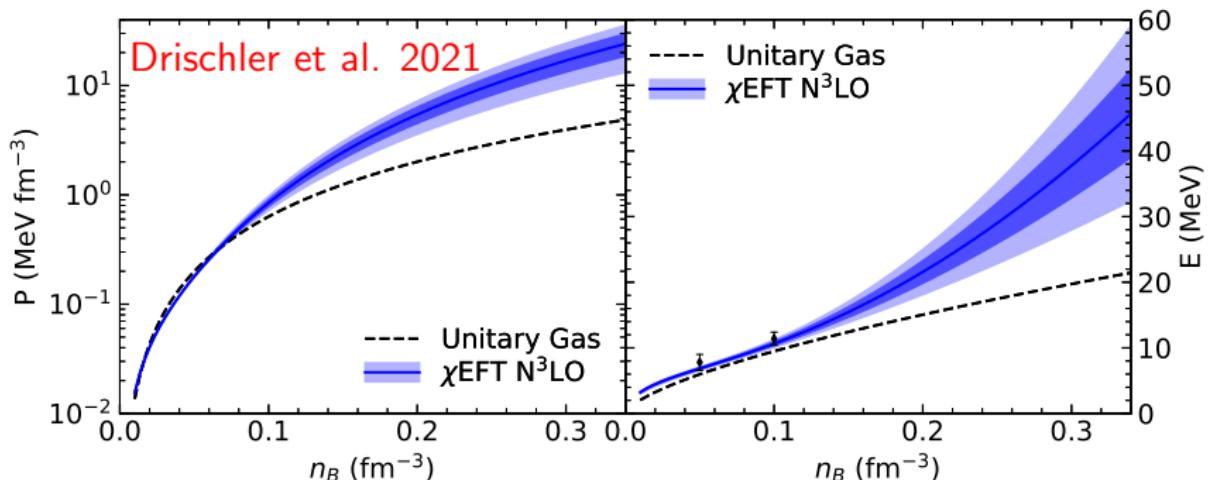


Correlations from 261 Forces vs. Mass Fitting



Theoretical Neutron Matter Studies

Recently developed chiral effective field theory allows a systematic expansion of nuclear forces at low energies based on the symmetries of quantum chromodynamics. It exploits the gap between the pion mass (the pseudo-Goldstone boson of chiral symmetry-breaking) and the energy scale of short-range nuclear interactions established from experimental phase shifts. It provides the only known consistent framework for estimating energy uncertainties.



Symmetry Parameters From Neutron Matter

Pure neutron matter calculations are more reliable than for symmetric matter.

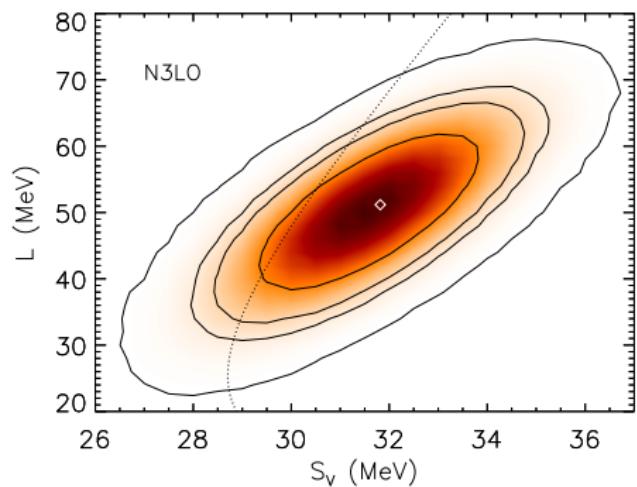
Symmetric matter emerges from a delicate cancellation sensitive to short- and intermediate-range three-body interactions at N^2LO that are Pauli-blocked in pure neutron matter.

N^3LO symmetric matter calculations don't saturate within empirical ranges for n_s and B . We infer symmetry parameters from $E_N(n_s)$ and $P_N(n_s)$ using

$$S_V = E_N(n_s) + B$$

$$L = 3P_N(n_s)/n_s$$

and include uncertainties in E_N , P_N , n_s and B .



Bounds From The Unitary Gas Conjecture

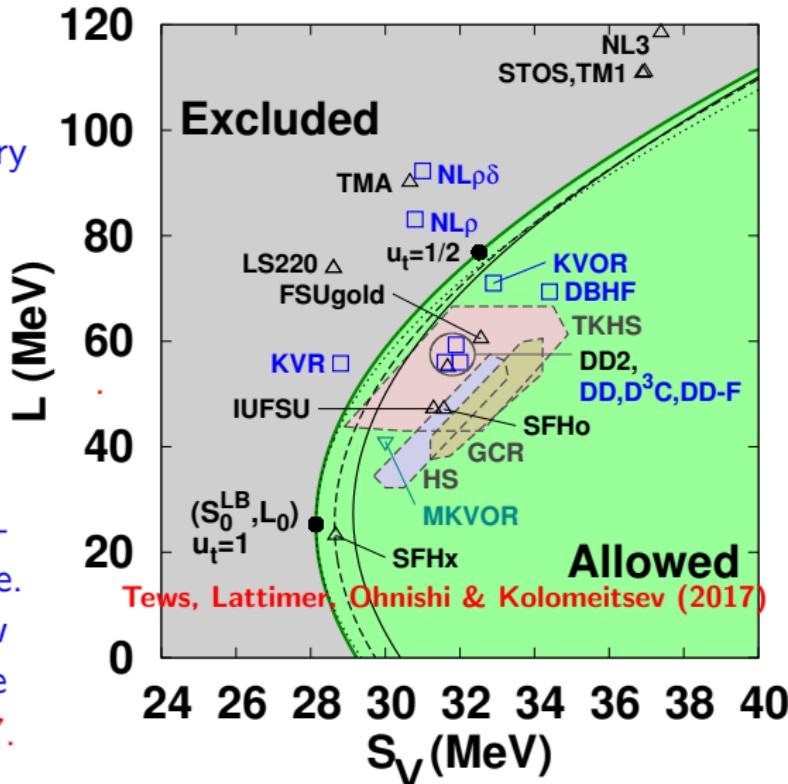
The Conjecture:

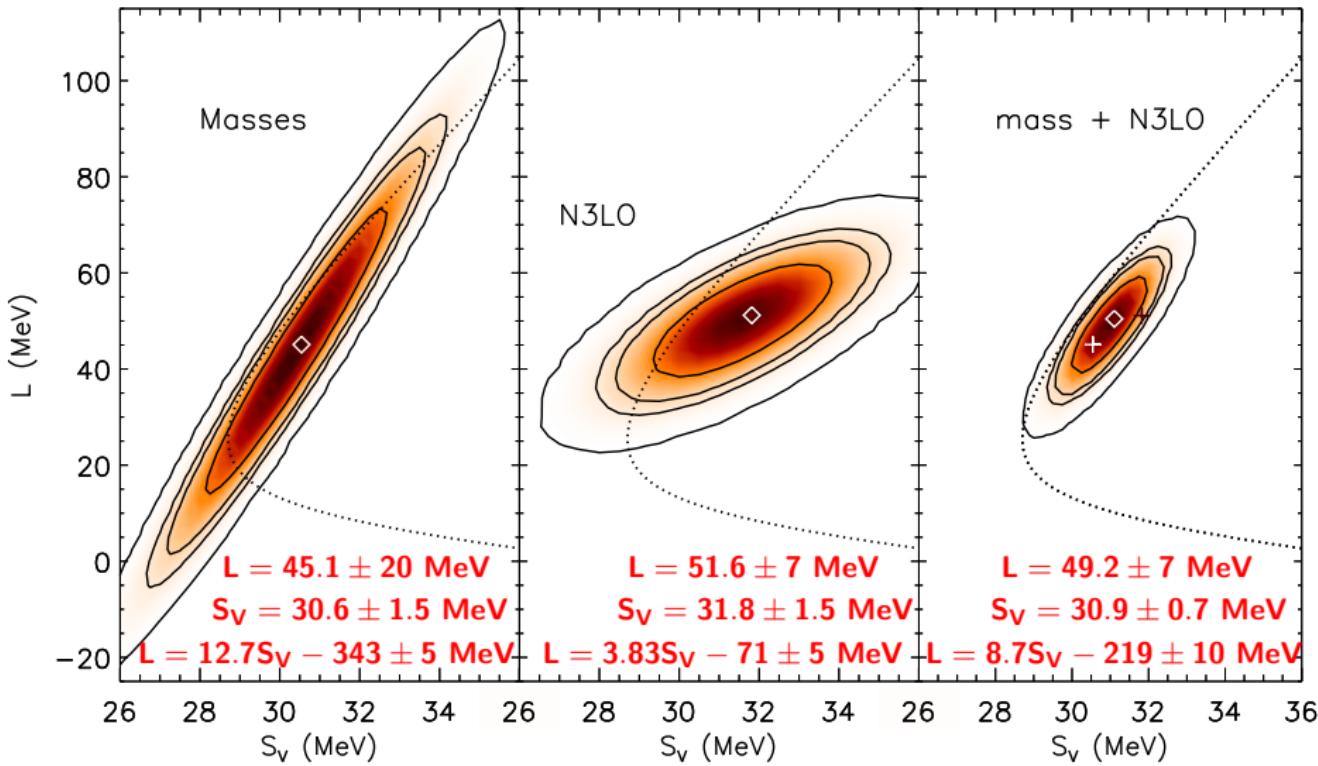
Neutron matter energy is larger than that of the unitary gas $E_{UG} = \xi_0(3/5)E_F$, or

