Asymptotics, Holography and Fluids

Marios Petropoulos

HEP2021 conference – Thessaloniki

June 2021
1 The big picture and our motivations

2 The local symmetries of relativistic fluids

3 Asymptotics and fluid/gravity correspondence

4 Back to BMS

5 Summary
Sixties – the dawn of GR renaissance

Key experimental and theoretical breakthroughs
Quasars, Shapiro effect, gravitational redshift, CMB, black-hole physics, Penrose paper on singularity formation etc.

1962 Bondi, van der Burg, Metzner and Sachs
Asymptotic symmetry group of a strict asymptotically flat spacetime in four dimensions

\[
\text{Lorentz } \ltimes \text{Supertranslations } \rightarrow 6 + \infty \equiv \text{BMS}_4
\]

not

\[
\text{Poincaré } \equiv \text{Lorentz } \ltimes \text{Translations } \rightarrow 6 + 4
\]

Intriguing but in line with general covariance
Recognized as a valuable tool (Ashtekar, Komar, Penrose and many others)

- Classical: taming of solution space, computing the conserved charges and algebra for a gravitational field, etc.
- Quantum: scattering of particles escaping at null infinity should be encoded in a BMS$_4$-invariant $S$ matrix
  - new handle on the infrared issues (dressed states)
  - includes quantum gravity

Recent revival for the quest of BMS-invariant field theories [Harvard school among others]
2000s – the advent of anti de Sitter

Anti de Sitter
Maximally symmetric Einstein spacetime with negative curvature (cosmological constant)

AdS/CFT holographic correspondence
Type IIB string theory in the bulk and $N = 4$ super-Yang–Mills on the boundary – AdS$_5$ soon extended to arbitrary dimension

Diverse asymptotic symmetries
e.g. for strict AdS$_n$ asymptotics

\[
\text{conformal group in } n - 1 \text{ dimensions } SO(n - 1, 2) \rightarrow n(n+1)/2
\]

Less exotic but can also become infinite
Phenomenological branch of holography: fluid/gravity correspondence [Bhattacharyya, Hubeny, Minwalla, Rangamani ’07; Haack, Yarom ’08; etc.]

- Ignore the microscopic structure of the boundary theory – quantum many-body state in the hydrodynamic regime
- Keep only gravity in the bulk – possibly Einstein–Maxwell

Holography: the boundary relativistic hydrodynamics encodes the gravitational bulk dynamics

Central and timely questions

- How do the symmetries of the boundary fluid dynamics match the asymptotic bulk symmetries? What about the widely (un)known hydrodynamic-frame invariance?
- What happens when the bulk AdS asymptotics is flattened? How does BMS symmetry emerge on the boundary?

Subject of the talk
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Equations of motion \[ \nabla_\mu T^{\mu \nu} = 0 \] plus Gibbs–Duhem & equation of state (conformal)

- Energy density \( \varepsilon = \frac{1}{k^2} T_{\nu \mu} u^\mu u^\nu \) thermodynamic pressure \( p \)
- Heat current and viscous stress tensor \( q^\mu \), \( \tau^{\mu \nu} \) — transverse normally expressed as \( u^\mu \)- and \( T \)-derivative expansions with transport coefficients – constitutive relations ignored here
Fluid equations covariant – diffeomorphism invariance

Diffeomorphisms are generated by vector fields \((i, j = 1, \ldots, d)\)

\[
\xi = f \partial_t + Y^i \partial_i
\]

\(f(x)\) and \(Y^i(x)\) \(d + 1\) functions of time and space

\[
\delta \xi = -\mathcal{L}_\xi
\]

Fluid equations invariant under arbitrary rescaling of the metric (conformal fluids)

\[
\delta \omega g_{\mu\nu} = -2\omega g_{\mu\nu} \quad \delta \omega u^\mu = \omega u^\mu
\]

\(\omega(x)\) arbitrary function of time and space

\[
\delta \omega = w \omega
\]
Landau–Lifshitz’s statement for non-perfect fluids

First of all, however, we must discuss more closely the concept of the velocity \( u^\mu \) itself. In relativistic mechanics, an energy flux necessarily involves a mass flux. Hence, when there is (e.g.) a heat flux, the definition of the velocity in terms of the mass flux density (as in non-relativistic fluid dynamics) has no direct meaning.

In short: \( u^\mu \) is not physical – a book-keeping device

Translation in the formalism: another gauge invariance

Arbitrary transformations of \( u^\mu \) (local Lorentz transformation) can be compensated by appropriate modifications of \( T, \varepsilon, p, q^\mu, \tau^{\mu\nu} \) such that \( T^{\mu\nu} \) and the entropy current \( S^\mu \) remain invariant.

