Bulk viscosity from Urca processes: $npe\mu$ matter in the neutrino-trapped regime

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Outline

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  - Bulk viscosity of $npe\mu\nu$ matter
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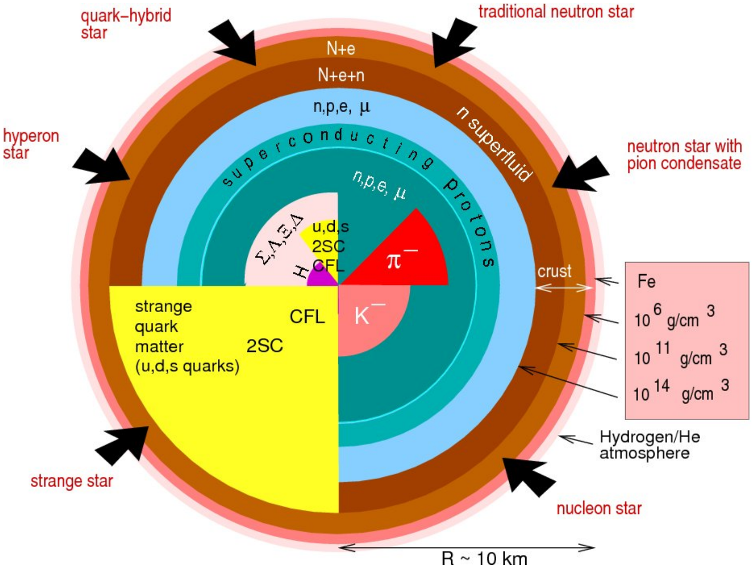
The structure of a neutron star



Compact star binaries

- Compact stars are natural laboratories which allow us to study the properties of nuclear matter under extreme physical conditions (strong gravity, strong magnetic fields, etc.).
- The recent detection of gravitational and electromagnetic waves originating from black hole or neutron star mergers motivates studies of compact binary systems.
- Various physical processes in the compact binary systems can be modelled in the framework of general-relativistic hydrodynamics simulations.
- Transport coefficients are key inputs in hydrodynamic modelling of compact star mergers as they measure the energy dissipation rate in hydrodynamic evolution of matter.
- The bulk viscosity might affect the hydrodynamic evolution of mergers by damping the density oscillations which can affect the form of the gravitational signal.

Aim and novelty of the work

- We extend our previous study of bulk viscosity of hot *npe* matter from weak processes to include muons which appear in significant amounts above the nuclear saturation density.
- We improve pur previous calculations including properly all relativistic corrections to the spectrum of nucleons in the relevant β-equilibration rates.
- At temperatures $T \geq 5$ MeV neutrinos/natineutrinos are trapped in neutron star matter and they affect strongly the bulk viscosity.

Relativistic hydrodynamics and bulk viscosity

- Hydrodynamic state of a relativistic system is described by means of the energy-momentum tensor and the particle current which obey the conservation laws

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu N^\mu = 0.$$

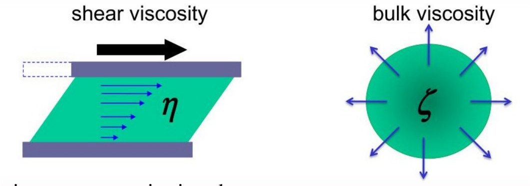
- For ideal, *i.e.,* non-dissipative fluids

$$T_0^{\mu\nu} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu}, \qquad N_0^\mu = nu^\mu,$$

and the system of conservation laws is closed by an equation of state $p = p(\epsilon, n)$.

- For dissipative fluids with velocity gradients

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}, \qquad N^\mu = nu^\mu.$$



- The bulk viscous pressure is given by

$$\Pi = -\zeta\theta, \qquad \theta = \partial_\mu u^\mu.$$

- Thus, the bulk viscosity ζ describes dissipation in the case where pressure falls out of equilibrium on uniform expansion/contraction.

