Factorisation as a Unified Framework for the Description of Neutrino-Nucleus Interactions

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Neutrino-Nucleus Interactions
in the Standard Model and Beyond
CERN, January 19, 2022

OUTLINE

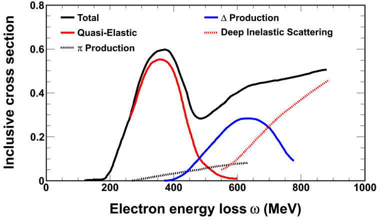
- The trouble with neutrino-nucleus interactions
- The factorisation *ansatz*
  - Initial state
  - Interaction vertex
  - Final state
- Advanced implementations of factorisation
- Applications to electron- and neutrino-nucleus scattering
- Outlook

The Trouble with Flux Average

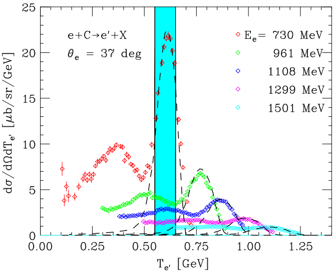
- ⋆ Flux average hampers the determination of the energy transfer to the nuclear target, which is the main factor the reaction mechanism
- ⋆ consider the process

$$e + {}^{12}\mathrm{C} \to e' + X$$

- ▶ fixed beam energy $E_e \sim 1$ GeV



- ▶ Beam energy $0.7 \lesssim E_e \lesssim 1$ GeV



OUTSTANDING ISSUES

- List of suggested issues for discussion at the mini-workshop in preparation for the white paper "Theoretical tools for neutrino scattering" (August 24-25, 2021)
- Using different models in different kinematical regimes unavoidably involves a degree of inconsistency
- Factorisation provides a unified framework, allowing to link different kinematical regimes using the *same* nuclear model

Discussion: Ab Initio

Main Points:

Linking different kinematical regions

Linking different nuclear models

Connection to LQCD

Precision requirements

5 - 10 year goals

The Lepton-Nucleus x-Section

⋆ Consider, for example, the cross section of the process $\ell + A \to \ell' + X$ at fixed beam energy

$$d\sigma_A \propto L_{\mu\nu} W_A^{\mu\nu}$$

- $L_{\mu\nu}$ is fully specified by the lepton kinematical variables
- The nuclear response tensor

$$W_A^{\mu\nu} = \sum_X \langle 0|J_A^{\mu\,\dagger}|X\rangle\langle X|J_A^{\nu}|0\rangle\delta^{(4)}(P_0 + k - P_X - k')$$

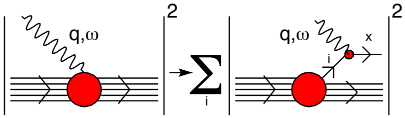
involves

- the target ground state, $|0\rangle$, largely non relativistic
- hadronic final state, $|X\rangle$, carrying momentum $q = k - k'$ and possibly involving hadrons other than nucleons
- the nuclear current operator, explicitly depending on q

$$J_A^{\mu} = \sum_i j_i^{\mu} + \sum_{j>i} j_{ij}^{\mu}$$

IMPULSE APPROXIMATION AND FACTORISATION

- ⋆ for $\lambda \sim 1/|\mathbf{q}| \ll d_{\rm NN} \sim 1.6$ fm, the average nucleon-nucleon distance in the target nucleus, nuclear scattering reduces to the incoherent sum of scattering processes involving individual nucleons



- ⋆ Basic assumptions
  - ▷ $J^\mu_A(q) \approx \sum_i j^\mu_i(q)$: single-nucleon coupling
  - ▷ $|X\rangle \to |\mathbf{p}\rangle \otimes |n_{(A-1)}, \mathbf{p_n}\rangle$: factorisation of the final state
- ⋆ Corrections arising from te occurrence of Final State Interactions (FSI) and processes involving two-nucleon Meson-Exchange Currents (MEC) can be consistently accounted for (more on this later)

The IA Cross Section

- ⋆ Factorisation allows to rewrite the nuclear transition matrix element as

$$\langle X|J_A^\mu|0\rangle \to \sum_i \int d^3k \; M_n(\mathbf{k}) \langle \mathbf{k}+\mathbf{q}|j_i^\mu|\mathbf{k}\rangle$$

  - ▸ The nuclear amplitude $M_n = \langle n|a_{\mathbf{k}}|0\rangle$ is independent of momentum transfer. It can be obtained from non relativistic many-body theory
  - ▸ The matrix element of the current between free-nucleon states can be computed exactly using the fully relativistic expression