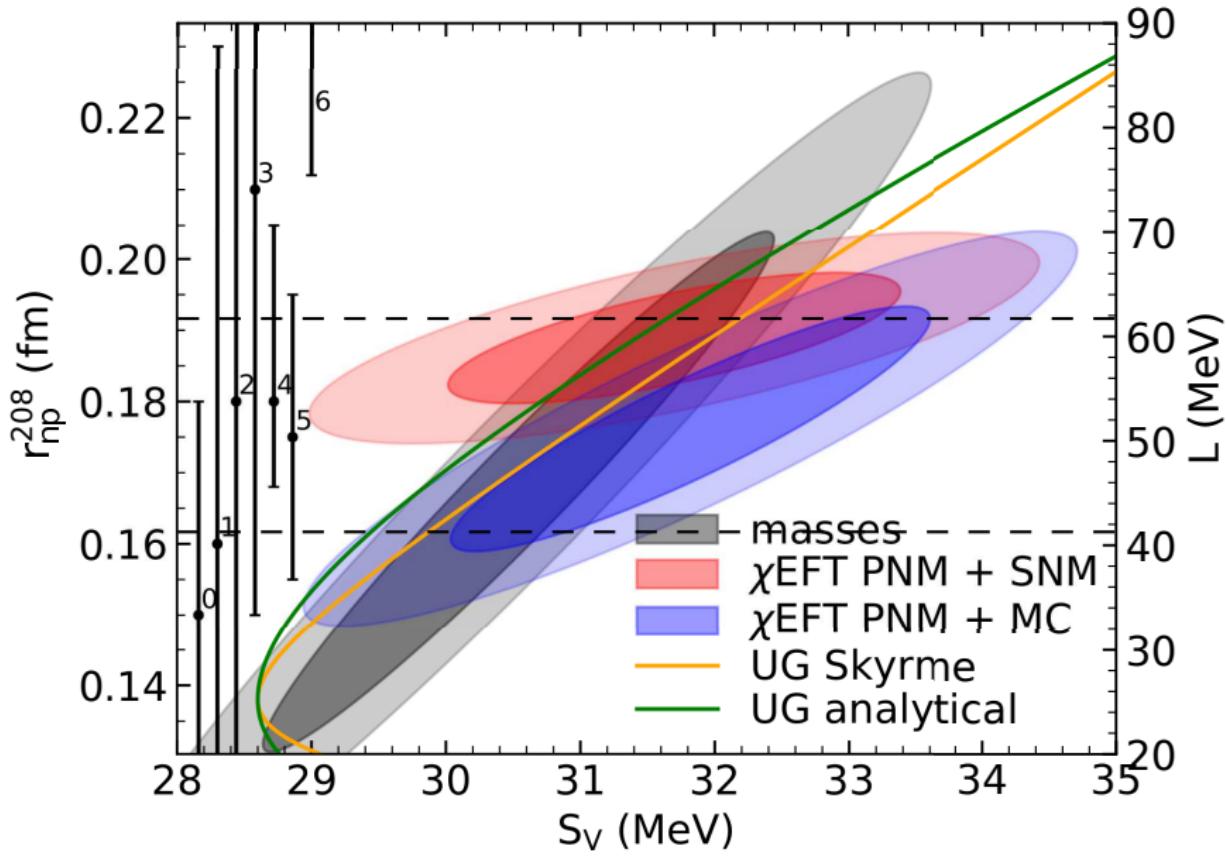
$$E_{UG} \simeq 12.6 \left(\frac{n}{n_s} \right)^{2/3} \text{ MeV}$$

The unitary gas consists of fermions interacting via a pairwise short-range s-wave interaction with infinite scattering length and zero range. Cold atom experiments show a universal behavior with the Bertsch parameter $\xi_0 \simeq 0.37$.

$$S_V \geq 28.6 \text{ MeV}; L \geq 25.3 \text{ MeV}; P_N(n_s) \geq 1.35 \text{ MeV fm}^{-3}; R_{1.4} \geq 9.7 \text{ km}$$







Neutron Skin Thickness

The difference between the mean neutron and proton radii in the liquid droplet model is $t_{np} = R_n - R_p$

$$t_{np} = \frac{2r_o I}{3} \frac{S_s}{S_V} \left[1 + S_s A^{-1/3} / S_V \right]^{-1}$$

$$r_{np} = \sqrt{\langle R_n \rangle^2 - \langle R_p \rangle^2}$$

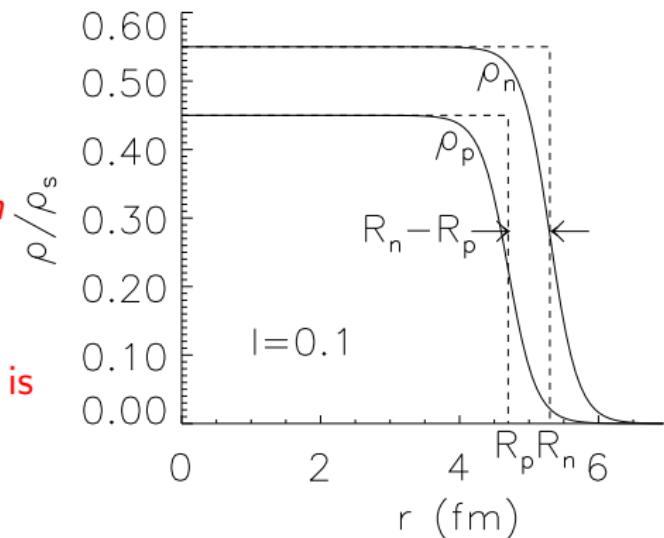
As for masses, this implies an $L - S_V$ correlation.

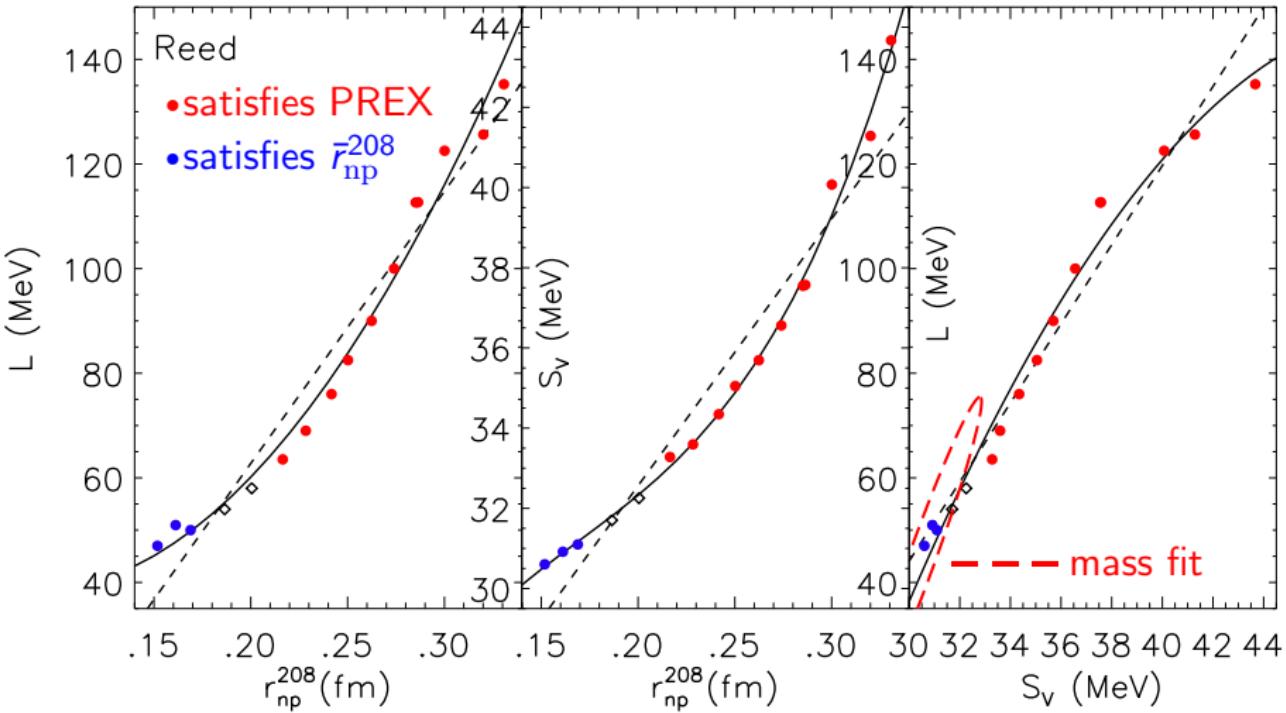
Additionally, Brown found the correlation between S and dS/dn was strongest for $n \sim 2n_s/3$.

