Effective Action for Scalar Leptoquarks

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Outline

- Introduction
- Functional Matching for Scalar Leptoquarks
- Summary

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Matching in EFTs

Effective Lagrangian structure:

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \sum_{\mathcal{D}} C_i^{\mathcal{D}} \frac{\mathcal{O}_i^{\mathcal{D}}}{\Lambda^{\mathcal{D}-4}}$$

The idea of matching

Given a UV Lagrangian $\mathcal{L}_{\text{UV}}[S,\, f]$, with a hierarchy in mass scales $M_S \gg m_f$,

$$C_i^{\mathcal{D}} = ?$$

so that EFT and UV give the same physical predictions.

There are two approaches:

- Amplitude matching, using Feynman diagrams
- Functional matching, using the method of Supertraces
  T. Cohen, X. Lu, Z. Zhang: 2011.02484

Feynman Diagrammatic Matching

Amplitude Matching

- Compute amplitudes in th UV

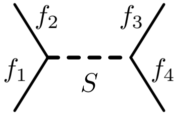
$$\mathcal{L}_{\text{UV}}[S,\, f] \quad \xrightarrow[p_{\text{ext}}=0]{p_i \ll M_S} \quad \mathcal{A}_{\text{UV}}(p_i)$$

- Choose a basis (Warsaw, Green etc.) and compute EFT amplitudes

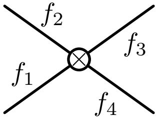
$$\mathcal{L}_{\text{EFT}}[f] \quad \longrightarrow \quad \mathcal{A}_{\text{EFT}}(p_i)$$

- Equate to get Wilson coefficients in a given dimension, $\{C_i^{\mathcal{D}}\}$

- UV tree amplitude



- EFT amplitude



Functional Matching

Equate generating functionals,

$$\Gamma_{\rm EFT}[f] = \Gamma_{\rm L,UV}[f]$$

- Tree level:

$$\mathcal{L}^{\rm (tree)}_{\rm EFT}[f] = \mathcal{L}_{\rm UV}[S,\, f]\big|_{S=S_c[f]} \;,$$

where S_c solves the classical EOMs: $\delta\mathcal{S}_{\rm UV}/\delta S|_{S=S_c[f]}$.

- One loop:

$$\int d^dx\, \mathcal{L}^{\rm (1\text{-}loop)}_{\rm EFT}[f] = \frac{i}{2}\log {\rm Sdet}\left(- \left.\frac{\delta^2 \mathcal{S}_{\rm UV}}{\delta\varphi_i \delta\varphi_j}\right|_{S=S_c}\right)\Bigg|_{\rm hard} \;,$$

where φ_i is a multiplet containing all independent fields and "hard" denotes regions of momenta where $q \sim M_S \gg m_f$

Master Formula

The functional derivative has the general form,

$$-\frac{\delta^2 \mathcal{S}_{\mathrm{UV}}}{\delta\varphi_i \delta\varphi_j} = \mathbf{K} - \mathbf{X} = \mathbf{K}\left(1 - \mathbf{K}^{-1}\mathbf{X}\right) \ .$$

- $\mathbf{K}^{-1}$ contains propagators, derived from the second derivative of the relevant kinetic terms
- $\mathbf{X}$ is the interaction matrix, derived by taking the second derivative of every other term in the Lagrangian

Plugging back in and Taylor expanding

Master One Loop Formula

$$\int d^d x \, \mathcal{L}^{(\text{1-loop})}_{\mathrm{EFT}}[f] = \frac{i}{2} \, \mathrm{STr} \log \mathbf{K}\big|_{\mathrm{hard}} - \frac{i}{2} \sum_{n=1}^{\infty} \frac{1}{n} \, \mathrm{STr}\left[(\mathbf{K}^{-1}\mathbf{X})^n\right]\Big|_{\mathrm{hard}}$$

