Towards an all-orders calculation of the electroweak bubble wall velocity

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Outline

- Motivations for first-order electroweak phase transition
- Relativistic bubble wall velocity calculations: 1-to-1
- Relativistic bubble wall velocity calculations: 1-to-2
- Relativistic bubble wall velocity to all orders
- Summary

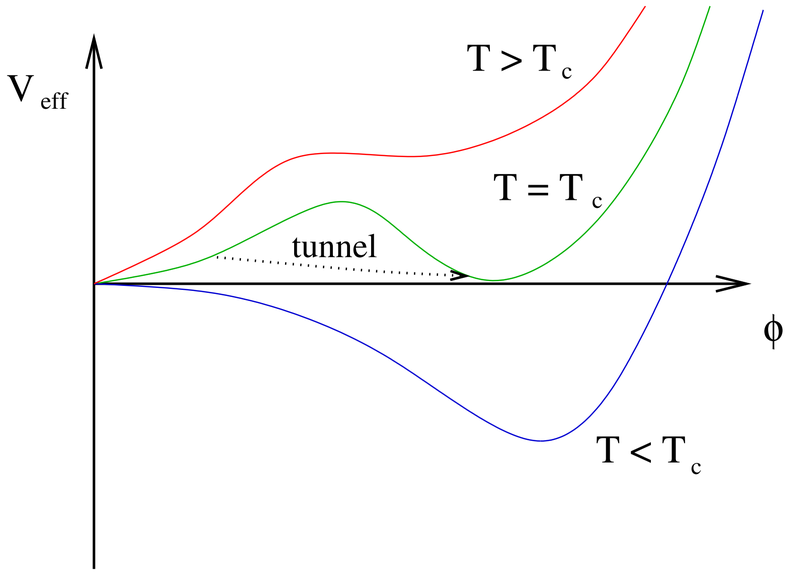
First order electroweak phase transition

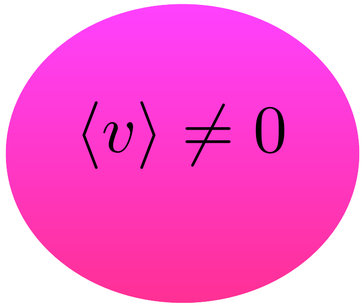
Within the Standard Model the EWPT is a crossover D'Onofrio & Rummukainen (2015)

Minimal new physics → first order PT Anderson & Hall (1992)

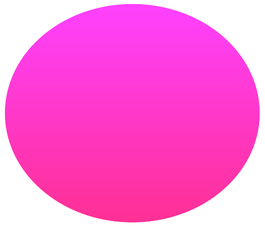
- Matter-antimatter asymmetry Kuzmin, Rubakov & Shaposhnikov (1985)
- Topological defects Achucarro & Vachaspati (2000)
- Primordial magnetic fields Vachaspati (1991)
- Stochastic gravitational wave background Kamionkowski, Kosowsky & Turner (1993)

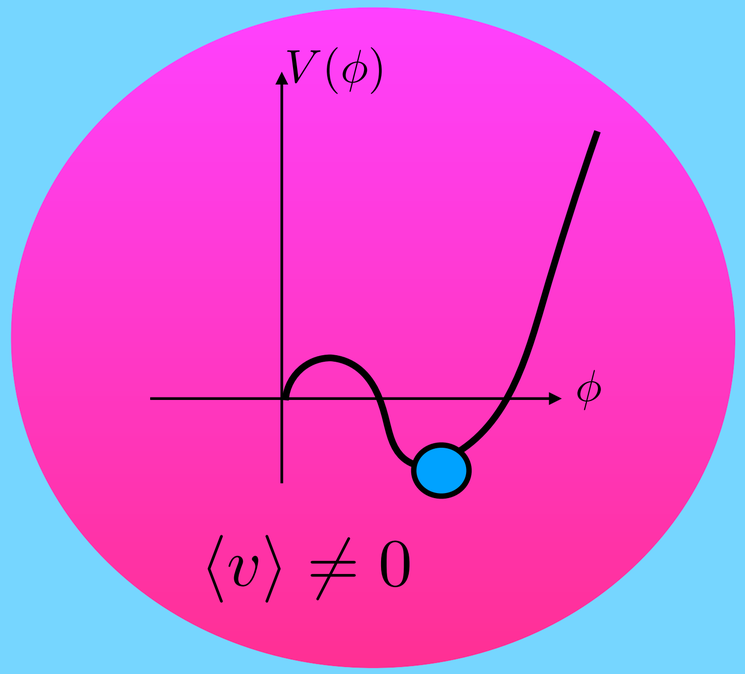
First order electroweak phase transition



