Embeddings of integrable models in supergravity and their perturbative stability

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Thessaloniki, June 18, 2021

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HELLENIC REPUBLIC
National and Kapodistrian University of Athens
EST. 1837

Ευρωπαϊκή Ένωση
European Social Fund

Operational Programme
Human Resources Development, Education and Lifelong Learning
Co-financed by Greece and the European Union

ΕΣΠΑ 2014-2020
ανάπτυξη - εργασία - αλληλεγγύη

Motivation

- Our story starts with the λ-model which is an integrable deformation of a CFT interpolating between a WZW model and the non-Abelian T-dual of the PCM [K. Sfetsos, '13]
- Various λ-models on group and coset spaces have been embedded to the type-II supergravity [K. Sfetsos, D.C. Thompson, '14; S. Demulder, K. Sfetsos & D.C. Thompson, '15; B. Hoare & A.A. Tseytlin, '15; R.Borsato, A.A.Tseytlin & L.Wulff, '16; Y. Chervonyi & O. Lunin, '16]
- Embeddings including undeformed AdS spaces have been constructed only recently [G. I. & K. Sfetsos, '19]
- Our desire is to study these new solutions in the context of the AdS/CFT correspondence
- Lack of supersymmetry suggests that the stability of these solutions must be analysed
- This is also related to the Ooguri-Vafa conjecture [H. Ooguri & C. Vafa, '16]

Gravity and Scalars

- Consider a theory of gravity in D-dimensions with n scalars:

$$S(\mathfrak{g}, X) = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{|\mathfrak{g}|} \Big(\mathfrak{R} - \gamma_{ij} \partial X^i \cdot \partial X^j - V(X) \Big)$$
$$i, j = 1, \ldots, n$$

- The equations of motion for the metric and the scalars are:

$$\nabla_{\mathfrak{g}}^2 X^i - \frac{1}{2} \gamma^{ij} \partial_j V(X) = 0$$
$$\mathfrak{R}_{\mu\nu} - \gamma_{ij} \partial_\mu X^i \partial_\nu X^j - \frac{\mathfrak{g}_{\mu\nu}}{D-2} V(X) = 0$$

- We focus on AdS solutions with constant scalars:

$$ds_D^2 = \bar{\mathfrak{g}}_{\mu\nu} dx^\mu dx^\nu = L^2 \left(r^2 \eta_{\alpha\beta} dx^\alpha dx^\beta + \frac{dr^2}{r^2} \right), \quad \bar{X}^i = \mathrm{const}$$

- The potential in this case must satisfy:

$$V(\bar{X}) = -\frac{(D-1)(D-2)}{L^2} \quad \& \quad \partial_i V(\bar{X}) = 0\,, \quad \forall\, i = 1, \ldots, n$$

- We are interested in studying the fluctuations around the background:

$$\mathfrak{g}_{\mu\nu} = \bar{\mathfrak{g}}_{\mu\nu} + \delta\mathfrak{g}_{\mu\nu}\,, \qquad X^i = \bar{X}^i + \delta X^i\,, \qquad i = 1, \ldots, n$$

- The linearised equations of motion read:

$$\nabla^2_{\bar{\mathfrak{g}}}\delta X^i - (M^2)^i{}_j \delta X^j = 0 \quad \& \quad \delta\mathfrak{R}_{\mu\nu} + \frac{D-1}{L^2}\delta\mathfrak{g}_{\mu\nu} = 0$$

where:

$$\boxed{(M^2)^i{}_j = \frac{1}{2}\gamma^{ik}\partial_j\partial_k V(X)\Big|_{X=\bar{X}}}$$

- The scaling dimensions for the scalar fluctuations are extracted from their asymptotic behaviour at the boundary of *AdS*
- Requiring reality of the scaling dimensions implies that the eigenvalues of M^2 satisfy the *Breitenlohner-Freedman* (BF) bound [P. Breitenlohner & D. Z. Freedman, '82]:

$$\boxed{d_i \geqslant -\left(\frac{D-1}{2L}\right)^2 , \quad \forall\ i = 1, \ldots, n}$$

