Asymptotics, holography and fluids

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Highlights

Sixties – the dawn of GR renaissance

Key experimental and theoretical breakthroughs

Quasars, Shapiro effect, gravitational redshift, CMB, black-hole physics, Penrose paper on singularity formation etc.

1962 Bondi, van der Burg, Metzner and Sachs

Asymptotic symmetry group of a *strict* asymptotically flat spacetime in four dimensions

$$\boxed{\text{Lorentz} \ltimes \text{Supertranslations} \longrightarrow 6 + \infty \equiv \text{BMS}_4}$$

not

$$\text{Poincaré} \equiv \text{Lorentz} \ltimes \text{Translations} \longrightarrow 6 + 4$$

Intriguing but in line with general covariance

Recognized as a valuable tool (Ashtekar, Komar, Penrose and many others)

- Classical: taming of solution space, computing the conserved charges and algebra for a gravitational field, etc.
- Quantum: scattering of particles escaping at null infinity should be encoded in a BMS_4-invariant S matrix
  - new handle on the infrared issues (dressed states)
  - includes quantum gravity

Recent revival for the quest of BMS-invariant field theories [Harvard school among others]

2000s – the advent of anti de Sitter

Anti de Sitter

Maximally symmetric Einstein spacetime with negative curvature (cosmological constant)

AdS/CFT holographic correspondence

Type IIB string theory in the *bulk* and $N = 4$ super-Yang–Mills on the *boundary* – AdS_5 soon extended to arbitrary dimension

Diverse asymptotic symmetries

e.g. for *strict* AdS_n asymptotics

$$\boxed{\text{conformal group in } n-1 \text{ dimensions } SO(n-1,2) \longrightarrow {}^{n(n+1)}/_{2}}$$

Less exotic but can also become infinite

Phenomenological branch of holography: fluid/gravity correspondence [Bhattacharyya, Hubeny, Minwalla, Rangamani '07; Haack, Yarom '08; etc.]

- Ignore the microscopic structure of the boundary theory – quantum many-body state in the hydrodynamic regime
- Keep only gravity in the bulk – possibly Einstein–Maxwell

Holography: the boundary relativistic hydrodynamics encodes the gravitational bulk dynamics

Central and timely questions

- How do the symmetries of the boundary fluid dynamics match the asymptotic bulk symmetries? What about the widely (un)known hydrodynamic-frame invariance?
- What happens when the bulk AdS asymptotics is flattened? How does BMS symmetry emerge on the boundary?

Subject of the talk

Highlights

Equations of motion [Eckart '40; Landau and Lifshitz '60]

- Ignoring matter current and chemical potential
- On arbitrary (non-flat) geometry $g_{\mu\nu}$ of dim $d+1$

$\nabla_\mu T^{\mu\nu} = 0$ plus Gibbs–Duhem & equation of state (conformal)

- $\|\mathrm{u}\|^2 = -k^2 \quad h^{\mu\nu} = g^{\mu\nu} + \frac{u^\mu u^\nu}{k^2}$

$$\boxed{T^{\mu\nu} = \varepsilon \frac{u^\mu u^\nu}{k^2} + p h^{\mu\nu} + \tau^{\mu\nu} + \frac{u^\mu q^\mu}{k^2} + \frac{u^\nu q^\mu}{k^2}}$$

- energy density $\varepsilon = \frac{1}{k^2} T_{\mu\nu} u^\mu u^\nu$ thermodynamic pressure p
- heat current and viscous stress tensor q^μ, $\tau^{\mu\nu}$ – transverse
  normally expressed as u^μ- and T-derivative expansions with transport coefficients – constitutive relations ignored here

General covariance and Weyl invariance

Fluid equations covariant – diffeomorphism invariance

Diffeomorphisms are generated by vector fields ($i, j = 1, \ldots, d$)

$$\xi = f\partial_t + Y^i\partial_i$$

$f(x)$ and $Y^i(x)$ $d+1$ functions of time and space

$$\delta_\xi = -\mathscr{L}_\xi$$

Fluid equations invariant under arbitrary rescaling of the metric (conformal fluids)

$$\delta_\omega g_{\mu\nu} = -2\omega g_{\mu\nu} \quad \delta_\omega u^\mu = \omega u^\mu$$

$\omega(x)$ arbitrary function of time and space

$$\delta_\omega = w\omega$$

The hydrodynamic-frame invariance

Landau–Lifshitz's statement for non-perfect fluids

[Theoretical Physics vol. 6 §136]

First of all, however, we must discuss more closely the concept of the velocity u^μ itself. In relativistic mechanics, an energy flux necessarily involves a mass flux. Hence, when there is (e.g.) a heat flux, the definition of the velocity in terms of the mass flux density (as in non-relativistic fluid dynamics) has no direct meaning.