Weak processes in $npe\mu\nu$ matter

- The simplest weak-interaction processes among baryons are the direct Urca processes

$$
\begin{aligned}
n &\rightleftarrows p + e^- + \bar{\nu}_e \quad (\text{neutron e}-\text{decay}),\\
p + e^- &\rightleftarrows n + \nu_e \quad (\text{electron capture}),\\
n &\rightleftarrows p + \mu^- + \bar{\nu}_\mu \quad (\text{neutron } \mu-\text{decay}),\\
p + \mu^- &\rightleftarrows n + \nu_\mu \quad (\text{muon capture}).
\end{aligned}
$$

- In addition, we have the following leptonic processes

$$
\begin{aligned}
\mu^- &\rightleftarrows e^- + \bar{\nu}_e + \nu_\mu \quad (\text{muon decay}),\\
\mu^- + \nu_e &\rightleftarrows e^- + \nu_\mu \quad (\text{neutrino scattering}),\\
\mu^- + \bar{\nu}_\mu &\rightleftarrows e^- + \bar{\nu}_e \quad (\text{antineutrino scattering}).
\end{aligned}
$$

- In β-equilibrium we have $\mu_n + \mu_{\nu_l} = \mu_p + \mu_l$. Out of equilibrium there is a chemical imbalance

$$
\mu_{\Delta_l} \equiv \mu_n + \mu_{\nu_l} - \mu_p - \mu_l \neq 0, \quad l = \{e, \mu\}.
$$

Rates of the Urca processes

- The rate of the neutron decay $n \to p + l^- + \bar{\nu}_l$ is given by $[\bar{f}(k) = 1 - f(k)]$

$$\Gamma_{n\to pl\bar{\nu}} = \int d\Omega_k |\mathcal{M}_{\rm Urca}|^2 f(k_n)\bar{f}(k_{\bar{\nu}_l})\bar{f}(k_l)\bar{f}(k_p)\,(2\pi)^4\,\delta^{(4)}(k_p + k_l + k_{\bar{\nu}_l} - k_n).$$

- The rate of the lepton capture $p + l^- \to n + \nu_l$ is given by

$$\Gamma_{pl\to n\nu} = \int d\Omega_k |\mathcal{M}_{\rm Urca}|^2 f(k_l)f(k_p)\bar{f}(k_n)\bar{f}(k_{\nu_l})\,(2\pi)^4\,\delta^{(4)}(k_p + k_l - k_{\nu_l} - k_n).$$

- The squared matrix element of Urca processes is $\left[G^2 = G_F^2\cos^2\theta_c(1+3g_A^2)\right]$

$$\sum |\mathcal{M}_{\rm Urca}|^2 \simeq 32G_F^2\cos^2\theta_c\left[(1+g_A)^2(k_e\cdot k_p)(k_{\nu_l}\cdot k_n)\right].$$

- In β-equilibrium $\Gamma_{n\to pl\bar{\nu}} = \Gamma_{pl\bar{\nu}\to n} \equiv \Gamma_{n\leftrightarrow pl\bar{\nu}}, \Gamma_{pl\to n\nu} = \Gamma_{n\nu\to pl} \equiv \Gamma_{pl\leftrightarrow n\nu}$.
- For small departures from equilibrium $\mu_{\Delta_l} \ll T$

$$\Gamma_{n\to pl\bar{\nu}} - \Gamma_{pl\bar{\nu}\to n} = \lambda_{n\leftrightarrow pl\bar{\nu}}\,\mu_{\Delta_l}, \quad \Gamma_{n\nu\to pl} - \Gamma_{pl\to n\nu} = \lambda_{pl\leftrightarrow n\nu}\,\mu_{\Delta_l},$$

with the expansion coefficients

$$\lambda_{n\leftrightarrow pl\bar{\nu}} = \frac{\Gamma_{n\leftrightarrow pl\bar{\nu}}}{T}, \quad \lambda_{pl\leftrightarrow n\nu} = \frac{\Gamma_{pl\leftrightarrow n\nu}}{T}.$$

Density oscillations in neutron-star matter

- Consider now small-amplitude density oscillations in baryonic matter with frequency ω

$$n_B(t) = n_{B0} + \delta n_B(t), \quad n_{L_l}(t) = n_{L_l 0} + \delta n_{L_l}(t), \quad \delta n_B(t),\, \delta n_{L_l}(t) \sim e^{i\omega t}.$$