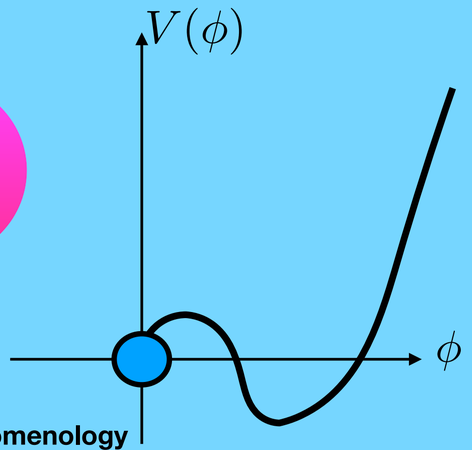


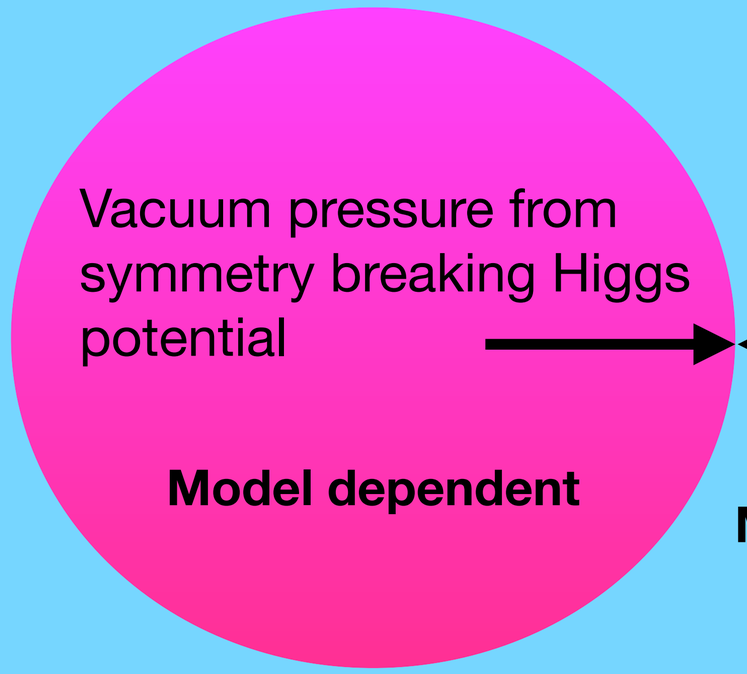
$$\langle v \rangle = 0$$





$$\langle v \rangle = 0$$

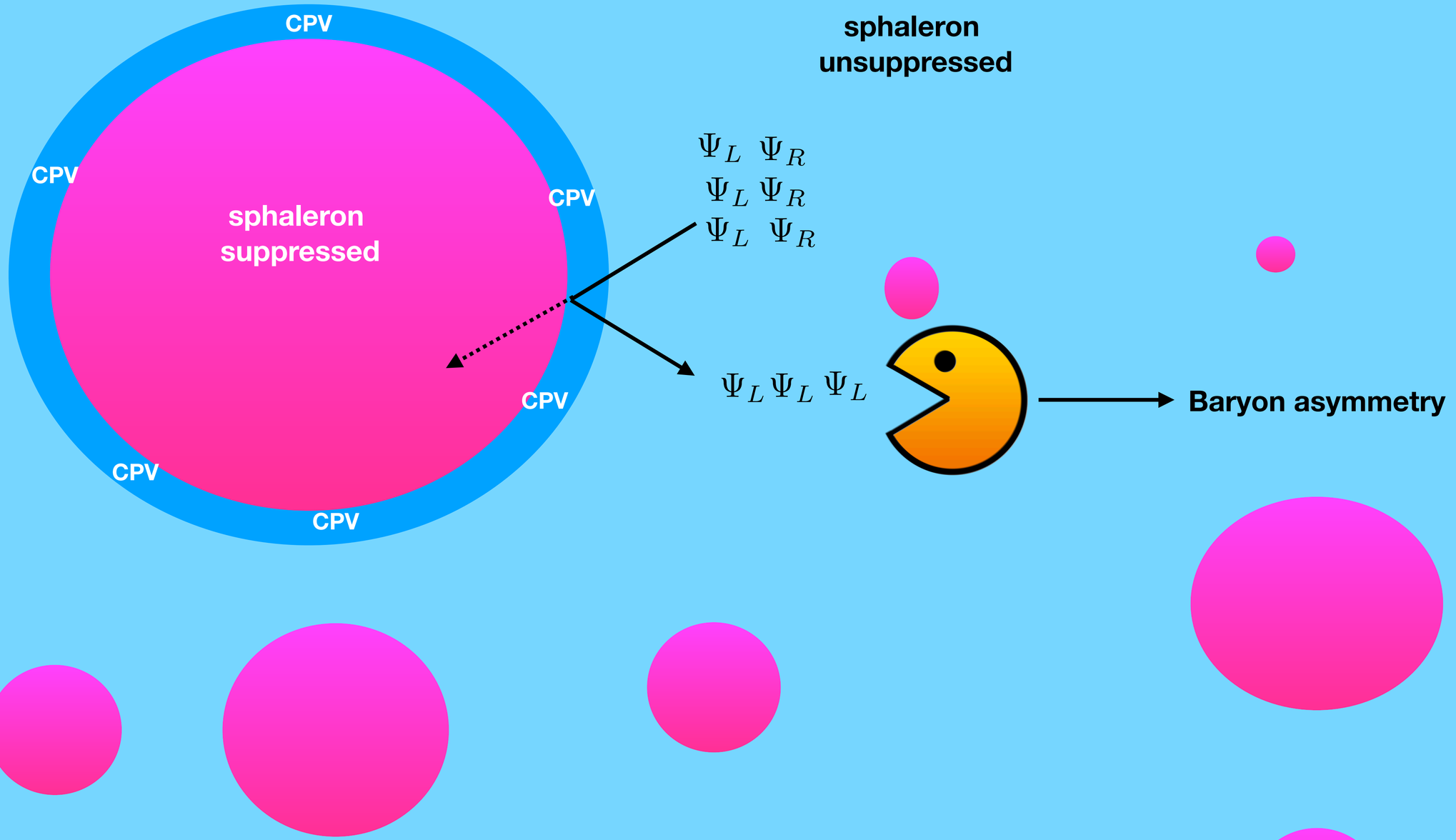




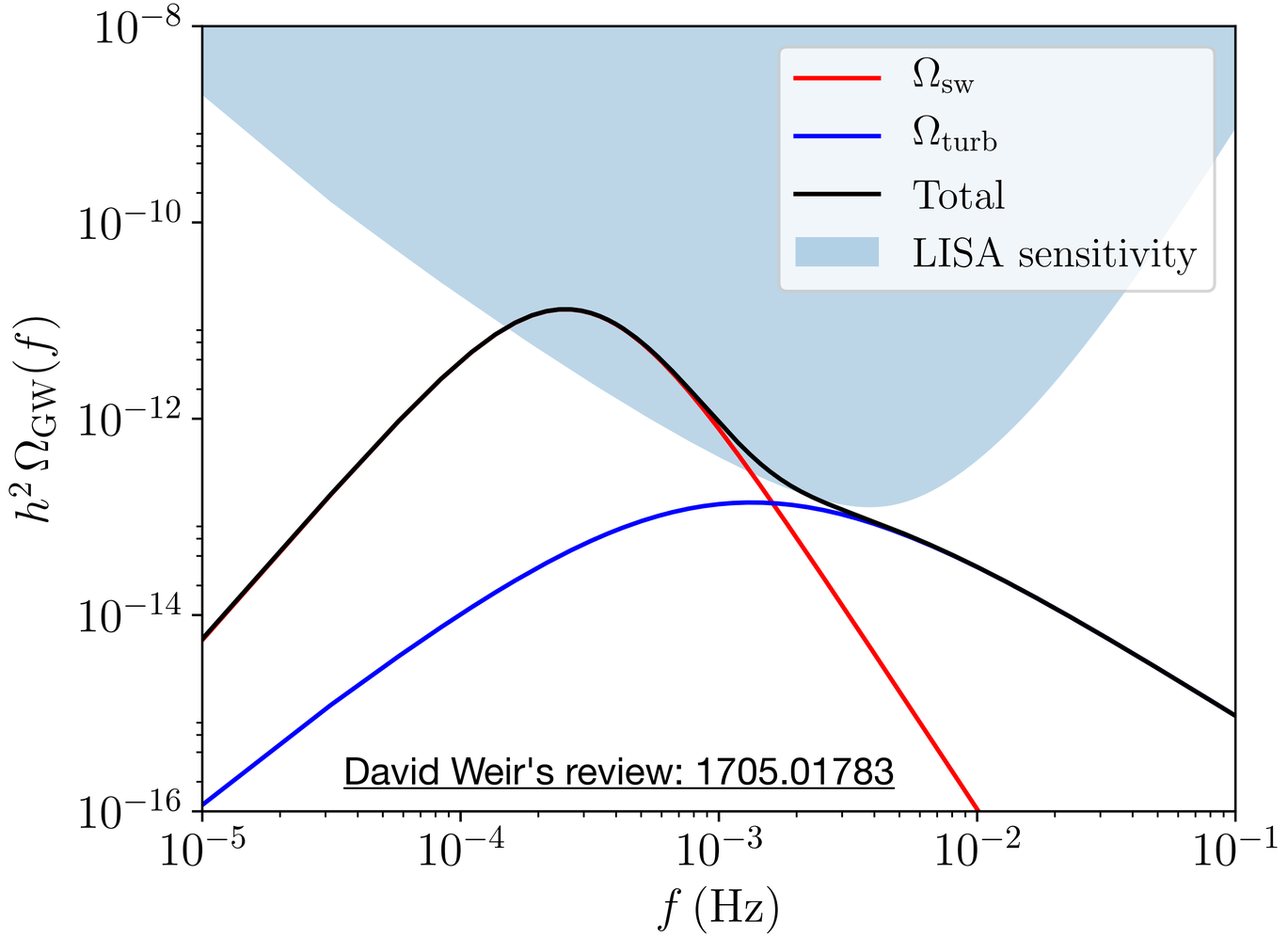
Thermal pressure, resulting from interactions of wall with the plasma particles

Model independent(ish)

Electroweak Baryogenesis



Gravitational waves generated from 1st order PT



- Velocity affects SGWB spectrum
- EWBG and SGWB in tension

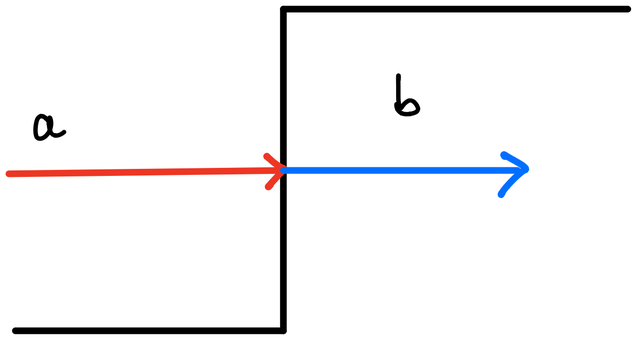
Kinematics and Assumptions

z-translation invariance broken by bubble wall

$$p_{a,z,s} = \sqrt{E_a^2 - |p_{a,\perp}|^2 - m_{a,s}^2}$$
$$p_{a,z,h} = \sqrt{E_a^2 - |p_{a,\perp}|^2 - m_{a,h}^2}$$

$s \equiv$ symmetric phase

$h \equiv$ broken phase



Throughout we assume infinitely thin walls:
$L_w \to 0$

Transverse momentum conserved:

$$p_{a,\perp} = p_{b,\perp}$$

Energy conserved:

$$E_a = E_b$$

1-to-1 pressure calculation for relativistic bubble walls

1-to-1 calculation argument

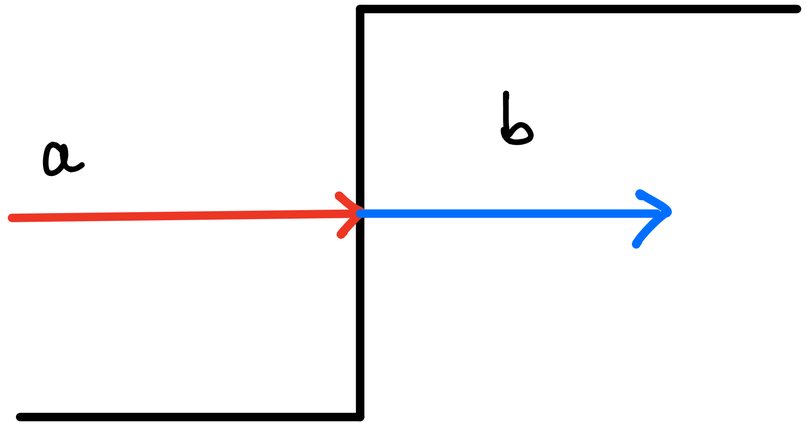
Friction → scattering particles that couple to Higgs condensate

Arnold (1993)

Bodeker & Moore (2009)

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

Lorentz factor of the wall



$$\Delta p_{\rm wall} \equiv p_{a,z,s} - p_{b,z,h} = \sqrt{E_a^2 - m_{a,s}^2 - p_{a,\perp}^2} - \sqrt{E_b^2 - m_{b,h}^2 - p_{b,\perp}^2}$$

$$E_a^2 \sim p_{a,z,s}^2 \sim \gamma^2 T^2 \gg m_{a,s}^2, m_{b,h}^2, p_{a,\perp}^2 \Longrightarrow \Delta p_{\rm wall} \approx \frac{m_{b,h}^2 - m_{a,s}^2}{2E_a} = \frac{m_{b,h}^2 - m_{a,s}^2}{2\gamma T}$$

$$\mathcal{P}_{1\to 1} = \frac{[\text{ force }]}{[\text{ area }]} = \frac{\Delta[\text{ momentum }]}{[\text{ area }]\times[\text{ time }]} = [\text{ flux }]\times\Delta[\text{ momentum }]$$

$$\sim \gamma T^3 \times \frac{\Delta m^2}{2\gamma T} = \frac{\gamma^0 T^2 \Delta m^2}{2}$$

- 1-to-1: no flavour change. Assume no particles reflected

Arnold (1993)

Bodeker & Moore (2009)

Flux incoming particle

Momentum transfer "observable"

$$\mathcal{P}_{1\to 1} = \sum_a \int \mathrm{d}\mathcal{F}_a \sum_b \int \mathrm{d}\mathbb{P}_{a\to b} \Delta p_z \left(1 \pm f_b\right)$$

Differential probability

Quantum Statistical factors

$$\mathrm{d}\mathbb{P}_{a\to b} = \frac{d^3 p_b}{(2\pi)^3} \frac{1}{2E_b} \times (2\pi)^3 \delta^2 \left(p_{a,\perp} - p_{b,\perp}\right) \delta\left(E_a - E_b\right) \left(2p_{b,z,h}\right) \delta_{ab}$$

$$\mathcal{P}_{1\to 1} = \sum_a \nu_a \int \frac{d^3 p_a}{(2\pi)^3} \frac{d^3 p_b}{(2\pi)^3} \frac{p_{b,z,s}}{E_b} f_a \left(1 \pm f_b\right) \left(p_{a,z,s} - p_{b,z,h}\right) (2\pi)^3 \delta^2 \left(p_{a,\perp} - p_{b,\perp}\right) \delta\left(E_a - E_b\right)$$

$$\frac{d^3 p_b}{(2\pi)^3 2E_b} = \frac{d^2 p_{b,\perp}}{(2\pi)^3} \frac{dE_b}{2E_b} \frac{E_b}{p_{b,z,s}}$$

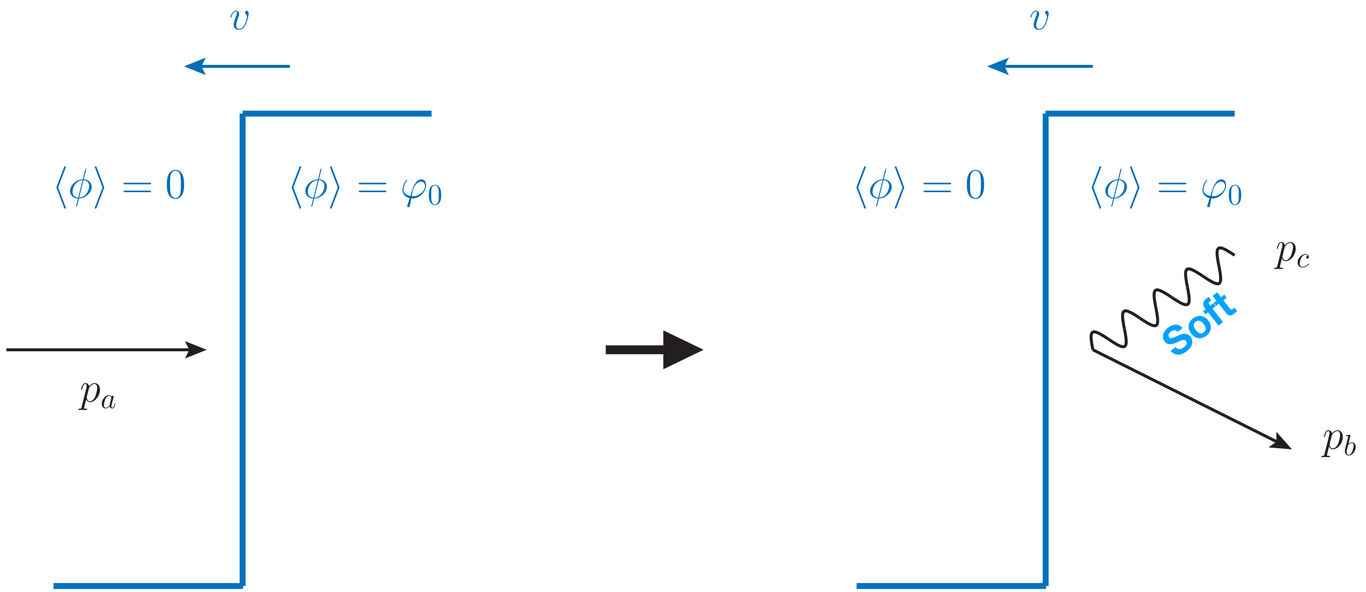
$$\mathcal{P}_{1\to 1} = \sum_a \nu_a \int \frac{d^3 p_a}{(2\pi)^3} f_a \left(1 \pm f_b\right) \Delta p_{\mathrm{wall}} \approx \gamma^0 T^2 \Delta m^2$$

8 years later …

1-to-2 pressure calculation for relativistic bubble walls

1-to-2 calculation

Bodeker & Moore (2017)



Incident particle's energy → mass second particle + transverse momentum

$\Delta p_{1\to 1} < \Delta p_{1\to 2}$ unless $p_\perp = 0$ and $m_c = 0$

1-to-2 calculation

Bodeker & Moore (2017)

Flux incoming particle

Quantum Statistical factors

Momentum transfer "observable"

