Progress on Feynman Integrals for $2 \to 3$ scattering at NNLO

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based on work in collaboration with D. Canko, C. Papadopoulos, C. Wever, A. Kardos and A. Smirnov

arXiv:2009.13917 [hep-ph] (JHEP)
arXiv:2012.10635 [hep-ph] (JHEP)

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Recent Developments in High Energy Physics and Cosmology
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Motivation

- Indications for the need of physics Beyond the Standard Model mostly from Cosmology (e.g. is dark matter a new particle/ particles?).
- Absence of a clear signal for BSM physics from the LHC → Collider Physics at the Precision Frontier[1].
- Measured cross sections are being compared to improved theoretical predictions looking for deviations from the SM → constraints on BSM physics.
- LHC Run 3 and HL-LHC Run will require at least NNLO corrections.
- NNLO → two-loop Feynman Diagrams → two-loop Feynman Integrals (FI).
- Numerical approaches not efficient (physical kinematics) → need for analytical results.

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[1] G. Heinrich, arXiv:2009.00516 [hep-ph]

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Computing Feynman Integrals

Introduce a family of two-loop FI for a specific scattering process (e.g. $2 \to 2$, $2 \to 3$):

$$F_{a_1,\ldots,a_N}(\{p_j\},\epsilon) = \int \left(\prod_{r=1}^{2} \frac{\mathrm{d}^d k_r}{i\pi^{d/2}} \right) \frac{\mathrm{e}^{2\epsilon\gamma_E}}{D_1^{a_1} \ldots D_N^{a_N}}, \quad (1)$$

$$D_i = (c_{ij} k_j + f_{ij} p_j)^2, \quad d = 4 - 2\epsilon$$

Two pillars

1. Integrals of total derivatives wrt loop momenta vanish within DR. (IBP identities)
2. FI satisfy differential equations (DE) derived wrt kinematic invariants.

Computing Master Integrals

- IBP reduction identifies a minimal set of Master Integrals, called basis of MI **G**.
- Use DE to compute them.

$$\frac{\partial}{\partial s_{ij}}\mathbf{G} = \mathbf{A}(\{s_{ij}\},\epsilon)\mathbf{G} \tag{2}$$

- Simplified DE (SDE)[2]: introduce an external parameter x and differentiate wrt it.

$$\partial_x\mathbf{G} = \mathbf{A}(\{s_{ij}\},x,\epsilon)\mathbf{G} \tag{3}$$

- In general the matrix **A** can be very complicated.
- Instead of **G** use a special basis[3] $\mathbf{g} = \mathbf{TG}$

[2] C. G. Papadopoulos, JHEP **07** (2014), 088

[3] J. M. Henn, Phys. Rev. Lett. **110** (2013), 251601

Computing Master Integrals

- Canonical SDE

$$\partial_x \mathbf{g} = \epsilon\, \mathbf{M}(\{s_{ij}\}, x)\mathbf{g} \tag{4}$$

- ϵ is fully factorised
- Differential matrix $\mathbf{M}(\{s_{ij}\}, x)$ is Fuchsian, i.e. it has simple poles in x (for all cases considered in this talk).
- Solved with recursive iterations in terms of Goncharov PLs (GPLs)

$$\mathcal{G}(a_1, a_2, \ldots, a_n; x) = \int_0^x \frac{\mathrm{dt}}{t - a_1} \mathcal{G}(a_2, \ldots, a_n; t) \tag{5}$$

$$\mathcal{G}(0, \ldots, 0; x) = \frac{1}{n!} \log^n(x) \tag{6}$$

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$2 \to 3$ with one massive leg (e.g. W+2 jets)

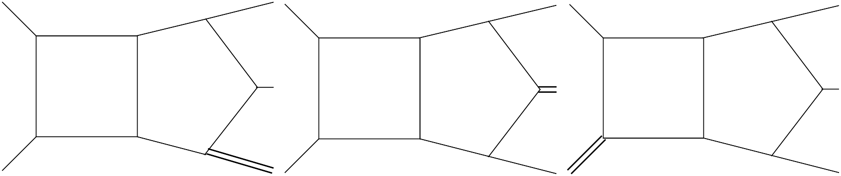

Figure: The two-loop diagrams representing the top-sector of the planar pentabox family P_1(74 MI), P_2(75 MI) and P_3(86 MI). All external momenta are incoming.

- IBP tools: FIRE6, KIRA2

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Traditional approach

Semi-numerical approach

S. Abreu, H. Ita, F. Moriello, B. Page, W. Tschernow and M. Zeng, JHEP **11** (2020), 117

Kinematics

- External momenta q_i, $i = 1 \ldots 5$
- $\sum_1^5 q_i = 0$, $q_1^2 \equiv p_{1s}$, $q_i^2 = 0$, $i = 2 \ldots 5$
- $\{q_1^2, s_{12}, s_{23}, s_{34}, s_{45}, s_{15}\}$, with $s_{ij} := (q_i + q_j)^2$

$$d\mathbf{g} = \epsilon \sum_a d \log (W_a) \tilde{\mathbf{M}}_a \mathbf{g} \tag{7}$$

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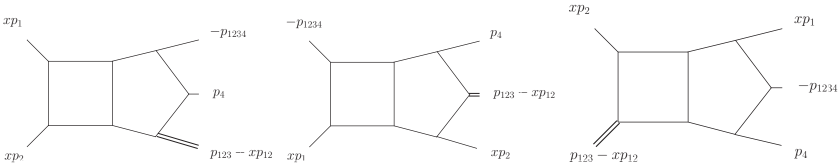
Very difficult to solve analytically!

