Gravitational Singularities and Modification of Gravity with Perimeter Action

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1. *The appearance of singularities in General Relativity pointed out to the limitations of the predictive power of the theory and has similarity with the difficulties that appeared in the past in attempts to describe the spectrum of hydrogen atom within the realms of classical electromagnetism and mechanics.*

2. *The late acceleration of the Universe and the appearance of the positive cosmological constant is another challenging problem within the realms of quantum field theory of vacuum.*

Curing Gravitational Singularities. The Limiting Curvature Hypothesis

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Y. I. ANINI

In order to actually implement the limiting curvature hypothesis we are lead to postulate the following gravitational action [6]

$$S_g = k \int \left[(R - \Lambda/2) + \frac{\Lambda}{2} \sqrt{1 - \frac{R^2}{\Lambda^2}} \right] \sqrt{-g}\, d^4x \qquad (1)$$

Modification of Gravitational Interaction

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$$I_{\mathrm{BI}} = -1/2\,\lambda^2 \int \mathrm{d}^4x\ \left\{-|g_{\mu\nu} + \lambda F_{\mu\nu}|\right\}^{\frac{1}{2}} \ .$$

$$I_{\mathrm{EDD}} = \int \mathrm{d}^4x |R_{(\mu\nu)}(\Gamma)|^{\frac{1}{2}}$$

$$I_{\mathrm{G}} = \int \mathrm{d}^4x \{-|a g_{\mu\nu} + b R_{\mu\nu} + c X_{\mu\nu}|\}^{\frac{1}{2}} \ .$$

$X_{\mu\nu}(R)$ quadratic or higher in curvature.

Modified gravity by the f(R) term …

The higher-order curvature corrections to gravitational action

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Alternative Modification of Gravitational Interaction

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Mod. Phys. Lett. A 11 (1996), 1379-1396

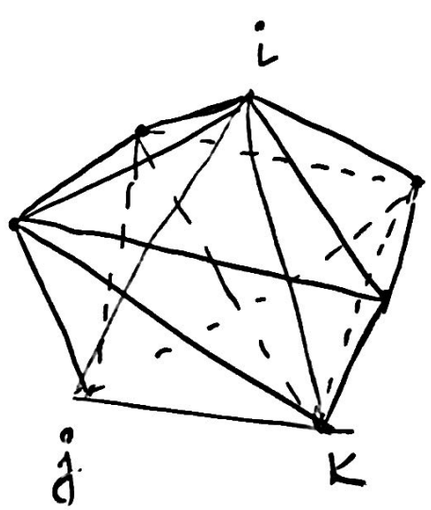
In General Relativity the action is proportional to the Area of the Universe

$$S_A = -\frac{c^3}{16\pi G}\int R\sqrt{-g}d^4x.$$

R ~ 1/cm^2 $\sqrt{-g}d^4x$ ~ cm^4 $\int R\sqrt{-g}d^4x$ ~cm^2

The Discrete Area Action of Regge

$$S_A = \sum_{<ijk>} \sigma_{ijk} \cdot \omega^{(2)}_{ijk}$$



M_4

the four-dimensional simplex

the *area* of the triangle σ_{ijk} cm^2

the corresponding deficit angle $\omega^{(2)}_{ijk}$ dimensionless

The Perimeter Action is proportional to the size of the Universe

$$S_P = \sum_{<ijk>} \lambda_{ijk} \cdot \omega^{(2)}_{ijk}$$

the perimeter λ_{ijk} of the triangle ~cm

the corresponding deficit angle $\omega^{(2)}_{ijk}$ dimension less

Perimeter action suppresses the lower-dimensional fluctuations that can grow out of the 4D Space-Time manifold

The question is: what is the continuum form of the S_P *?*

One can suggest the following form for the Perimeter action

$$S_P = \gamma\sqrt{\hbar}\,\sqrt{\frac{c^3}{16\pi G}}\int\sqrt{I_1 + (1-\epsilon)I_2}\,\sqrt{-g}d^4x.$$

$$I_1 = -\frac{1}{80\pi}R_{\mu\nu\lambda\rho;\sigma}R^{\mu\nu\lambda\rho;\sigma}, \qquad I_2 = +\frac{1}{16\pi}R_{\mu\nu\lambda\rho}\Box R^{\mu\nu\lambda\rho}$$ ~1/cm^6

