Quantum fluctuations of baryon number density

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Motivation:

The Joint Institute for Computational Fundamental Science

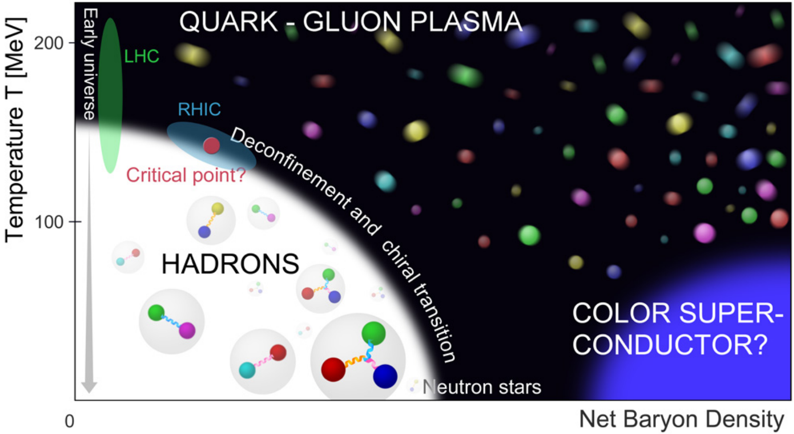

Figure: Theoretical diagram based on Lattice QCD simulations depicting expected quark-gluon phase transition.

Motivation:

Fluctuations of various physical quantities play a very important role in all fields of physics, as they reveal the information about

- possible phase transitions

- formation of structures in the Early Universe

- dissipative phenomena

Most common fluctuations we deal with are those arising from quantum uncertainty relation or those present in thermodynamic systems.

Motivation:

We discuss fluctuations of the baryon number density in a hot and dense relativistic gas of fermions.

Our analysis is relevant for relativistic heavy-ion physics, in particular, in the context of the beam energy scan (BES).

Hunt for the conjectured critical endpoint in the QCD phase diagram has triggered vast theoretical and experimental studies of many fluctuation observables.

Study of fluctuations of baryon number might provide an excellent opportunity to study the critical phenomena.

Basic concepts and definitions:

We consider the fluctuation of the baryon number in the subsystem S_a of the thermodynamic system S_V described by the grand canonical ensemble characterized by the temperature (T) and the baryon chemical potential ( ).

The volume V of the larger system S_V is larger than the characteristic volume of the subsystem S_a.

We derive a compact formula that defines quantum fluctuations of the baryon number operator in subsystems of a hot and dense relativistic gas.

Then we apply this formula to get physical insights into situations expected in relativistic heavy-ion collisions.

Basic concepts and definitions:

A quantum field operator for spin-$\frac{1}{2}$ particle has the standard form:

$$\psi(t,x) = \sum_r \int \frac{d^3k}{(2\pi)^3 \sqrt{2\omega_k}} \left(U_r(k) a_r(k) e^{-ik\cdot x} + V_r(k) b_r^\dagger(k) e^{ik\cdot x} \right),$$

where $a_r(k)$ and $b_r^\dagger(k)$ are annihilation and creation operators for particles and antiparticles, respectively, satisfying the canonical commutation relations $\{a_r(k), a_s^\dagger(k')\} = (2\pi)^3 \delta_{rs} \delta^{(3)}(k - k')$ and $\{b_r(k), b_s^\dagger(k')\} = (2\pi)^3 \delta_{rs} \delta^{(3)}(k - k')$, whereas $\omega_k = \sqrt{k^2 + m^2}$ is the energy of a particle.

To perform thermal averaging, it is sufficient to know the expectation values of the products of two and four creation and/or annihilation operators

$$\langle a_r^\dagger(k) a_s(k') \rangle = (2\pi)^3 \delta_{rs} \delta^{(3)}(k - k') f(\omega_k);$$

$$\langle a_r^\dagger(k) a_s^\dagger(k') a_{r'}(p) a_{s'}(p') \rangle = (2\pi)^6 \Big(\delta_{rs'} \delta_{r's} \delta^{(3)}(k - p') \delta^{(3)}(k' - p) - \delta_{rr'} \delta_{ss'} \delta^{(3)}(k - p) \delta^{(3)}(k' - p') \Big) f(\omega_k) f(\omega_{k'}).$$

Here $f(\omega_k) = 1/(\exp(\beta(\omega_k - \mu)) + 1)$ is the Fermi–Dirac distribution function for particles. For antiparticles, the Fermi–Dirac distribution function differs by the sign of the baryon chemical potential μ, i.e. $\bar{f}(\omega_k) = 1/(\exp(\beta(\omega_k + \mu)) + 1)$.