Energy and transverse momentum conserved

$$\mathcal{P}_{1\to2} = \sum \nu_{a,b,c} \int [dp_a]\,[dp_b]\,[dp_c]\, f\,(p_a,p_b,p_c)\, \Delta p_z (2\pi)^3 \delta^2\,(p_{a,\perp} - p_{c,\perp} - p_{b,\perp})\,\delta\,(E_a - E_c - E_b)\,|\mathcal{M}|^2$$

Combination of PS + "observable"

$$\mathcal{P}_{1\to2} = \sum_{abc} \nu_a \int \frac{d^3p_a}{(2\pi)^3 2E_a} \int \frac{d^2p_{c,\perp}}{(2\pi)^2} \frac{dE_c}{(2\pi)2E_c} f(p_a,p_b,p_c) \Delta p_z \frac{1}{2p_{b,z,s}} \frac{E_c}{p_{c,z,s}} |\mathcal{M}|^2$$

B&M region of interest:

Ingoing hard	$p_{a,\perp} \sim T$	$p_{a,z,s} \sim E_a \sim \gamma_w T$	$m_{a,s}, m_{a,h} \ll \gamma_w T$
Outgoing hard	$p_{b,\perp} \sim \max[T, m_c]$	$p_{b,z,s} \sim E_b \sim \gamma_w T$	$m_{b,s}, m_{b,h} \ll \gamma_w T$
Outgoing soft	$p_{c,\perp} \sim \max[T, m_c]$	$p_{c,z,s} \sim \max[T, m_c]$	$m_{c,s}, m_{c,h} \ll \gamma_w T$

1-to-2 calculation

Bodeker & Moore (2017)

$$\mathcal{P}_{1\to 2} = \sum_{abc} \nu_a \int \frac{d^3p_a}{(2\pi)^3 2E_a} \int \frac{d^2p_{c,\perp}}{(2\pi)^2} \frac{dE_c}{(2\pi)2E_c} f(p_a, p_b, p_c) \Delta p_z \frac{1}{2p_{b,z,s}} \frac{E_c}{p_{c,z,s}} |\mathcal{M}|^2$$

B&M region of interest:

$$p_{c,z}(z) = E_c \sqrt{1 - \underbrace{\frac{m_{c,z}^2 + p_{c,\perp}^2}{E_c^2}}_{\epsilon}} \approx E_c \left(1 - \frac{\epsilon}{2} + \mathcal{O}(\epsilon^2) + \cdots\right)$$

Collinearity: ϵ

$$p_{b,z} = \sqrt{E_b^2 - p_{\perp,b}^2 - m_b^2(z)} \approx E_a(1-x)$$

Softness: $x = \frac{E_c}{E_a}$

From PS

"Observable"

$$(p_{a,z,s} - p_{c,z,h} - p_{b,z,h}) \frac{1}{2p_{b,z,s}} \frac{E_c}{p_{c,z,s}} \approx \underbrace{\frac{1}{2E_a} \frac{m_c^2(z) + p_{c,\perp}^2}{2E_c}}_{\sim x\epsilon + O(\epsilon^2 x) + \ldots}$$

B&M 1-to-2 master equation:

$$\mathcal{P}_{1\to 2} = \sum \nu_a \int \frac{d^3 p_a}{(2\pi)^3 2E_a} \int \frac{d^2 p_{c,\perp}}{(2\pi)^2} \frac{dE_c}{(2\pi) 2E_c} f(p_a, p_b, p_c) \frac{m_{c,h}^2(z) + p_{c,\perp}^2}{2E_c} |\mathcal{M}|^2$$

Still need to determine matrix element squared

Bodeker & Moore (2017)

The bubble wall is invariant in time and the transverse directions

$$\langle p_c p_b | T | p_a \rangle = \int d^4x \, \langle p_c p_b \, | H_{\rm int} | \, p_a \rangle = (2\pi)^3 \delta^2 \left(p_{a,\perp} - p_{b,\perp} - p_{c,\perp} \right) \delta \left(E_a - E_b - E_c \right) \mathcal{M}$$

$$\mathcal{M} \equiv \int dz \, \chi^*_{p_c}(z) \chi^*_{p_b}(z) V(z) \chi_{p_a}(z)$$

Mode functions are treated using the WKB approximation:

$$\chi_{p_c}(z) \simeq \sqrt{\frac{p_{c,z,s}}{p_{c,z}(z)}} \exp\left(i \int_0^z p_{c,z}\left(z'\right) dz' \right)$$

Mode function quick summary

Bodeker & Moore (2017)

scalar interacting with wall

$$\mathcal{L} = \sum_{f=a,b,c} \left[\frac{1}{2} \left(\partial_\mu \phi_f \right)^2 - \frac{1}{2} m_f^2(z) \phi_f^2 \right]$$

Z-varying mass parametrises spatial inhomogeneity

KG field equation

$$\Box \phi_f + m_f^2(z) \phi_f = 0$$

solve with an **homogeneous** mass parameter,
solutions can be can labeled by a 3-vector $\overrightarrow{p}$

$$\chi_f(\vec{p}, x) = e^{-iE_f(\vec{p})t} e^{i\vec{p}\cdot\vec{x}} \quad \text{with} \quad E_f(\vec{p}) \equiv \sqrt{|\vec{p}|^2 + m_f^2}$$

$$\phi_f(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \tilde{\phi}_f(\vec{p}) \chi_f(\vec{p}, x)$$

Mode function quick summary

Bodeker & Moore (2017)

But we have **inhomogeneous** mass term, make
Ansatz for solution to KG equation

$$\phi_f(x) = \int \frac{d^2\vec{p}_\perp}{(2\pi)^2}\frac{dp_{z,s}}{(2\pi)}\tilde{\phi}_f\left(\vec{p}_\perp, p_{z,s}\right)\chi_f\left(p_{z,s}, z\right)e^{-iE_f(\vec{p}_\perp,p_z,s)t}e^{i\vec{p}_\perp\cdot\vec{x}_\perp}$$

Substitute into Klein Gordon $\rightarrow$ WKB solution for a particle with inhomogeneous mass

$$\chi_p(z) = \sqrt{\frac{p_{z,s}}{p_z(z)}}\exp\left(i\int_0^z p_z\left(z'\right)dz'\right)$$

Amplitude Phase

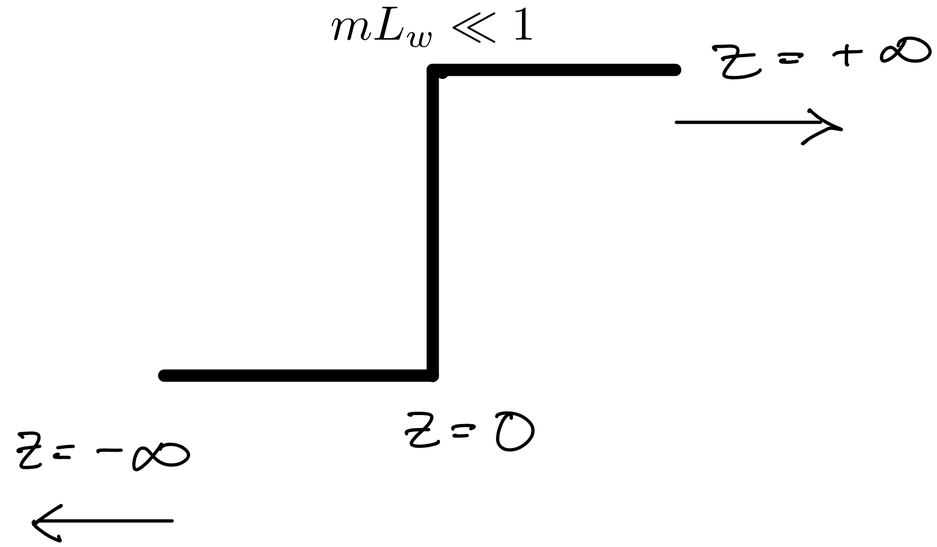
Analogous 1D-scattering off a potential well. Normally there would be a wave function with a negative phase (reflected) but here all particles transmitted

Bodeker & Moore (2017)

$$\mathcal{M} = \int dz\, \chi_b\left(p_{b,z,s}, z\right)^* \chi_c\left(p_{c,z,s}, z\right)^* \chi_a\left(p_{a,z,s}, z\right) V(z)$$

$$\mathcal{M} = V_s \int_{-\infty}^{0} dz \exp\left[iz\frac{A_s}{2E_a}\right] + V_h \int_{0}^{\infty} dz \exp\left[iz\frac{A_h}{2E_a}\right] = 2iE_a\left(\frac{V_h}{A_h} - \frac{V_s}{A_s}\right)$$

$$A_s = E_a\left(p_{a,z,s} - p_{b,z,s} - p_{c,z,s}\right)$$
$$A_h = E_a\left(p_{a,z,h} - p_{b,z,h} - p_{c,z,h}\right)$$



1/A terms resemble propagators, but they only propagate in the z-direction!

Vertex Function

Bodeker & Moore (2017)

$$|\mathcal{M}|^2 \simeq 4E_a^2|V|^2\frac{\left(A_h - A_s\right)^2}{A_h^2 A_s^2}$$

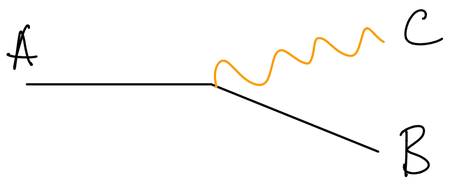
$a(p) \to b(k)c(p-k)$	$\lvert V\rvert^2$
$S \to V_T S$ $F \to V_T F$ $V \to V_T V$	$4g^2 C_2[R]\frac{1}{x^2}k_\perp^2$
$S \to V_L S$ $F \to V_L F$ $V \to V_L V$	$4g^2 C_2[R]\frac{1}{x^2}m^2$
$F \to F V_T$	$2g^2 C_2[R]\frac{1}{x}\left(k_\perp^2 + m_b^2\right)$
$V \to F F$	$2g^2 T[R]\frac{1}{x}\left(k_\perp^2 + m_b^2\right)$
$S \to S V_T$	$4g^2 C_2[R]k_\perp^2$
$F \to S F$	$y^2\left(k_\perp^2 + 4m_a^2\right)$
$S \to S S$	$\lambda^2\varphi^2$

$$k_\perp \equiv p_{c,\perp}$$

These are splitting functions up to the normalisation $P_{b\leftarrow a}(x) = |V|^2 x(1-x)/16\pi^2 k_\perp^2$

Quick Recap on splitting functions

Gribov & Lipatov (1972)
Dokshitzer (1977)
Altarelli & Parisi (1977)



P_{CA} is probability that A emits (collinear) C which carries $x = E_C/E_A$ energy fraction of parent particle

$$P_{BA}(z) = \frac{1}{2} z(1-z) \sum_{\text{spins}} \frac{|V_{A\to B+C}|^2}{p_\perp^2}$$

$$\sum_{\text{spin}} |V_{A\to B+C}|^2 = \frac{1}{2} C_2(R)\, \text{Tr}\left(\not{k}_B \gamma_\mu \not{k}_A \gamma_\nu\right) \sum_{\text{pol}} \epsilon^{*\mu}\epsilon_\mu$$

$$\sum_{\text{spin}} |V_{A\to B+C}|^2 = \frac{2p_\perp^2}{x(1-x)} \frac{1+(1-x)^2}{x} C_2(R)$$