SDE approach

Re-parametrize external momenta in terms of a dimensionless parameter x.

$$q_1 \to p_{123} - xp_{12},\ q_2 \to p_4,\ q_3 \to -p_{1234},\ q_4 \to xp_1 \qquad (8)$$



Kinematics

- Underline momenta $p_i,\ i = 1 \dots 5$
- $\sum_1^5 p_i = 0,\ p_i^2 = 0,\ i = 1 \dots 5$, with $p_{i \dots j} := p_i + \dots + p_j$
- $\{S_{12}, S_{23}, S_{34}, S_{45}, S_{51}, x\}$, with $S_{ij} := (p_i + p_j)^2$

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SDE approach

- Much simpler canonical DE

$$\partial_x \mathbf{g} = \epsilon \sum_b \frac{1}{x - l_b} \mathbf{M}_b \, \mathbf{g} \tag{9}$$

- Naturally expressed in terms of GPLs.
- Boundary terms: need $x \to 0$ limit, Expansion-By-Regions[4] (Chris Wever, Adam Kardos (Gsuite)).
- Re-write $\mathbf{g} = \mathbf{T}\,\mathbf{G}$
- Asymptotic expansion around $x \to 0$ for $\mathbf{G}$.

[4] B. Jantzen, A. V. Smirnov and V. A. Smirnov, Eur. Phys. J. C **72** (2012), 2139

Analytic solution up to $\mathcal{O}(\epsilon^4)$

$$
\begin{aligned}
\mathbf{g} = \epsilon^0 \mathbf{b}_0^{(0)} &+ \epsilon\left(\sum \mathcal{G}_a \mathbf{M}_a \mathbf{b}_0^{(0)} + \mathbf{b}_0^{(1)}\right) \\
&+ \epsilon^2\left(\sum \mathcal{G}_{ab} \mathbf{M}_a \mathbf{M}_b \mathbf{b}_0^{(0)} + \sum \mathcal{G}_a \mathbf{M}_a \mathbf{b}_0^{(1)} + \mathbf{b}_0^{(2)}\right) \\
&+ \epsilon^3\left(\sum \mathcal{G}_{abc} \mathbf{M}_a \mathbf{M}_b \mathbf{M}_c \mathbf{b}_0^{(0)} + \sum \mathcal{G}_{ab} \mathbf{M}_a \mathbf{M}_b \mathbf{b}_0^{(1)} + \sum \mathcal{G}_a \mathbf{M}_a \mathbf{b}_0^{(2)} + \mathbf{b}_0^{(3)}\right) \\
&+ \epsilon^4\left(\sum \mathcal{G}_{abcd} \mathbf{M}_a \mathbf{M}_b \mathbf{M}_c \mathbf{M}_d \mathbf{b}_0^{(0)} + \sum \mathcal{G}_{abc} \mathbf{M}_a \mathbf{M}_b \mathbf{M}_c \mathbf{b}_0^{(1)}\right. \\
&\left. + \sum \mathcal{G}_{ab} \mathbf{M}_a \mathbf{M}_b \mathbf{b}_0^{(2)} + \sum \mathcal{G}_a \mathbf{M}_a \mathbf{b}_0^{(3)} + \mathbf{b}_0^{(4)}\right)
\end{aligned}
\tag{10}
$$

$$\mathcal{G}_{ab\ldots} := \mathcal{G}(l_a, l_b, \ldots; x)$$

- Pure solution: residue matrices $\mathbf{M}_i$ are independent of the kinematics.
- Universally transcendental.
- Transcendental weight $\sim$ number of integrations.

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Explicit expressions up to $\mathcal{O}(\epsilon^1)$

$$gb_{72}^{P1} = \frac{3}{2} + \epsilon\Bigg[\mathcal{G}\left(\frac{S_{45}}{S_{12}}, x\right) - \mathcal{G}\left(\frac{S_{12}-S_{34}}{S_{12}}, x\right) - 3\mathcal{G}(0,x) + \mathcal{G}(1,x) - \log\left(S_{12}-S_{34}\right) - 2\log\left(-S_{51}\right)\Bigg] \tag{11}$$

$$gb_{73}^{P2} = \epsilon\Bigg[3\mathcal{G}\left(\frac{S_{12}+S_{23}}{S_{12}}, x\right) - 3\mathcal{G}\left(\frac{S_{12}-S_{34}}{S_{12}}, x\right) - 3\mathcal{G}\left(\frac{S_{45}}{S_{12}}, x\right) + 3\mathcal{G}(1,x) - 3\log\left(S_{12}-S_{34}\right) + 3\log\left(-S_{51}\right)\Bigg] \tag{12}$$