Contents of Matching

- **Field Multiplet**.
  For example, a complex scalar and a Dirac fermion organize into

$$\varphi_S = \begin{pmatrix} S \\ S^* \end{pmatrix}\ , \quad \varphi_f = \begin{pmatrix} f \\ f^c \end{pmatrix} \qquad \longrightarrow \qquad \varphi = \begin{pmatrix} \varphi_S \\ \varphi_f \end{pmatrix}$$

- **Propagator** matrix $\mathbf{K}^{-1} \rightarrow \delta^2 \mathcal{L}_{\mathrm{UV}} \supset \frac{1}{2}\,\delta\bar{\varphi}\,\mathbf{K}\,\delta\varphi$

$$K_i = \begin{cases} P^2 - M_i^2 & \text{spin-0} \\ \not{P} - m_i & \text{spin-1/2} \end{cases}$$

  where $P_\mu = iD_\mu$ the hermitian covariant derivative.

- **Interaction Matrix X** $\rightarrow \delta^2 \mathcal{L}_{\mathrm{UV}} \supset -\frac{1}{2}\,\delta\bar{\varphi}\,\mathbf{X}\,\delta\varphi$

$$X_{ij} = U_{ij} + \left(P_\mu Z^\mu_{ij} + \bar{Z}^\mu_{ij} P_\mu + \ldots\right)\ .$$

  Indices i, j denote fields from the full field multiplet φ.
  e.g. X_{Sf}, X_{fS} refers to the interaction matrix of the scalar S with the fermion f.

Supertraces

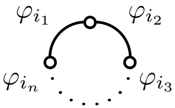
- **Log-type**
  Matrix $\mathbf{K}$ is diagonal, therefore,

$$\frac{i}{2}\ \mathrm{STr}\log\mathbf{K}\big|_{\mathrm{hard}} = \frac{i}{2}\sum_{i}\mathrm{STr}\log K_i$$

- **Power-type**
  The power expansion of the master formula becomes,

$$-\frac{i}{2}\,\frac{1}{n}\,\left[(\mathbf{K}^{-\mathbf{1}}\mathbf{X})^n\right] = -\frac{i}{2}\,\frac{1}{n}\sum_{i_1,\ldots i_n}\mathrm{STr}\left[\frac{1}{K_{i_1}}X_{i_1 i_2}\ldots\frac{1}{K_{i_n}}X_{i_n i_1}\right]\ ,$$

  and admits a diagrammatic representation of order n,



$$\equiv -\frac{i}{2}\,\frac{1}{r}\,\mathrm{STr}\left[\frac{1}{K_{i_1}}X_{i_1 i_2}\ldots\frac{1}{K_{i_n}}X_{i_n i_1}\right]\Bigg|_{\mathrm{hard}}$$

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Setup of the Lagrangian

There are 5 different LQ representations,

$$S_1 \sim (\bar{3}, 1, \frac{1}{3}) , \qquad \tilde{S}_1 \sim (\bar{3}, 1, \frac{4}{3}) , \qquad S_2 \sim (3, 2, \frac{7}{6}) ,$$
$$\tilde{S}_2 \sim (3, 2, \frac{1}{6}) , \qquad S_3 \sim (\bar{3}, 3, \frac{1}{3}) .$$

General interactions,

$$\begin{aligned}
\mathcal{L}_{\text{S-f}} &= \bar{F}^c \boldsymbol{\lambda}_i^L F S_i + \bar{f}^c \boldsymbol{\lambda}_i^R f S_i + \bar{f} \tilde{\boldsymbol{\lambda}}_i F S_i + \text{h.c.} , \\
\mathcal{L}_{\text{S-H}} &= (A_{ij} H^\dagger S_i S_j + \text{h.c.}) + \lambda_{Hi} (S_i^\dagger S_i)(H^\dagger H) + (\lambda_{3\text{S}} S_i S_j S_k H^\dagger + \text{h.c.}) + \ldots \\
\mathcal{L}_{\text{S}} &= -M_i^2 |S_i|^2 + A'_{ijk} (S_i^\dagger S_j S_k) + c_{ijkl} (S_i^\dagger S_j)(S_k S_l) + \ldots
\end{aligned}$$

where $F = \{q, \ell\}$, $f = \{u, d, e\}$ and $S_i = \{S_1, \tilde{S}_1, S_2, \tilde{S}_2, S_3\}$.