- The equation for the metric fluctuations greatly simplifies at the transverse-traceless gauge:

$$\nabla^{\mu}_{\bar{\mathfrak{g}}} \delta\mathfrak{g}_{\mu\nu} = 0\,, \qquad \mathfrak{g}^{\mu\nu}\, \delta\mathfrak{g}_{\mu\nu} = 0$$

where it reduces to:

$$\nabla^2_{\bar{\mathfrak{g}}} \delta\mathfrak{g}_{\mu\nu} + \frac{2}{L^2} \delta\mathfrak{g}_{\mu\nu} = 0$$

The supergravity solution

- The solution of our interest [G. I. & K. Sfetsos, '19] has metric:

$$ds^2 = \frac{2}{\ell}\Big(- r^2 dt^2 + r^2 dx^2 + \frac{dr^2}{r^2} + d\theta_1^2 + \sin^2\theta_1\, d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2\, d\theta_3^2 \Big)$$
$$+ e^{2\phi_y}\left(\lambda_+^2\, dy_1^2 + \lambda_-^2\, dy_2^2\right) + e^{2\phi_z}\left(\lambda_+^2\, dz_1^2 + \lambda_-^2\, dz_2^2\right)$$

where

$$\phi_y(y) = -\frac{1}{2}\ln\left(1 - y_1^2 - y_2^2\right) \quad \& \quad \phi_z(z) = -\frac{1}{2}\ln\left(z_1^2 + z_2^2 - 1\right)$$

- The dilaton is:

$$\Phi(y,z) = \phi_y(y) + \phi_z(z)$$

- The RR sector contains only a self-dual five-form:

$$F_5 = 2\,k\, dz_1 \wedge dy_2 \wedge \left(\sqrt{\frac{\ell-\mu}{2}}\mathrm{Vol}(AdS_3) + \sqrt{\frac{\ell+\mu}{2}}\mathrm{Vol}(S^3)\right)$$
$$- 2\,k\, dz_2 \wedge dy_1 \wedge \left(\sqrt{\frac{\ell+\mu}{2}}\mathrm{Vol}(AdS_3) + \sqrt{\frac{\ell-\mu}{2}}\mathrm{Vol}(S^3)\right)$$

- The rest of the fields are trivial

$$H_3 = F_1 = F_3 = 0$$

- We also define:

$$\lambda_\pm = \sqrt{k\frac{1\pm\lambda}{1\mp\lambda}}\,, \qquad \mu = \frac{4\lambda}{k(1-\lambda^2)}\,, \qquad \lambda\in[0,1)\,, \quad \boxed{\ell \geqslant \mu}$$

- Dilaton and Einstein equations:

$$R + 4\,\nabla^2\Phi - 4\left(\partial\Phi\right)^2 - \frac{1}{12}H_3^2 = 0\,,$$

$$R_{MN} + 2\,\nabla_M\nabla_N\Phi - \frac{1}{4}\left(H_3^2\right)_{MN} = \frac{e^{2\Phi}}{2}\left[\left(F_1^2\right)_{MN} + \frac{1}{2}\left(F_3^2\right)_{MN} + \frac{1}{48}\left(F_5^2\right)_{MN} - G_{MN}\left(\frac{1}{2}F_1^2 + \frac{1}{12}F_3^2\right)\right]$$

- Bianchi and flux equations:

$$dH_3 = 0\,, \qquad dF_1 = 0\,, \qquad dF_3 = H_3\wedge F_1\,, \qquad dF_5 = H_3\wedge F_3$$

$$d\star F_1 + H_3\wedge\star F_3 = 0 = 0\,, \qquad d\star F_3 + H_3\wedge F_5$$

$$d\left(e^{-2\Phi}\star H_3\right) - F_1\wedge\star F_3 - F_3\wedge F_5 = 0$$

Stability analysis

- We will study the perturbative stability of our solution from a lower dimensional point of view
- We adopt the following reduction ansatz for the metric:

$$d\hat{s}^2 = e^{2A}\Big[ds^2_{\mathcal{M}_3} + \frac{2e^{2\psi}}{\ell}\left(d\theta_1^2 + \sin^2\theta_1\, d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2\, d\theta_2^2\right)$$
$$+ e^{2\phi_y}\left(\lambda_+^2\, e^{2\chi_1}\, dy_1^2 + \lambda_-^2\, e^{2\chi_2}\, dy_2^2\right) + e^{2\phi_z}\left(\lambda_+^2\, e^{2\chi_3}\, dz_1^2 + \lambda_-^2\, e^{2\chi_4}\, dz_2^2\right)\Big]\,,$$
$$ds^2_{\mathcal{M}_3} = g_{\mu\nu}(x)\, dx^\mu\, dx^\nu$$