The dimension of the integrant invariant is $1/cm^3$

the integral has the dimensions of cm

γ is a dimensionless coupling constant $\qquad 0 \leq \epsilon < 1$

γ expressing the mass parameter in terms of Plank mass units,

$$M = \gamma\ M_P, \qquad M_P = \sqrt{\hbar c/16\pi G},$$

The action we shall consider is a sum

$$S = -\frac{c^3}{16\pi G}\int R\sqrt{-g}d^4x + \gamma\sqrt{\hbar}\,\sqrt{\frac{c^3}{16\pi G}}\int\sqrt{I_1 + (1-\epsilon)I_2}\,\sqrt{-g}d^4x.$$

The full equation has the form

$$\frac{\delta S}{\delta g^{\mu\nu}} = \frac{\delta S_A}{\delta g^{\mu\nu}} + \gamma\,\frac{\delta S_P}{\delta g^{\mu\nu}} = -\frac{c^3}{16\pi G}(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) + \tag{3.15}$$

$$+\gamma\sqrt{\hbar}\,\sqrt{\frac{c^3}{16\pi G}}\Big(\frac{1}{2}\frac{1}{\sqrt{I_1 + (1-\epsilon)I_2}}(\frac{\delta I_1}{\delta g^{\mu\nu}} + (1-\epsilon)\frac{\delta I_2}{\delta g^{\mu\nu}}) - \frac{1}{2}\sqrt{I_1 + (1-\epsilon)I_2}\;g_{\mu\nu}\Big) = 0.$$

We shall solve these equations using perturbation theory in γ

Let us consider the perturbation of the Schwarzschild solution

The Schwarzschild solution has the form

$$ds^2 = (1 - \frac{r_g}{r})c^2 dt^2 - (1 - \frac{r_g}{r})^{-1} dr^2 - r^2 d\Omega^2 \ ,$$

where $g_{00} = 1 - \frac{r_g}{r}$, $g_{11} = -(1 - \frac{r_g}{r})^{-1}$, $g_{22} = -r^2$, $g_{33} = -r^2 \sin^2\theta$, and

$$r_g = \frac{2GM}{c^2}, \qquad \sqrt{-g} = r^2 \sin\theta.$$

$$I_0 \ = \ \frac{1}{12} R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} \ = \ (\frac{r_g}{r^3})^2$$

and shows the location of the curvature singularity at $r = 0$.

On the Schwarzschild solution the invariants are

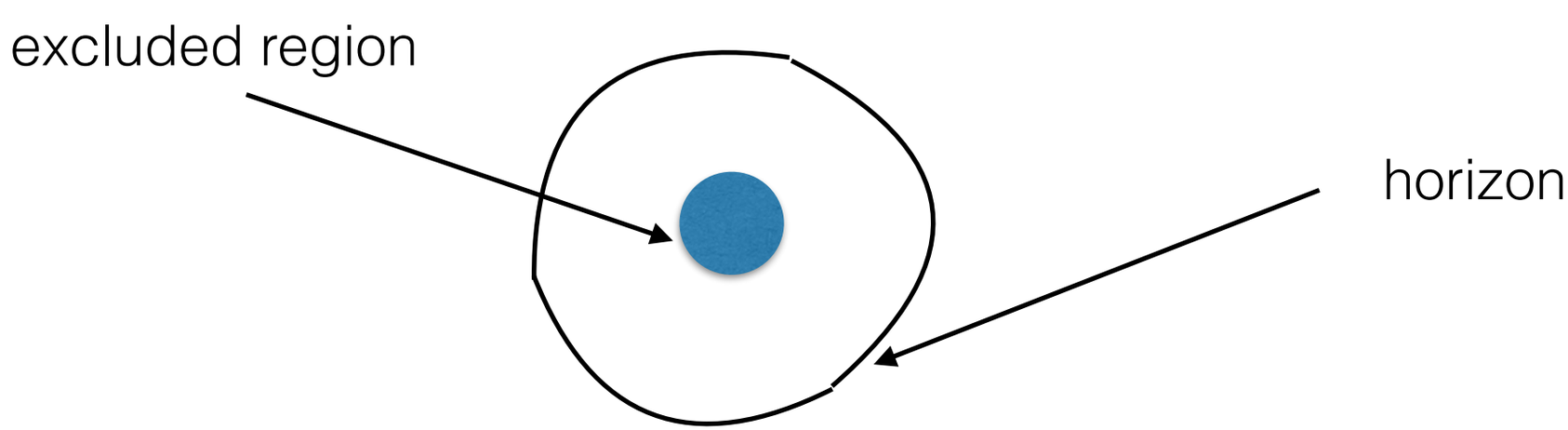
$$I_1 = \frac{9\pi}{4}\frac{r_g^2(r-r_g)}{r^9}, \qquad I_2 = \frac{9\pi}{4}\frac{r_g^3}{r^9} \ ,$$