Tree Level Matching

- First expand $S_{i,c}$ into inverse powers of the heavy mass M_i

$$S_{i,c} = S_{i,c}^{(3)} + S_{i,c}^{(4)} + \dots$$

- Collect terms with operator dimension 3 and 4

$$(S_{i,c}^{(3)})^\dagger = \frac{1}{M_i^2}(\bar{F}^c \boldsymbol{\lambda}_i^L F S_i + \bar{f}^c \boldsymbol{\lambda}_i^R f S_i + \bar{f}\tilde{\boldsymbol{\lambda}}_i F S_i) \,,$$

$$(S_{i,c}^{(4)})^\dagger = \frac{1}{M_i^2} A_{ij} H^\dagger S_{j,c}^{(3)}$$

- Plug back in the Lagrangian to obtain operators up to dim-7

$$\boxed{\mathcal{L}_{\text{EFT}}^{(\text{tree})} = M_i^2 (S_{i,c}^{(3)})^\dagger (S_{i,c}^{(3)}) + (A_{ij} H^\dagger S_{i,c}^{(3)} S_{j,c}^{(3)} + \text{h.c.})}$$

One Loop Matching

The interaction matrix connects the field multiplets, through the second variation of the action,

$$-\frac{1}{2}\begin{pmatrix}\delta\bar{\varphi}_S & \delta\bar{\varphi}_L\end{pmatrix}\begin{bmatrix}\mathbf{X}_{SS} & \mathbf{X}_{SL}\\ \mathbf{X}_{LS} & \mathbf{X}_{LL}\end{bmatrix}\begin{pmatrix}\delta\varphi_S\\ \delta\varphi_L\end{pmatrix}$$

- $\mathbf{X}_{SS}$: LQ-only variations

$$\mathbf{X}_{SS} \rightarrow \quad \{U_{S_i S_j}\}$$

- $\mathbf{X}_{SL(LS)}$: LQ-Light fields (Light fields-LQ) variations

$$\mathbf{X}_{LS(SL)} \rightarrow \quad \{U_{S_i f(fS_i)},\, U_{S_i H(HS_i)},\, X_{S_i V(VS_i)}\}$$

- $\mathbf{X}_{LL}$: Light-Light fields variations

$$\mathbf{X}_{LL} \rightarrow \quad \{U_{ff},\, U_{HH},\, U_{VV},\, U_{Hf(fH)},\, U_{fV(Vf)},\, U_{VH(HV)}\}$$

Constructing Supertraces

The rules for Supertraces

- **Propagators**:

$$S_i = \frac{1}{P^2 - M_i^2} \qquad f = \frac{1}{\not{P}}$$

$$H = \frac{1}{P^2} \qquad V = \frac{-\eta_{\mu\nu}}{P^2}$$

- **Vertices**:

$$S_i \; S_j = U^{[1,2,3,4,6]}_{S_i S_j} \qquad f \; H = U^{[3/2]}_{fH}$$

$$S_i \; f = U^{[3/2]}_{S_i f} \qquad f \; V = U^{[3/2]}_{fV}$$

$$S_i \; H = U^{[3,4,6]}_{S_i H} \qquad H \; H' = U^{[2,6]}_{HH'}$$

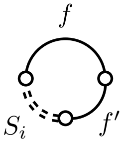
$$S_i \; V = X^{[3,4]}_{S_i V} \qquad H \; V = U^{[2]}_{HV}$$

$$f \; f' = U^{[1,3]}_{ff'} \qquad V \; V' = U^{[2,6]}_{VV'}$$

Supertraces Examples

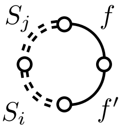
Let's pick some matrices and make up traces for heavy-light contributions