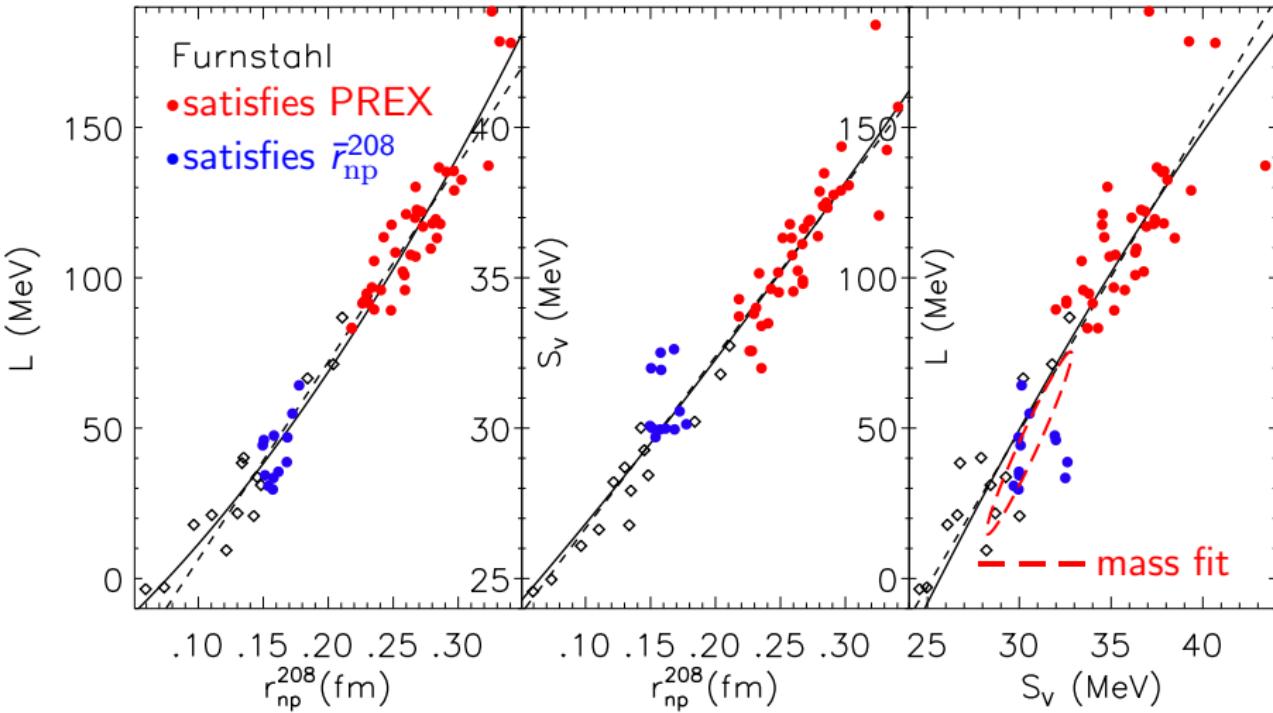
$$\tilde{L} = 3ndS/dn, r_{np} \propto \tilde{L}(2n_s/3).$$

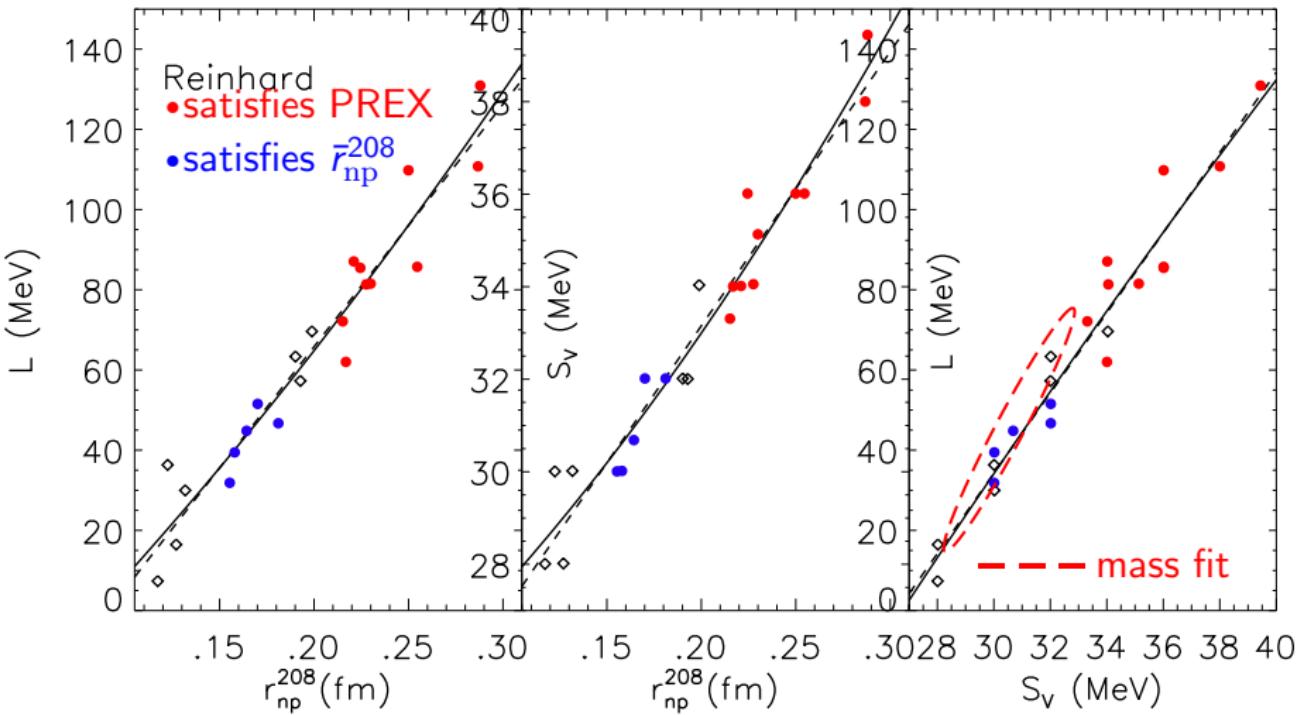
$S(u) \simeq S_V + (u - 1)L/3$, so this is the same correlation.