- The baryon and lepton number conservation laws in the co-moving frame imply

$$i\omega\delta n_B + \theta n_{B0} = 0, \qquad i\omega\delta n_{L_l} + \theta n_{L_l 0} = 0, \qquad \theta = \mathrm{div}\, \boldsymbol{v}.$$

- The shifts in the particle densities $n_j(t) = n_{j0} + \delta n_j(t)$ lead to chemical imbalances

$$\begin{aligned} \mu_{\Delta_e} &= (A_{nn} - A_{pn})\delta n_n - (A_{pp} - A_{np})\delta n_p - A_{ee}\delta n_e + A_{\nu_e\nu_e}\delta n_{\nu_e}, \quad A_{ij} = \partial\mu_i/\partial n_j, \\ \mu_{\Delta_\mu} &= (A_{nn} - A_{pn})\delta n_n - (A_{pp} - A_{np})\delta n_p - A_{\mu\mu}\delta n_\mu + A_{\nu_\mu\nu_\mu}\delta n_{\nu_\mu}. \end{aligned}$$

  The off-diagonal susceptibilities A_{np} and A_{pn} are non-zero because of the cross-species strong interaction between neutrons and protons.

- Let us clarify how how the lepton reactions affect the bulk viscosity from the Urca processes. Typically, we deal with two limiting cases:
  - fast lepton-equilibration limit, where the lepton process rates are much higher than Urca process rates $\lambda_{\mathrm{lep}} \gg \lambda_{\mathrm{Urca}}$; this implies $\mu_{\Delta_e} = \mu_{\Delta_\mu} \equiv \mu_\Delta \quad \Rightarrow$ problem with one degree of freedom.
  - slow lepton-equilibration limit, where the lepton process rates are much lower than Urca process rates: $\lambda_{\mathrm{lep}} \ll \lambda_{\mathrm{Urca}}$; in this case $\mu_{\Delta_e} \neq \mu_{\Delta_\mu} \quad \Rightarrow$ problem with two degrees of freedom.

Fast lepton-equilibration limit $\lambda_{\rm lep} \gg \lambda_{\rm Urca}$

- The rate equations which take into account the loss and gain of particles read

$$
\begin{aligned}
i\omega\delta n_n(t) + \theta n_{n0} &= -(\lambda_e + \lambda_\mu)\mu_\Delta(t), \\
i\omega\delta n_e(t) + \theta n_{e0} &= +\lambda_e\mu_\Delta(t) + I_L,
\end{aligned}
$$

  where $\lambda_e = \lambda_{n\leftrightarrow pe\bar{\nu}} + \lambda_{pe\leftrightarrow n\nu}$ and $\lambda_\mu = \lambda_{n\leftrightarrow p\mu\bar{\nu}} + \lambda_{p\mu\leftrightarrow n\nu}$.
- The quantity I_L is the summed rate of the three lepton reactions, which arises as a result of an almost vanishing shift $\mu_{\Delta_e} - \mu_{\Delta_\mu} \ll \mu_\Delta$ but cannot be neglected because the relevant λ-coefficient can be very large.
- Solving these equations we can compute the pressure out of equilibrium

$$
p = p(n_j) = p(n_{j0} + \delta n_j) = p_{\rm eq} + \delta p',
$$

  where the non-equilibrium part of the pressure - the bulk viscous pressure, is given by

$$
\Pi \equiv \delta p' = \sum_j \frac{\partial p}{\partial n_j}\delta n_j' = \sum_{ij} n_{i0}A_{ij}\delta n_j'.
$$

- The bulk viscosity is then identified from $\Pi = -\zeta\theta$

$$
\boxed{\zeta(\omega) = \frac{C^2}{B}\frac{\gamma}{\omega^2 + \gamma^2}}
$$

  with susceptibilities $B = \left.\frac{\partial\mu_\Delta}{\partial n_n}\right|_{n_B}$, $C = n_B\left.\frac{\partial\mu_\Delta}{\partial n_B}\right|_{Y_n}$, and relaxation rate $\gamma = (\lambda_e + \lambda_\mu)B$.