Calculated in light-like Axial gauge in Altarelli Parisi (AP) paper

Sudakov parametrisation used in Altarelli & Parisi's paper

$$k_A = (P, P, 0)$$

$$k_B = \left((1-x)P + \frac{p_\perp^2}{2(1-x)P}, (1-x)P, -p_\perp\right)$$

$$k_C = \left(xP + \frac{p_\perp^2}{2xP}, xP, p_\perp\right)$$

$$A_s \approx 2E_a \times \left(-\frac{|p_{a,\perp}|^2 + m_{a,s}^2}{2E_a} + \frac{|p_{b,\perp}|^2 + m_{b,s}^2}{2E_b} + \frac{|p_{c,\perp}|^2 + m_{c,s}^2}{2E_c} \right) \approx \frac{|p_{c,\perp}|^2 + m_{c,s}^2}{E_c/E_a}$$

$$A_h \approx 2E_a \times \left(-\frac{|p_{a,\perp}|^2 + m_{a,h}^2}{2E_a} + \frac{|p_{b,\perp}|^2 + m_{b,h}^2}{2E_b} + \frac{|p_{c,\perp}|^2 + m_{c,h}^2}{2E_c} \right) \approx \frac{|p_{c,\perp}|^2 + m_{c,h}^2}{E_c/E_a}$$

$$|\mathcal{M}|^2 = 4E_a^2 \, |V_s|^2 \, \frac{(A_s - A_h)^2}{A_s^2 A_h^2}$$

$$\approx 4E_a \, \frac{|V_s|^2}{\underbrace{(E_c/E_a)}_{x}{}^{-2}} \, \frac{\left(m_{c,h}^2 - m_{c,s}^2\right)^2}{\left(|p_{c,\perp}|^2 + m_{c,s}^2\right)^2 \left(|p_{c,\perp}|^2 + m_{c,h}^2\right)^2}$$

Lets keep track of what cancels where…

$$|\mathcal{M}|^2 = 4E_a^2 \left|V_s\right|^2 x^2 \frac{m_{c,h}^4}{\left|p_{c,\perp}\right|^4 \left(\left|p_{c,\perp}\right|^2 + m_{c,h}^2\right)^2}$$

$$= 16g^2 C_2[R] \frac{\left|p_{c,\perp}\right|^2}{\cancel{x^2}} \cancel{x^2} \frac{m_{c,h}^4}{\left|p_{c,\perp}\right|^4 \left(\left|p_{c,\perp}\right|^2 + m_{c,h}^2\right)^2}$$

$$= 16g^2 C_2[R] E_a^2 \frac{m_{c,h}^4}{\left|p_{c,\perp}\right|^2 \left(\left|p_{c,\perp}\right|^2 + m_{c,h}^2\right)^2}$$

$m_{c,s} \ll m_{c,h}$

Combine matrix element squared with momentum transfer observable:

$$|\mathcal{M}|^2 \times \Delta p_z \approx 16E_a^2 g^2 C_2[R] \frac{m_{c,h}^4}{\left|p_{c,\perp}\right|^2 \left(\left|p_{c,\perp}\right|^2 + m_{c,h}^2\right)^2} \times \frac{m_{c,h}^2 + p_{c,\perp}^2}{2E_c}$$

$$\approx \frac{8g^2 E_a}{x} C_2[R] \frac{m_{c,h}^4}{\left|p_{c,\perp}\right|^2 \left(\left|p_{c,\perp}\right|^2 + m_{c,h}^2\right)}$$

Two pieces left: the integration of PS of incoming “a” gives the flux. We also need to integrate over phase space of particle “c” our soft emission

$$\int_{g^2T^2}^{m^2} \frac{dp_{c,\perp}^2}{(2\pi)^2 \left|p_{c,\perp}\right|^2 \left(\left|p_{c,\perp}\right|^2 + m_{c,h}^2\right)} \sim \frac{1}{24\pi m_{c,h}^2}$$

$$\int \frac{dE_c}{E_c^2} \sim \frac{1}{m_{c,h}}$$

B&M assume $g^2T^2 \ll m^2$

The thermal mass would cut of the integral and you’d get some (possibly large) $\log\left(m_{c,h}^2/m_{c,s}^2\right)$ see talk by Azatov

Gamma dependence enters from flux

$$\mathcal{P}_{1\to 2} \sim \gamma T^3 \frac{1}{m^2} \frac{1}{m} m^4$$

Mode Function

Transverse momentum integration

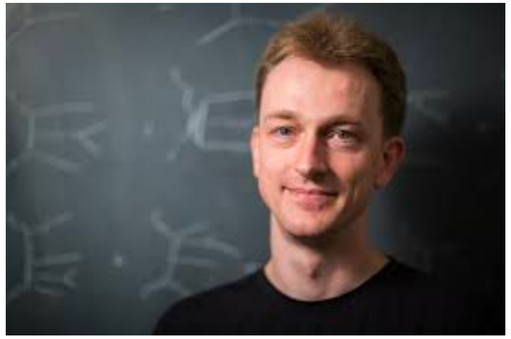
Energy integration

$$\mathcal{P}_{1\to 2} \sim m\gamma T^3$$

- $\mathscr{P}_{1\to 2} \propto \gamma$ while $\mathscr{P}_{1\to 1} \propto \gamma^0$. Since, vacuum pressure does not grow in λ, a terminal velocity will be reached.
- $\mathscr{P} \propto m^2 \implies$ no phase change pressure goes to zero. This comes from integrating over the mode functions
- $\mathscr{M}$: WKB and vertex. The vertex part is dominated in the soft regime.
- B&M cut off the $k_\perp$ and k^0 integration by gauge boson mass.
- This interaction looks like a collider/scattering experiment where the collision occurs between the ingoing particle and the wall $\implies$ the centre of mass energy will be large $\implies$ many soft emission.

1-to-n pressure calculation for relativistic bubble walls

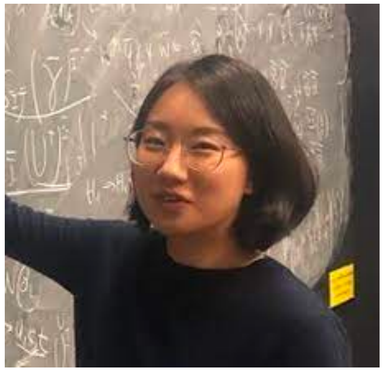
2007.10343 (JCAP 2103 (2021) 009)



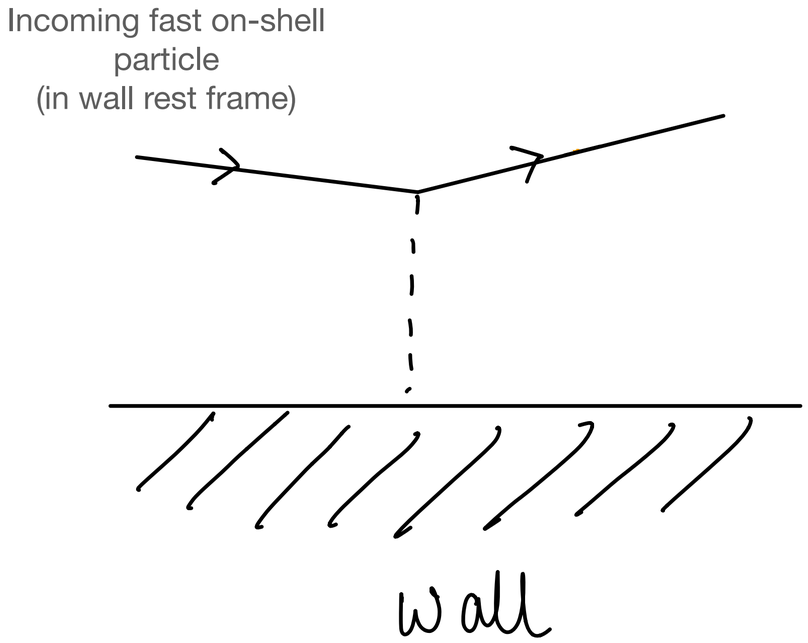
Stefan Höche

Jonathan Kozaczuk

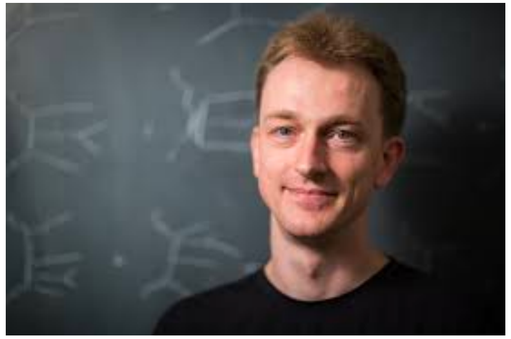
Andrew Long



Yikun Wang



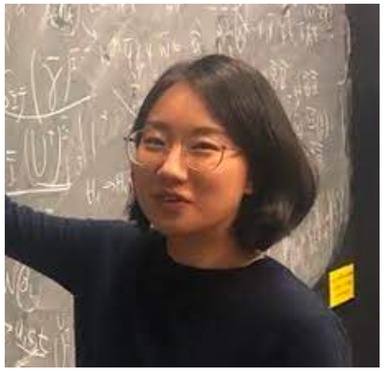
B&M investigated 1-to-1
which we will treat as leading order
process.



Stefan Höche

Jonathan Kozaczuk

Andrew Long



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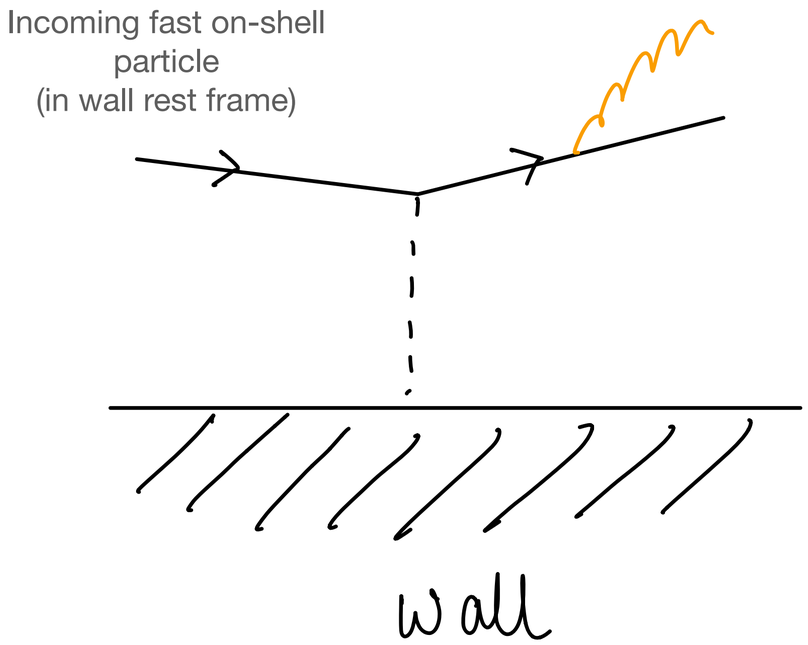
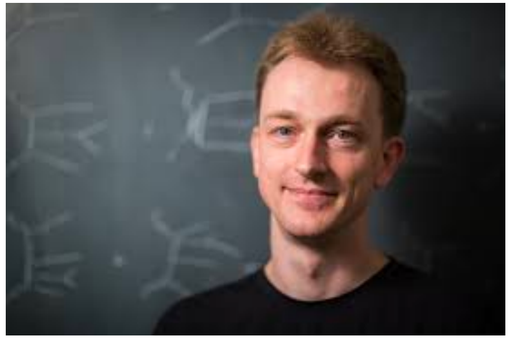


Treat 1-to-2 as quantum correction to 1-to-1 process.