$$r_{np}^{208} \simeq \frac{(dS/dn)_{0.1}}{(882 \pm 32) \text{ MeV fm}^{-2}}$$

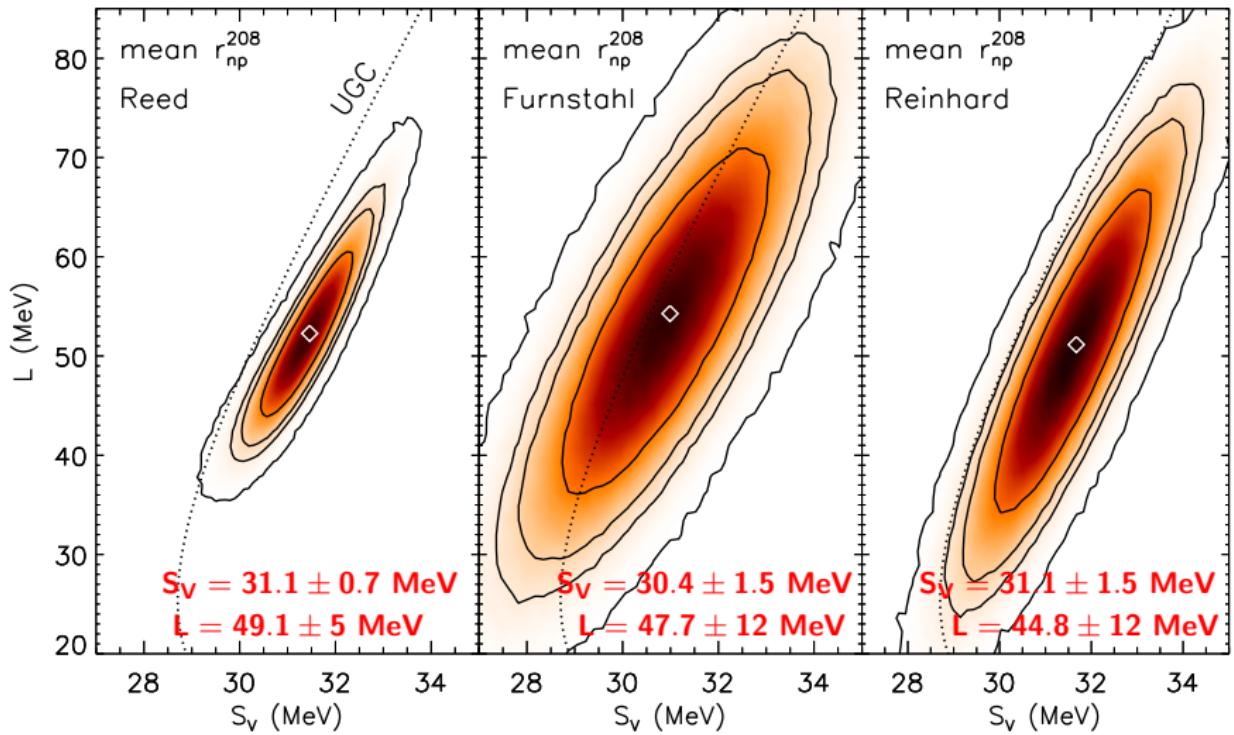




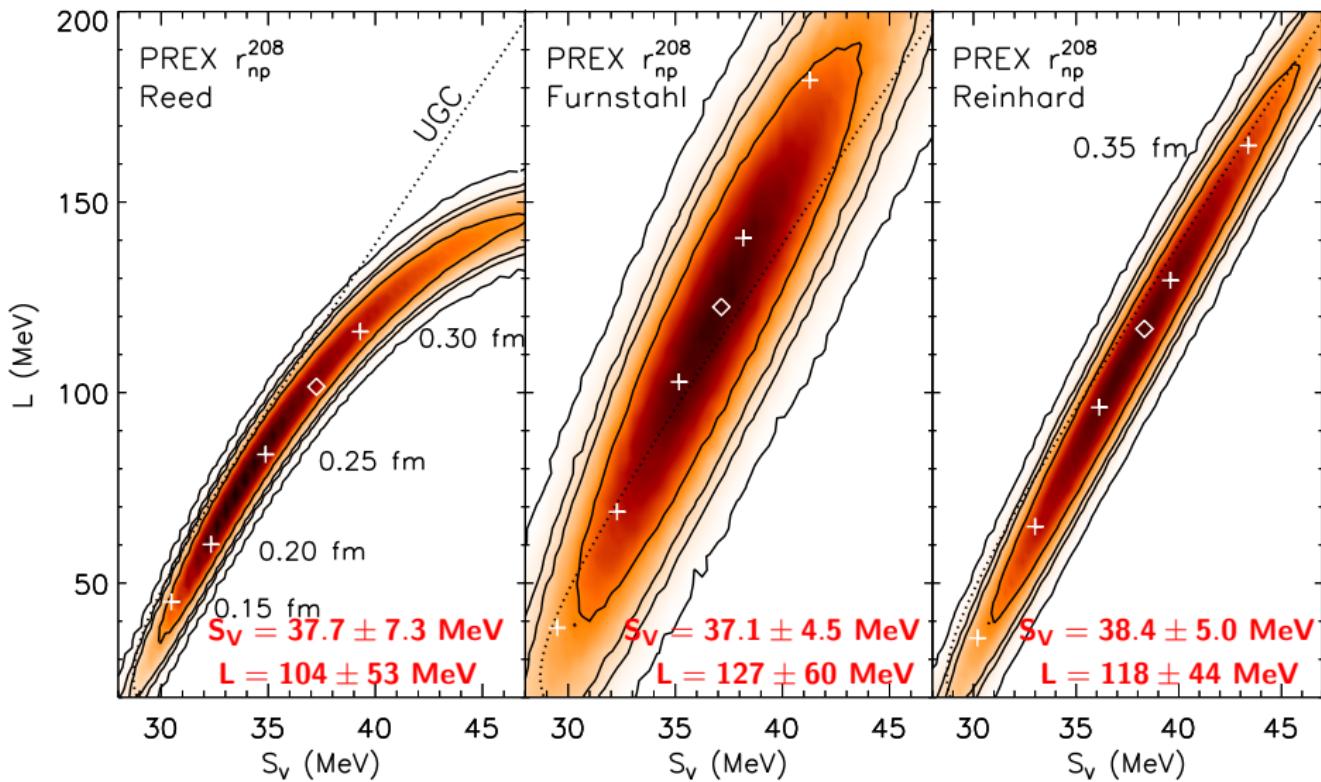




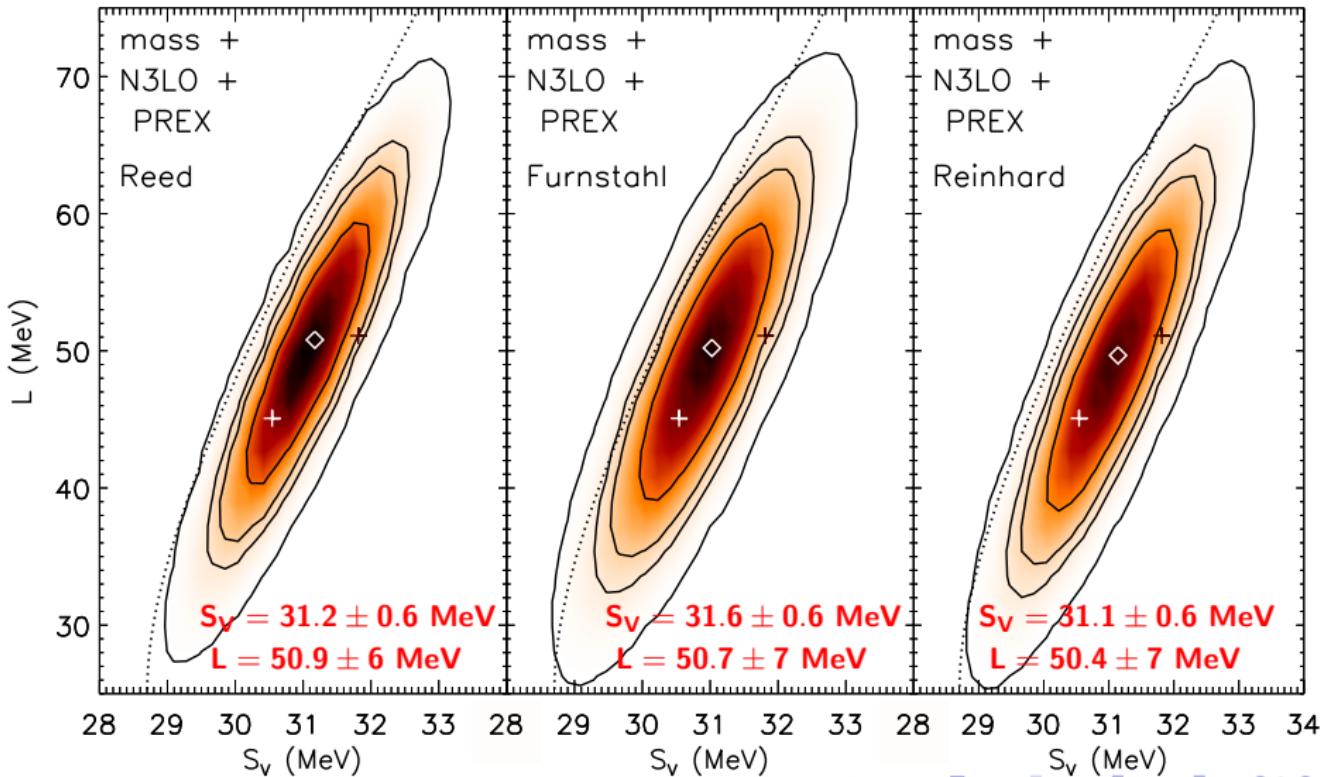
Inferred $L - S_V$ with mean r_{np}^{208}



Inferred $L - S_V$ with PREX r_{np}^{208}



Masses + N3LO + PREX



Electric Dipole Polarizability

Trippa et al. (2005) found central energy of giant dipole resonance of ^{208}Pb has highest correlation with symmetry energy at $n_1 = 0.1 \text{ fm}^{-3}$, with $S(n_1) = 24.1 \pm 0.8 \text{ MeV}$.

Zhang et al. (2015) found dipole polarizability has highest correlation at $n_2 = 0.05 \text{ fm}^{-3}$, and $S(n_2) = 16.54 \pm 1.0 \text{ MeV}$.

$$\Rightarrow L = 10.9S_V - 287.3 \pm 8.9 \text{ MeV}, \quad L = 8.05S_V - 206.9 \pm 9.6 \text{ MeV}$$

Hashimoto (2015) Sn: $\alpha_D^{120} = 8.59 \pm 0.37 \text{ fm}^3$

Tamii (2012) Pb: $\alpha_D^{208} = 19.6 \pm 0.6 \text{ fm}^3$

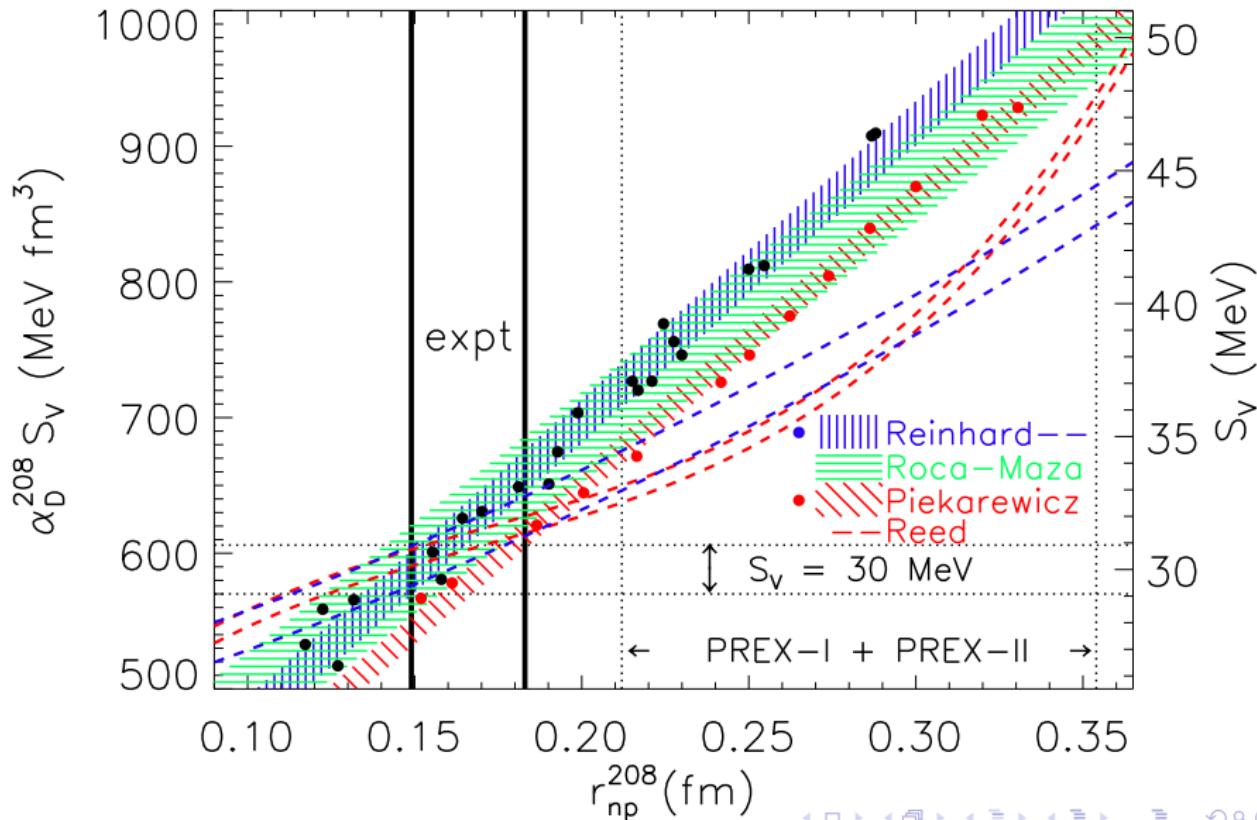
Birkhan (2017) Ca: $\alpha_D^{48} = 2.07 \pm 0.22 \text{ fm}^3$