- ⋆ Nuclear x-section

$$d\sigma_A = \int d^3k dE \; d\sigma_N \; P_h(\mathbf{k}, E)$$

- ⋆ The lepton-nucleon cross section $d\sigma_N$ can be obtained—at least in principle—from proton and deuteron data, theoretical models or LQCD
- ⋆ The spectral function $P_h(\mathbf{k}, E)$ describes the probability of removing a nucleon of momentum $\mathbf{k}$ from the nuclear ground state, leaving the residual system with excitation energy E

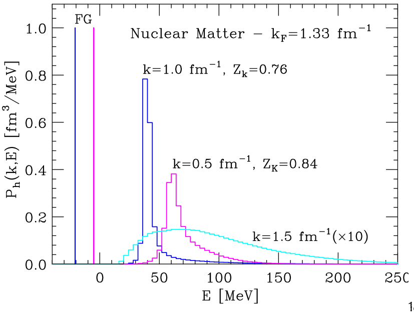
Analytic Structure of the Spectral Function

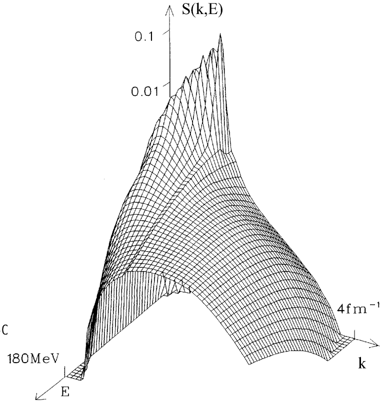
- The spectral function, trivially related to the Green's function

$$P_h(\mathbf{k}, E) = \frac{1}{\pi} G_h(\mathbf{k}, E) = \sum_n |\langle n | a_{\mathbf{k}} | 0 \rangle|^2 \delta(E - E_0 + E_n)$$

  can be split into pole and continuum contributions

- Isospin-symmetric matter at equilibrium, NPA 505, 267 (1989)

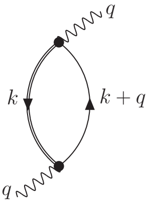




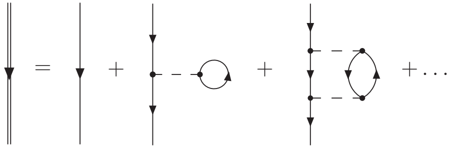
REACTION MECHANISMS IN THE 0π CHANNEL

- Including the effect of interactions in the initial state

$W_A^{\mu\nu} =$



$G_h(k, E) =$



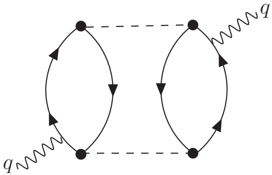
- Warning: the *bare* nucleon-nucleon interaction cannot be used for perturbation theory in the basis of eigenstates of the non-interacting system

- Interactions couple the 1h states of the residual nucleon to 2h1p states, in which one of the spectator nucleons is excited to the continuum. This mechanism leads to the appearance of 2p2h final states. For a ^{12}C target

$$|X\rangle = |pp, {}^{10}\mathrm{B}\rangle \ , \ |np^{10}\mathrm{C}\rangle \ldots$$

- In addition, in the presence of correlations 2p2h states appear through their coupling to the ground state

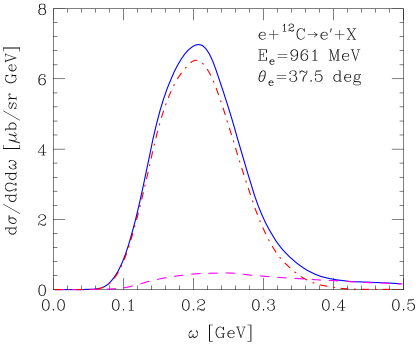
$$W_A^{\mu\nu} =$$



- These contributions exhibit a specific energy dependence
- Note: in interacting many body systems the excitation of 2p2h states *does not* require a two-nucleon current

CORRELATION EFFECTS ON THE QE CROSS SECTION

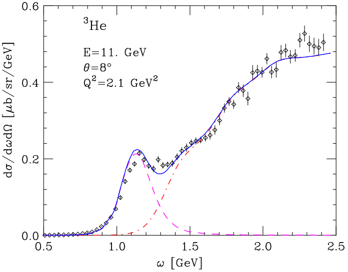
- ⋆ Correlations move strength from the 1p1h sector—in which the residual system bound state—to the 2p2h sector—in which one spectator nucleon excited to the continuum—leading to a quenching of the peak and to the appearance of a tail extending to large energy loss



