Quantum fluctuations of baryon number density

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Motivation:

The Joint Institute for Computational Fundamental Science

Figure: Theoretical diagram based on Lattice QCD simulations depicting expected quark-gluon phase transition.
Motivation:

Fluctuations of various physical quantities play a very important role in all fields of physics, as they reveal the information about

• possible phase transitions

• formation of structures in the Early Universe

• dissipative phenomena

Most common fluctuations we deal with are those arising from quantum uncertainty relation or those present in thermodynamic systems.
Motivation:

- We discuss fluctuations of the baryon number density in a hot and dense relativistic gas of fermions.

- Our analysis is relevant for relativistic heavy-ion physics, in particular, in the context of the beam energy scan (BES).

- Hunt for the conjectured critical endpoint in the QCD phase diagram has triggered vast theoretical and experimental studies of many fluctuation observables.

- Study of fluctuations of baryon number might provide an excellent opportunity to study the critical phenomena.
Basic concepts and definitions:

- We consider the fluctuation of the baryon number in the subsystem $S_a$ of the thermodynamic system $S_V$ described by the grand canonical ensemble characterized by the temperature ($T$) and the baryon chemical potential ($\mu$).

- The volume $V$ of the larger system $S_V$ is larger than the characteristic volume of the subsystem $S_a$.

- We derive a compact formula that defines quantum fluctuations of the baryon number operator in subsystems of a hot and dense relativistic gas.

- Then we apply this formula to get physical insights into situations expected in relativistic heavy-ion collisions.
A quantum field operator for spin-$\frac{1}{2}$ particle has the standard form:

$$\psi(t, x) = \sum_r \int \frac{d^3 k}{(2\pi)^3 \sqrt{2\omega_k}} \left( U_r(k) a_r(k) e^{-ik\cdot x} + V_r(k) b_r^\dagger(k) e^{ik\cdot x} \right),$$

where $a_r(k)$ and $b_r^\dagger(k)$ are annihilation and creation operators for particles and antiparticles, respectively, satisfying the canonical commutation relations

$$\{ a_r(k), a_s^\dagger(k') \} = (2\pi)^3 \delta_{rs} \delta(3)(k - k')$$

and

$$\{ b_r(k), b_s^\dagger(k') \} = (2\pi)^3 \delta_{rs} \delta(3)(k - k'),$$

whereas $\omega_k = \sqrt{k^2 + m^2}$ is the energy of a particle.
Basic concepts and definitions:

To perform thermal averaging, it is sufficient to know the expectation values of the products of two and four creation and/or annihilation operators

\[ \langle a_{r}^{\dagger}(k) a_{s}(k') \rangle = (2\pi)^3 \delta_{rs} \delta^{(3)}(k - k') f(\omega_{k}), \]
\[ \langle a_{r}^{\dagger}(k) a_{s}^{\dagger}(k') a_{r'}(p) a_{s'}(p') \rangle = (2\pi)^6 \left( \delta_{rs'} \delta_{r's} \delta^{(3)}(k - p') \delta^{(3)}(k' - p) \right. \]
\[ \left. - \delta_{rr'} \delta_{ss'} \delta^{(3)}(k - p) \delta^{(3)}(k' - p') \right) f(\omega_{k}) f(\omega_{k'}). \]

Here \( f(\omega_{k}) = 1/(\exp(\beta(\omega_{k} - \mu)) + 1) \) is the Fermi–Dirac distribution function for particles. For antiparticles, the Fermi–Dirac distribution function differs by the sign of the baryon chemical potential \( \mu \), i.e.

\( \bar{f}(\omega_{k}) = 1/(\exp(\beta(\omega_{k} + \mu)) + 1). \)
Basic concepts and definitions:

We define the baryon number density operator $\hat{J}_a^0$, associated with the conserved baryon current in a subsystem $S_a$ using a smooth Gaussian profile placed at the origin of the coordinate system

$$\hat{J}_a^0 = \frac{1}{(a\sqrt{\pi})^3} \int d^3x \, \hat{J}_a^0(x) \exp \left(-\frac{x^2}{a^2}\right)$$

where $\hat{J}_0 = \psi^\dagger \psi$.