$$g_{84}^{P3} = \frac{1}{2} + \epsilon\Bigg[\frac{5}{2}\mathcal{G}\left(\frac{S_{45}}{S_{12}}, x\right) - \frac{3}{2}\mathcal{G}\left(\frac{S_{12}-S_{34}}{S_{12}}, x\right) - \frac{5}{2}\mathcal{G}\left(-\frac{S_{45}}{S_{23}-S_{45}}, x\right) - 2\mathcal{G}(0,x) + \frac{5}{2}\mathcal{G}(1,x) - \log\left(-S_{12}\right) - \frac{3}{2}\log\left(S_{12}-S_{34}\right) + \frac{3}{2}\log\left(-S_{51}\right)\Bigg]$$

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Numerics (N=32 digits, 1.9, 3.3, 2 sec) in GiNaC

P_1	g_{72}	ϵ^0: 3/2 ϵ^1: -2.2514604753379400332169314784961 ϵ^2: -17.910593443812320786572184851867 ϵ^3: -26.429770706459534336624681550003 ϵ^4: 21.437938934510558345847354772412
P_2	g_{73}	ϵ^1: 2.8124788185742741402751457351382 ϵ^2: 5.4813042746593704203645729908938 ϵ^3: 11.590234540689191439870956817546 ϵ^4: -5.9962816226829136730734255754596
P_3	g_{84}	ϵ^0: 1/2 ϵ^1: 3.2780415861887284967738281876762 ϵ^2: 0.11455863130537720411162743574627 ϵ^3: -16.979642659429606120982671925458 ϵ^4: -48.101985355625914648042310964575

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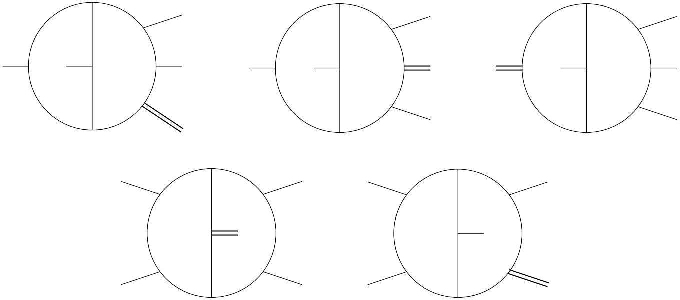
Summary

- Fully analytic results for all 2-loop planar families.
- Pentagon (1-loop 5-point) with one massive leg to all orders.
- Results used for 2-loop QCD corrections to $Wb\bar{b}$ production[5].
- Push to compute all MI for $2 \to 3$ with up to one massive leg at two loops. (Planar: known, Non-Planar: work in progress).
- Fully analytic results for first non-planar family (numerical[6]).
- Automated framework for NNLO predictions following the NLO paradigm. (talk by D. Canko)

[5] S. Badger, H. B. Hartanto and S. Zoia, [arXiv:2102.02516 [hep-ph]]
[6] C. G. Papadopoulos and C. Wever, JHEP **02** (2020), 112

Future work

Compute remaining non-planar MI for two-loop $2 \to 3$ with one massive leg[7],



[7] Pure bases from S. Abreu, H. Ita, B. Page, W. Tschernow

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Thank you for your attention!

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Automated tools

- Ginac[8] for the numerical calculation of GPLs.
- PolyLogTools[9] for the algebraic manipulation of GPLs.
- FIRE6[10] and KIRA2[11] for the IBP reduction.
- FIESTA4[12] for Expansion-By-Regions.
- pySecDec[13] and FIESTA4 for numerical computation of FI, used for cross-checking our results.

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[8] J. Vollinga and S. Weinzierl, Comput. Phys. Commun. **167** (2005), 177
[9] C. Duhr and F. Dulat, JHEP **08** (2019), 135
[10] A. V. Smirnov and F. S. Chuharev, arXiv:1901.07808 [hep-ph]
[11] J. Klappert, F. Lange, P. Maierhöfer and J. Usovitsch, arXiv:2008.06494 [hep-ph]
[12] A. V. Smirnov, Comput. Phys. Commun. **204** (2016), 189-199
[13] S. Borowka, G. Heinrich, S. Jahn, S. P. Jones, M. Kerner, J. Schlenk and T. Zirke, Comput. Phys. Commun. **222** (2018), 313-326

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Scattering kinematics

- Results in Euclidean region (all initial kinematic invariants are negative).
- GPLs and solutions are real there.
- Analytic continuation to get results in physical regions for phenomenology.
- Fibration basis techniques (exploit symbol algebra and coproduct to analytically continue GPLs).
- Numerically: $\{S_{ij}, x\} \rightarrow \{S_{ij} + i\delta_{ij}\eta, x + i\delta\eta\}$ for $\eta \rightarrow 0$.
- Constraints on $\delta_{ij},\ \delta_x$ from one-scale integrals and second graph polynomial of top sector FI.

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