- $\mathcal{O}\left(U^{[3/2]}_{S_i f} U^{[1,3]}_{f f'} U^{[3/2]}_{f' S_i}\right) \longrightarrow$ Operators dimensions $[4,6]$:



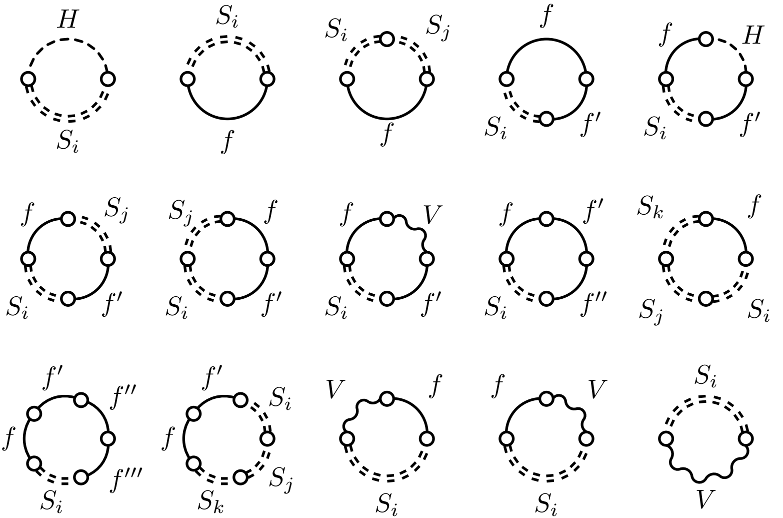
$$= -\frac{i}{2}\mathrm{STr}\left[\frac{1}{P^2 - M_i^2} U_{S_i f} \frac{1}{\not{P}} U_{f f'} \frac{1}{\not{P}} U_{f' S_i}\right]\Bigg|_{\text{hard}}$$

- $\mathcal{O}\left(U^{[1,2]}_{S_i S_j} U^{[3/2]}_{S_j f} U^{[1]}_{f f'} U^{[3/2]}_{f' S_i}\right) \longrightarrow$ Operator dimension $[5,6]$



$$= -\frac{i}{2}\mathrm{STr}\left[\frac{1}{P^2 - M_i^2} U_{S_i S_j} \frac{1}{P^2 - M_j^2} U_{S_j f} \frac{1}{\not{P}} U_{f f'} \frac{1}{\not{P}} U_{f' S_i}\right]\Bigg|_{\text{hard}}$$

Heavy-Light Supertrace Diagrams



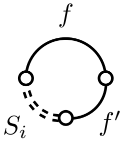
Calculating Supertraces

We use the MATHEMATICA package STrEAM to evaulate Supertraces + some manual intervention

T. Cohen, X. Lu, Z. Zhang: 2012.07851

J. Fuentes-Martin, M. Konig, J. Pages, A.E. Thomsen, F. Wilsch: 2012.08506

Example:



$\longrightarrow$

$$
\begin{aligned}
&-\frac{1}{2M_i^2}\left(\frac{1}{2}+\log\frac{\mu^2}{M_i^2}\right)\operatorname{tr}\left\{U_{S_i f}(P^2U_{ff'})U_{f'S_i}\right\} \\
&+\frac{1}{2M_i^2}\operatorname{tr}\left\{U_{S_i f}U_{ff'}(P^2U_{f'S_i})\right\}\ , \\
&+\frac{1}{2M_i^2}\operatorname{tr}\left\{U_{S_i f}(P_\mu U_{ff'})(P^\mu U_{f'S_i})\right\}\ , \\
&-\frac{1}{4M_i^2}\left(\frac{3}{2}+\log\frac{\mu^2}{M_i^2}\right)\operatorname{tr}\left\{U_{S_i f}U_{ff'}i\sigma_{\mu\nu}G'^{\mu\nu}_{f'}U_{f'S_i}\right\}\ , \\
&-\frac{1}{4M_i^2}\left(\frac{1}{2}+\log\frac{\mu^2}{M_i^2}\right)\operatorname{tr}\left\{U_{S_i f}i\sigma_{\mu\nu}G'^{\mu\nu}_{f}U_{ff'}U_{fS_i}\right\} \\
&+\frac{1}{2M_i^2}\operatorname{tr}\left\{U_{S_i f}i\sigma_{\mu\nu}(P^\mu U_{ff'})(P^\mu U_{f'S_i})\right\}\ .
\end{aligned}
$$