Roca-Maza et al. (2015) showed (using liquid droplet model as justification) $\alpha_D S_V \propto r_{np}$:

$$\alpha_D^{120} S_V = (1234 \pm 93) (r_{np}^{120}/\text{fm}) + 115 \pm 36 \text{ MeV fm}^3,$$

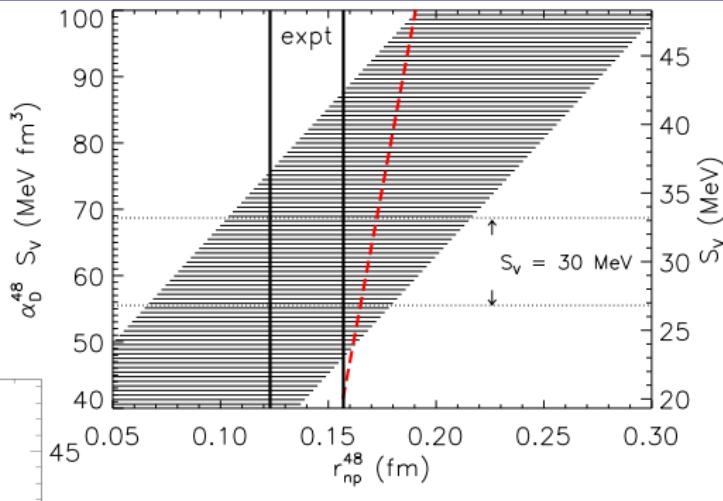
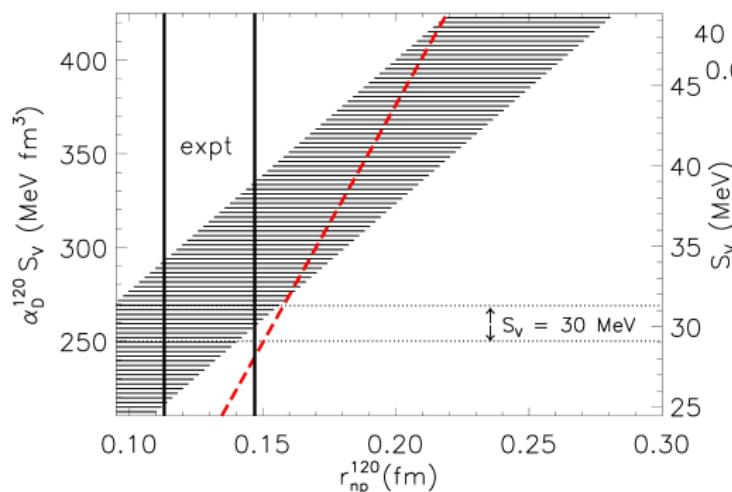
$$\alpha_D^{208} S_V = (1922 \pm 73) (r_{np}^{208}/\text{fm}) + 301 \pm 32 \text{ MeV fm}^3,$$

Dipole Polarizability and Skin of ^{208}Pb



Dipole Polarizability and Skins of ^{120}Sn and ^{48}Ca

--- Chen et al. 2010



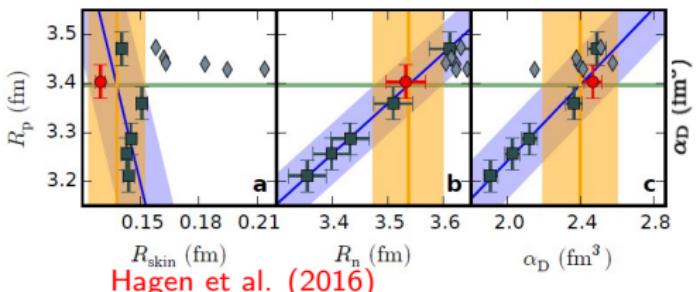
Dipole Polarizability and Skin of ^{48}Ca

Theoretically, $r_{np}^{48} \simeq r_{np}^{208}$.

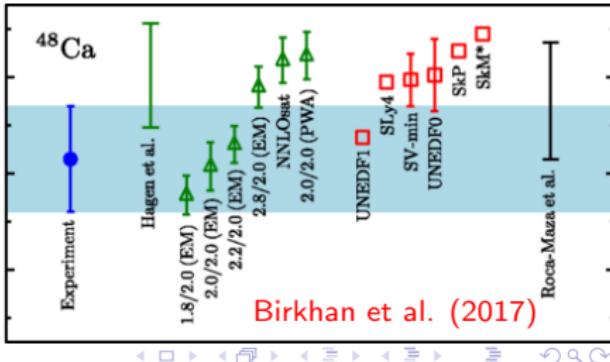
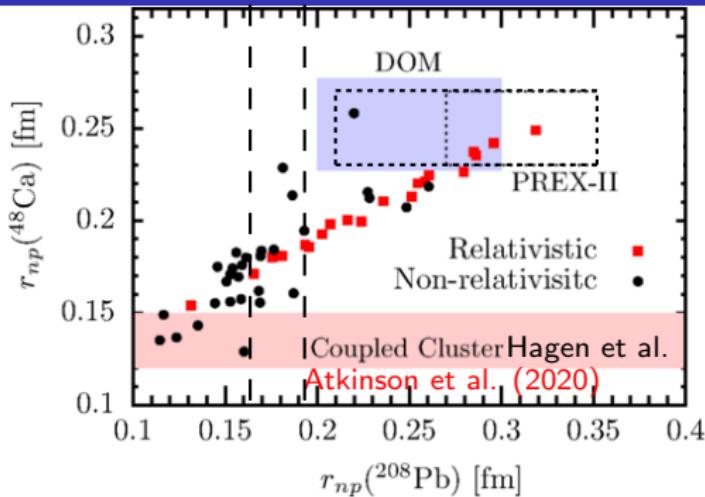
Ab-initio calculations give smallish $r_{np}^{48} = 0.12\text{-}0.15$ fm.

PREX r_{np}^{208} suggests r_{np}^{48} larger than most theoretical or ab-initio calculations.

CREX plans to announce results for r_{np}^{48} in October.



Hagen et al. (2016)



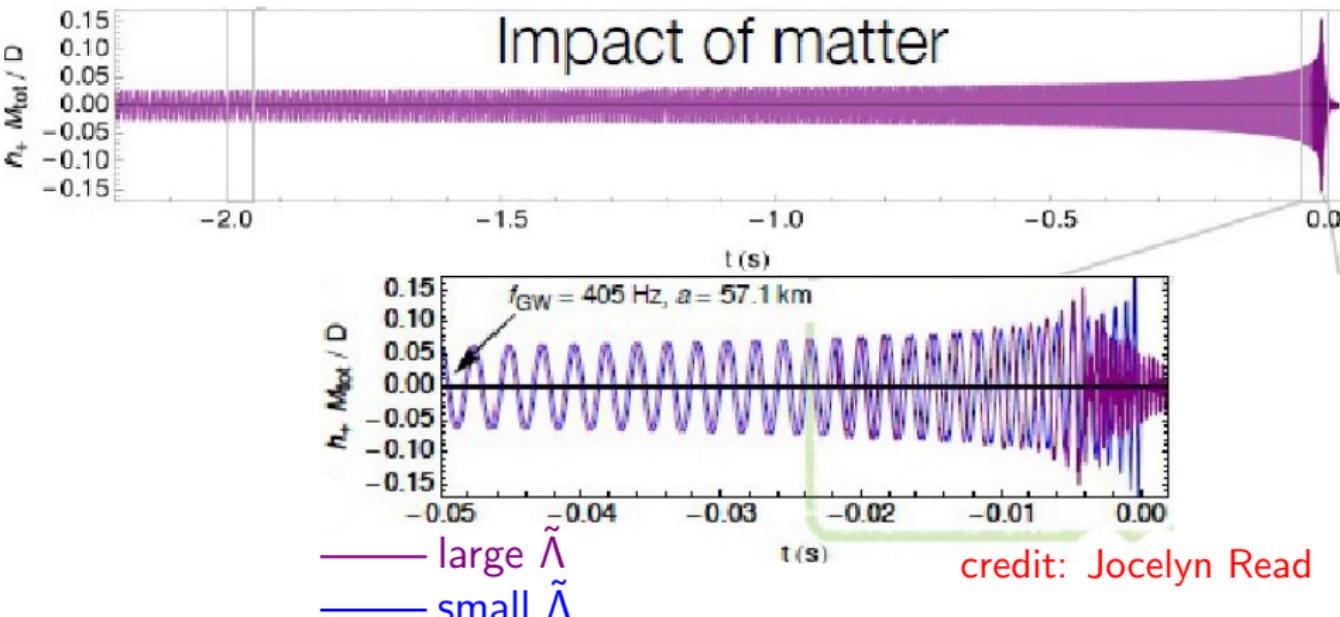
Measuring Neutron Star Masses and Radii

- ▶ Pulsar timing can accurately measure masses in binaries.