and the perimeter action takes the form

$$S_P = \gamma\sqrt{\frac{\hbar c^5}{G}}\int \frac{3\pi}{2}\sqrt{1-\epsilon\frac{r_g}{r}}\ \frac{r_g}{r}\frac{dr}{r}dt \ , \qquad r < \epsilon r_g, \qquad 0 \leq \epsilon < 1$$

the expression under the square root becomes negative at

$$r < \epsilon r_g, \qquad 0 \leq \epsilon < 1$$



This amplitude is represented by the path integral

$$A_E(r < \varepsilon r_g) = \int_0^{r<\varepsilon r_g} d^3\vec{r} \int_{x'}^{x} e^{\frac{i}{\hbar}(S[g]+S[g,x])} \mathcal{D}g\mathcal{D}x \quad ;$$

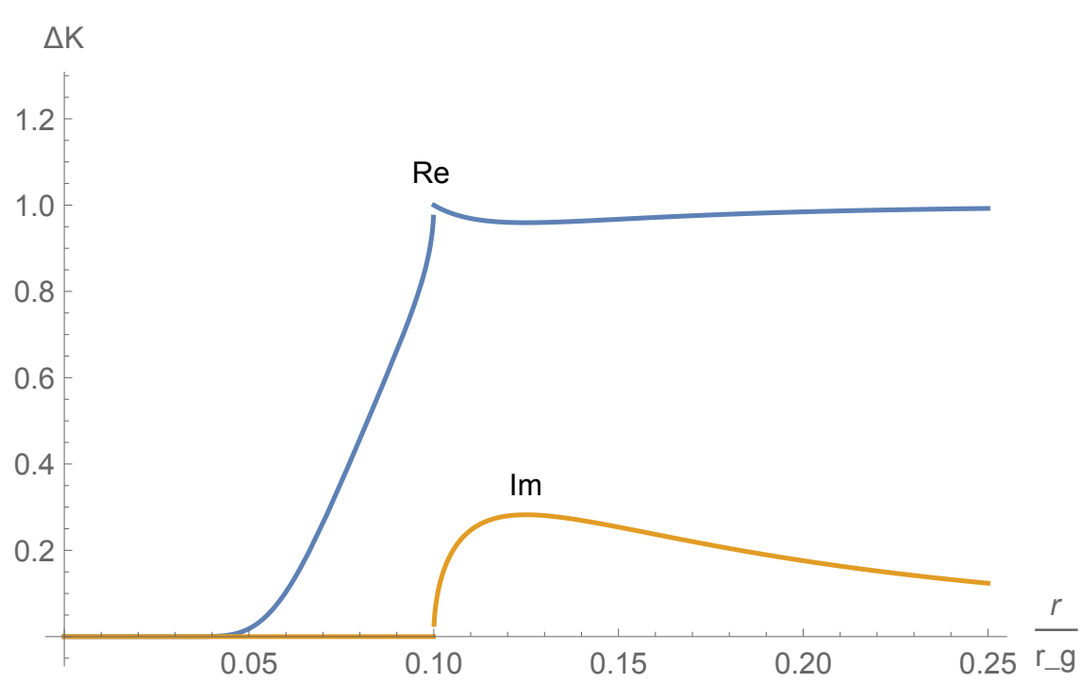
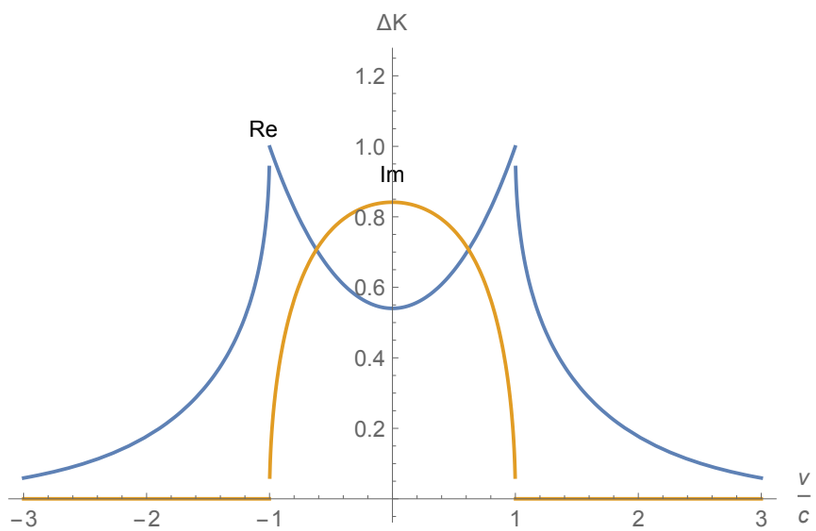