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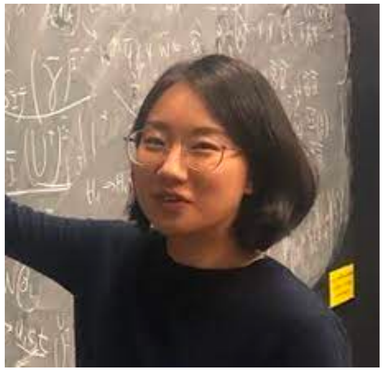
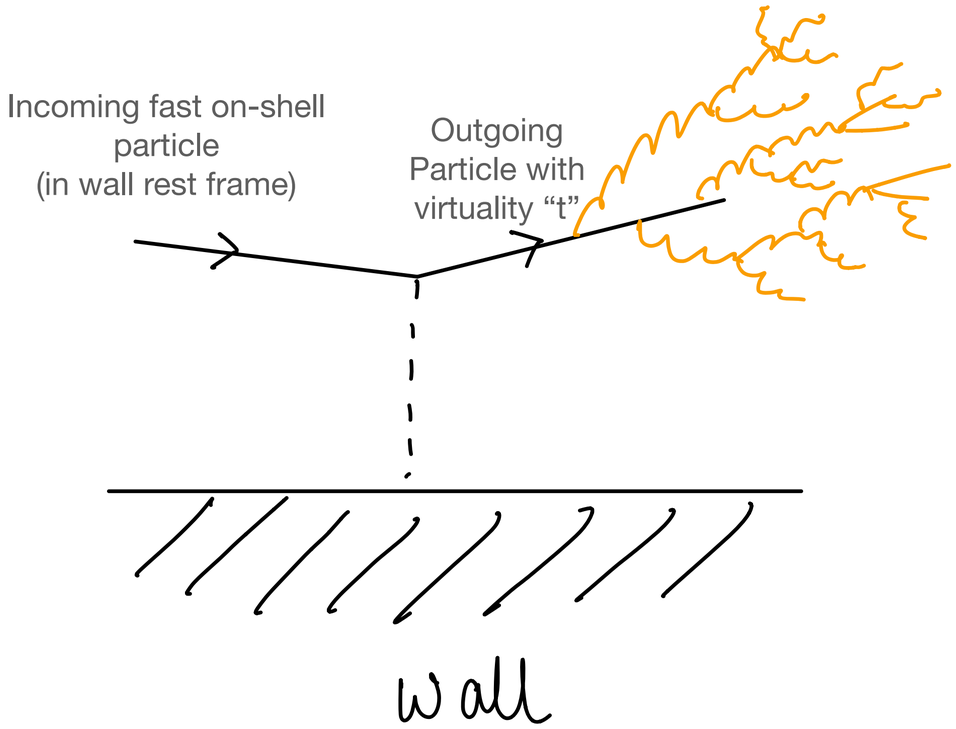


Yikun Wang



If 1-to-2 is important, will 1-to-n be even more important?

Reformulation of matrix element

B&M

$$A_s = -2E_a\left(p_{a,z,s} - p_{b,z,s} - p_{c,z,s}\right)$$

Ultrarelativistic incoming particle

$$\approx p_{a,\perp}^2 + m_{a,s}^2 - \frac{p_{b,\perp}^2 + m_{b,s}^2}{E_b/E_a} - \frac{k_\perp^2 + m_{c,s}^2}{E_c/E_a}$$

Thermal mass incoming particle negligible. Frame where incoming particle is "headon"

$$= -\frac{k_\perp^2 + m_{b,s}^2}{(1-x)} - \frac{k_\perp^2 + m_{c,s}^2}{x}$$

Soft limit

$$\approx -\frac{k_\perp^2 + m_{c,s}^2}{x} \quad (x \ll 1)$$

$$A_h = -2E_a\left(p_{a,z,h} - p_{b,z,h} - p_{c,z,h}\right)$$

$$\approx p_{a,\perp}^2 + m_{a,h}^2 - \frac{p_{b,\perp}^2 + m_{b,h}^2}{E_b/E_a} - \frac{k_\perp^2 + m_{c,h}^2}{E_c/E_a}$$

$$= -\frac{k_\perp^2 + m_{b,h}^2}{(1-x)} - \frac{k_\perp^2 + m_{c,h}^2}{x}$$

$$\approx -\frac{k_\perp^2 + m_{c,h}^2}{x} \quad (x \ll 1)$$

JT & friends

$$A_s = -2p_{a,s}\cdot p_{c,s}$$

$$\approx -\frac{k_\perp^2 + m_{c,s}^2}{x(1-x)} - x m_{a,s}^2$$

$$\approx -\frac{k_\perp^2 + m_{c,s}^2}{x} \quad x \ll 1$$

$$A_h = -2p_{b,h}\cdot p_{c,h}$$

$$\approx -\frac{k_\perp^2}{x(1-x)} - \frac{1-x}{x}m_{c,h}^2 - \frac{x}{1-x}m_{b,h}^2$$

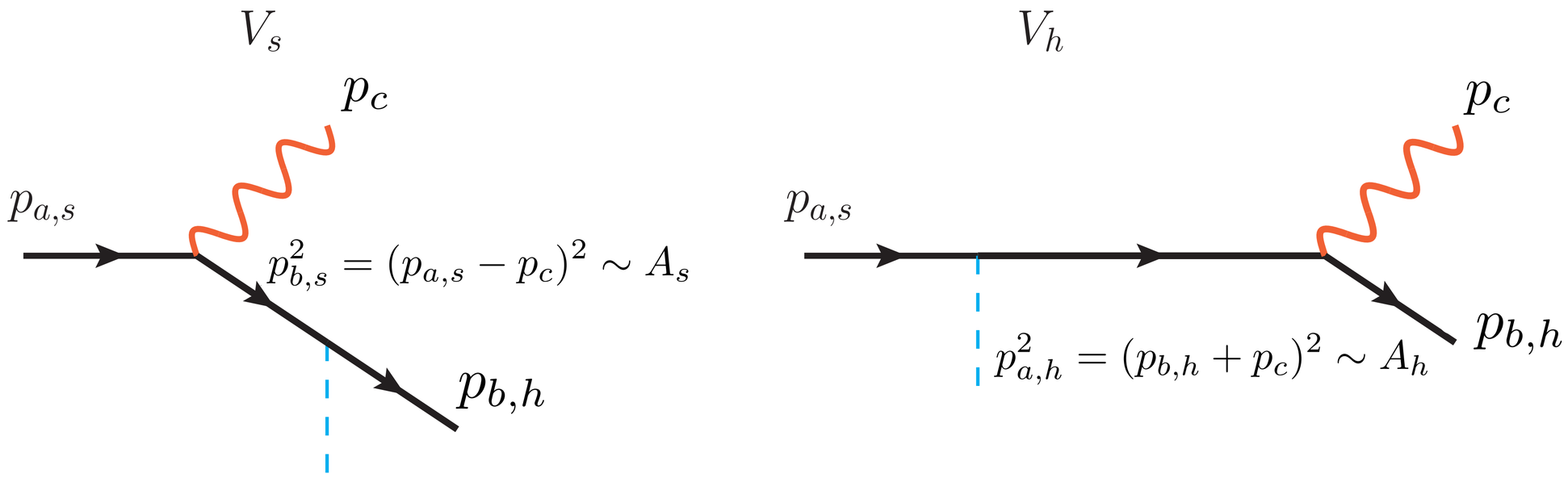
$$\approx -\frac{k_\perp^2}{x(1-x)} - \frac{1-x}{x}m_{c,h}^2 - \frac{x}{1-x}m_{b,h}^2$$

$$\approx -\frac{k_\perp^2 + m_{c,h}^2}{x} \quad x \ll 1$$

Two expressions same in the soft-limit but not same outside of soft limit.

Why the redefinition of A's?

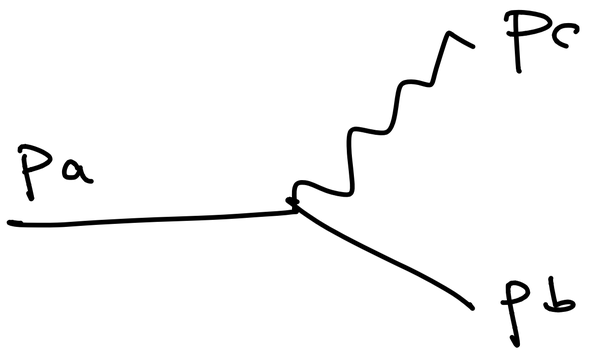
Treat A's as covariant propagators. Wall provides spatial discontinuity off of which incoming particle scatters.



Let's study the case where the soft emission is massless and the phase change comes form the hard fermion leg. This way the polarisation sum is well defined on either side of the wall and we can treat V_s and V_h separately.

Reformulation of the Matrix Element

$$V_s = (-ig)\bar{u}_s\left(p_{b,s}\right)\gamma^\mu\varepsilon^*_\mu\left(p_{c,s}\right)u_s\left(p_{a,s}\right)$$
$$V_h = (-ig)\bar{u}_h\left(p_{b,h}\right)\gamma^\mu\varepsilon^*_\mu\left(p_{c,h}\right)u_h\left(p_{a,h}\right)$$



$$\sum_\lambda \epsilon\epsilon^* = -g^{\mu\nu} + \zeta\left(-\frac{n^2 p_c^\mu p_c^\nu}{\left(n\cdot p_c\right)^2} + \frac{n^\nu p_c^\mu + n^\mu p_c^\nu}{n\cdot p_c}\right)$$

$$V_s^* V_s = 2g^2\left(\left(1+\frac{2\zeta}{x^2}\right)k_\perp^2 - 2(1-\zeta)m_{b,s}^2 + 2\zeta m_c^2\frac{1-x}{x^2}\right)$$

$$V_s^* V_h = 2g^2\left(\left(1+\frac{2\zeta}{x^2}\right)k_\perp^2 - (1-\zeta)\left(m_{b,s}^2 + m_{b,h}^2\right) + 2\zeta m_c^2\frac{1-x}{x^2}\right)$$

$$V_h^* V_s = 2g^2\left(\left(1+\frac{2\zeta}{x^2}\right)k_\perp^2 - (1-\zeta)\left(m_{b,s}^2 + m_{b,h}^2\right) + 2\zeta m_c^2\frac{1-x}{x^2}\right)$$

$$V_h^* V_h = 2g^2\left(\left(1+\frac{2\zeta}{x^2}\right)k_\perp^2 - 2(1-\zeta)m_{b,h}^2 + 2\zeta m_c^2\frac{1-x}{x^2}\right)$$

$$\zeta = 1 \implies |V|^2 \propto \frac{k_\perp^2}{x^2}$$

from A&P and same as B&M

Reformulation of the Matrix Element

Feynman gauge: $\zeta = 0$

$$
\begin{aligned}
V_s^* V_s &= 2g^2 k_\perp^2 \\
V_s^* V_h &= 2g^2 \left(k_\perp^2 - m_{b,h}^2\right) \\
V_h^* V_s &= 2g^2 \left(k_\perp^2 - m_{b,h}^2\right) \\
V_h^* V_h &= 2g^2 \left(k_\perp^2 - 2m_{b,h}^2\right)
\end{aligned}
$$

Our A's

Soft and collinear limit

$k_\perp \to \lambda k_\perp,$

$x \to \lambda x$

$$
\begin{aligned}
\frac{V_s^* V_s}{A_s^2} &= O(1) \\
\frac{V_s^* V_h}{A_s A_h} &= -2g^2 \frac{1}{\lambda^2 k_\perp^2} \frac{x^2}{x^2 + k_\perp^2/m_{b,h}^2} + O(1/\lambda) \\
\frac{V_h^* V_s}{A_h A_s} &= -2g^2 \frac{1}{\lambda^2 k_\perp^2} \frac{x^2}{x^2 + k_\perp^2/m_{b,h}^2} + O(1/\lambda) \\
\frac{V_h^* V_h}{A_h^2} &= -2g^2 \frac{2}{\lambda^2 x^2 m_{b,h}^2} \left(\frac{x^2}{x^2 + k_\perp^2/m_{b,h}^2}\right)^2 + O(1/\lambda)
\end{aligned}
$$

$$
|\mathcal{M}|^2 = 4g^2 \frac{1}{\lambda^2 k_\perp^2} \left(\frac{x^2}{x^2 + k_\perp^2/m_{b,h}^2}\right)^2 + O(1/\lambda)
$$