Most are between $1.2M_{\odot}$ and $1.5M_{\odot}$. The lowest well-measured mass is $1.174 \pm 0.004M_{\odot}$, the highest are $2.08 \pm 0.07M_{\odot}$ and $2.01 \pm 0.04M_{\odot}$. Those with higher estimated masses have large uncertainties.
- ▶ X-ray observations yield radii, but are uncertain to a few km.
 - ▶ Quiescent binary sources in globular clusters
 - ▶ Thermonuclear explosions leading to photospheric radius expansion bursters on accreting neutron stars in binaries
 - ▶ Pulse profile modeling of hot spots on rapidly rotating neutron stars.
- ▶ Gravitational waves from merging binary neutron stars (BNS) measure their masses and tidal deformabilities.

The Effect of Tides

Tides accelerate the inspiral and produce a phase shift compared to the case of two point masses.



$$\delta\Phi_t = -\frac{117}{256} \frac{(1+q)^4}{q^2} \left(\frac{\pi f_{GW} G M}{c^3} \right)^{5/3} \tilde{\Lambda} + \dots$$

Tidal Deformability

The tidal deformability λ is the ratio of the induced dipole moment Q_{ij} to the external tidal field E_{ij} , $Q_{ij} \equiv -\lambda E_{ij}$.

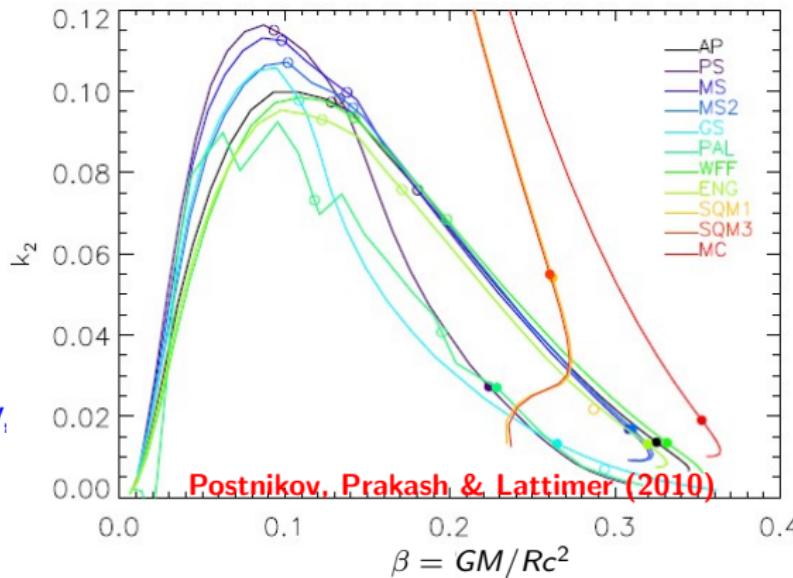
We use the dimensionless quantity

$$\Lambda = \frac{\lambda c^{10}}{G^4 M^5} \equiv \frac{2}{3} k_2 \left(\frac{R c^2}{G M} \right)^5$$

k_2 is the dimensionless Love number.

For a neutron star binary, the mass-weighted $\tilde{\Lambda}$ is the relevant parameter:

$$\tilde{\Lambda} = \frac{16}{13} \frac{(1 + 12q)\Lambda_1 + (12 + q)q^4\Lambda_2}{(1 + q)^5}, \quad q = M_2/M_1 \leq 1$$



Λ is Highly Correlated With M and R

$$R = 13.9 \left(\frac{M}{1.4M_{\odot}} \right) \left(\frac{\Lambda}{800} \right)^{1/6} \text{ km}$$

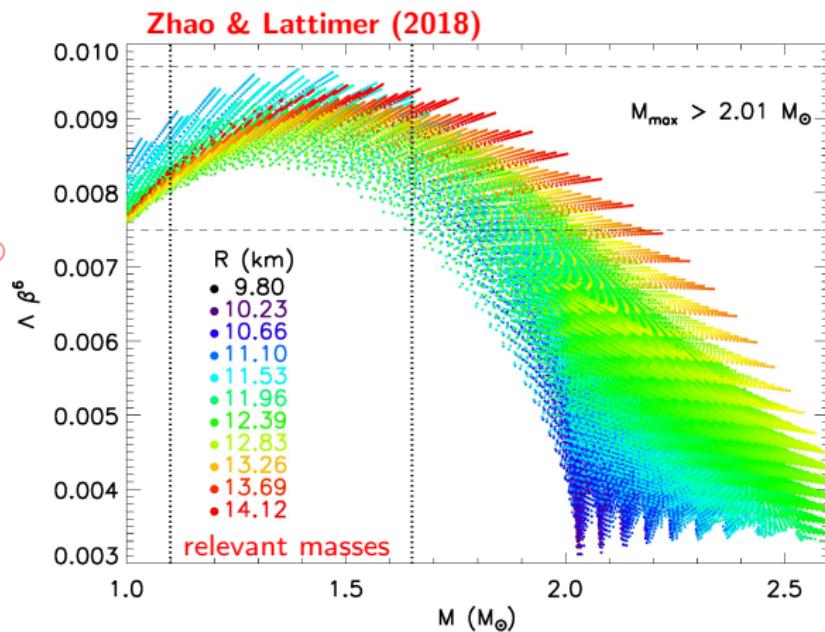
- ▶ $\Lambda = a\beta^{-6}$
 $\beta = GM/Rc^2$

$$a = 0.0086 \pm 0.0011$$

for

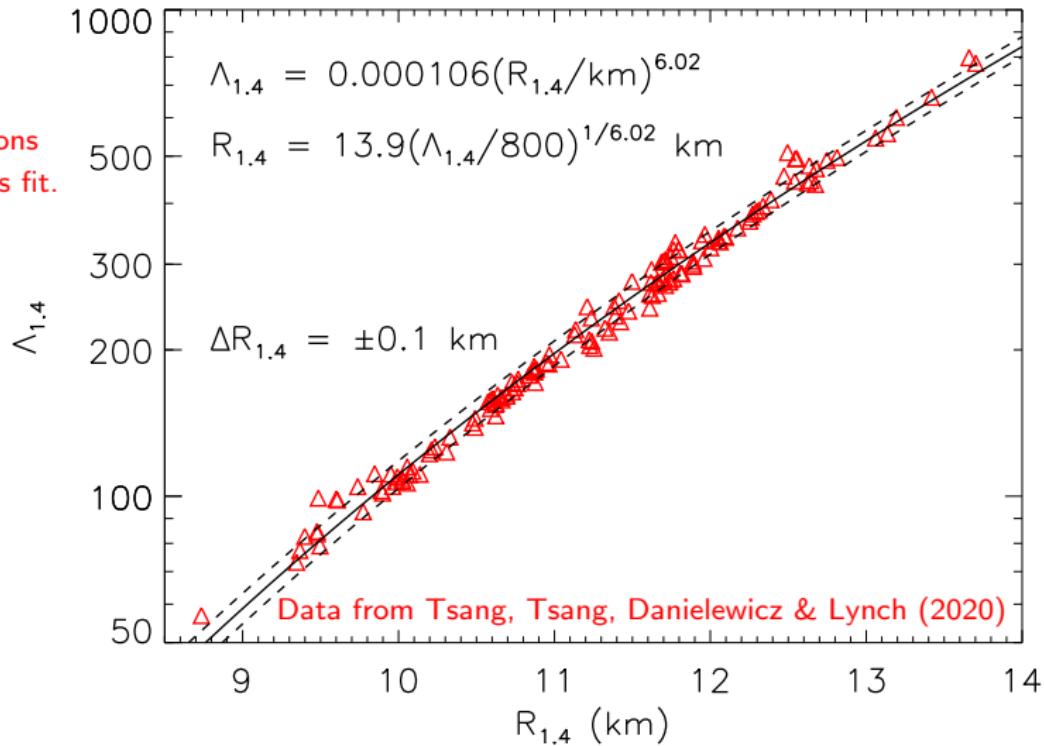
$$M = (1.35 \pm 0.25) M_{\odot}$$

- ▶ If $R_1 \simeq R_2 \simeq R_{1.4}$,
it follows that
 $\Lambda_2 \simeq q^{-6}\Lambda_1$.



186 Skyrme Interactions

RMF interactions
also obey this fit.