Basic concepts and definitions:

We define the baryon number density operator $\hat{J}^0_a$, associated with the conserved baryon current in a subsystem S_a using a smooth Gaussian profile placed at the origin of the coordinate system

$$\hat{J}^0_a = \frac{1}{(a\sqrt{\pi})^3} \int d^3x \; \hat{J}^0(x) \; \exp\left(-\frac{x^2}{a^2}\right)$$

where $\hat{J}^0 = \psi^\dagger \psi$.

To determine the baryon number fluctuation of the subsystem S_a, we consider the variance

$$\sigma^2(a; m; T; \mu) = \langle : \hat{J}^0_a :: \hat{J}^0_a : \rangle - \langle : \hat{J}^0_a : \rangle^2$$

and the normalized standard deviation as

$$\sigma_n(a; m; T; \mu) = \frac{\left(\langle : \hat{J}^0_a :: \hat{J}^0_a : \rangle - \langle : \hat{J}^0_a : \rangle^2\right)^{1/2}}{\langle : \hat{J}^0_a : \rangle}$$

Using the thermal averaging of two creation and/or annihilation operators, the thermal expectation value of $:\hat{J}^0_a:$ has the form

$$\langle :\hat{J}^0_a: \rangle = 2 \int dK \left[f(\omega_k) - \bar{f}(\omega_k) \right]$$

This expression agrees with the standard kinetic-theory definition, with the factor of 2 accounting for the spin degeneracy.

$$
\begin{aligned}
\sigma^2(a; m; T; \mu) &= \langle :\hat{J}^0_a::\hat{J}^0_a: \rangle - \langle :\hat{J}^0_a: \rangle^2 \\
&= \int \frac{dK}{\omega_k}\frac{dK'}{\omega_{k'}}(\omega_k \omega_{k'} + \mathbf{k}\cdot\mathbf{k}' + m^2) e^{-\frac{a^2}{2}(\mathbf{k}-\mathbf{k}')^2} \\
&\qquad f(\omega_k)(1 - f(\omega_{k'})) + \bar{f}(\omega_k)(1 - \bar{f}(\omega_{k'})) \\
&\quad - \int \frac{dK}{\omega_k}\frac{dK'}{\omega_{k'}}(\omega_k \omega_{k'} + \mathbf{k}\cdot\mathbf{k}' - m^2) e^{-\frac{a^2}{2}(\mathbf{k}+\mathbf{k}')^2} \\
&\qquad f(\omega_k)(1 - \bar{f}(\omega_{k'})) + \bar{f}(\omega_k)(1 - f(\omega_{k'}))
\end{aligned}
$$

Variation of normalized baryon density fluctuation:

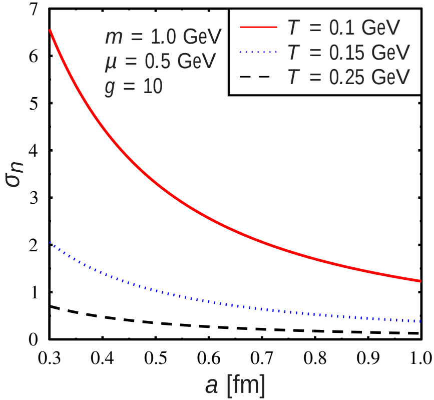

Figure: Variation of normalized fluctuation $_n$ in the subsystem S_a with the scale a for different values of the temperature T and fixed particle mass $m = 1{:}0$ GeV and baryon chemical potential $= 0{:}5$ GeV.

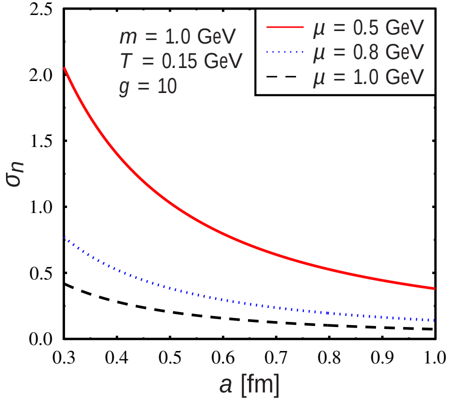

Figure: Variation of normalized fluctuation $\ _n$ in the subsystem S_a with the scale a for different values of the baryon chemical potential   and fixed particle mass $m = 1{:}0$ GeV and temperature $T = 0{:}15$ GeV.