Figure 2: The graph of the real and imaginary parts of the amplitude in gravity with perimeter action (2.9) $\triangle K = e^{i\gamma\sqrt{\frac{c^5}{\hbar G}}\frac{3}{2}\sqrt{1-\epsilon\frac{r_g}{r}}\ \frac{r_g}{r}\frac{\triangle r}{r}\triangle t}$, here $\epsilon = 0.1$ and $\gamma\sqrt{\frac{c^5}{\hbar G}}\frac{3}{2}\frac{\triangle r}{r_g}\triangle t = 0.01$.

Analogy with the Relativistic Particle Action

$$S = -mc \int ds = -mc^2 \int \sqrt{1 - \frac{\vec{v}^2}{c^2}}\, dt.$$

$$K(t_b, x_b; t_a, x_a) = \int_{x_a}^{x_b} e^{\frac{i}{\hbar} mc^2 \int_{t_a}^{t_b} \sqrt{1 - \frac{\dot{x}^2}{c^2}} dt} \mathcal{D}x(t).$$



The graphic of the rial and imaginary parts of the amplitude $\triangle K = e^{\frac{i}{\hbar} mc^2 \sqrt{1 - \frac{v^2}{c^2}} \triangle t}$.

The full equation has the form

$$\frac{\delta S}{\delta g^{\mu\nu}} = \frac{\delta S_A}{\delta g^{\mu\nu}} + \gamma \, \frac{\delta S_P}{\delta g^{\mu\nu}} = -\frac{c^3}{16\pi G}(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) +$$
$$+\gamma\sqrt{\hbar}\,\sqrt{\frac{c^3}{16\pi G}}\Big(\frac{1}{2}\frac{1}{\sqrt{I_1 + (1-\epsilon)I_2}}(\frac{\delta I_1}{\delta g^{\mu\nu}} + (1-\epsilon)\frac{\delta I_2}{\delta g^{\mu\nu}}) - \frac{1}{2}\sqrt{I_1 + (1-\epsilon)I_2}\; g_{\mu\nu}\Big) \;= 0.$$