In short: u^μ is not physical – a book-keeping device

Translation in the formalism: another gauge invariance

Arbitrary transformations of u^μ (local Lorentz transformation) can be compensated by appropriate modifications of T, ε, p, q^μ, $\tau^{\mu\nu}$ such that $T^{\mu\nu}$ *and* the entropy current S^μ remain invariant

Note: These are *not* Lorentz isometries (generally absent) but tangent-space *local* transformations generated by Z^i (d boosts), S_{ij} antisymmetric ($d(d-1)/2$ rotations)

Highlights

Asymptotic symmetries in dimension n

The gauge-fixing approach

- Fix a gauge: n conditions
- Determine the residual diffeomorphisms
- Set "reasonable" fall-offs/boundary conditions

The *asymptotic symmetry group* (ASG) is the subset of residual diffeomorphisms compatible with the adopted fall-offs

Einstein spacetimes with $\Lambda < 0$ in n dimensions

[see Brussels school; Poole, Skenderis, Taylor '19; Compère, Fiorucci, Ruzziconi '19]

The Newman–Unti gauge in coordinates r, u, x^i

$(i = 1, \ldots, n-2)$

- Gauge conditions: $G_{rr} = 0$, $G_{ru} = -1$, $G_{ri} = 0$

$$\mathrm{d}s^2 = \frac{V}{r}\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + G_{ij}\left(\mathrm{d}x^i - U^i\mathrm{d}u\right)\left(\mathrm{d}x^j - U^j\mathrm{d}u\right)$$

  V, G_{ij}, U^i functions of *all* coordinates
- Residual diffeomorphisms: $\omega(u, \mathbf{x})$, $f(u, \mathbf{x})$, $Y^i(u, \mathbf{x})$
- Several options for fall-offs/boundary conditions
  - mildest boundary conditions ($G_{ij} = O(r^2)$) $\to$ infinite-dim Weyl, supertranslations and superrotations
  - Dirichlet boundary conditions e.g. locked $G_{ij} \simeq r^2 S^{n-2}$ $\to$ $\omega = 0$, f restricted, Y^i conformal $\to$ $SO(n-1, 2)$ finite conformal group at $n-1$ dimensions

Incomplete Newman–Unti gauge fixing [Ciambelli, Marteau, Petropoulos, Ruzziconi '20; Ciambelli, Marteau, Petkou, Petropoulos, Rivera, Ruzziconi, Siampos]

- Gauge conditions: $G_{rr} = 0$, $G_{ru} = -1$, $G_{ri} \neq 0$
- Residual diffeomorphisms: $\omega(u, \mathbf{x})$, $f(u, \mathbf{x})$, $Y^i(u, \mathbf{x})$ plus $Z^i(u, \mathbf{x})$, $S_{ij}(u, \mathbf{x})$ antisymmetric
- Mild boundary conditions ($G_{ij} = O\left(r^2\right)$) $\rightarrow$ infinite-dim Weyl, supertranslations, superrotations & local $SO(n-2, 1)$

Solution space with incomplete Newman–Unti gauge and mild boundary conditions – the boundary and its fluid

$n^2 - 3$ arbitrary functions of $u, \mathbf{x} \to$ emergence of the boundary

$\mu, \nu = 0, 1, \ldots, n-2$

- $g_{\mu\nu}$ symmetric $\leftarrow \frac{n(n-1)}{2}$
  boundary metric
- $T_{\mu\nu}$ symmetric and traceless $\leftarrow \frac{n(n-1)}{2} - 1$
  energy–momentum tensor for a boundary conformal fluid
- normalized $u^\mu \leftarrow n - 2$
  arbitrary boundary fluid velocity

remaining bulk Einstein's equations

$$\boxed{\nabla_\mu T^{\mu\nu} = 0}$$

Back to the symmetries

- The bulk Einstein space in the incomplete Newman–Unti gauge is uniquely reconstructed from the boundary data
- The relativistic boundary fluid symmetries are rescalings, diffeomorphisms and local Lorentz transformations

The boundary fluid symmetries precisely coincide with the infinite-dim bulk asymptotic symmetry group

Weyl, supertranslations, superrotations plus local $SO(n-2,1)$

Locking a hydrodynamic frame – e.g. Landau–Lifshitz – discards the local $SO(n-2,1)$ transformations and completes the bulk gauge fixing – yes but …

Yes but ...