Fully Functional Result

- We need to include the **UOLEA** as well.
- UOLEA contains 19-unique terms.
- Captures contributions of only the heavy particle runs in the loop.

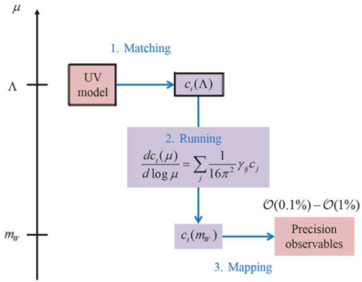
Complete Lagrangian:

$$\boxed{\mathcal{L}_{\text{EFT}}^{(\text{1-loop})} = \mathcal{L}_{\text{UOLEA}} + (\textit{Supertrace Results})}$$

Two applications of this formula:

- Two LQ model $S_1 + S_3$, full agreement with the Feynman rule approach.
  V. Gherardi, D. Marzocca, E. Venturini:2003.12525v4
- Two LQ model $S_1 + \tilde{S}_2$, full derivation of the spectrum of dim-6 operators in the Green basis.
- Second model, phenomenology rich, e.g. radiative neutrino masses, proton decay, $(g-2)_\mu$,...

The Whole Picture



B. Henning, X. Lu, H. Murayama: 1412.1837

Some preliminary results in the $S_1 + \tilde{S}_2$

- $(g-2)_\mu$:

$$\Delta a_\mu^{S_1+\tilde{S}_2} = -\frac{m_\mu m_t}{4\pi^2 M_1^2}\left[\log\frac{m_t^2}{M_1^2} + \frac{7}{4}\right]\mathrm{Re}(\hat{\lambda}^{1L*}_{t\mu}\hat{\lambda}^{1R}_{t\mu}) - \frac{m_\mu^2}{32\pi^2 M_1^2}(\hat{\lambda}^{1L*}_{t\mu}\hat{\lambda}^{1L}_{t\mu} + \hat{\lambda}^{1R*}_{t\mu}\hat{\lambda}^{1R}_{t\mu}) + \frac{m_\mu^2}{32\pi^2 \tilde{M}_2^2}\hat{\tilde{\lambda}}^*_{t\mu}\hat{\tilde{\lambda}}_{t\mu}$$

In agreement with
M. Bauer, M. Neubert: 1511.01900

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Robust Way of Functional Matching Procedure

Given a UV Lagrangian one can decouple any particle with any structure of interactions following the steps below:

- Identify the heavy degrees of freedom.
- Write down equations of motion for the heavy fields.
- Organize all fields in the a great multiplet and derive **K** and **X** matrices.
- Identify the dimensions of given **X**'s and draw the relevant Supertraces up to the desired mass dimension.
- Evaluate these graphs through automated packages.
- Finally, evaluate the resulting traces and extract operators.

Conclusions:

- Two distinct advantages of functional methods over traditional Feynman diagrammatic technique
  - In all steps of the calculation we work with explicitly gauge covariant quantities.
  - Possibility of obtaining universal results, with widely applicable master formulas
- Derived the full 1-loop Lagrangian for the decoupling of all leptoquarks
- Extracted the complete spectrum of up to dimension-6 operators for two LQ models, currently exploring the phenomenology of the model

Thank you for your attention!