We will search the solution of this equations in the following standard spherically symmetric form:

$$ds^2 = e^{\nu(r)}c^2dt^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

the Schwarzschild metric is:

$$\nu_0(r) = \log(1 - \frac{r_g}{r}), \quad \lambda_0(r) = -\log(1 - \frac{r_g}{r}).$$

we will represent the solution $g_{\mu\nu}$ in the form:

$$g^{\mu\nu} = g_0^{\mu\nu} + \gamma g_1^{\mu\nu}, \quad \nu \to \nu_0 + \gamma\nu_1, \quad \lambda = \lambda_0 + \gamma\lambda_1,$$

The perturbation of the Schwarzschild solution

$$g_{11} = e^{\lambda_0+\lambda_1} = \frac{-1}{1-\frac{r_g}{r}}\, e^{\lambda_1},$$

where

$$\lambda_1 = \gamma \frac{l_P}{r-r_g}\Big[\ \log\frac{1+\sqrt{1-\varepsilon\frac{r_g}{r}}}{1-\sqrt{1-\varepsilon\frac{r_g}{r}}} + \sqrt{1-\varepsilon\frac{r_g}{r}} - (1-\varepsilon\frac{r_g}{r})^{3/2}\ + C\Big]$$

$$g_{00} = e^{\nu_0+\nu_1} = (1-\frac{r_g}{r})e^{\nu_1}$$

$$\nu_1 = -\lambda_1 + \gamma\frac{l_P}{r_g}\Big[\sqrt{1-\varepsilon\frac{r_g}{r}} - (1-\varepsilon\frac{r_g}{r})^{3/2}\Big]$$

the behaviour of the temporal component of the metric at large distances $r \gg r_g$:

$$g_{00} = 1 + 2\frac{\phi}{c^2} \simeq 1 - \frac{r_g}{r} - \gamma \frac{l_P}{r}\Big(\underline{a_1 \log \frac{r}{r_g}} + a + c_1\Big),$$

The logarithmically growing potential

$$g_{00}(M_\odot) \simeq 1 - 10^{-8} - \gamma\ 10^{-43},$$

$$g_{00}(M_{MW}) \simeq 1 - 10^{-5} - \gamma\ 10^{-54},$$

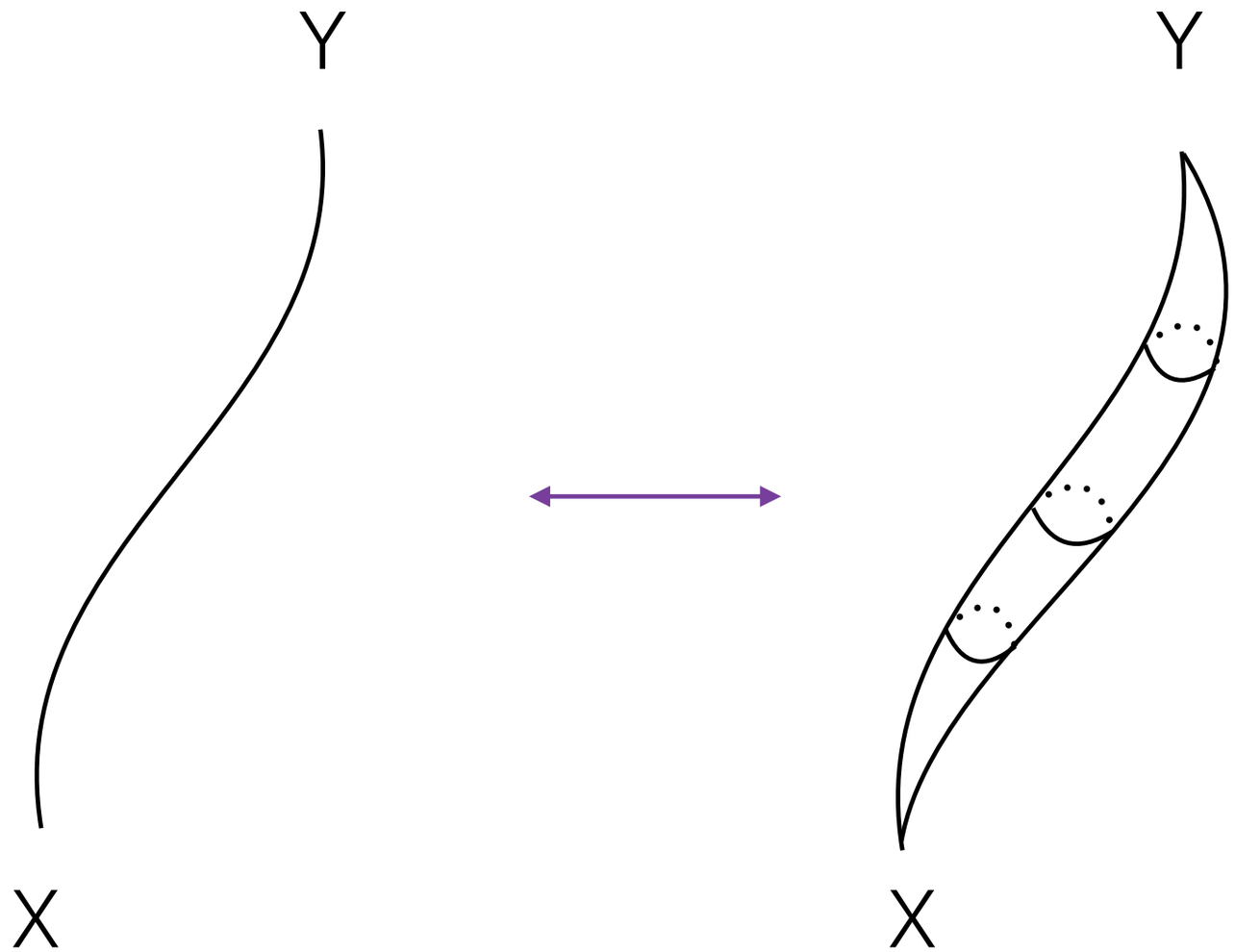
The advance precession of the perihelion $\delta\phi$ expressed in radians per revolution is