Slow lepton-equilibration limit $\lambda_{\rm lep} \ll \lambda_{\rm Urca}$

- In this case $I_L \simeq 0$, and the rate equations read

$$
\begin{aligned}
i\omega\delta n_n(t) + \theta n_{n0} &= -\lambda_e \mu_{\Delta_e}(t) - \lambda_\mu \mu_{\Delta_\mu}(t),\\
i\omega\delta n_e(t) + \theta n_{e0} &= +\lambda_e \mu_{\Delta_e}(t).
\end{aligned}
$$

- Solving these equations and computing the off equilibrium-pressure leads to

$$
\zeta(\omega) = \frac{\lambda_e\lambda_\mu\left[\lambda_e(C_1 - a_1C_2)^2 + \lambda_\mu(C_2 - a_2C_1)^2\right] + \omega^2(\lambda_e C_1^2 + \lambda_\mu C_2^2)/(A_n+A_p)^2}{\left[\lambda_e\lambda_\mu(A_n+A_p)(a_1a_2-1) - \omega^2/(A_n+A_p)\right]^2 + \omega^2(\lambda_e a_1 + \lambda_\mu a_2)^2},
$$

  with $a_1 = 1 + (A_e + A_{\nu_e})/(A_n + A_p)$ and $a_2 = 1 + (A_\mu + A_{\nu_\mu})/(A_n + A_p)$.
- If the muon contribution is neglected ($\lambda_\mu = 0$) this result reduces to

$$
\zeta_e(\omega) = \frac{C_1^2}{A_1}\frac{\gamma_e}{\omega^2+\gamma_e^2}, \quad \gamma_e = \lambda_e A_1.
$$

  contribution by electrons and muons, respectively.
- In the neutrino-transparent matter we deal with the low-frequency limit $\omega \ll \lambda_i A_i$ (typically, $\omega \simeq 1$ kHz in neutron star mergers)

$$
\zeta = \frac{\lambda_e(C_1 - a_1C_2)^2 + \lambda_\mu(C_2 - a_2C_1)^2}{\lambda_e\lambda_\mu(A_n+A_p)^2(a_1a_2-1)^2}.
$$

Beta-equilibrated nuclear matter

We use the density functional theory approach to the nuclear matter, which is based on phenomenological baryon-meson Lagrangians of the type proposed by Walecka and others.

The Lagrangian density of matter is given by

$$\mathcal{L} = \sum_N \bar{\psi}_N \left[\gamma^\mu \left(i\partial_\mu - g_{\omega N}\omega_\mu - \frac{1}{2} g_{\rho N} \boldsymbol{\tau}\boldsymbol{\rho}_\mu \right) - m_N^* \right] \psi_N + \sum_l \bar{\psi}_l (i\gamma^\mu \partial_\mu - m_l)\psi_l$$
$$+ \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - U(\sigma) - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4}\boldsymbol{\rho}^{\mu\nu}\boldsymbol{\rho}_{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}^\mu \boldsymbol{\rho}_\mu .$$

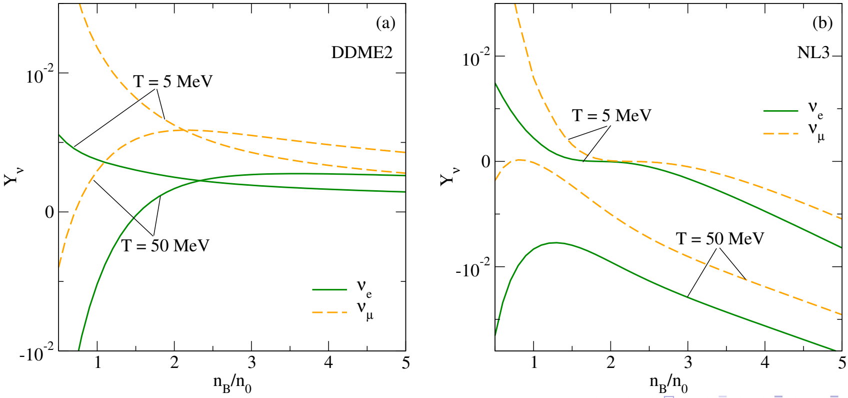
The pressure of baryonic matter is given by