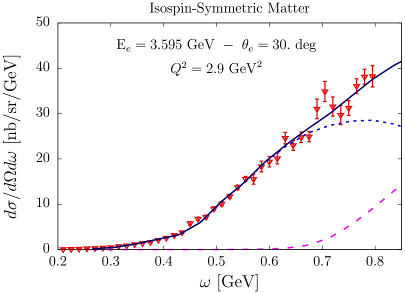
ELECTRON SCATTERING BEYOND THE QE SECTOR

- ⋆ elastic and inelastic (RES + DIS) processes consistently taken into account (Bodek & Ritchie parametrisation)
- ⋆ no adjustable parameters involved

- ▶ SLAC data
  Day *et al*, PRL 43,1143 (1979)

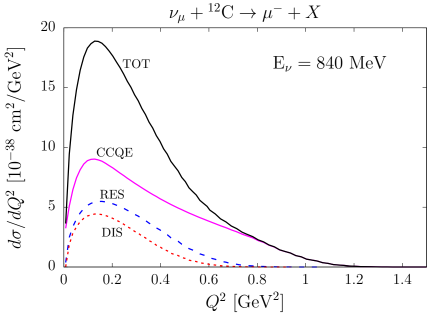


- ▶ Extrapolation of SLAC data
  Day *et al*, PRC 40, 1011 (1989)



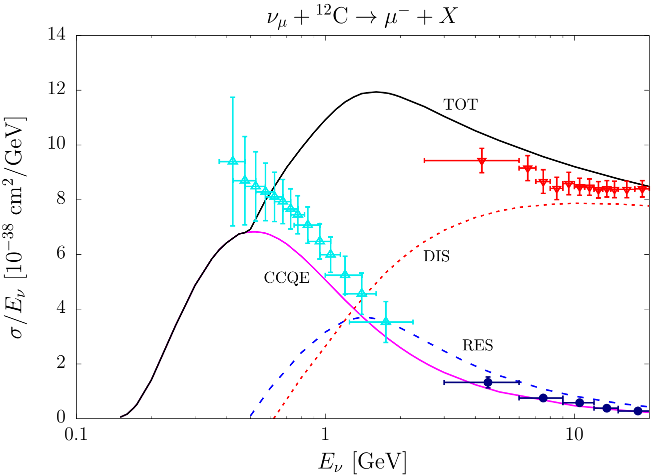
Q^2 DISTRIBUTION AT FIXED NEUTRINO ENERGY

- E. Vagnoni, OB, and D. Meloni, PRL 118, 142502 (2017)
- Nucleon structure functions:
  - CCQE: BBBA vector form factors + dipole fit of the axial form factor
  - RES: model of Lalakulich, Paschos, and Sakuda
  - DIS: parton distributions of Glück, Reya, and Vogt



- Note: $E_\nu = 840$ MeV is the average energy of the T2K flux

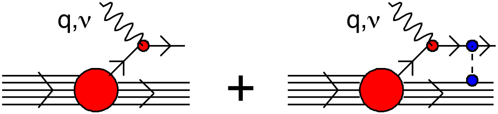
Total ν-Carbon Cross Section



- σ_{CCQE}: NOMAD, PLB 660, 19 (2008), MiniBooNE, PRD 81, 092005 (2010)
- σ_{TOT}: NOMAD, EPJC 63, 555 (2009)

Final State Interactions (FSI)

- The measured $(e, e'p)$ x-sections provide overwhelming evidence of the occurrence of significant FSI effects in the QE sector



$$d\sigma_A = \int d^3k dE \, d\sigma_N \, P_h(\mathbf{k}, E) P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$$

- the particle-state spectral function $P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$ describes the propagation of the struck particle in the final state
- the IA is recovered replacing

$$P_p(|\mathbf{k}+\mathbf{q}|, \omega - E) \to \delta(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2})$$