Binary Deformability and the Radius

$$\tilde{\Lambda} = \frac{16}{13} \frac{(1+12q)\Lambda_1 + q^4(12+q)\Lambda_2}{(1+q)^5} \simeq \frac{16a}{13} \left(\frac{R_{1.4}c^2}{GM} \right)^6 \frac{q^{8/5}(12-11q+12q^2)}{(1+q)^{26/5}}$$

- $\tilde{\Lambda} = a'(R_{1.4}c^2/GM)^6$
 $a' = 0.0035 \pm 0.0006$

for

$$M = (1.2 \pm 0.2) M_\odot$$

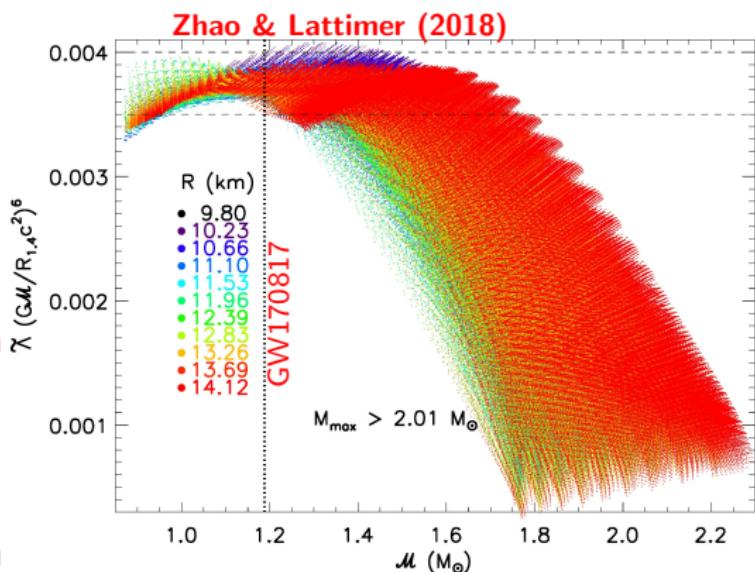
- $R_{1.4} =$

$$(11.5 \pm 0.3) \frac{M}{M_\odot} \left(\frac{\tilde{\Lambda}}{800} \right)^{1/6} \text{ km}$$

- For GW170817:

$$a' = 0.00375 \pm 0.00025$$

$$R_{1.4} = (13.4 \pm 0.1) \left(\frac{\tilde{\Lambda}}{800} \right)^{1/6} \text{ km}$$



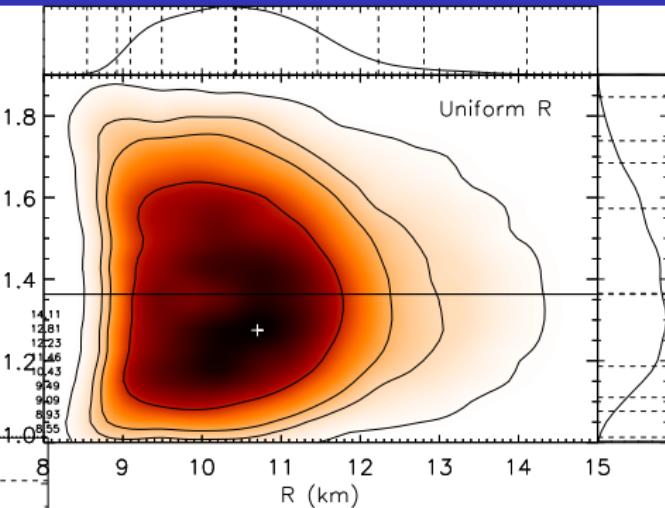
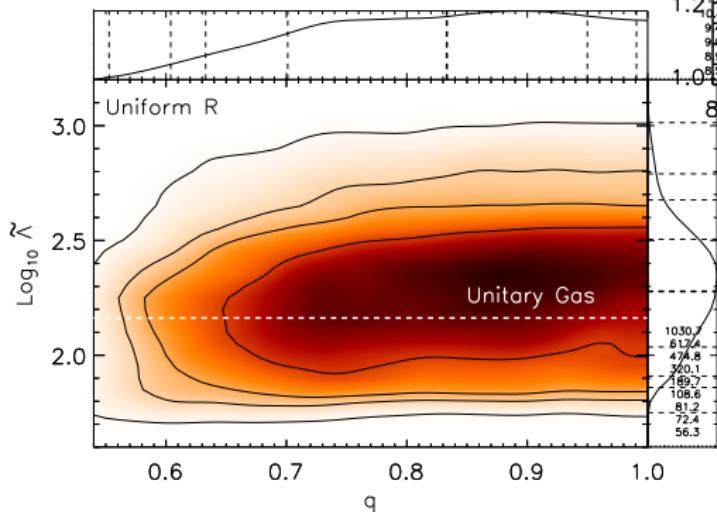
68.3%, 90%, 95.4% and 99.7% Confidence Bounds

Waveform analysis by De et al. (2018)

Zhao and Lattimer (2021)

Uniform $\ln \Lambda$ and causality priors

$$R = 10.4^{+1.1}_{-0.9} \text{ km}$$



$$\tilde{\Lambda} = 190^{+130}_{-81}$$

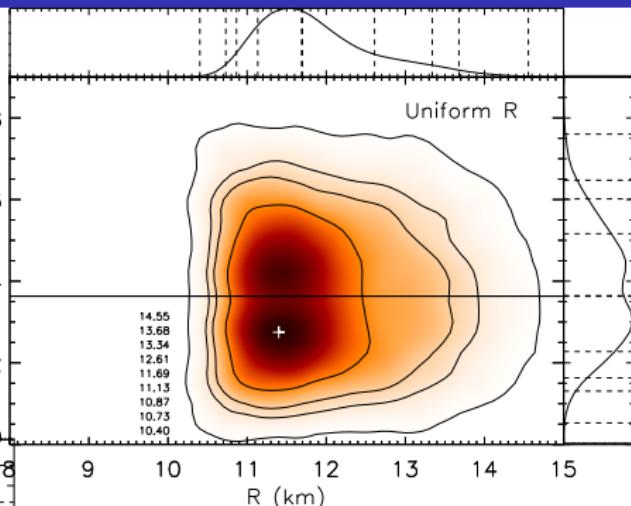
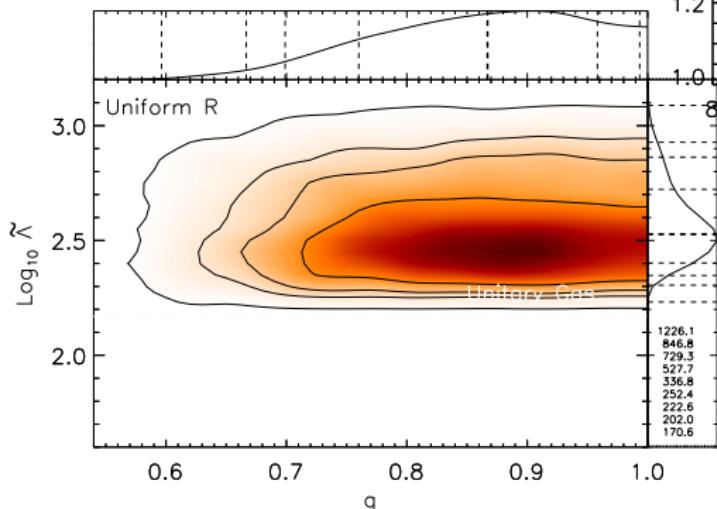
68.3%, 90%, 95.4% and 99.7% Confidence Bounds

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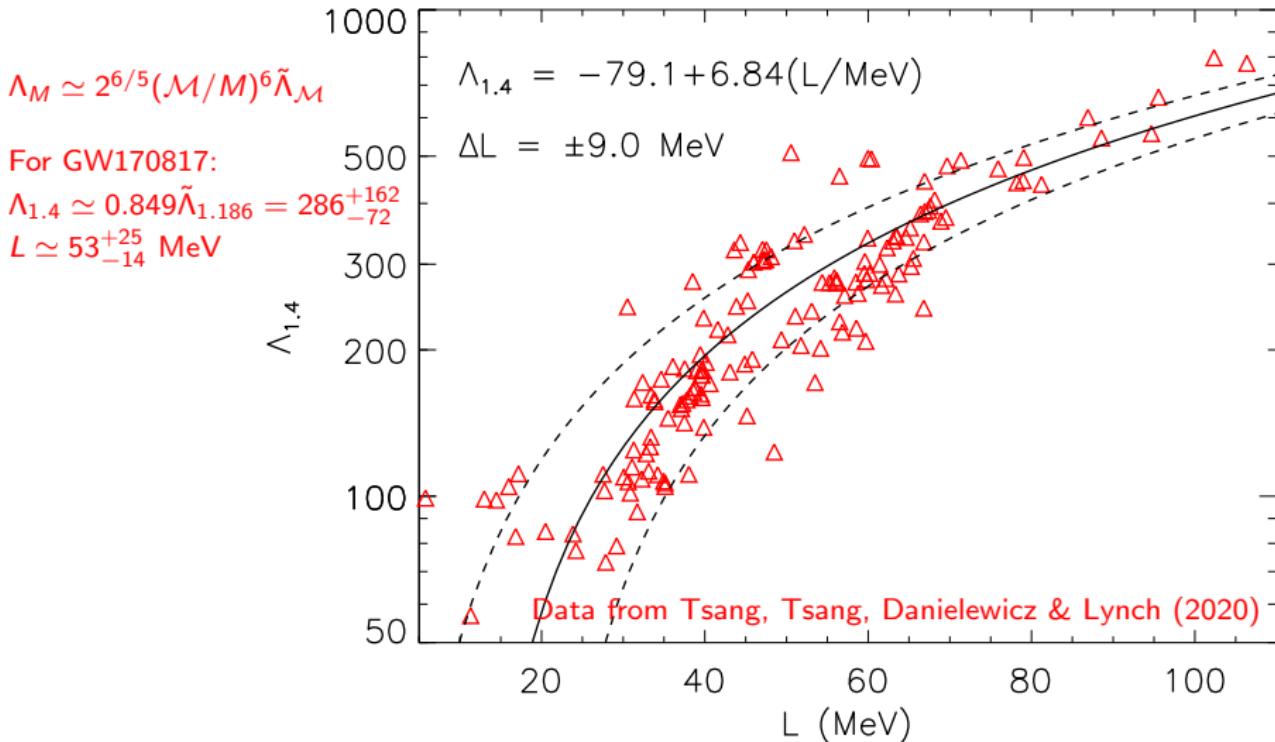
Uniform $\ln \Lambda$ and Unitary Gas Conjecture priors