- For the dilaton and the five-form we take:

$$\widehat{\Phi}(x,y,z) = 4A(x) + \phi_y(y) + \phi_z(z)\,,$$
$$\widehat{F}_5 = dz_1 \wedge dy_2 \wedge \left(c_1 e^{\chi_2-\chi_1+\chi_3-\chi_4-3\psi}\,\mathrm{Vol}(\mathcal{M}_3) + c_2 \mathrm{Vol}(S^3)\right)$$
$$- dz_2 \wedge dy_1 \wedge \left(c_2 e^{\chi_1-\chi_2-\chi_3+\chi_4-3\psi}\mathrm{Vol}(\mathcal{M}_3) + c_1 \mathrm{Vol}(S^3)\right)$$

- The rest of the fields are taken to be zero
- The various functions that enter in the ansatz are taken to depend only on the coordinates of $\mathcal{M}_3$

- The constants c_1 and c_2 are:

$$c_1 = 2k\sqrt{\frac{\ell - \mu}{2}}, \qquad c_2 = 2k\sqrt{\frac{\ell + \mu}{2}}$$

- To recover our solution we set:

$$\boxed{\begin{aligned} & ds^2_{\mathcal{M}_3} = \bar{g}_{\mu\nu}\, dx^\mu\, dx^\nu = \frac{2}{\ell}\left(r^2 \eta_{\alpha\beta} dx^\alpha dx^\beta + \frac{dr^2}{r^2}\right), \\ & \bar{A} = \bar{\psi} = \bar{\chi}_1 = \bar{\chi}_2 = \bar{\chi}_3 = \bar{\chi}_4 = 0 \end{aligned}}$$

- The reduction ansatz satisfies the Bianchi and flux equations
- From the dilaton and Einstein equations we obtain differential equations for the functions $A, \psi, \chi_1, \ldots, \chi_4$ and the metric $g_{\mu\nu}$

The dilaton equation:

$$R_g + \#\nabla_g^2 \text{scalars} + \#\partial\text{scalars} \cdot \partial\text{scalars} + \text{exps of scalars} = 0$$

Einstein equations on $\mathcal{M}_3$:

$$R_{\mu\nu}^g + \#\nabla_\mu \nabla_\nu \text{scalars} + \#\partial_\mu \text{scalars}\ \partial_\nu \text{scalars}$$
$$+ g_{\mu\nu}\left[\#\nabla_g^2 \text{scalars} + \#\partial\text{scalars} \cdot \partial\text{scalars} + \text{exps of scalars}\right] = 0$$

- The two can be combined to eliminate R_g from the dilaton equation:

$$\nabla_g^2 \text{scalars} + \#\partial\text{scalars} \cdot \partial\text{scalars} + \text{exps of scalars} = 0$$

- From the transverse directions we obtain 5 more equations of the same form and 2 first order equations:

$$\partial_\mu(2A + \chi_1 + \chi_2) = 0\,, \qquad \partial_\mu(2A + \chi_3 + \chi_4) = 0$$

- We eliminate χ_2 & χ_4 and we are left with $A\,,\psi\,,\chi_1\,,\chi_3\,,g_{\mu\nu}$ and an equal number of equations for each object
- We move to a more convenient frame where:

$$\boxed{g_{\mu\nu} = e^{8A-6\psi}\,\mathfrak{g}_{\mu\nu}}$$

- In this frame the equations for $A\,,\psi\,,\chi_1\,,\chi_3\,,\mathfrak{g}_{\mu\nu}$ can be derived from a $3D$ action of gravity with scalars with:

$$\gamma_{ij} = \begin{pmatrix} 32 & -12 & 2 & 2 \\ -12 & 12 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 2 & 0 & 0 & 2 \end{pmatrix}$$

and

$$V(X) = -\,3\,\ell\,e^{8A-8\psi} - 2\,e^{8A-6\psi}\left(\frac{e^{-2\chi_1}}{\lambda_+^2} + \frac{e^{4A+2\chi_1}}{\lambda_-^2} - \frac{e^{-2\chi_3}}{\lambda_+^2} - \frac{e^{4A+2\chi_3}}{\lambda_-^2}\right) + \frac{e^{12A-12\psi}}{2k^2}\left(c_1^2\,e^{2\chi_3-2\chi_1} + c_2^2\,e^{2\chi_1-2\chi_3}\right)$$