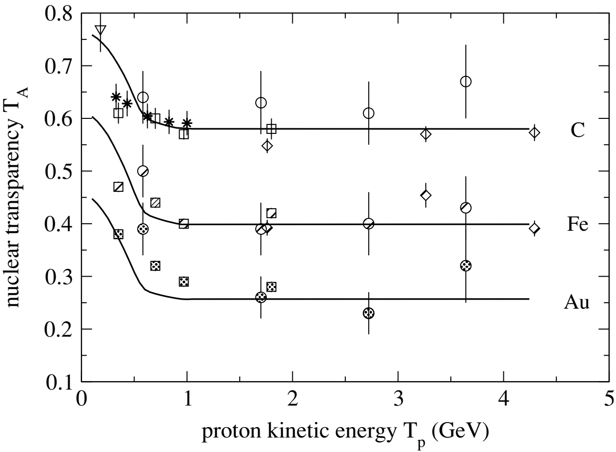
- effects of FSI on the inclusive cross section
  - shift in energy transfer due to the mean field of the spectator nucleons
  - redistributions of the strength due to rescattering of the knocked out nucleon
- high energy (eikonal) approximation
  - the struck nucleon moves along a straight trajectory with constant velocity
  - the fast struck nucleon "sees" the spectator system as a collection of fixed scattering centers

$$\delta(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2}) \to \sqrt{T_{|\mathbf{k}+\mathbf{q}|}}\,\delta(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2})$$
$$+(1 - \sqrt{T_{|\mathbf{k}+\mathbf{q}|}})\,f(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2}))$$

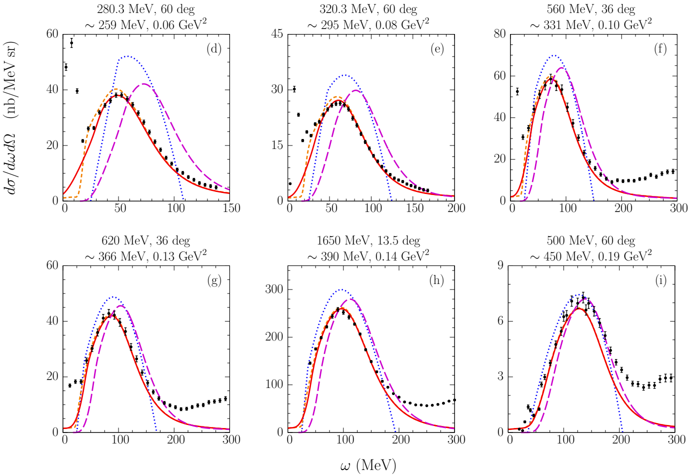
- the nuclear transparency T is measured by $(e, e'p)$ experiments, and the folding function f can be computed within nuclear many-body theory using as input nucleon-nucleon scattering data
- complex pattern of significant medium effects

GAUGING FSI: NUCLEAR TRANSPARENCY FROM $(e, e'p)$

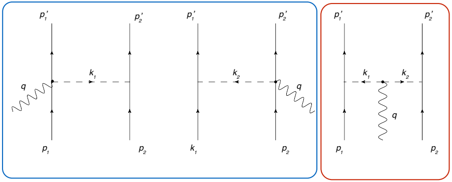
- Nuclear transparency, measured by the ratio $\sigma_{\text{exp}}/\sigma_{\text{IA}}$. PRC 72, 054602 (2005)



⋆ $e + {}^{12}\mathrm{C} \to e' + X$ quasi elastic cross section computed within the IA including FSI, PRD 91, 033005 (2015)

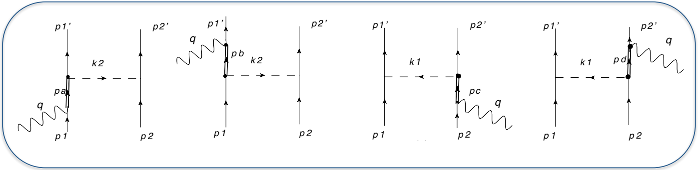


Meson-Exchange Currents



Seagull
or
contact
term

Pion
in
flight
term



The Extended Factorisation *ansatz*

- Highly accurate and consistent calculations of processes involving MEC can be carried out in the non relativistic regime
- Fully relativistic MEC used mainly within the Fermi gas model
- Using relativistic MEC and a realistic description of the nuclear ground state requires the extension of the IA scheme to two-nucleon emission amplitudes
  - Rewrite the hadronic final state $|n\rangle$ in the factorized form

$$|n\rangle \to |\mathbf{p}, \mathbf{p}'\rangle \otimes |n_{(A-2)}\rangle = |n_{(A-2)}, \mathbf{p}, \mathbf{p}'\rangle$$

$$\langle X|j^{\mu}_{ij}|0\rangle \to \int d^3k d^3k' M_n(\mathbf{k}, \mathbf{k}') \langle \mathbf{p}\mathbf{p}'|j^{\mu}_{ij}|\mathbf{k}\mathbf{k}'\rangle \, \delta(\mathbf{k}+\mathbf{k}'+\mathbf{q}-\mathbf{p}-\mathbf{p}')$$

