Primordial Black Holes And Gravitational Waves Based On No-Scale Supergravity

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This presentation is based on these works:

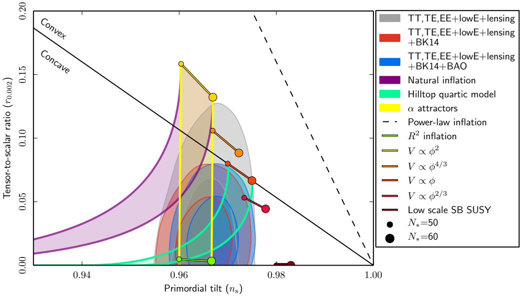
- Primordial Black Holes From No-Scale Supergravity (10.1103/PhysRevD.102.083536)
- Mechanisms of Producing Primordial Black Holes By Breaking The SU(2,1)/SU(2)×U(1) Symmetry (10.1103/PhysRevD.103.083512)
- Gravitational Waves From No-Scale Supergravity (in preparation)

Why do we study the production of PBHs and GWs ?

- The detection of Gravitational Waves (GWs) by a binary black hole merge opens a new window in physics of primordial black holes(PBHs).
- As a result there are numerous of recent studies which show that the origin of PBHs can explain a fraction of Dark Matter in the Universe.
- The signal of the GWs are expected to be detected by future space-based GW interferometers such as LISA, BBO and DECIGO.
- Both the generation of PBHs and GWs can be explained in the framework of inflation. It is proposed that an amplification in scalar power spectrum can explain both PBHs and GWs.
- Significant peaks in scalar power spectrum, which can interpreted by the production of PBHs & GWs can be produced by a near inflection point in effective scalar potential.

Constraints of the inflationary models

- The new theoretical models which have been proposed in the literature for explaining the generation of PBHs and GWs have to be in accordance with observable constraints on inflation released by Planck collaboration.



- Models based on Starobinsky-like potential give acceptable vallues for the spectral index n_s and tensor-to-scalar r.
- Models which leads to Starobinsky-like effective scalar potential can be found through no-scale supergravity theory.

Basic Aspects of SU(1,1) Symmetry

The general Lagrangian in effective field theory is:

$$\mathcal{L} = K_i^{\bar{j}} \partial_\mu \Phi^i \partial^\mu \bar{\Phi}_{\bar{j}} - V(\Phi, \bar{\Phi}). \tag{1}$$

The F-term of scalar potential is given as follows:

$$V = e^K \left(D_\Phi W K^{\bar{\Phi}\Phi} D_{\bar{\Phi}} \bar{W} - 3|W|^2 \right) \tag{2}$$

where the Kähler covariant derivative is:

$$D_\Phi W = \frac{\partial W}{\partial \Phi} + \frac{\partial K}{\partial \Phi} W$$

The cosmological constant vanishes due to the identity:

$$K^{\Phi\bar{\Phi}} K_\Phi K_{\bar{\Phi}} = 3. \tag{3}$$

A flat potential can be found by the following form of Kähler potential:

$$K = -3 \ln(\Phi + \bar{\Phi}) \tag{4}$$

and the corresponding kinetic term of the Lagrangian is:

$$\mathcal{L}_{kin} = \frac{3}{(\Phi + \bar{\Phi})} \partial^\mu \Phi \partial_\mu \bar{\Phi}.$$

Basic Aspects of SU(1,1) Symmetry

We consider $\Phi = (y+1)/(y-1)$:

$$K = -3\ln(1 - \frac{|y|^2}{3}) \tag{5}$$

and the corresponding kinetic term of the Lagrangian:

$$\mathcal{L}_{kin} = \frac{3}{(1-|y|^2)^2}\partial^\mu y \partial_\mu \bar{y}$$

which is invariant under the transformation of:

$$y \to \frac{\alpha y + \beta}{\bar{\beta} y + \bar{\alpha}}, \quad |\alpha|^2 - |\beta|^2 = 1. \tag{6}$$

This definies the non-compact group SU(1,1).
In order to derive Starobinsky-like effective scalar potential, we consider an extension of this group: SU(2,1)/SU(2)$\times$U(1) and we have two chiral field the inflaton and the modulo.