Variation of normalized baryon density fluctuation:

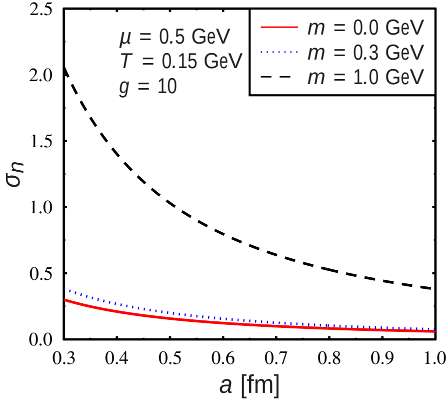

Figure: Variation of normalized fluctuation $_n$ in the subsystem S_a with the scale a for different values of the particle mass and fixed temperature $T = 0{:}15$ GeV and baryon chemical potential $= 0{:}5$ GeV.

Variation of normalized baryon fluctuation:

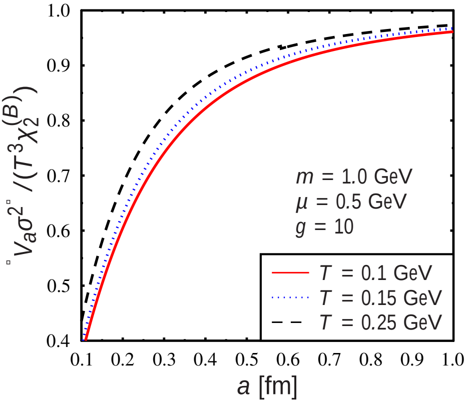

Figure: Variation of normalized fluctuation $V_a\ ^2=(T^3\ _2^{(B)})$ for different values of temperature (T) but with fixed baryon chemical potential ( ) and particle mass (m).

Variation of normalized baryon fluctuation:

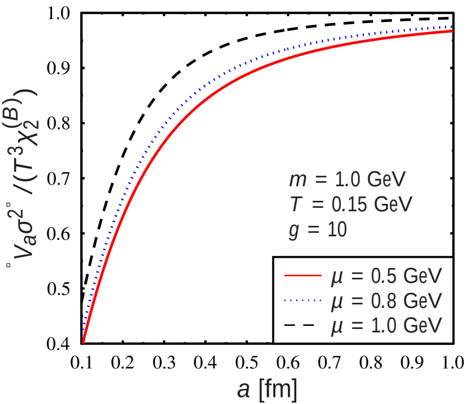

Figure: Variation of normalized fluctuation V_a ^{2}=(T^3 $_2^{(B)}$) for different values of baryon chemical potential ( ) but with fixed temperature (T) and particle mass (m).

Variation of normalized baryon fluctuation:

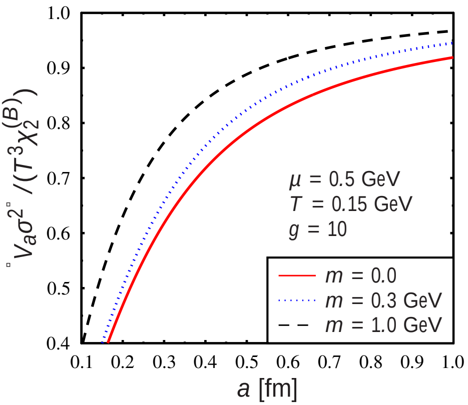

Figure: Variation of normalized fluctuation $V_a\ {}^2 = (T^3\ {}_2^{(B)})$ for different values of particle mass (m) but with fixed temperature (T) and baryon chemical potential ( ).

Summary:

We have analyzed quantum baryon-number fluctuations in subsystems of a hot and dense relativistic gas of fermions

And found that they diverge for small system sizes.

Our results agree with the results known from statistical physics for sufficiently large system size a.

In this way, we have delivered a useful formula that accounts for both statistical and quantum features of the fluctuations.

The numerical results obtained here can be useful to interpret and shed new light on the heavy-ion experimental data.