Reformulation of the Matrix Element

Axial gauge: $\zeta = 1$ and choose auxiliary vector $n = p_{a,s}$

$$V_s = 0 \qquad \Longrightarrow \qquad |V_h|^2 = \mathrm{Tr}\left[(\not{p}_{b,h} - m_{b,h})\,\gamma^\mu\,(\not{p}_{b,h} - m_{b,h})\,\gamma^\nu\right]\left(-g^{\mu\nu} + \frac{p_c^\mu p_{a,s}^\nu + p_c^\nu p_{a,s}^\mu}{p_{a,s}p_c}\right)$$

$$= 8p_{b,h}^\mu p_{b,h}^\nu \left(-g^{\mu\nu} + \frac{p_c^\mu p_{a,s}^\nu + p_c^\nu p_{a,s}^\mu}{p_{a,s}p_c}\right)$$

$$= 8\left(2p_{b,h}p_c\frac{p_{a,s}p_{b,h}}{p_{a,s}p_c} - m_{b,h}^2\right)$$

$$\frac{\sum |V_h|^2}{A_h^2} = \frac{2p_{a,s}p_{b,h}}{p_{a,s}p_c p_{b,h}p_c} - \frac{m_{b,h}^2}{(p_{b,h}p_c)^2} \longrightarrow 4g^2\frac{1}{\lambda^2 k_\perp^2}\left(\frac{x^2}{x^2 + k_\perp^2/m_{b,h}^2}\right)^2 + O(1/\lambda)$$

Same as Feynman gauge

Reformulation of the Matrix Element

Our matrix element has the following form:

$$\left|M^{(0)}_{a\to bc}\right|^2 = 4E_a^2|g|^2\left(\frac{2p_{a,s}p_{b,h}}{p_{a,s}p_c p_{b,h}p_c} - \frac{m^2_{a,s}}{\left(p_{a,s}p_c\right)^2} - \frac{m^2_{b,h}}{\left(p_{b,h}p_c\right)^2}\right)$$

This is a gauge invariant matrix element squared that corresponds to decelerated charges emitting soft gauge field quanta.

Reformulation of the Matrix Element

$$\left|M^{(0)}_{a\to bc}\right|^2 = 4E_a^2 \frac{|V_s|^2}{A_s^2}\bigg|^2_{n^\mu = p^\mu_{b,h}}$$

$m^2_{a,s} \ll k^2_\perp, m^2_{b,h}$:

$$8E_a^2|g|^2 \frac{2x^2}{k^2_\perp}\left(\frac{k^2_\perp + xm^2_{b,h}}{k^2_\perp + x^2 m^2_{b,h}}\right)^2 \xrightarrow[m^2_{a,s}, m^2_{b,h} \ll k^2_\perp]{} 8E_a^2|g|^2\frac{2x^2}{k^2_\perp}$$

$k^2_\perp \ll x^2 m^2_{b,h}$:

$$4|g|^2\frac{k^2_\perp}{x^2}$$

$$\frac{1}{\left|\mathcal{M}^{(0)}_{a\to b}\right|^2}\int \frac{d^3p_c}{(2\pi)^3 2E_c}\left|M^{(0)}_{a\to bc}\right|^2 \approx \begin{cases} \dfrac{\alpha}{2\pi}\displaystyle\int \frac{dk^2_\perp}{k^2_\perp} C_{abc}\log\frac{m^2_{b,h}}{k^2_\perp} & \text{if} \quad m^2_{a,s} \ll k^2_\perp \ll m^2_{b,h} \\ \dfrac{\alpha}{2\pi}\displaystyle\int \frac{dk^2_\perp}{k^2_\perp} C_{abc} & \text{if} \quad m^2_{a,s}, m^2_{b,h} \ll k^2_\perp \end{cases}$$

Small $k_\perp$ →

Large $k_\perp$ →

Analytic Resummation

Banfi, Salam & Zanderighi (2005)

$R(v)$ probability for decay $a \to bc$. For this splitting to produce momentum transfer of "V" we require that it did not produce a larger momentum transfer before that splitting.

$$R(v) = \int [dk] \left| M^2(k) \right| \Theta[V(\{p\}, k) - v]$$

$$V\left(p_a, p_b, p_c\right) = \frac{\Delta p_z}{\gamma T} \approx \frac{k_\perp^2 / \left(2E_a^2\right)}{x(1-x)}$$

$$R_{abc}(V) = C_{abc}|g|^2 \int \frac{d^3 p_c}{(2\pi)^3 2E_c} \left(\frac{2p_{b,h}p_{a,s}}{p_{a,s}p_c p_{b,h}p_c} + O\left(\frac{m_{a,s}^2}{k_\perp^2}, \frac{m_{b,h}^2}{k_\perp^2} \right) \right) \Theta\left(V\left(p_a, p_b, p_c\right) - V\right) \Theta\left(p_{b,z,h}\right) \Theta\left(p_{c,z,h}\right)$$

Rewrite phase space and matrix element squared in terms of observable V

$$R_{abc}(V) = C_{abc} \frac{\alpha}{2\pi} \int_V^1 \frac{\mathrm{d}V'}{V'} \int_0^1 \mathrm{d}x 2x \Theta\left(\frac{1}{1+V'} - x \right) \Theta\left(x - \frac{V'}{1+V'} \right)$$

$$R_{abc}(V) = \frac{\alpha}{2\pi} C_{abc} \left(L + 2\log\left(1 + e^{-L}\right) \right) \text{ where } L = \log \frac{1}{V}$$

Sudakov form factor ↘

Banfi, Salam & Zanderighi (2005)

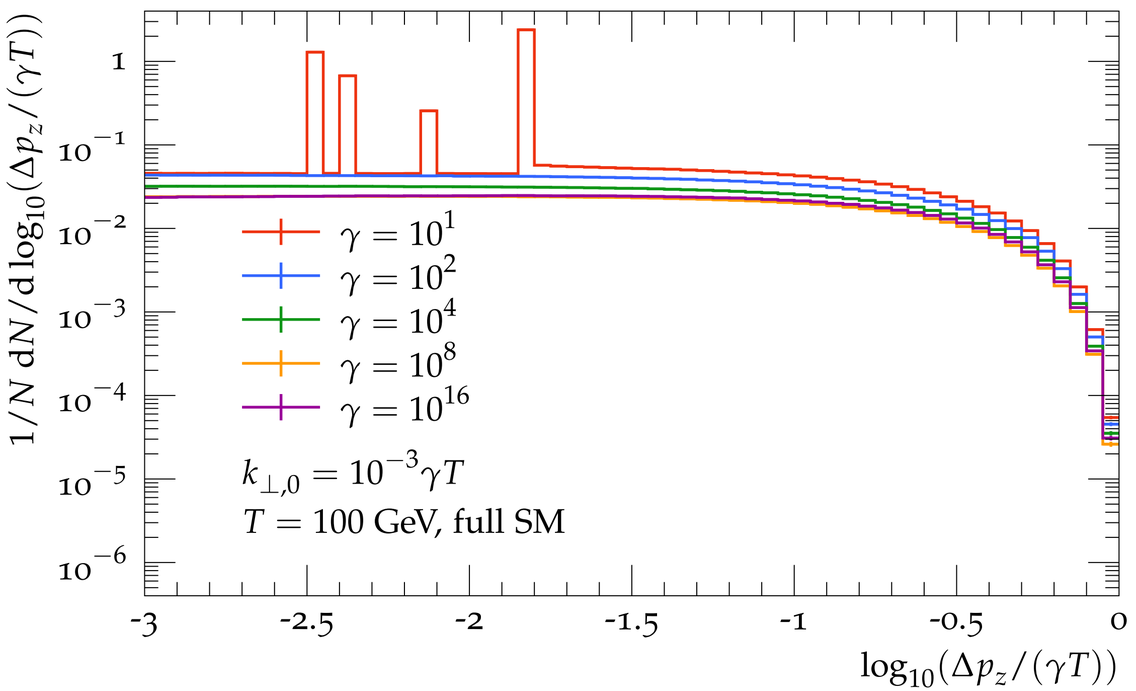
$$\Delta_a(V) = \exp\left\{-\sum_b R_{ab}(V)\right\}, \qquad \text{where} \qquad R_{ab}(V) = \sum_c R_{abc}(V)$$

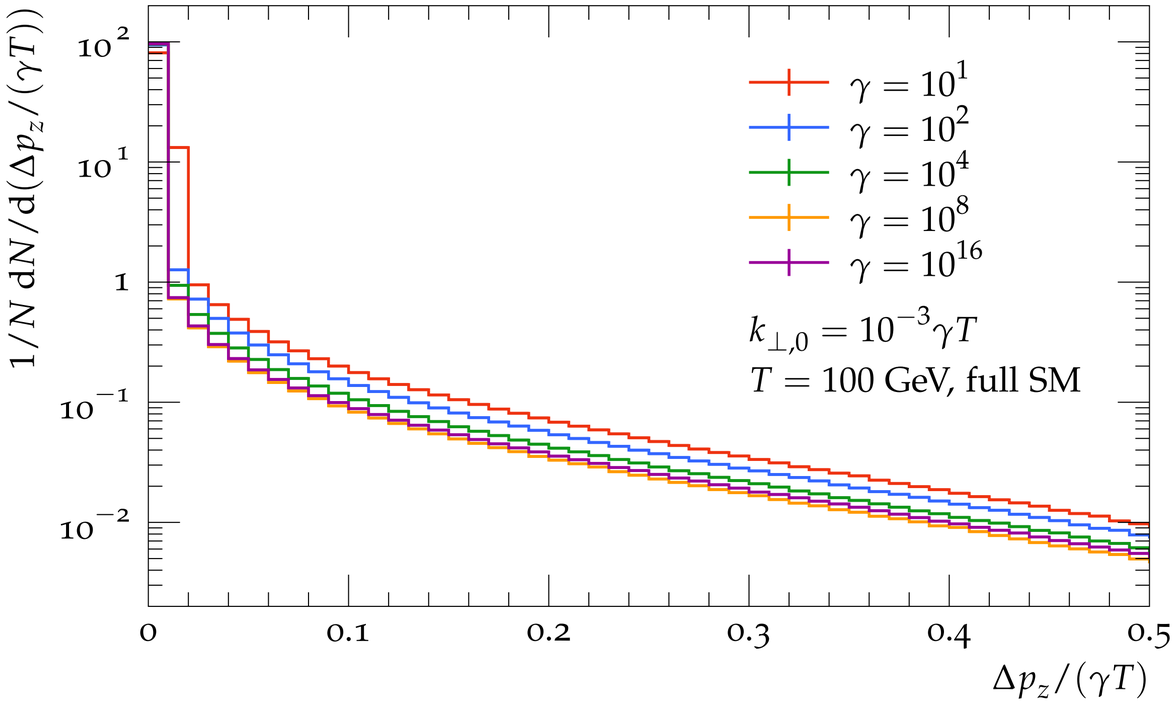
$$\left\langle \frac{\Delta p_z}{\gamma T} \right\rangle = \int_0^1 \mathrm{d}V V \frac{\mathrm{d}}{\mathrm{d}V} \prod_{a\in\mathcal{S}} \Delta_a(V)$$

$$\left\langle \frac{\Delta p_z}{\gamma T} \right\rangle_{\mathrm{FC}} = \int_0^\infty \mathrm{d}L e^{-L} \frac{(\alpha C)_\Sigma}{2\pi} \frac{e^L - 1}{e^L + 1} \exp\left\{-\frac{(\alpha C)_\Sigma}{2\pi}\left(L + 2\log\left(1 + e^{-L}\right)\right)\right\} \approx \zeta(\log 4 - 1) \qquad \zeta = \frac{(\alpha C)_\Sigma}{2\pi}$$

$$\langle \Delta p_z \rangle \sim \gamma T$$

Numerical Resummation





$$\left\langle \frac{\Delta p_z}{\gamma T} \right\rangle = 0.89(17)\,\% - 0.14(3)\,\% \log_{10}\gamma \implies P \propto \gamma^2 T^4$$

Comparing analytic and numerics

Particle	n_f	ν_a	analytic $(\langle \Delta p_z/(\gamma T)\rangle_{\mathrm{FC}})$	numeric $(\langle \Delta p_z/(\gamma T)\rangle_{\mathrm{FC}})$
$l^{\pm}$	2×3	2	0.17%	0.17%
u	2×3	2×3	0.46%	0.44%
d	2×3	2×3	0.46%	0.44%
$W^{\pm}$	2	2	0.52%	0.52%
Z	1	2	0.41%	0.40%
h	1	1	0.22%	0.21%
$G_{W^{\pm}}$	2	1	0.22%	0.21%
G_Z	1	1	0.22%	0.21%

Good agreement between analytics and numerics. Largest pressure contribution

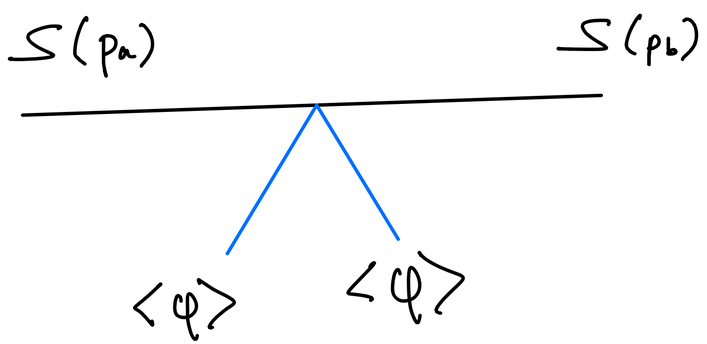
From QCD processes

Points for discussion and future work

Where does the dependence in the phase change enter?