The SU(2,1)/SU(2)×U(1) Symmetry

Two equivalent form of SU(2,1)/SU(2)×U(1) Symmetry:

$$K = -3\ln(1 - \frac{|y_1|^2}{3} - \frac{|y_2|^2}{3}) \quad or \quad K = -3\ln(T + \bar{T} - \frac{|\varphi|^2}{3}) \tag{7}$$

The complex fields (y_1, y_2) are related to (T, φ) by the following expressions:

$$y_1 = \left(\frac{2\varphi}{1+2T}\right), \quad y_2 = \sqrt{3}\left(\frac{1-2T}{1+2T}\right)$$

and the inverse relations by:

$$T = \frac{1}{2}\left(\frac{1-y_2/\sqrt{3}}{1+y_2/\sqrt{3}}\right), \quad \varphi = \left(\frac{y_1}{1+y_2/\sqrt{3}}\right).$$

The superpotential transforms as:

$$W(T,\varphi) \to \bar{W}(y_1, y_2) = (1 + y_2/\sqrt{3})^3 W.$$

Two equivalent form for Superpotential which lead to Starobinsky-like effective scalar potential:

$$W_{WZ} = \left(\frac{\hat{\mu}}{2}\left(y_1^2 + \frac{y_1^2 y_2}{\sqrt{3}}\right) - \lambda\frac{y_1^3}{3}\right) \iff W'_{WZ} = \frac{\hat{\mu}}{2}\varphi^2 - \frac{\lambda}{3}\varphi^3 \text{ "Wess-Zumino"}$$

$$W_C = m\left(-y_1 y_2 + \frac{y_2 y_1^2}{l\sqrt{3}}\right) \iff W'_C = \sqrt{3}m\varphi\left(T - \frac{1}{2}\right) \text{ "Cecotti"}$$

The SU(2,1)/SU(2)$\times$U(1) coset space is parametrized by the matrix U:

$$U = \begin{bmatrix} \alpha & \beta & 0 \\ -\beta^* & \alpha^* & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8}$$

The superpotentials in (y_1, y_2) basis keep the transformations laws:

$$y_1 \to \alpha y_1 + \beta y_2, \quad y_2 \to -\beta^* y_1 + \alpha^* y_2. \tag{9}$$

where $\alpha, \beta \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 = 1$.
Refs.(JHEP 03 (2019) 099))

Modifying Superpotential

An inflection point in effective scalar potential can be achieved by a modification in superpotential such as:

$$W_1 = \left(\frac{\hat{\mu}}{2}\left(y_1^2 + \frac{y_1^2 y_2}{\sqrt{3}}\right) - \lambda\frac{y_1^3}{3}\right)(1 + g_1(y_1)) \ \&$$
$$W_2 = m\Big(-y_1 y_2 + \frac{y_2 y_1^2}{l\sqrt{3}}\Big)(1 + g_2(y_1)).$$

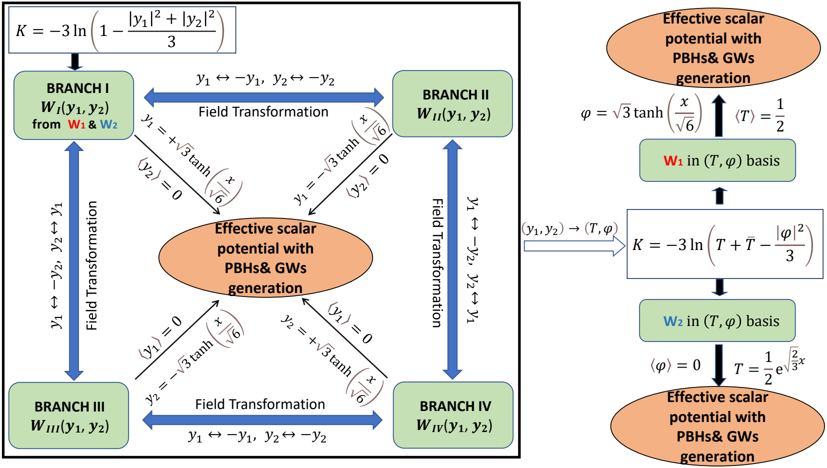
Aim: Find the proper function of g_1, g_2, in order to

- Derive the same effective scalar potential in both (y_1, y_2) and (T, φ) basis.
- Conserve the transformation laws for the SU(2,1)/SU(2)×U(1).
- Have an inflection point in the effective scalar potential.
- Explain the production of PBHs & GWs.