Appendix

Full Results 1/4

$$\underline{\mathcal{O}\left(U^2\right)}$$

$$\left(1+\log\frac{\mu^2}{M_i^2}\right)\mathrm{tr}\left\{U_{S_iH}U_{HS_i}\right\}\ ,$$

$$\frac{1}{2}\left(\frac{1}{2}+\log\frac{\mu^2}{M_i^2}\right)\mathrm{tr}\left\{U_{S_if}\gamma_\mu(P^\mu U_{fS_i})\right\}\ ,$$

$$\frac{1}{12M_i^2}\,\mathrm{tr}\left\{U_{S_if}\gamma^\mu(P^2P_\mu U_{fS_i})+U_{S_if}\gamma^\mu(P_\mu P^2U_{fS_i})\right\}\ ,$$

$$-\frac{1}{18M_i^2}\,\mathrm{tr}\left\{U_{S_if}\gamma_\nu U_{fS_i}(P_\mu G'^{\mu\nu}_{S_i})\right\}\ ,$$

$$-\frac{1}{3M_i^2}\left(\frac{7}{12}+\log\frac{\mu^2}{M_i^2}\right)\mathrm{tr}\left\{U_{S_if}\gamma_\nu(P^\mu G'^{\mu\nu}_f)U_{fS_i}\right\}\ ,$$

$$\frac{1}{2M_i^2}\,\mathrm{tr}\left\{(P^\mu U_{S_if})\tilde{G}'_{\mu\nu}\gamma^\nu\gamma_5U_{fS_i}+U_{S_if}\tilde{G}'_{\mu\nu}\gamma^\nu\gamma_5(P^\mu U_{fS_i})\right\}\ .$$

Full Results 2/4

$$\underline{\mathcal{O}\left(U^3\right)}$$

$$\left(1+\log\frac{\mu^2}{M_i^2}\right)\ \mathrm{tr}\left\{U_{S_i f}U_{ff'}U_{f'S_i}\right\}+\frac{2\Delta_{ij}^2+(M_i^2+M_j^2)\log M_j^2/M_i^2}{4(\Delta_{ij}^2)^2}\ \mathrm{tr}\left\{(P^\mu U_{S_iS_j})U_{S_jf}\gamma_\mu U_{fS_i}\right\}\ ,$$

$$\frac{1}{4\Delta_{ij}^2}\log\frac{M_j^2}{M_i^2}\ \mathrm{tr}\left\{U_{S_iS_j}U_{S_jf}\gamma_\mu(P^\mu U_{fS_i})-U_{S_iS_j}(P^\mu U_{S_jf})\gamma_\mu U_{fS_i}\right\}\ ,$$

$$-\frac{1}{2M_i^2}\left(\frac{1}{2}+\log\frac{\mu^2}{M_i^2}\right)\ \mathrm{tr}\left\{U_{S_if}(P^2U_{ff'})U_{f'S_i}\right\}+\frac{1}{2M_i^2}\ \mathrm{tr}\left\{U_{S_if}U_{ff'}(P^2U_{f'S_i})\right\}\ ,$$

$$\frac{1}{2M_i^2}\ \mathrm{tr}\left\{U_{S_if}(P_\mu U_{ff'})(P^\mu U_{f'S_i})\right\}\ ,$$

$$\frac{1}{4M_i^2}\left(\frac{3}{2}+\log\frac{\mu^2}{M_i^2}\right)\ \mathrm{tr}\left\{U_{S_if}U_{ff'}i\sigma_{\mu\nu}G'^{\mu\nu}_{f'}U_{f'S_i}\right\}\ ,$$

$$\frac{1}{4M_i^2}\left(\frac{1}{2}+\log\frac{\mu^2}{M_i^2}\right)\ \mathrm{tr}\left\{U_{S_if}i\sigma_{\mu\nu}G'^{\mu\nu}_{f}U_{ff'}U_{fS_i}\right\}+\frac{1}{2M_i^2}\ \mathrm{tr}\left\{U_{S_if}i\sigma_{\mu\nu}(P^\mu U_{ff'})(P^\mu U_{f'S_i})\right\}\ .$$