$$\delta\phi = \frac{3\pi m^2 c^2 r_g^2}{2L^2}\Big(1 + 2\gamma\frac{l_P}{r_g}(8a_1 + a + c_1)\Big) = \frac{6\pi GM}{c^2 l(1-e^2)}\Big(1 + 2\gamma\frac{l_P}{r_g}(8a_1 + a + c_1)\Big),$$

$\gamma\frac{l_P}{r_g}$ is tiny

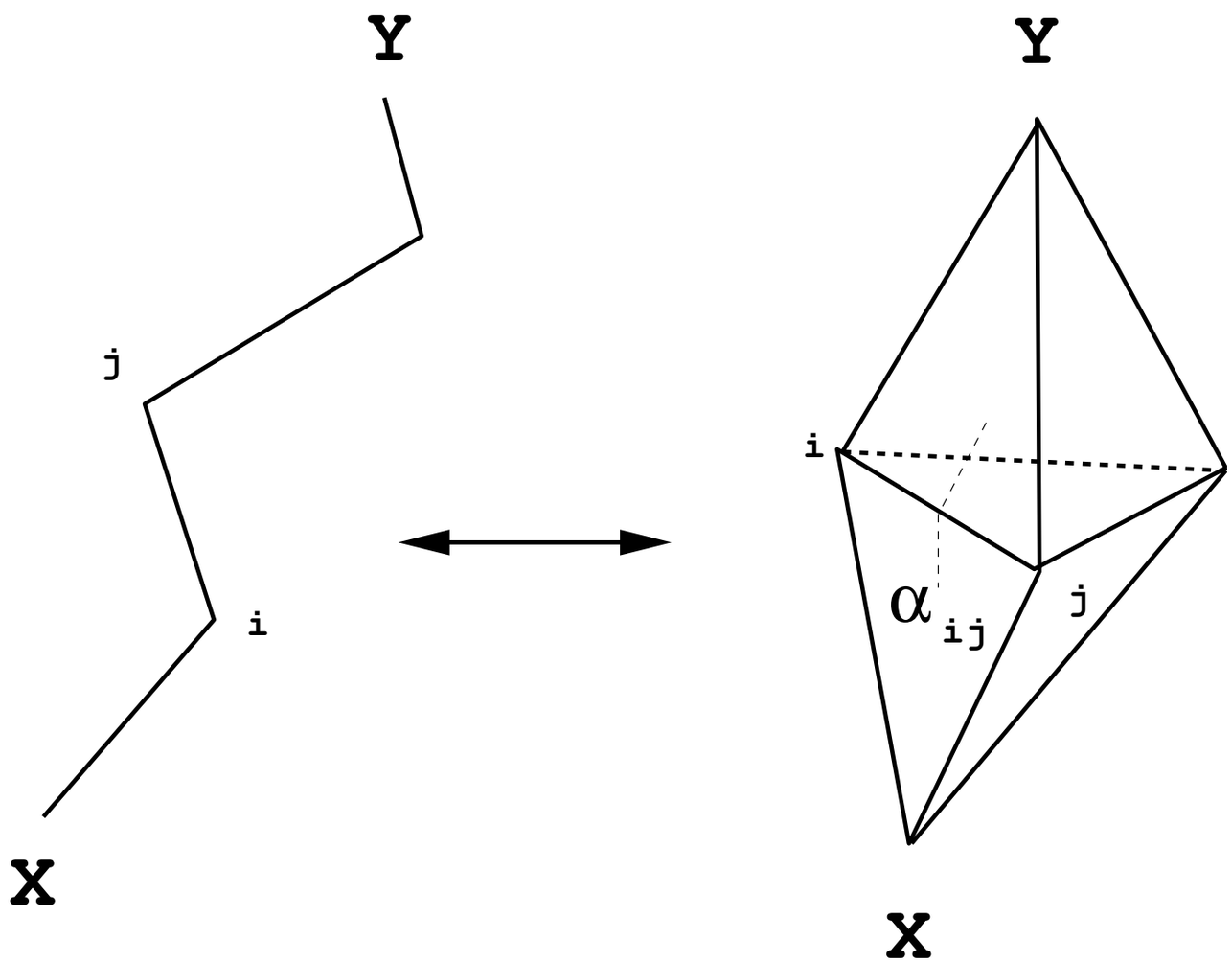
$$l_P = \sqrt{\hbar\frac{16\pi G}{c^3}} \simeq 11.3 \times 10^{-33} cm,$$

Thank You !



$$\sum_{paths} e^{i\frac{mc}{h}A_{xy}} \longleftrightarrow \sum_{surfaces} e^{i\frac{mc}{h}A_{xy}}$$

$$[A_{xy}] = cm$$



$$\mathbf{A}_{xy}$$

$$\sum_{\langle ij \rangle}^{m} \lambda_{ij} \longleftrightarrow \sum_{\langle ij \rangle}^{m} \lambda_{ij} \left(\pi - \alpha_{ij}\right)^{\zeta}$$

Tensionless string action, there is no Nambu area term

$$A(M) = m \sum_{<ij>} \lambda_{ij} |\pi - \alpha_{ij}| \qquad \rightarrow \qquad m \sum_{<ij>} \lambda_{ij}. \qquad \sim\text{cm}$$

Tensionless string action for the smooth surfaces

$$A(M) = m \int d^2\zeta \sqrt{g} \sqrt{(\Delta(g) X^\mu)^2} \qquad \rightarrow \qquad m \int ds,$$

where $g_{ab} = \partial_a X^\mu \partial_b X^\mu$ is the induced metric and $\Delta(g)$ is a Laplace operator.

Linear string action suppresses spices that can grow out of the surface

The perturbation of the Schwarzschild solution

$$S_P = \gamma\sqrt{\frac{\hbar c^5}{G}}\int \frac{3\pi}{2}\sqrt{1-\epsilon\frac{r_g}{r}}\;\frac{r_g}{r}\frac{dr}{r}dt\ , \qquad r < \epsilon r_g, \qquad 0 \le \epsilon < 1$$

it is similar of the relativistic particle action

$$S = -mc\int ds = -mc^2\int\sqrt{1-\frac{\vec{v}^2}{c^2}}\;dt.$$

the region where the action became complex seems unreachable by the test particles. The size of the region depends on the parameter ϵ and is smaller than the gravitational radius r_g. This observation may have a profound consequences on the gravitational singularity at $r = 0$. In the suggested scenario it seems possible to eliminate the singularities from the theory based on the fundamental principles of quantum mechanics.