Superpotentials

$$W_1 = \left(\frac{\hat{\mu}}{2}\left(y_1^2 + \frac{y_1^2 y_2}{\sqrt{3}}\right) - \lambda\frac{y_1^3}{3}\right)\left(1 + e^{-b_1 y_1^2}(c_1 y_1^2 + c_2 y_1^4)\right)\&$$

$$W_2 = m\left(-y_1 y_2 + \frac{y_2 y_1^2}{l\sqrt{3}}\right)\left(1 + c_3 e^{-b_2 y_1^2} y_1^2\right)$$



Modifying the Kinetic Term

It is possible to achieve enhancement in scalar power spectrum by modifying the Kähler potential.

Two More Schemes for PBHs and GWs:

$$K_1 = -3\ln(T + \bar{T} - \frac{\varphi\bar{\varphi}}{3} + ce^{-b_3(\varphi+\bar{\varphi})^2}(\varphi + \bar{\varphi})^4) \quad (10)$$

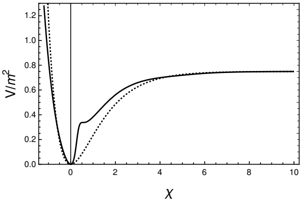
$$W_{WZ} = \frac{\hat{\mu}}{2}\varphi^2 - \frac{\lambda}{3}\varphi^3$$ "Wess-Zumino superpotential"

$$K_2 = -3\ln(T + \bar{T} - \frac{\varphi\bar{\varphi}}{3} + F(T + \bar{T}, \varphi + \bar{\varphi})) \quad (11)$$

$$W_C = \sqrt{3}m\varphi\left(T - \frac{1}{2}\right)$$ "Cecotti superpotential"

Proposed models with an inflection point

- We have four district schemes in order to have an inflection point in effective scalar potential, which produces PBHs & GWs:
  - Two by modifying superpotential and conserve the transformation laws of SU(2,1)/SU(2)×U(1) symmetry.
  - Two by modifying Kähler potential and break SU(2,1)/SU(2)× U(1) symmetry.



- All models are in complete consistence with Planck constraints.

Scalar Power Spectrum

The equation of motion of the inflaton:

$$\chi'' + 3\chi' - \frac{1}{2}\chi'^3 + \left(3 - \frac{1}{2}\chi'^2\right)\frac{d\ln V(\chi)}{d\chi} = 0. \quad (12)$$

Considering the perturbation of the field is given as $\chi + \delta\chi$:

$$\delta\chi'' = -\left(3 - \frac{1}{2}\chi'^2\right)\delta\chi' - \frac{1}{H^2}\frac{d^2V}{d\chi^2}\delta\chi - \frac{k^2}{a^2H^2}\delta\chi + 4\Psi'\chi' - \frac{2\Psi}{H^2}\frac{dV}{d\chi}. \quad (13)$$

The Bardeen potential Ψ is considered by the equation:

$$\Psi'' = -\left(7 - \frac{1}{2}\chi'^2\right)\Psi' - \left(2\frac{V}{H^2} + \frac{k^2}{a^2H^2}\right)\Psi - \frac{1}{H^2}\frac{dV}{d\chi}\delta\chi \quad (14)$$

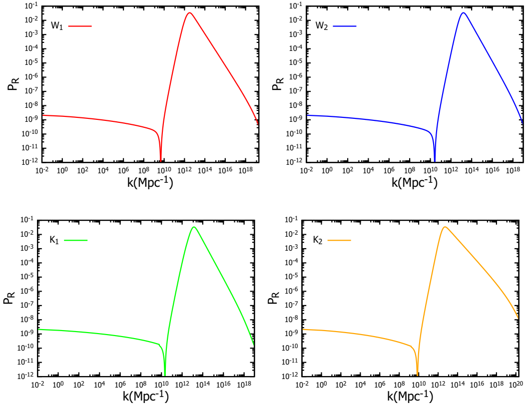
where with primes we denote the derivative in efold time, k is the comoving wavenumber and H is the Hubble parameter.
The power spectrum :

$$P_R = \frac{k^3}{2\pi^2}\left|R_k\right|^2, \quad (15)$$

where R_k is the comoving curvature perturbation:

$$R_k = \Psi + \frac{\delta\phi}{\phi'}.$$

The power spectra for the cases of W_1 & W_2 (modyfying superpotential), K_1 & K_2 (modyfying Kähler potential):



We notice that the scalar power spectum has a significant enhancement due to inflection point.