To determine the baryon number fluctuation of the subsystem $S_a$, we consider the variance

$$\sigma^2(a, m, T, \mu) = \langle : \hat{J}_a^0 :: \hat{J}_a^0 : \rangle - \langle : \hat{J}_a^0 : \rangle^2$$

and the normalized standard deviation as

$$\sigma_n(a, m, T, \mu) = \left(\frac{\langle : \hat{J}_a^0 :: \hat{J}_a^0 : \rangle - \langle : \hat{J}_a^0 : \rangle^2}{\langle : \hat{J}_a^0 : \rangle}\right)^{1/2}$$
Mean value for baryon number density operator:

Using the thermal averaging of two creation and/or annihilation operators, the thermal expectation value of $\hat{J}_a^0$ has the form

$$\langle : \hat{J}_a^0 : \rangle = 2 \int dK \left[ f(\omega_k) - \bar{f}(\omega_k) \right]$$

This expression agrees with the standard kinetic-theory definition, with the factor of 2 accounting for the spin degeneracy.
Quantum fluctuation expression:

\[
\sigma^2(a, m, T, \mu) = \langle : \hat{J}_a^0 :: \hat{J}_a^0 : \rangle - \langle : \hat{J}_a^0 : \rangle^2
\]

\[
= \int \frac{dK}{\omega_k} \frac{dK'}{\omega_{k'}} (\omega_k \omega_{k'} + k \cdot k' + m^2) e^{-\frac{a^2}{2}(k-k')^2} \times
\]

\[
\left[ f(\omega_k)(1 - f(\omega_{k'})) + \bar{f}(\omega_k)(1 - \bar{f}(\omega_{k'})) \right]
\]

\[
- \int \frac{dK}{\omega_k} \frac{dK'}{\omega_{k'}} (\omega_k \omega_{k'} + k \cdot k' - m^2) e^{-\frac{a^2}{2}(k+k')^2} \times
\]

\[
\left[ f(\omega_k)(1 - \bar{f}(\omega_{k'})) + \bar{f}(\omega_k)(1 - f(\omega_{k'})) \right]
\]
Variation of normalized baryon density fluctuation:

Figure: Variation of normalized fluctuation $\sigma_n$ in the subsystem $S_a$ with the scale $a$ for different values of the temperature $T$ and fixed particle mass $m = 1.0$ GeV and baryon chemical potential $\mu = 0.5$ GeV.
Variation of normalized baryon density fluctuation:

Figure: Variation of normalized fluctuation $\sigma_n$ in the subsystem $S_a$ with the scale $a$ for different values of the baryon chemical potential $\mu$ and fixed particle mass $m = 1.0$ GeV and temperature $T = 0.15$ GeV.
Variation of normalized baryon density fluctuation:

Figure: Variation of normalized fluctuation $\sigma_n$ in the subsystem $S_a$ with the scale $a$ for different values of the particle mass and fixed temperature $T = 0.15$ GeV and baryon chemical potential $\mu = 0.5$ GeV.
Variation of normalized baryon fluctuation:

Figure: Variation of normalized fluctuation $V_a \sigma^2 / (T^3 \chi_2^{(B)})$ for different values of temperature ($T$) but with fixed baryon chemical potential ($\mu$) and particle mass ($m$).
Variation of normalized baryon fluctuation:

Figure: Variation of normalized fluctuation $V_a \sigma^2 / (T^3 \chi_2^{(B)})$ for different values of baryon chemical potential ($\mu$) but with fixed temperature ($T$) and particle mass ($m$).
Variation of normalized baryon fluctuation:

\[
\frac{V_{a}\sigma^2}{(T^3\chi_B^2)}
\]

Figure: Variation of normalized fluctuation \( V_{a}\sigma^2/(T^3\chi_B^2) \) for different values of particle mass \( (m) \) but with fixed temperature \( (T) \) and baryon chemical potential \( (\mu) \).
Summary:

- We have analyzed quantum baryon-number fluctuations in subsystems of a hot and dense relativistic gas of fermions.

- And found that they diverge for small system sizes.

- Our results agree with the results known from statistical physics for sufficiently large system size $a$.

- In this way, we have delivered a useful formula that accounts for both statistical and quantum features of the fluctuations.

- The numerical results obtained here can be useful to interpret and shed new light on the heavy-ion experimental data.
We are a product of quantum fluctuations in the very early universe.

John C. Lennox

Thank you for your attention!