To perform a resummation or even define what the quantum correction is, we need to define what the leading order process is (aka Born-level process/hard function)

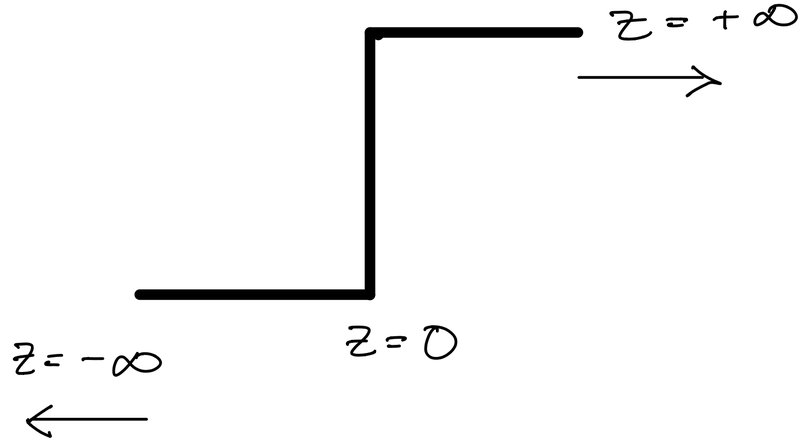
Lets consider our hard function to be a scalar interacting with the wall (fermions follow analogously). Leading order process is calculated from VEV-insertion:

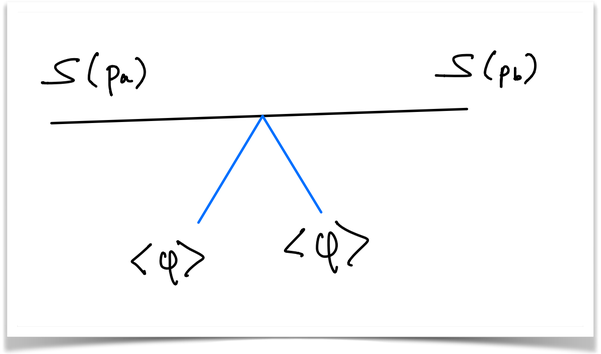


$$L_{\text{int}} \supset -\frac{\lambda}{4} S^2 \Phi^2$$

$$\Phi = \varphi(x,t) + \hat{\varphi}$$

$$\varphi(z) = \varphi_0 \Theta(z)$$





$$\mathcal{M}^{(0)}_{a\to b} = i\frac{\lambda}{4}\left\langle p_b \left| T\int d^4x S(x,t)^2\varphi(x,t)^2 \right| p_a \right\rangle$$

Fourier transform
step function profile of the wall

$$\varphi(x,t)^2 = \varphi_0^2\int\frac{dk_z}{2\pi}\int\frac{dk_z'}{2\pi}e^{i\left(k_z+k_z'\right)z}\left(\frac{\delta\left(k_z\right)}{2}-\frac{i}{2\pi}PV\frac{1}{k_z}\right)\left(\frac{\delta\left(k_z'\right)}{2}-\frac{i}{2\pi}PV\frac{1}{k_z'}\right) \xrightarrow{\textbf{Perform integrations}} \frac{1}{4\pi^3}\frac{1}{\Delta p_z}$$

Scalar quantised usual way

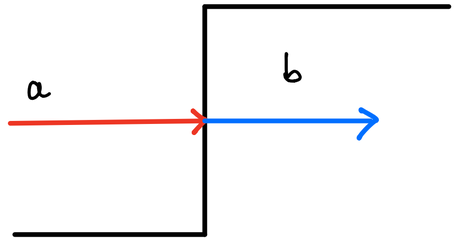
$$S(x,t) = \int\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\left(a_p e^{-ip\cdot x}+a_p^\dagger e^{ip\cdot x}\right) \longrightarrow \left\langle p_b\left|S(x,t)^2\right|p_a\right\rangle = \langle p_b|S(x,t)|0\rangle\langle 0|S(x,t)|p_a\rangle = e^{i(p_a-p_b)\cdot x}$$

Put it all together

$$P_{a\to b} = \frac{\lambda\varphi_0^2}{8\pi^3}\delta\left(E_a-E_b\right)\delta^2\left(p_{a,\perp}-p_{b,\perp}\right)\frac{1}{\Delta p_z} \xrightarrow{E_a^2\gg m^2} \Delta p_z = \frac{m_{b,h}^2-m_{a,s}^2}{2E_a}\approx\frac{m_{b,h}^2}{2E_a} \quad (m_{a,s}\ll m_{b,h})$$

$$P_{a\to b} = \frac{\lambda\varphi_0^2}{8\pi^3}\delta\left(E_a-E_b\right)\delta^2\left(p_{a,\perp}-p_{b,\perp}\right)\frac{1}{\Delta p_z} \xrightarrow{m_{b,h}^2\gg m_{a,s}} P_{a\to b} = \frac{m_{b,h}^2}{8\pi^3}\delta\left(E_a-E_b\right)\delta^2\left(p_{a,\perp}-p_{b,\perp}\right)\frac{E_a}{m_{b,h}^2}$$

Recall the B&M 1-to-1 calculation



$$\mathrm{d}\mathbb{P}_{a\to b} = \frac{d^3p_b}{(2\pi)^3}\frac{1}{2E_b} \times (2\pi)^3\delta^2\left(p_{a,\perp} - p_{b,\perp}\right)\delta\left(E_a - E_b\right)\left(2p_{b,z,h}\right)\delta_{ab}$$

$$\mathcal{P}_{1\to 1} = \sum_a \nu_a \int \frac{d^3p_a}{(2\pi)^3} f_a\left(1 \pm f_b\right)\Delta p_{\mathrm{wall}} \approx \gamma^0 T^2 \Delta m^2$$

Mass dependence comes from observable not from matrix element

Summary: treat 1-to-1 as leading order process. As long as the ingoing particles gains a mass it will interact with a probability of 1 following from VEV-insertion method assuming an infinitely thin wall. This hard function/LO process normalises our NLO process. This procedure of defining LO process that normalises NLO process is tried and tested (> 30 years of collider data) and follows from the universal factorisation properties of gauge theories in the soft limit.

Summary

- 1st order EWPT has many interesting physical consequences such as baryogenesis & GW production. Both quantitatively depend on the velocity of the bubble wall.

- Bubble wall velocity is a force balancing exercise: pressure from Higgs potential versus frictional pressure from plasma.

- We reformulated the calculation of the latter in a GI way and calculated the average pressure to all orders. This has a physical correspondence with a charged particle decelerating and emitting a soft radiation pattern.

- We find pressure $\propto \gamma^2$ also massless GB contribute the largest pressure of all SM. Numerical and analytic resumption agree to 10% level.

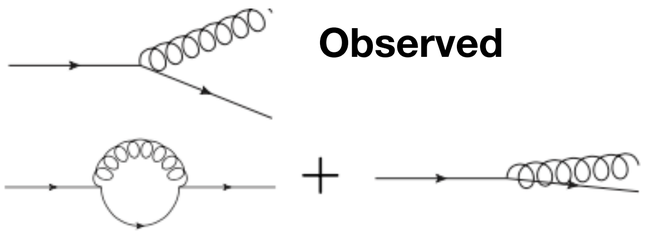
- Still many interesting avenues to explore.

Thank you for your time

Backup slides

Radiative corrections as branching processes

Marchesini & Webber (1983)
Sjöstrand (1985)



Observed

Virtual correction

Unresolved Emission

$$P_{\text{no em}} + P_{em} = 1$$

Unobserved processes captured by Sudakov factor

Expression for change in population:

$N = \text{population}$

$$dN = -\lambda N dt$$

$\lambda = \text{decay constant}$

Survival probability at time "t": $e^{-\lambda t}$ where $t \leftrightarrow$ energy scale $(\log(1/\text{v}))$

Change in population analogous to boson emission probability

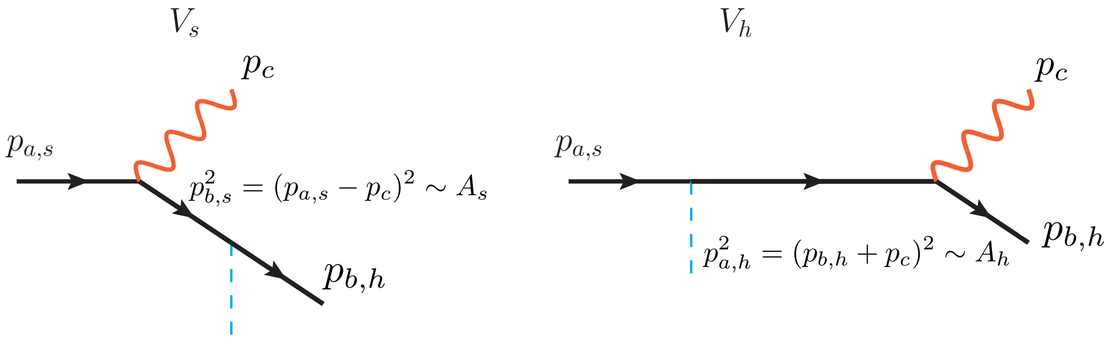
$$\lambda N dt = \int [dk] M^2(k) \Theta(v - V(\{\tilde{p}\}, k)) \quad \Rightarrow \quad [\text{ virt. } + \text{ unres. }] = e^{-\int [dk] M^2(k) \Theta(V(\{\tilde{p}\},k) - v)}$$

Probability of not emitting bosons above "v" (Sudakov)

Kinematics

$$
\begin{aligned}
p_a^\mu &= \left(E_a, \overrightarrow{0}, \sqrt{E_a^2 - m_a^2}\right) \approx \left(E_a, \overrightarrow{0}, E_a\left(1 - \tfrac{m_a^2}{2E_a^2}\right)\right) \\
p_b^\mu &= \left((1-x)E_a, -\vec{k}_\perp, \sqrt{(1-x)^2E_a^2 - \vec{k}_\perp^2 - m_b^2}\right) \approx \left((1-x)E_a, -\vec{k}_\perp, (1-x)E_a\left(1 - \tfrac{\vec{k}_\perp^2 + m_b^2}{2(1-x)^2E_a^2}\right)\right) \\
p_c^\mu &= \left(xE_a, \vec{k}_\perp, \sqrt{x^2E_a^2 - \vec{k}_\perp^2 - m_c^2}\right) \approx \left(xE_a, \vec{k}_\perp, xE_a\left(1 - \tfrac{\vec{k}_\perp^2 + m_c^2}{2x^2E_a^2}\right)\right)
\end{aligned}
$$