$$P_N = \sum_N \frac{2J_N + 1}{6\pi^2} \int_0^\infty \frac{k^4\, dk}{(k^2 + m_N^{*2})^{1/2}} \left[f(E_k^N - \mu_N^*) + f(E_k^N + \mu_N^*) \right] - \frac{m_\sigma^2 \sigma^2}{2} - U(\sigma)$$
$$+ \frac{m_\omega^2 \omega_0^2}{2} + \frac{m_\rho^2 \rho_{03}^2}{2} + \sum_l \frac{g_l}{6\pi^2} \int_0^\infty \frac{k^4\, dk}{(k^2 + m_l^2)^{1/2}} \left[f(E_k^l - \mu_l) + f(E_k^l + \mu_l) \right] + \rho_B \Sigma_r .$$

Here $m_N^* = m_N - g_{\sigma N}\sigma$ and $\mu_N^* = \mu_N - g_{\omega N}\omega_0 - g_{\rho N}\rho_{03} I_3 - \Sigma_r$ are the nucleon effective mass and effective chemical potentials; I_3 is the third component of nucleon isospin and σ, ω_0 and ρ_{03} are the mean values of the meson fields, Σ_r is the rearrangement self-energy.

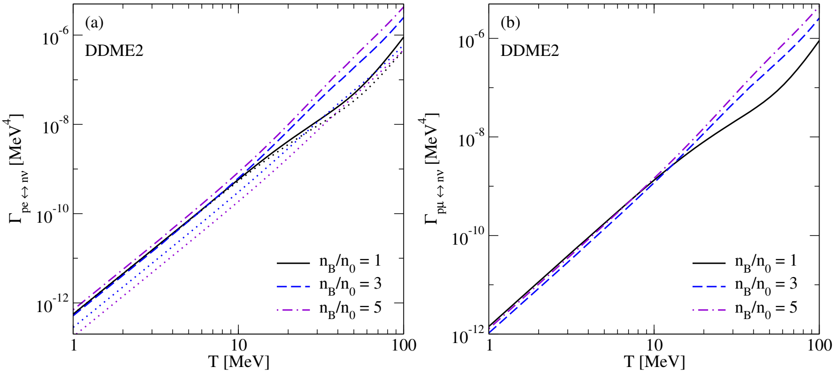
Particle fractions in equilibrium (DDME2 & NL3)

- Equilibrium state of matter is found from conditions $\mu_n + \mu_{\nu_l} = \mu_p + \mu_l$, the charge neutrality condition $n_p = n_e + n_\mu$, the baryon conservation $n_B = n_n + n_p$, and the lepton conservation $n_l + n_{\nu_l} = Y_{L_l} n_B$.
- In neutron star merger matter we heve typically $Y_{L_e} = Y_{L_\mu} = 0.1$, which implies $Y_n \simeq 0.8$, $Y_p \simeq 0.2$, and $Y_e \simeq Y_\mu \simeq 0.1$ for models DDME2 and NL3.
- The trapped species are neutrinos in the DDEM2 matter, whereas these are antineutrinos in the case of the NL3 model.



Urca process rates (DDME2)

$p + e^- \rightleftarrows n + \nu_e$ $\quad\quad$ $p + \mu^- \rightleftarrows n + \nu_\mu$