- **A relevant boundary question**
  Is the choice of hydrodynamic frame innocuous?
- **Our perspective**
  A boundary hydrodynamic frame transformation is a bulk residual diffeomorphism within an incomplete gauge fixing
- **Recast in the bulk**
  Are different-fluid-velocity subspaces of the Einstein solution space equivalent?
- **Method**
  Determine the charge algebra for the solution subspaces
- **Answer**
  No – the algebras are different – typical avatar of global gauge issues [proven in $n = 3$ by Campoleoni, Ciambelli, Marteau, Petropoulos, Siampos; '19]

Highlights

A cartoon summary

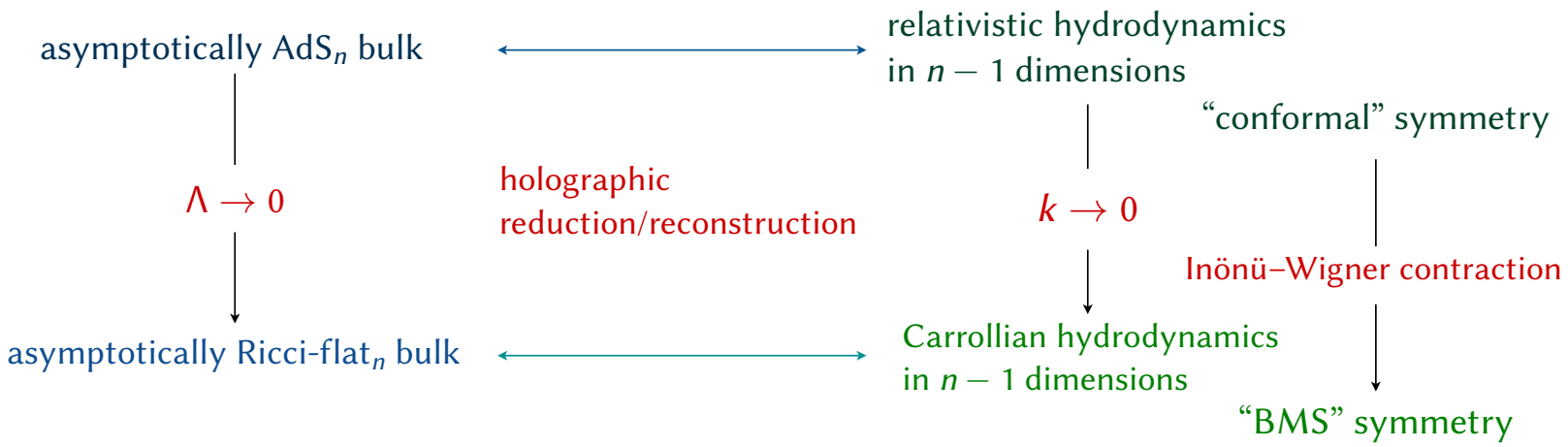
AdS holography: remarkable relationship

bulk cosmological constant $\Lambda \leftrightarrow$ boundary velocity of light k

Flat holography: $\Lambda = -\frac{(n-1)(n-2)}{2}k^2 \to 0$

Carrollian limit on the boundary (null infinity) [Lévy-Leblond '65]

- Solution space $\leftrightarrow$ conformal Carrollian boundary data
- Bulk ASG $\to$ Inönü–Wigner contraction: BMS_n
- Boundary symmetries: $\mathrm{CCarrroll}_{n-1} \equiv \mathrm{BMS}_n$



Highlights

Satisfactory picture of asymptotic symmetries and their fluid/gravity holographic realization

- Valid for asymptotically AdS or flat spacetimes
- Asymptotic symmetry group $\supset$ boundary hydrodynamic-frame transformations i.e. local $SO(n-2,1)$
- $\exists$ a BMS corner with its Carrollian fluids – in line with the "big picture:" the quest of BMS-invariant field theories

Better understanding of hydrodynamic-frame invariance

- Three bulk dimensions: $SO(1,1)$ has non-vanishing charge consequences for the fluid side: hydrodynamic-frame invariance is *only local*
- Higher dimensions: hydrodynamic-frame invariance is a handle for defining the fluid charges – gravity techniques