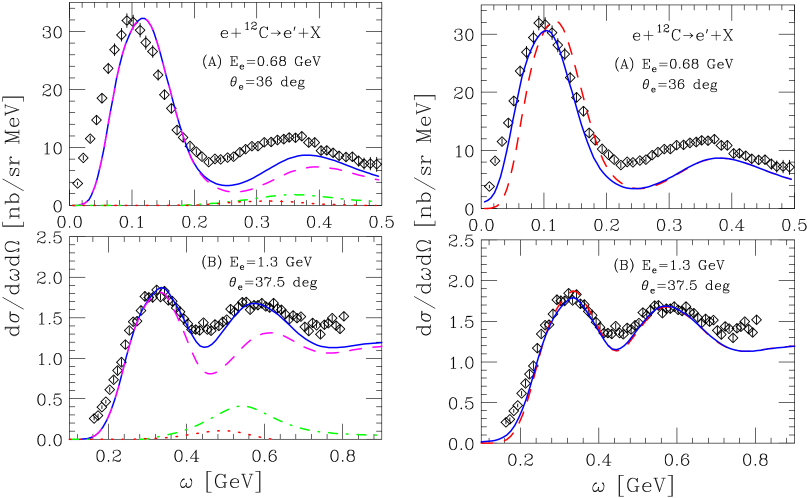
The amplitude

$$M_n(\mathbf{k}, \mathbf{k}') = \langle n_{(A-2)}, \mathbf{k}, \mathbf{k}'|0\rangle$$

is independent of q , and can be obtained from non relativistic many-body theory

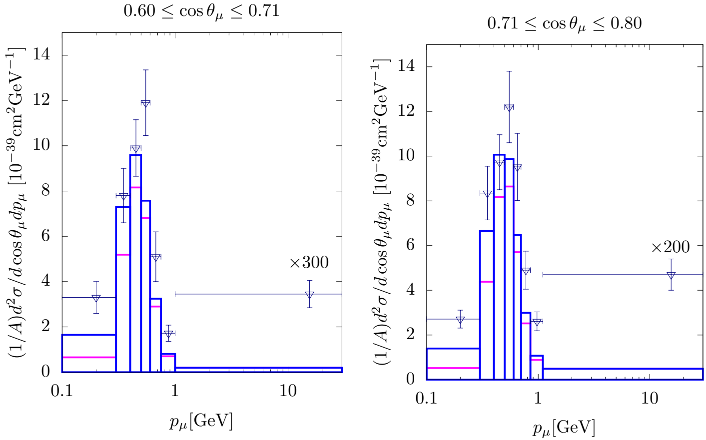
Pinning down FSI & MEC

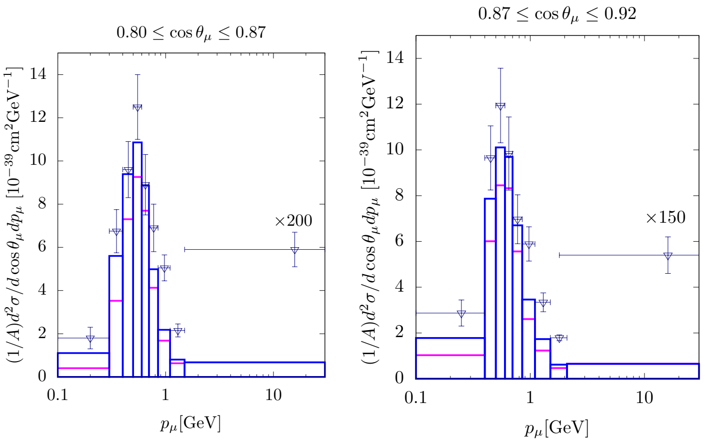
- N. Rocco, OB, and A. Lovato, PRL 116, 192501 (2016)



PRELIMINARY: FLUX-AVERAGED INCLUSIVE CROSS SETION

- ⋆ Comparison with the inclusive flux=integrated ν_μ-Carbon CC cross section measured by the T2K collabortion [PRD **98**. 012004 (2018)]. Inelastic structure functions provided by T. Sato, No MEC, no FSI.