Gauge dependence



$$\begin{aligned}
\tilde{V}_s &= \bar{u}\left(p_{b,h}\right) u\left(p_{b,s}\right) \bar{u}\left(p_{b,s}\right) \gamma^\mu u\left(p_{a,s}\right) \varepsilon_\mu^*\left(p_c, q\right) \\
&= \bar{u}\left(p_{b,h}\right) \left(\left[\not{p}_{a,s} - \not{p}_c\right] + m_{c,s}\right) \gamma^\mu u\left(p_{a,s}\right) \varepsilon_\mu^*\left(p_c, q\right) \\
\tilde{V}_h &= \bar{u}\left(p_{b,h}\right) \gamma^\mu u\left(p_{a,h}\right) \bar{u}\left(p_{a,h}\right) u\left(p_{a,s}\right) \varepsilon_\mu^*\left(p_c, q\right) \\
&= \bar{u}\left(p_{b,h}\right) \gamma^\mu \left(\left[\not{p}_{c,h} + \not{p}_c\right] + m_{a,h}\right) u\left(p_{a,s}\right) \varepsilon_\mu^*\left(p_c, q\right)
\end{aligned}$$

Feynman Gauge

$$\sum_\lambda \varepsilon_\nu^\lambda \varepsilon_\nu^\lambda * = -g_{\mu\nu}$$

$$\sum \left|\tilde{V}_s\right|^2 = -8p_c^2 \left(p_{a,s} \cdot p_{b,h}\right) - 16p_{a,s}^2 \left(p_c \cdot p_{b,h}\right) + 16 \left(p_{a,s} \cdot p_c\right) \left(p_c \cdot p_{b,h}\right) + 8p_{a,s}^2 \left(p_{a,s} \cdot p_{b,h}\right)$$

$$\begin{aligned}
\sum \left|\tilde{V}_h\right|^2 = &-32m_{b,h}^2 \left(p_{a,s} \cdot p_c\right) - 24m_{b,h}^2 \left(p_{a,s} \cdot p_{b,h}\right) - 8p_c^2 \left(p_{a,s} \cdot p_{b,h}\right) \\
&+ 16 \left(p_{a,s} \cdot p_c\right) \left(p_c \cdot p_{b,h}\right) + 16p_{b,h}^2 \left(p_{a,s} \cdot p_c\right) + 8p_{b,h}^2 \left(p_{a,s} \cdot p_{b,h}\right)
\end{aligned}$$

$$\begin{aligned}
\sum \tilde{V}_s \tilde{V}_h^* = \sum \tilde{V}_s^* \tilde{V}_h = &-16 \left(p_{a,s} \cdot p_{b,h}\right)^2 - 8m_{b,h}^2 \left(p_{a,s} \cdot p_c\right) + 8m_{b,h}^2 p_{a,s}^2 \\
&- 16 \left(p_{a,s} \cdot p_c\right) \left(p_{a,s} \cdot p_{b,h}\right) + 16 \left(p_{a,s} \cdot p_c\right) \left(p_c \cdot p_{b,h}\right) + 16 \left(p_{a,s} \cdot p_{b,h}\right) \left(p_c \cdot p_{b,h}\right)
\end{aligned}$$

Gauge dependence

Feynman Gauge

$$A_s = (p_{a,s} - p_c)^2 \approx 2p_{a,s}p_c \quad A_h = (p_{b,h} - p_c)^2 \approx 2p_{b,h}p_c$$

$$H \times |M|^2 = -\frac{4m_{b,h}^2 (p_{a,s} \cdot p_c)}{(p_c \cdot p_{b,h})^2} - \frac{4m_{b,h}^2 (p_{a,s} \cdot p_{b,h})}{(p_c \cdot p_{b,h})^2} + \frac{8 (p_{a,s} \cdot p_{b,h})^2}{(p_{a,s} \cdot p_c)(p_c \cdot p_{b,h})} + \frac{4m_{b,h}^2}{p_c \cdot p_{b,h}} - \frac{8 (p_{a,s} \cdot p_{b,h})}{p_{a,s} \cdot p_c} + \frac{4 (p_c \cdot p_{b,h})}{p_{a,s} \cdot p_c} + \frac{4 (p_{a,s} \cdot p_c)}{p_c \cdot p_{b,h}} + \frac{8 (p_{a,s} \cdot p_{b,h})}{p_c \cdot p_{b,h}} - 8$$

Axial gauge

$$\sum_\lambda \epsilon\epsilon^* = -\frac{n^2 p_c^\mu p_c^\nu}{(n \cdot p_c)^2} - g^{\mu\nu} + \frac{n^\nu p_c^\mu + n^\mu p_c^\nu}{n \cdot p_c}$$

n is just any arbitrary light-like vector

Turn the wheel ..

Gauge dependence

Axial gauge

$$\begin{aligned}\sum \tilde{V}_s \tilde{V}_h^* = &-\frac{8m_{c,h}^2 n^2 \left(p_{a,s}\cdot p_b\right)^2}{\left(n\cdot p_b\right)^2}+\frac{4m_{c,h}^2 n^2 p_{a,s}^2 p_b^2}{\left(n\cdot p_b\right)^2}+\frac{4m_{c,h}^2 n^2 p_b^2\left(p_{a,s}\cdot p_b\right)}{\left(n\cdot p_b\right)^2}+\frac{4n^2 p_b^2\left(p_{a,s}\cdot p_{c,h}\right)}{\left(n\cdot p_b\right)^2}+\\ &\frac{8n^2 p_{a,s}^2\left(p_b\cdot p_{c,h}\right)^2}{\left(n\cdot p_b\right)^2}-\frac{8n^2 p_b^2\left(p_{a,s}\cdot p_b\right)\left(p_{a,s}\cdot p_{c,h}\right)}{\left(n\cdot p_b\right)^2}\\ &-\frac{16n^2\left(p_{a,s}\cdot p_b\right)\left(p_{a,s}\cdot p_{c,h}\right)\left(p_b\cdot p_{c,h}\right)}{\left(n\cdot p_b\right)^2}+\frac{4n^2 p_{a,s}^2 p_b^2\left(p_b\cdot p_{c,h}\right)}{\left(n\cdot p_b\right)^2}+\frac{8n^2 p_b^2\left(p_{a,s}\cdot p_{c,h}\right)\left(p_b\cdot p_{c,h}\right)}{\left(n\cdot p_b\right)^2}\\ &+\frac{8n^2 p_{c,h}^2\left(p_{a,s}\cdot p_b\right)^2}{\left(n\cdot p_b\right)^2}-\frac{4n^2 p_{a,s}^2 p_b^2 p_{c,h}^2}{\left(n\cdot p_b\right)^2}-\frac{4n^2 p_b^2 p_{c,h}^2\left(p_{a,s}\cdot p_b\right)}{\left(n\cdot p_b\right)^2}+\frac{8\left(n\cdot p_{c,h}\right)\left(p_{a,s}\cdot p_b\right)^2}{n\cdot p_b}\\ &-\frac{8\left(n\cdot p_{a,s}\right)\left(p_b\cdot p_{c,h}\right)^2}{n\cdot p_b}-16\left(p_{a,s}\cdot p_{c,h}\right)^2+\frac{16m_{c,h}^2\left(n\cdot p_{a,s}\right)\left(p_{a,s}\cdot p_b\right)}{n\cdot p_b}\\ &-\frac{8m_{c,h}^2 p_b^2\left(n\cdot p_{a,s}\right)}{n\cdot p_b}-8m_{c,h}^2\left(p_{a,s}\cdot p_b\right)+\frac{16\left(n\cdot p_{c,h}\right)\left(p_{a,s}\cdot p_b\right)\left(p_{a,s}\cdot p_{c,h}\right)}{n\cdot p_b}-\frac{8p_{a,s}^2 p_b^2\left(n\cdot p_{c,h}\right)}{n\cdot p_b}\\ &-\frac{8p_b^2\left(n\cdot p_{c,h}\right)\left(p_{a,s}\cdot p_{c,h}\right)}{n\cdot p_b}+\frac{8p_b^2\left(n\cdot p_{a,s}\right)\left(p_{a,s}\cdot p_{c,h}\right)}{n\cdot p_b}-\frac{16p_{a,s}^2\left(n\cdot p_{c,h}\right)\left(p_b\cdot p_{c,h}\right)}{n\cdot p_b}\\ &+\frac{8\left(n\cdot p_{c,h}\right)\left(p_{a,s}\cdot p_b\right)\left(p_b\cdot p_{c,h}\right)}{n\cdot p_b}-\frac{8\left(n\cdot p_{a,s}\right)\left(p_{a,s}\cdot p_b\right)\left(p_b\cdot p_{c,h}\right)}{n\cdot p_b}\\ &+\frac{16\left(n\cdot p_{a,s}\right)\left(p_{a,s}\cdot p_{c,h}\right)\left(p_b\cdot p_{c,h}\right)}{n\cdot p_b}-\frac{16p_{c,h}^2\left(n\cdot p_{a,s}\right)\left(p_{a,s}\cdot p_b\right)}{n\cdot p_b}\\ &+\frac{8p_b^2 p_{c,h}^2\left(n\cdot p_{a,s}\right)}{n\cdot p_b}-8\left(p_{a,s}\cdot p_b\right)\left(p_{a,s}\cdot p_{c,h}\right)-8p_b^2\left(p_{a,s}\cdot p_{c,h}\right)\\ &+16\left(p_{a,s}\cdot p_b\right)\left(p_b\cdot p_{c,h}\right)+8\left(p_{a,s}\cdot p_{c,h}\right)\left(p_b\cdot p_{c,h}\right)+8p_{a,s}^2 p_{c,h}^2\end{aligned}$$

Put all this together with our definition of A's and put external particle On their mass-shell

Same matrix element as Feynman gauge

$$\begin{aligned}H\times|M|^2 = &-\frac{4m_{b,h}^2\left(p_{a,s}\cdot p_c\right)}{\left(p_c\cdot p_{b,h}\right)^2}-\frac{4m_{b,h}^2\left(p_{a,s}\cdot p_{b,h}\right)}{\left(p_c\cdot p_{b,h}\right)^2}+\frac{8\left(p_{a,s}\cdot p_{b,h}\right)^2}{\left(p_{a,s}\cdot p_c\right)\left(p_c\cdot p_{b,h}\right)}\\ &+\frac{4m_{b,h}^2}{p_c\cdot p_{b,h}}-\frac{8\left(p_{a,s}\cdot p_{b,h}\right)}{p_{a,s}\cdot p_c}+\frac{4\left(p_c\cdot p_{b,h}\right)}{p_{a,s}\cdot p_c}+\frac{4\left(p_{a,s}\cdot p_c\right)}{p_c\cdot p_{b,h}}+\frac{8\left(p_{a,s}\cdot p_{b,h}\right)}{p_c\cdot p_{b,h}}-8\end{aligned}$$

Gauge dependence

Our impression of B&M results

$$|\mathcal{M}|^2 = \left(\frac{|V_s|^2}{A_s^2} + \frac{|V_h|^2}{A_h^2} - \frac{(V_s V_h^* + V_s^* V_h}{A_s A_h} \right)$$

Use our vertex functions and B&M A's
Recall

$$A_s = E_a \left(p_{a,z,s} - p_{b,z,s} - p_{c,z,s} \right)$$
$$A_h = E_a \left(p_{a,z,h} - p_{b,z,h} - p_{c,z,h} \right)$$

$$|\mathcal{M}|^2\big|_{\zeta=1} - |\mathcal{M}|^2\big|_{\zeta=0} = f(m_{b,h}^2, k_\perp^2, ..) \neq 0$$