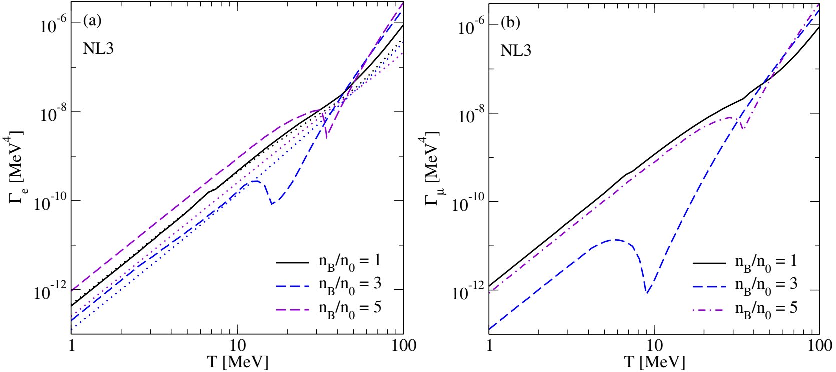


- The neutron decay processes involve antineutrinos $\Rightarrow$ their rates are suppressed compared to the lepton capture rates if the matter is neutrino-dominated.
- At moderate temperatures $T \leq 10$ MeV the lepton capture rates scale as $\Gamma_{pl\leftrightarrow n\nu} \propto T^3$.
- The electron and muon capture rates are similar both qualitatively and quantitatively.
- Non-relativistic approximation to the nucleon spectrum underestimates the equilibration rates by factors from 1 to 10.

Urca process rates (NL3)

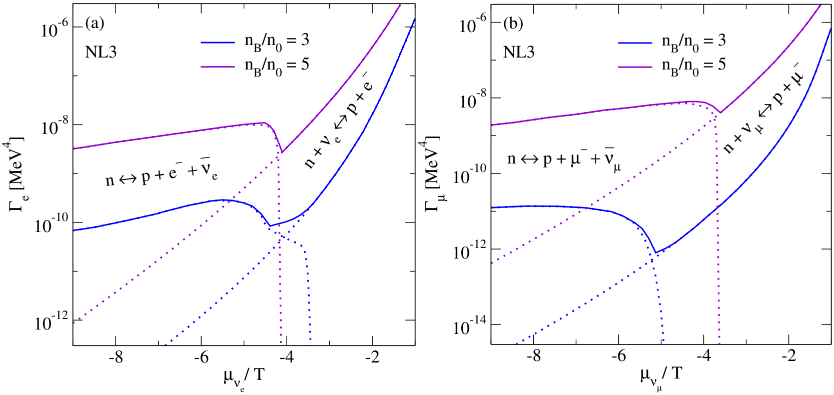
$$n \rightleftarrows p + e^- + \bar{\nu}_e \qquad\qquad n \rightleftarrows p + \mu^- + \bar{\nu}_\mu$$

$$p + e^- \rightleftarrows n + \nu_e \qquad\qquad p + \mu^- \rightleftarrows n + \nu_\mu$$



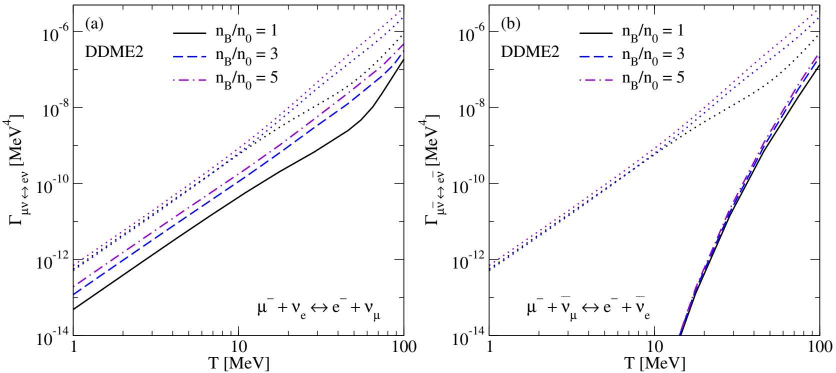
- The model NL3 is in the antineutrino-dominated regime at low temperatures, where the dominant equilibration process is the neutron decay ($\mu_{\nu_l}/T \leq -6$).
- The neutrino-dominated regime is realized at high temperatures, where the dominant equilibration process is the lepton capture ($\mu_{\nu_l}/T \simeq -3$).
- As a consequence, there is always a sharp minimum in the summed equilibration rate at the transition point between these two regimes.