Evaluating the production of PBHs

The present abundance of PBH is given by the integral:

$$f_{PBH} = \int d\ln M \frac{\Omega_{PBH}}{\Omega_{DM}}$$

$$\frac{\Omega_{PBH}}{\Omega_{DM}} = \frac{\beta(M_{PBH}(k))}{8\times 10^{-16}} \left(\frac{\gamma}{0.2}\right)^{3/2} \left(\frac{g}{106.75}\right)^{-1/4} \left(\frac{M_{PBH}(k)}{10^{-18} g}\right)^{-1/2} . \qquad (16)$$

The mass is given as a function of k mode:

$$M_{PBH}(k) = 10^{18} \left(\frac{\gamma}{0.2}\right) \left(\frac{g}{106.75}\right)^{-1/6} \left(\frac{k}{7\times 10^{13} Mpc^{-1}}\right)^{-2} g. \qquad (17)$$

The mass fraction β_{PS} is given by:

$$\beta_{PS}(M_{PBH}) = \frac{1}{\sqrt{2\pi\sigma^2(M)}} \int_{\delta_c}^{\infty} d\delta \; e^{\frac{-\delta^2}{2\sigma^2(M)}} = \frac{\Gamma\left(\frac{1}{2}, \frac{\delta_c^2}{2\sigma^2}\right)}{2\sqrt{\pi}}$$

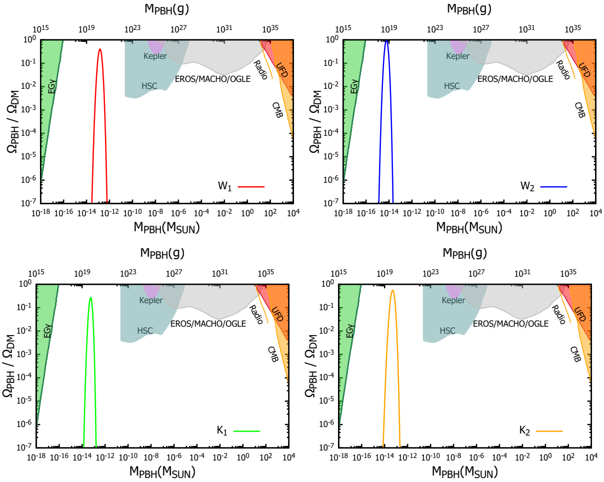
where the variance of curvature perturbation σ is related to the power spectrum:

$$\sigma^2(M_{PBH}(k)) = \frac{4(1+\omega)^2}{(5+3\omega)^2} \int \frac{dk'}{k'} \left(\frac{k'}{k}\right)^4 P_R(k') \tilde{W}^2 \left(\frac{k'}{k}\right)$$

where ω is the equation of state (in radiation dominated epoch is $\omega = 1/3$) and $\tilde{W}$ is a window function. We will use the Gaussian distribution for this function.

Results

The fractional abundance of PBH for the cases of W_1 & W_2 and K_1 & K_2.



Evaluating tensor perturbation from scalar power spectrum

The energy density of the GWs in terms of scalar power spectrum is given:

$$\Omega_{GW}(k) = \frac{\Omega_r}{36}\int_0^{\frac{1}{\sqrt{3}}} \mathrm{d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} \mathrm{d}s \left[\frac{(s^2-1/3)(d^2-1/3)}{s^2+d^2}\right]^2 \times P_R(kx)P_R(ky)(I_c^2+I_s^2) \tag{18}$$

where the radiation density $\Omega_r \approx 8.6 \times 10^5$.
The variables x and y are:

$$x = \frac{\sqrt{3}}{2}(s+d), \quad y = \frac{\sqrt{3}}{2}(s-d). \tag{19}$$

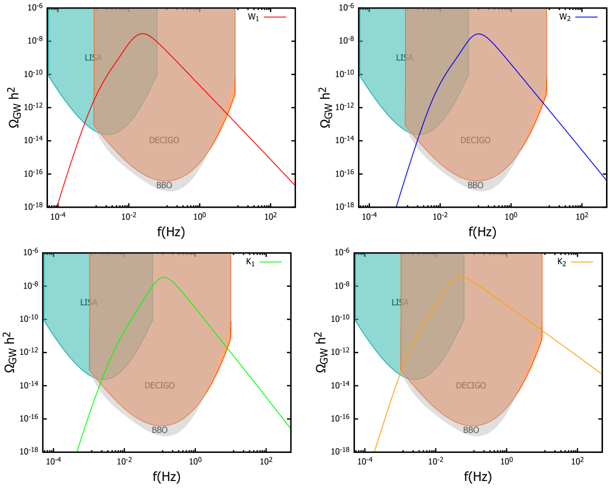
Finally, the functions I_c and I_s are given:

$$I_c = -36\pi\frac{(s^2+d^2-2)^2}{(s^2-d^2)^3}\theta(s-1) \tag{20}$$

$$I_s = -36\frac{(s^2+d^2-2)^2}{(s^2-d^2)^2}\left[\frac{(s^2+d^2-2)}{(s^2-d^2)}log\left|\frac{d^2-1}{s^2-1}\right|+2\right] \tag{21}$$

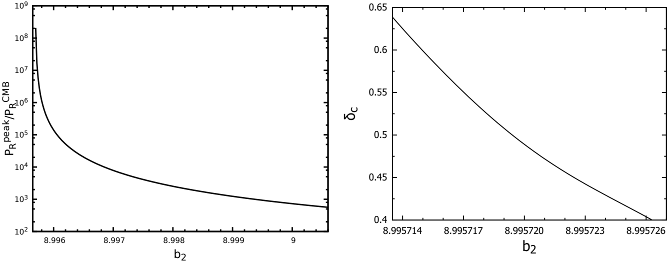
Results

The energy density of *GWs* for the cases of W_1 & W_2 (modyfying superpotential) and for the case of K_1 & K_2 (modyfying Kähler potential):



Fine-tuning in case of study the PBHs production

The parameter b_i demands fine-tuning in order to achieve the proper enhancement in power spectrum.



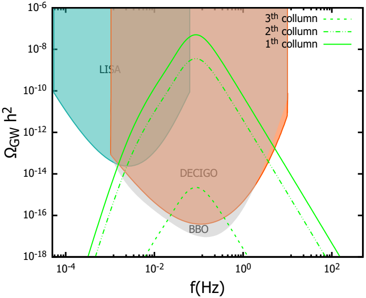
We evaluate the parameter Δ_b, which is the maximum value of the following logarithmic derivative:

$$\Delta_b = \mathrm{Max}\left|\frac{\partial ln(P_R^{PEAK})}{\partial ln(b_i)}\right| \tag{22}$$

In case of study PBHs we find $\Delta_b \approx 10^6$.

Fine-tuning in case of study the GWs production

The amount of fine-tuning in case of study GWs is decreased as a result that the value of the power spectrum's peak should not be at least $\mathcal{O}(10^{-2})$.



case	$(\Delta_b)_{PBHs}$	$(\Delta_b)_{GWs}$	$(\Delta_b)_{GWs}$
W_1	$7.9\ \times 10^5$	7.8×10^4	$3.5\ \times 10^3$
W_2	$9.8\ \times 10^5$	1.7×10^5	$3.6\ \times 10^3$
K_1	$1.6\ \times 10^5$	8.8×10^3	8.5×10^2
K_2	$4.2\ \times 10^6$	5.8×10^5	$5.6\ \times 10^4$

Conclusions-Perspectives

Conclusions:

- We provided a class of scalar potentials in order to generate PBHs & GWs which is derived by no-scale supergravity.
- We evaluated the production of PBHs in order to explain the dark matter in the Universe.
- We evaluated the abundances of GWs by using the scalar power spectra.
- We discussed the issue of fine-tuning of the parameters.

Perspectives:

The apparent drawback of such models is that fine-tuning is required, in order to achieve the desirable peaks in the power spectrum. One can move to the other theoretical approaches, which seem more natural and perform again the study in PBHs & GWs. We refer some of these theoretical approaches:

- A sharp feature in the effective scalar potential.
- Two fields models.

Thank you!