$0.80 \leq \cos\theta_\mu \leq 0.87$
$(1/A)d^2\sigma/d\cos\theta_\mu dp_\mu$ [10^{-39}cm^{2}GeV^{-1}]
×200
p_μ[GeV]
$0.87 \leq \cos\theta_\mu \leq 0.92$
$(1/A)d^2\sigma/d\cos\theta_\mu dp_\mu$ [10^{-39}cm^{2}GeV^{-1}]
×150
p_μ[GeV]

SUMMARY & OUTLOOK

- Despite the complexity of flux average, a consistent description of the neutrino-nucleus cross in both elastic and inelastic channels appears to be possible within the approach based on factorisation.
- Most theoretical models employed of neutrino-nucleus interactions involve some level of factorisation. However, to fully exploit its potential, this scheme must be implemented using spectral functions providing an accurate description of the initial state. Valuable new information will be provided by electron scattering experiments, e.g. the measurement of the $^{40}_{18}\mathrm{Ar}(e, e'p)$ cross section performed in Jlab Hall A
- A better understanding of the interaction vertices, involving vector and axial form factors, and structure functions in the resonance production and DIS regions, is needed.
- The present development of the treatment of FSI, while being adequate to describe inclusive processes need to be improved to describe exclusive processes, and pin down the relevant reaction mechanisms.
- Long-range effects and the breakdown of factorisation at low momentum transfer need to be carefully investigated

Backup slides

Two-Nucleon Spectral Function

- ⋆ Calculations have been carried out for uniform isospin-symmetric nuclear matter

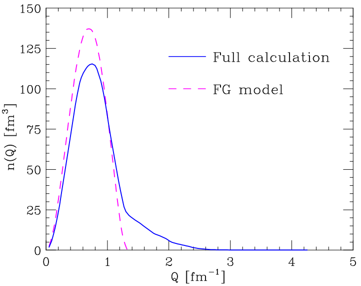
$$P(\mathbf{k}_1, \mathbf{k}_2, E) = \sum_n |M_n(k_1, k_2)|^2 \delta(E + E_0 - E_n)$$

$$n(\mathbf{k}_1, \mathbf{k}_2) = \int dE \, P(\mathbf{k}_1, \mathbf{k}_2, E)$$

- ⋆ Relative momentum distribution

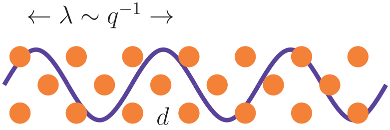
$$n(\mathbf{Q}) = 4\pi |\mathbf{Q}|^2 \int d^3q \, n\left(\frac{\mathbf{Q}}{2} + \mathbf{q}, \frac{\mathbf{Q}}{2} - \mathbf{q}\right)$$

$$\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2 \ , \quad \mathbf{Q} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}$$



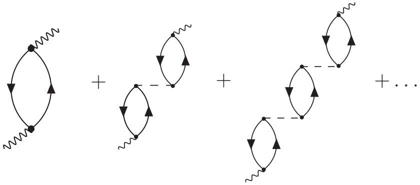
LONG-RANGE CORRELATIONS

- At low momentum transfer the space resolusion of the neutrino becomes much larger than the average NN separation distance (~ 1.5 fm), and the interaction involves many nucleons



- Write the nuclear final state as a superposition of 1p1h states (RPA scheme)

$$|n\rangle = \sum_{i=1}^{N} C_i \, |p_i h_i)$$



Tamm-Dancoff (ring) Approximation

- ⋆ Propagation of the particle-hole pair produced at the interaction vertex gives rise to a collective excitation. Replace

$$|ph\rangle \rightarrow |n\rangle = \sum_{i=1}^{N} C_i \, |p_i h_i)$$

- ⋆ The energy of the state $|n\rangle$ and the coefficients C_i are obtained diagonalizing the hamiltonian matrix

$$H_{ij} = (E_0 + e_{p_i} - e_{h_i})\delta_{ij} + (h_i p_i|V_{\rm eff}|h_j p_j)$$

$$e_k = \frac{k^2}{2m} + \sum_{\mathbf{k}'} \langle \mathbf{k}\mathbf{k}'|V_{\rm eff}|\mathbf{k}\mathbf{k}'\rangle_a$$

- ⋆ The appearance of an eigenvalue, ω_n, lying outside the particle-hole continuum signals the excitation of a collective mode

Beyond Factorisation: Long-Range Correlations

- $|\mathbf{q}|$-evolution of the density-response of isospin-symmetric nuclear matter, PLB 680, 305 (2009)

$|\mathbf{q}| \approx 480$ MeV

$|\mathbf{q}| \approx 300$ MeV

$|\mathbf{q}| \approx 60$ MeV