Both expressions contain the geometrical invariants which are not in general positive definite under the square root operation. In the relativistic particle case the expression under the root becomes negative for a particle moving with a velocity which exceeds the velocity of light. In that case the action develops an imaginary part and quantum mechanical superposition of amplitudes prevents a particle from exceeding the velocity of light A similar mechanism was implemented in the Born-Infeld modification of electrodynamics with the aim to prevent the appearance of infinite electric fields .

One can expect that in the case of the linear action there may appear a space-time regions which are unreachable by the test particles as in that regions the action develop an imaginary part. If these "locked" space-time regions happen to appear and if that space-time regions include singularities, then one can expect that the gravitational singularities are naturally excluded from the theory due to the fundamental principles of quantum mechanics. The question of consistency of the new action principle, if it is the right one, can only be decided by their physical consequences.

1 *Appendix*

The general form of the linear action has the form:

$$S_L = -m_P c \int \frac{3}{8\pi} \sqrt{\sum_1^3 \eta_i K_i + \sum_1^4 \chi_i J_i + \sum_1^9 \gamma_i I_i} \, \sqrt{-g} d^4x \ ,$$

where the curvature invariants have the form

$$\begin{aligned}
&I_0 = \frac{1}{12} R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho}, \quad I_1 = -\frac{1}{180} R_{\mu\nu\lambda\rho;\sigma} R^{\mu\nu\lambda\rho;\sigma}, \quad I_2 = +\frac{1}{36} R_{\mu\nu\lambda\rho} \Box R^{\mu\nu\lambda\rho} \ , \\
&I_3 = -\frac{1}{72} \Box (R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho}), \ I_4 = -\frac{1}{90} R_{\mu\nu\lambda\rho;\alpha} R^{\alpha\nu\lambda\rho;\mu}, \ I_5 = -\frac{1}{18} (R^{\alpha\nu\lambda\rho} R^{\mu}{}_{\nu\lambda\rho})_{;\mu;\alpha}, \\
&I_6 = -\frac{1}{18} (R^{\alpha\nu\lambda\rho} R^{\mu}{}_{\nu\lambda\rho})_{;\alpha;\mu} = I_3, \ I_7 = \frac{1}{18} R^{\alpha\nu\lambda\rho} R^{\mu}{}_{\nu\lambda\rho;\alpha;\mu}, \ I_8 = R^{\mu}{}_{\nu\lambda\rho;\mu} R^{\sigma\nu\lambda\rho}{}_{;\sigma} \ , \\
&I_9 = R^{\alpha\nu\lambda\rho} R^{\mu}{}_{\nu\lambda\rho;\mu;\alpha} \ , \quad I_3 = I_5 = I_6 = 5I_1 - I_2, \quad I_4 = I_1, \quad I_7 = I_2 \\
&J_0 = R_{\mu\nu} R^{\mu\nu} \ , \quad J_1 = R_{\mu\nu;\lambda} R^{\mu\nu;\lambda} \ , \quad J_2 = R^{\mu\nu} \Box R_{\mu\nu} \ , \quad J_3 = \Box (R^{\mu\nu} R_{\mu\nu}) \ , \quad J_4 = R_{\mu\sigma}{}^{;\mu} R^{\nu\sigma}{}_{;\nu} \\
&K_0 = R^2 \ , \quad K_1 = R_{;\mu} R^{;\mu} \ , \quad K_2 = R \Box R \ , \quad K_3 = \Box R^2 \ .
\end{aligned} \tag{1.1}$$

The η_i, χ_i and γ_i are free parameters. Some of the invariants can be expressed through others using Bianchi identities.