Relative rates of neutron decay and lepton capture processes (NL3)



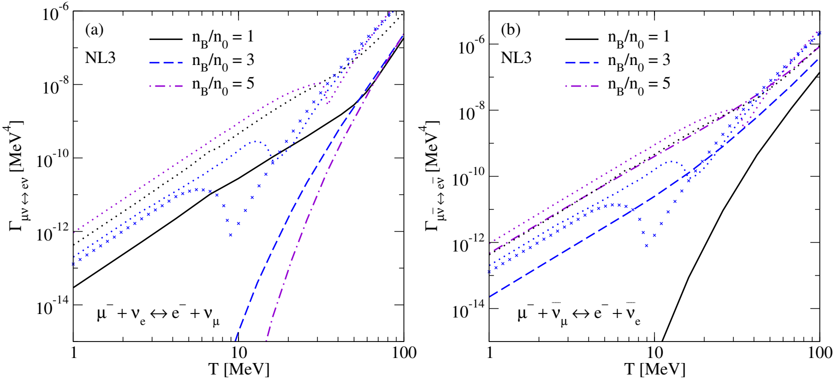
- The neutron decay and the lepton capture rates intersect at a value of the scaled chemical potential within the range $-5 \leq \mu_{\nu_l}/T \leq -3$.
- The non-relativistic approximation underestimates the exact rates by factors from 1 to 10 in the regions away from the minimum.
- Close to the minimum the exact relativistic rates are lower as there is no minimum in the non-relativistic approximation.
- Thus, we conclude that the minimum in the transition point it is purely relativistic effect.

Leptonic process rates (DDME2)



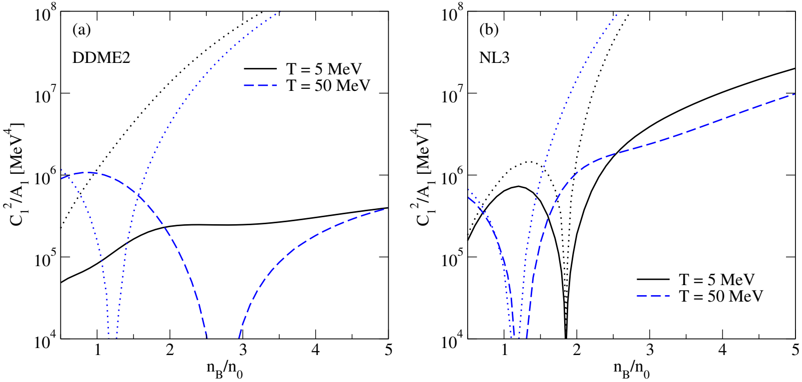
- The neutrino absorption rates are qualitatively similar to the lepton capture rates (shown by dotted lines), but are smaller on average by an order of magnitude.
- The antineutrino absorption rates are orders of magnitude smaller than the neutrino absorption rates.
- The muon decay rate is negligible because of the very small scattering phase space.
- Thus, within the DDME2 model the leptonic processes are always much slower than the Urca processes, putting the material in the "slow lepton equilibration" regime.

Leptonic process rates (NL3)



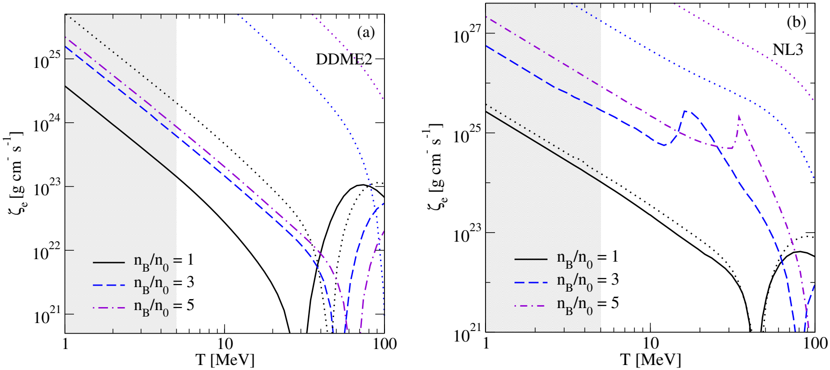
- In the NL3 model, the neutrino absorption is more efficient at low densities, whereas at the antineutrino absorption is operative at high densities.
- The material described by the NL3 model is almost always in the "slow lepton equilibration" regime.
- The only exception is the transition region where the muonic Urca process rate has a minimum. Here we have the "fast lepton equilibration" regime.