Full Results 3/4

$\underline{\mathcal{O}\left(U^4\right)}$

$$
\begin{aligned}
&-\frac{1}{4M_i^2}\left(\frac{3}{2}+\log\frac{\mu^2}{M_i^2}\right)\,\mathrm{tr}\left\{U_{S_if}\gamma^\mu U_{fH}U_{Hf'}\gamma_\mu U_{f'S_i}\right\}\ ,\\
&-\frac{\log M_i^2/M_j^2}{4\Delta_{ij}^2}\,\mathrm{tr}\left\{U_{S_if}\gamma^\mu U_{fS_j}U_{S_jf'}\gamma_\mu U_{f'S_i}\right\}\ ,\\
&-\frac{\log M_i^2/M_j^2}{\Delta_{ij}^2}\,\mathrm{tr}\left\{U_{S_iS_j}U_{S_jf}U_{ff'}U_{f'S_i}\right\}\ ,\\
&-\frac{1}{4M_i^2}\left(\frac{3}{2}+\log\frac{\mu^2}{M_i^2}\right)\,\mathrm{tr}\left\{U_{S_if}\gamma^\mu U_{fV}U_{Vf'}\gamma_\mu U_{f'S_i}\right\}\ ,\\
&-\frac{1}{4M_i^2}\,\mathrm{tr}\left\{U_{S_if}U_{ff'}U_{f'f''}\gamma_\mu(P^\mu U_{f''S_i})-(P^\mu U_{fS_i})U_{ff'}U_{f'f''}\gamma_\mu U_{f''S_i}\right\}\ ,\\
&-\frac{1}{2M_i^2}\left(1+\log\frac{\mu^2}{M_i^2}\right)\,\mathrm{tr}\left\{U_{fS_i}U_{ff'}(P^\mu U_{f'f''})\gamma_\mu U_{f''S_i}-U_{fS_i}(P^\mu U_{ff'})U_{f'f''}\gamma_\mu U_{f''S_i}\right\}\ ,\\
&\mathcal{I}[q^2]^{1112}_{ijk0}\,\mathrm{tr}\left\{U_{S_iS_j}U_{S_jS_k}U_{S_kf}\gamma^\mu(P_\mu U_{fS_i})-U_{S_iS_j}U_{S_jS_k}(P^\mu U_{S_kf})\gamma_\mu U_{fS_i}\right\}\ ,\\
&-\mathcal{I}[q^2]^{1211}_{ijk0}\,\mathrm{tr}\left\{U_{S_iS_j}(P^\mu U_{S_jS_k})U_{S_kf}\gamma_\mu U_{fS_i}-(P^\mu U_{S_iS_j})U_{S_jS_k}U_{S_kf}\gamma_\mu U_{S_if}\right\}\ .
\end{aligned}
$$

Full Results 4/4

$$\underline{\mathcal{O}\left(U^5\right)}$$

$$\frac{1}{M_i^2}\left(1+\log\frac{\mu^2}{M_i^2}\right)\mathrm{tr}\left\{U_{S_i f}U_{f f'}U_{f' f''}U_{f'' f'''}U_{f''' S_i}\right\} \ ,$$

$$4\mathcal{I}[q^2]^{1112}_{ijk0}\,\mathrm{tr}\left\{U_{S_i S_j}U_{S_j S_k}U_{S_k f}U_{f f'}U_{f' S_i}\right\} \ .$$

$$\underline{\mathcal{O}\left(Z^1, Z^2\right)}$$

$$\frac{1}{4}\left(\frac{3}{2}+\log\frac{\mu^2}{M_i^2}\right)\mathrm{tr}\left\{Z^{\mu}_{S_i V}U_{V f}\gamma_\mu U_{f S_i}+U_{S_i f}\gamma_\mu U_{f V}\bar{Z}^{\mu}_{V S_i}\right\} \ ,$$

$$\frac{M_i^2}{4}\left(\frac{3}{2}+\log\frac{\mu^2}{M_i^2}\right)\mathrm{tr}\left\{Z^{\mu}_{S_i V}\bar{Z}_{V S_i,\mu}\right\} \ .$$