- We have all the ingredients to compute the matrix M^2 and its eigenvalues
- The change of frame didn't affect the radius of AdS_3:

$$L = \sqrt{\frac{2}{\ell}}$$

- The BF bound now reads:

$$\boxed{d_i \geqslant -\frac{\ell}{2}\,, \quad \forall\, i = 1, \ldots, 4}$$

The undeformed case:

$$M^2 = 4\ell \begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -\frac{1}{\hat{\ell}} & 0 & \frac{1}{2} - \frac{1}{\hat{\ell}} & -\frac{1}{2} \\ \frac{1}{\hat{\ell}} & 0 & -\frac{1}{2} & \frac{1}{2} + \frac{1}{\hat{\ell}} \end{pmatrix}, \quad \begin{array}{ll} d_1 = 0\,, & d_2 = 2\,\ell\left(1 + \sqrt{1 + \frac{4}{\hat{\ell}^2}}\right) \\ d_3 = 4\,\ell\,, & d_4 = 2\,\ell\left(1 - \sqrt{1 + \frac{4}{\hat{\ell}^2}}\right) \end{array}$$

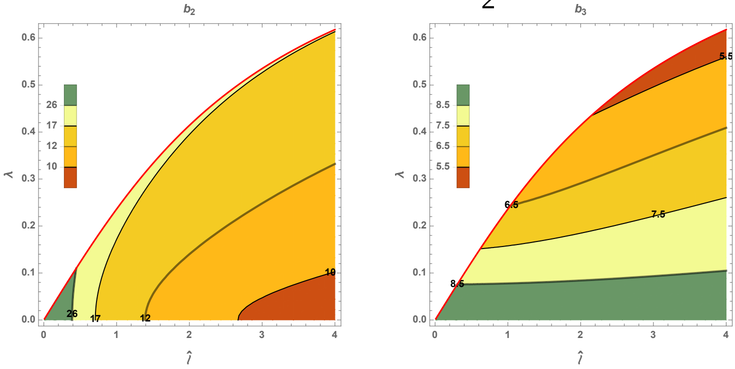
- The BF bound is not violated for:

$$\hat{\ell} \geqslant \frac{8}{3}\,, \qquad \hat{\ell} := k\ell$$

The deformed case:

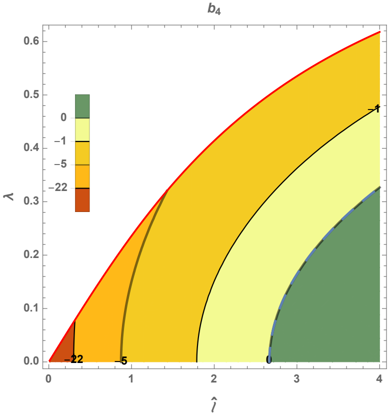
- One of the eignevalues of M^2 is zero ($d_1 = 0$) and the other three depend *non-trivially* on λ
- Violation of the BF bound means negative values for:

$$b_i := d_i + \frac{\ell}{2}$$



- The allowed region is: $\ell \geqslant \mu \quad \Rightarrow \quad \hat{\ell} \geqslant \frac{4\lambda}{1-\lambda^2}$

- The fourth eigenvalue of M^2 leads to a more interesting structure:



- The region that does not violate the BF bound is defined by:

$$\hat{\ell} \geqslant \frac{8}{3\sqrt{3}} \frac{\sqrt{3 + 22\,\lambda^2 + 3\,\lambda^4}}{1 - \lambda^2}$$

Conclusions

- We studied the perturbative stability of a non-supersymmetric solution of the type-IIB supergravity whose geometry contains an AdS_3, a round S^3 and two λ-deformed spaces.
- Our approach is based on the study of scalar fluctuations in a three-dimensional effective theory of gravity with scalars
- The analysis we performed revealed a sub-region in the allowed parametric space where the BF bound is not violated
- A more complete treatment requires the study of the full spectrum
- Non-perturbative instabilities should be also tested

Thank you!

Acknowledgment: The present work was co-funded by the European Union and Greek national funds through the Operational Program " Human Resources Development, Education and Lifelong Learning " (NSRF 2014-2020), under the call "Supporting Researchers with an Emphasis on Young Researchers - Cycle B" (MIS: 5047947)