The susceptibility C_1^2/A_1



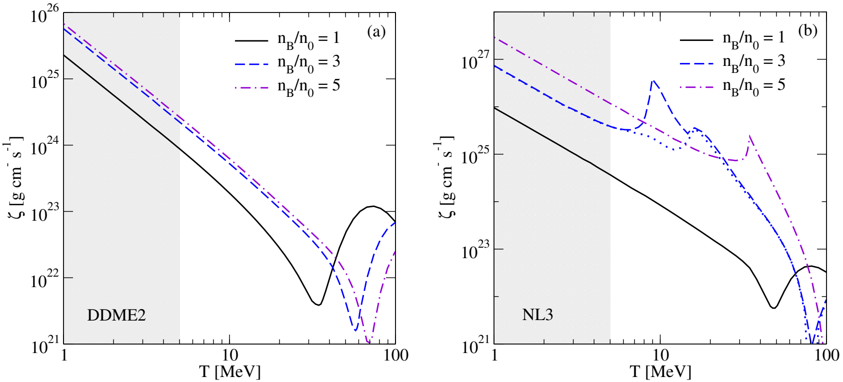
- At high temperatures $T \gtrsim 30$ MeV the susceptibility C crosses zero at certain values of density where the proton fraction attains a minimum.
- At that point the system becomes scale-invariant: it can be compressed and remain in beta equilibrium $\Rightarrow$ the bulk viscosity vanishes at that critical point.
- The non-relativistic approximation strongly overestimates the susceptibility even at low densities $n_B \leq 2n_0$ where the relativistic corrections to the nucleonic spectrum are small.

Bulk viscosity of *npeν* matter



- At low temperatures $T \leq 10$ MeV the bulk viscosity decreases as $\zeta \propto T^{-2}$.
- This scaling breaks down at high temperatures $T \geq 30$ MeV where the matter becomes scale-invariant and the bulk viscosity drops to zero.
- In the case of NL3 model, the bulk viscosity has a local maximum at high densities due to the transition from the antineutrino-dominated regime to the neutrino-dominated regime.
- The bulk viscosity decreases by orders of magnitude when the relativistic corrections are properly taken into account.

Bulk viscosity of *npeμν* matter



- The bulk viscosity of *npeμ* matter exceeds the one of *npe* matter by factors from 3 to 10 at the left side of the minimum.
- Above the minimum the bulk viscosities of these two cases are almost the same.
- At high temperatures the total bulk viscosity has a sharp minimum but does not drop to zero as $\zeta \simeq \zeta_e + \zeta_\mu$ in that regime, which drop to zero ar slightly different temperatures.
- In the case of NL3 model at $n_B/n_0 = 3$ we have the opposite regime of fast lepton-equilibration around the local maximum.

Summary and outlook

Summary

- We studied the bulk viscosity of neutrino-trapped $npe\mu$ matter from Urca processes under the conditions relevant to binary neutron star mergers.
- We improved our previous calculations of β-equilibration rates and the nuclear susceptibilities including the relativistic corrections to the nucleonic spectra.
- We dominant β-equilibration rate is the lepton capture in the neutrino-dominated matter and the neutron decay in the antineutrino-dominated matter.
- There is a transition point at around $-5 \leq \mu_{\nu_l}/T \leq -4$ between these two regimes where the summed equilibration rate has a sharp minimum.
- The analysis of relative rates of Urca processes and pure leptonic processes shows that the matter is in the "slow lepton equilibration regime" almost in the entire range of interest except the transition region where we have the oppisite limit.
- The bulk viscosity drops with the temperature and has sharp minimums at high temperatures where the system becomes scale-invariant.
- Bulk viscosity shows also local maxima at intermediate temperatures where the transition between the antineutrino- and neutrino-dominated regimes occurs.
- The numerical results show that muons enhance the bulk viscosity by factors $1\div10$.

THANK YOU FOR ATTENTION!