Note: These are not Lorentz isometries (generally absent) but tangent-space local transformations generated by \( Z^i \) (\( d \) boosts), \( S_{ij} \) antisymmetric (\( d(d-1)/2 \) rotations).
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Asymptotic symmetries in dimension $n$

**The gauge-fixing approach**

- Fix a gauge: $n$ conditions
- Determine the residual diffeomorphisms
- Set “reasonable” fall-offs/boundary conditions

The *asymptotic symmetry group* (ASG) is the subset of residual diffeomorphisms compatible with the adopted fall-offs
Einstein spacetimes with $\Lambda < 0$ in $n$ dimensions

[see Brussels school; Poole, Skenderis, Taylor ’19; Compère, Fiorucci, Ruzziconi ’19]

The Newman–Unti gauge in coordinates $r, u, x^i$

$(i = 1, \ldots, n - 2)$

- Gauge conditions: $G_{rr} = 0, G_{ru} = -1, G_{ri} = 0$

$$ds^2 = \frac{V}{r} du^2 - 2 du dr + G_{ij} (dx^i - U^i du) (dx^j - U^j du)$$

$V, G_{ij}, U^i$ functions of all coordinates

- Residual diffeomorphisms: $\omega(u, x), f(u, x), Y^i(u, x)$

- Several options for fall-offs/boundary conditions
  - mildest boundary conditions ($G_{ij} = O(r^2)$) $\rightarrow$ infinite-dim Weyl, supertranslations and superrotations
  - Dirichlet boundary conditions e.g. locked $G_{ij} \sim r^2 S^{n-2} \rightarrow \omega = 0, f$ restricted, $Y^i$ conformal $\rightarrow SO(n - 1, 2)$ finite conformal group at $n - 1$ dimensions
Incomplete Newman–Unti gauge fixing [Ciambelli, Marteau, Petropoulos, Ruzziconi ’20; Ciambelli, Marteau, Petkou, Petropoulos, Rivera, Ruzziconi, Siampos]

- **Gauge conditions:** \( G_{rr} = 0, \ G_{ru} = -1, \ G_{ri} \neq 0 \)
- **Residual diffeomorphisms:** \( \omega(u, x), f(u, x), Y^i(u, x) \) plus \( Z^i(u, x), S_{ij}(u, x) \) antisymmetric
- **Mild boundary conditions (\( G_{ij} = O(r^2) \)) → infinite-dim Weyl, supertranslations, superrotations & local \( SO(n - 2, 1) \)
Solution space with incomplete Newman–Unti gauge and mild boundary conditions — **The boundary and its fluid**

$n^2 - 3$ arbitrary functions of $u, x \rightarrow$ emergence of the boundary

$\mu, \nu = 0, 1, \ldots, n - 2$

- $g_{\mu\nu}$ symmetric $\leftarrow \frac{n(n-1)}{2}$ boundary metric

- $T_{\mu\nu}$ symmetric and traceless $\leftarrow \frac{n(n-1)}{2} - 1$
  energy–momentum tensor for a boundary conformal fluid

- normalized $u^{\mu} \leftarrow n - 2$
  arbitrary boundary fluid velocity

remaining bulk Einstein’s equations

$$\nabla_{\mu} T^{\mu\nu} = 0$$
Back to the symmetries

The bulk Einstein space in the incomplete Newman–Unti gauge is uniquely reconstructed from the boundary data.

The relativistic boundary fluid symmetries are rescalings, diffeomorphisms and local Lorentz transformations.

The boundary fluid symmetries precisely coincide with the infinite-dim bulk asymptotic symmetry group.

Weyl, supertranslations, superrotations plus local $SO(n - 2, 1)$

Locking a hydrodynamic frame – e.g. Landau–Lifshitz – discards the local $SO(n - 2, 1)$ transformations and completes the bulk gauge fixing – yes but …
Yes but …

- **A relevant boundary question**
  Is the choice of hydrodynamic frame innocuous?

- **Our perspective**
  A boundary hydrodynamic frame transformation is a bulk residual diffeomorphism within an incomplete gauge fixing

- **Recast in the bulk**
  Are different-fluid-velocity subspaces of the Einstein solution space equivalent?

- **Method**
  Determine the charge algebra for the solution subspaces

- **Answer**
  No – the algebras are different – typical avatar of global gauge issues [proven in $n = 3$ by Campoleoni, Ciambelli, Marteau, Petropoulos, Siampos; ’19]
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**A cartoon summary**

**AdS holography: remarkable relationship**

bulk cosmological constant $\Lambda \leftrightarrow$ boundary velocity of light $k$

**Flat holography:** $\Lambda = -\frac{(n-1)(n-2)}{2} k^2 \rightarrow 0$

Carrollian limit on the boundary (null infinity) [Lévy-Leblond '65]

- Solution space $\leftrightarrow$ conformal Carrollian boundary data
- Bulk ASG $\rightarrow$ Inönü–Wigner contraction: $\text{BMS}_n$
- Boundary symmetries: $\text{CCarrroll}_{n-1} \equiv \text{BMS}_n$

Asymptotically $\Lambda \rightarrow 0$

Asymptotically Ricci-flat $n$ bulk

Holographic reduction/reconstruction

Relativistic hydrodynamics in $n - 1$ dimensions

“Conformal” symmetry

Inönü–Wigner contraction

Carrollian hydrodynamics in $n - 1$ dimensions

“BMS” symmetry

$k \rightarrow 0$

$k = 0$
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Satisfactory picture of asymptotic symmetries and their fluid/gravity holographic realization

- Valid for asymptotically AdS or flat spacetimes
- Asymptotic symmetry group $\supset$ boundary hydrodynamic-frame transformations i.e. local $SO(n-2,1)$
- $\exists$ a BMS corner with its Carrollian fluids – in line with the “big picture:” the quest of BMS-invariant field theories

Better understanding of hydrodynamic-frame invariance

- Three bulk dimensions: $SO(1,1)$ has non-vanishing charge consequences for the fluid side: hydrodynamic-frame invariance is only local
- Higher dimensions: hydrodynamic-frame invariance is a handle for defining the fluid charges – gravity techniques