$$R = 11.7^{+0.9}_{-0.5} \text{ km}$$

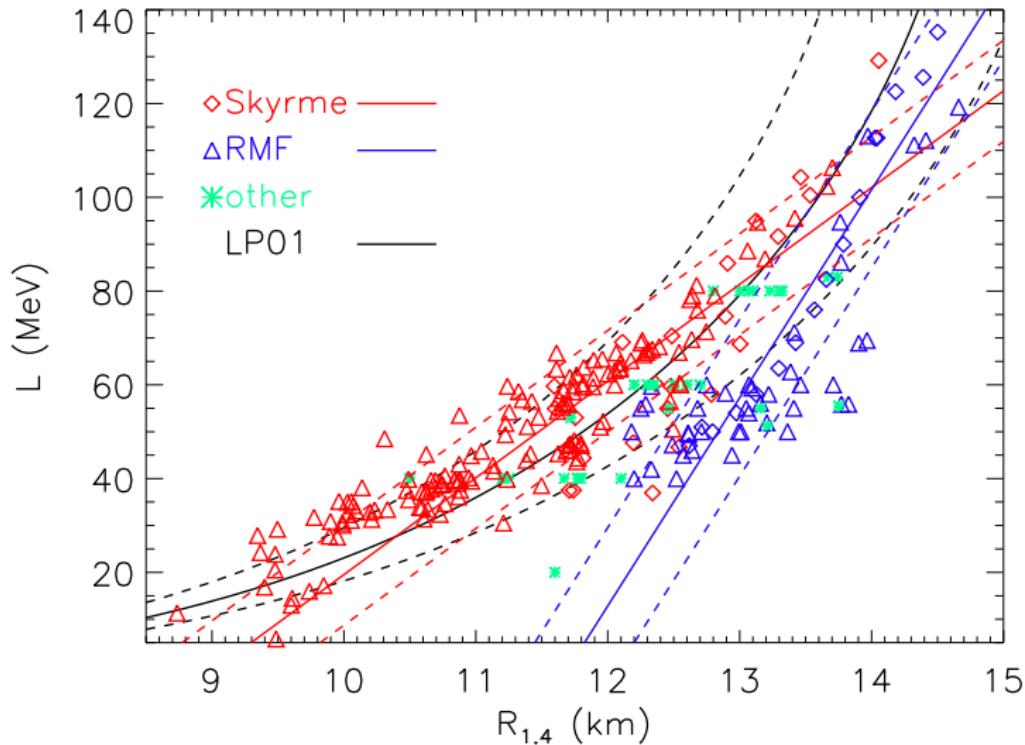


$$\tilde{\Lambda} = 337^{+191}_{-85}$$

186 Skyrme Interactions



Relation Between $R_{1.4}$ and L



Model-Dependence of $R_{1.4} - L$ Relation

Many studies show that $R_{1.4}$ is most sensitive to the pressure at about $2n_s$.

The usual energy expansion for dense matter ($u = n/n_s, x = n_p/n$) is

$$E(u, x \simeq 0) = -B + \frac{K_0}{18}(u-1)^2 + (1-2x)^2 \left[S_V + \frac{L}{3}(u-1) + \frac{K_{\text{sym}}}{18}(u-1)^2 + \dots \right],$$

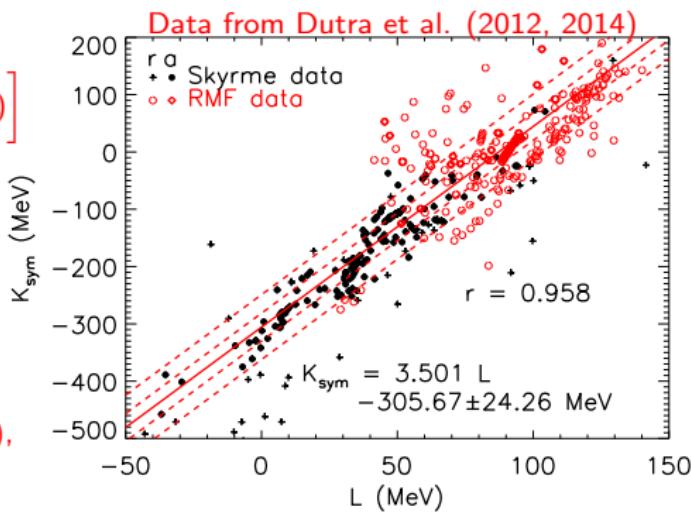
In β equilibrium, $x \ll 0.5$, so the pressure of neutron star matter is

$$P_{\text{NSM}}(u) = n_s u^2 \left[\frac{L}{3} + \frac{K_0 + K_{\text{sym}}}{9}(u-1) \right]$$

$$P_{\text{NSM}}(2) = \frac{4n_s}{3} \left[L + \frac{K_0 + K_{\text{sym}}}{3} \right]$$

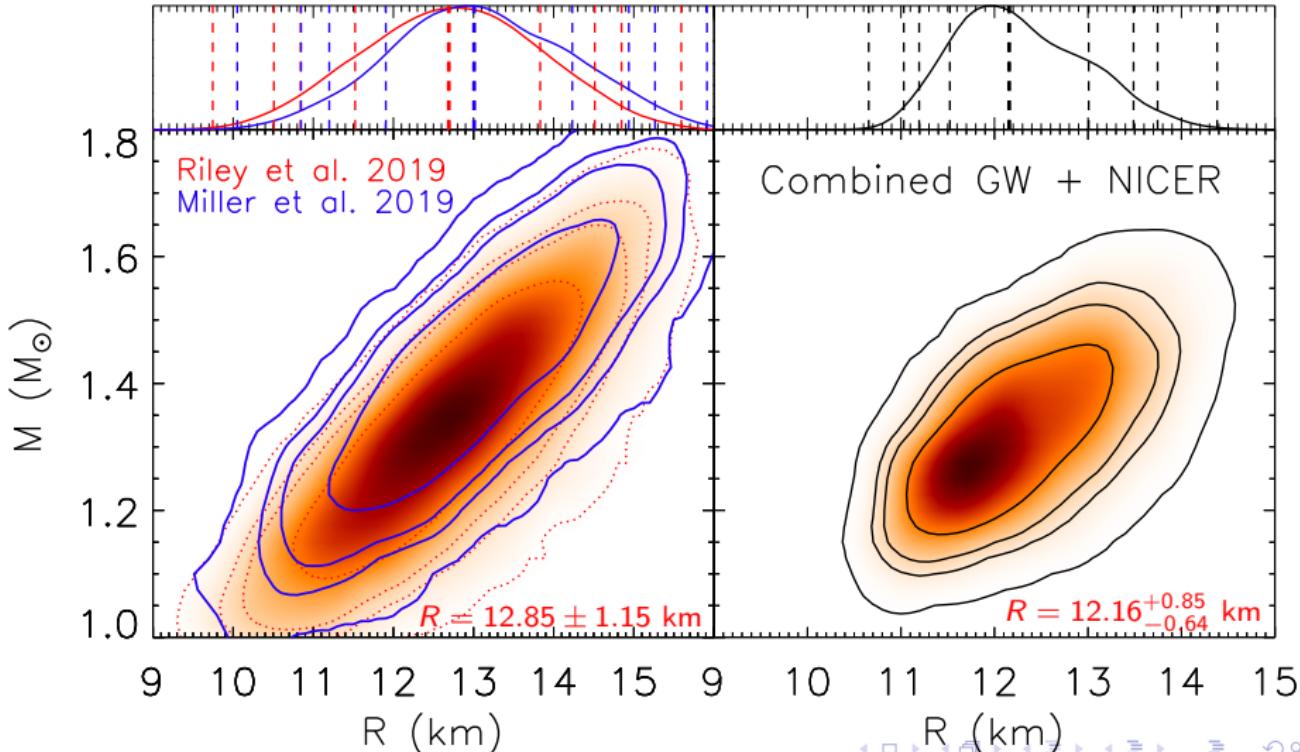
Around $L = 50$ MeV, Skyrme interactions have $K_{\text{sym}} \sim -100$ MeV, but RMF interactions can have $K_{\text{sym}} \sim 0$.

This produces $\sim 25\%$ increase in $P_{\text{NSM}}(2)$, and larger $R_{1.4}$ for the same L .



NICER Results For PSR J0030+0451

PSR J0030+0415 and GW170817 have neutron stars with similar masses $\simeq 1.4M_{\odot}$, but PSR J0740+6620 has a larger mass $\simeq 2.0M_{\odot}$, so don't include it in this analysis.



Summary of Constraints on S_V and L

