

# Coupled cluster theory for neutrino scattering

**Joanna Sobczyk**

In collaboration with  
Sonia Bacca  
Bijaya Acharya  
Gaute Hagen

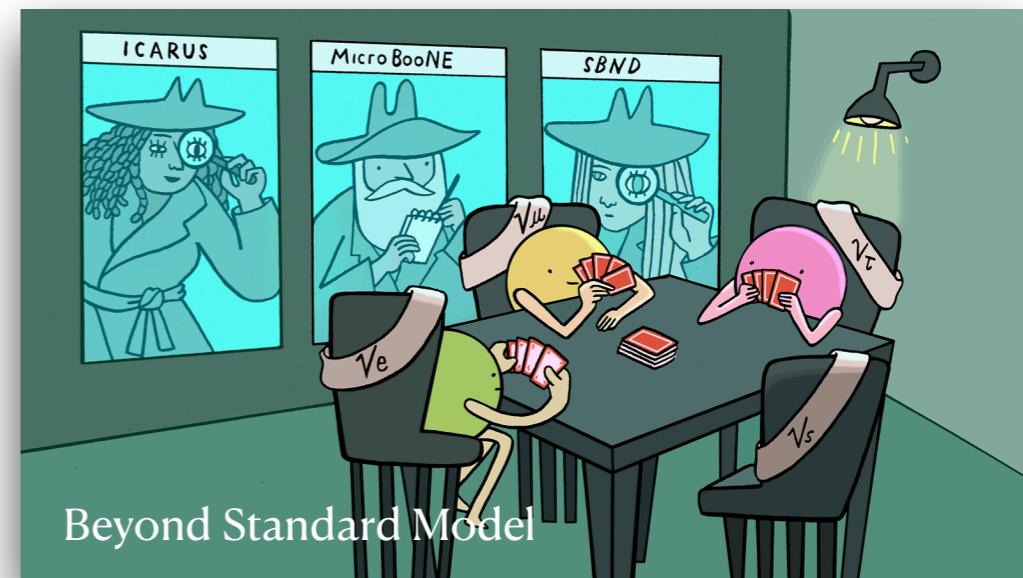
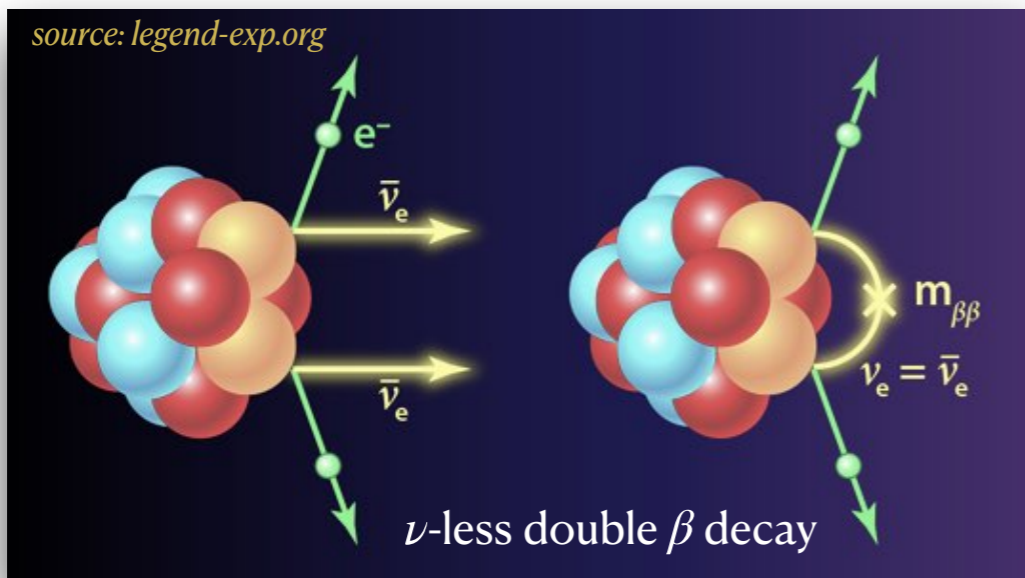
Neutrino–Nucleus Interactions in the Standard Model and Beyond  
CERN, 19/01/2022



JOHANNES GUTENBERG  
UNIVERSITÄT MAINZ

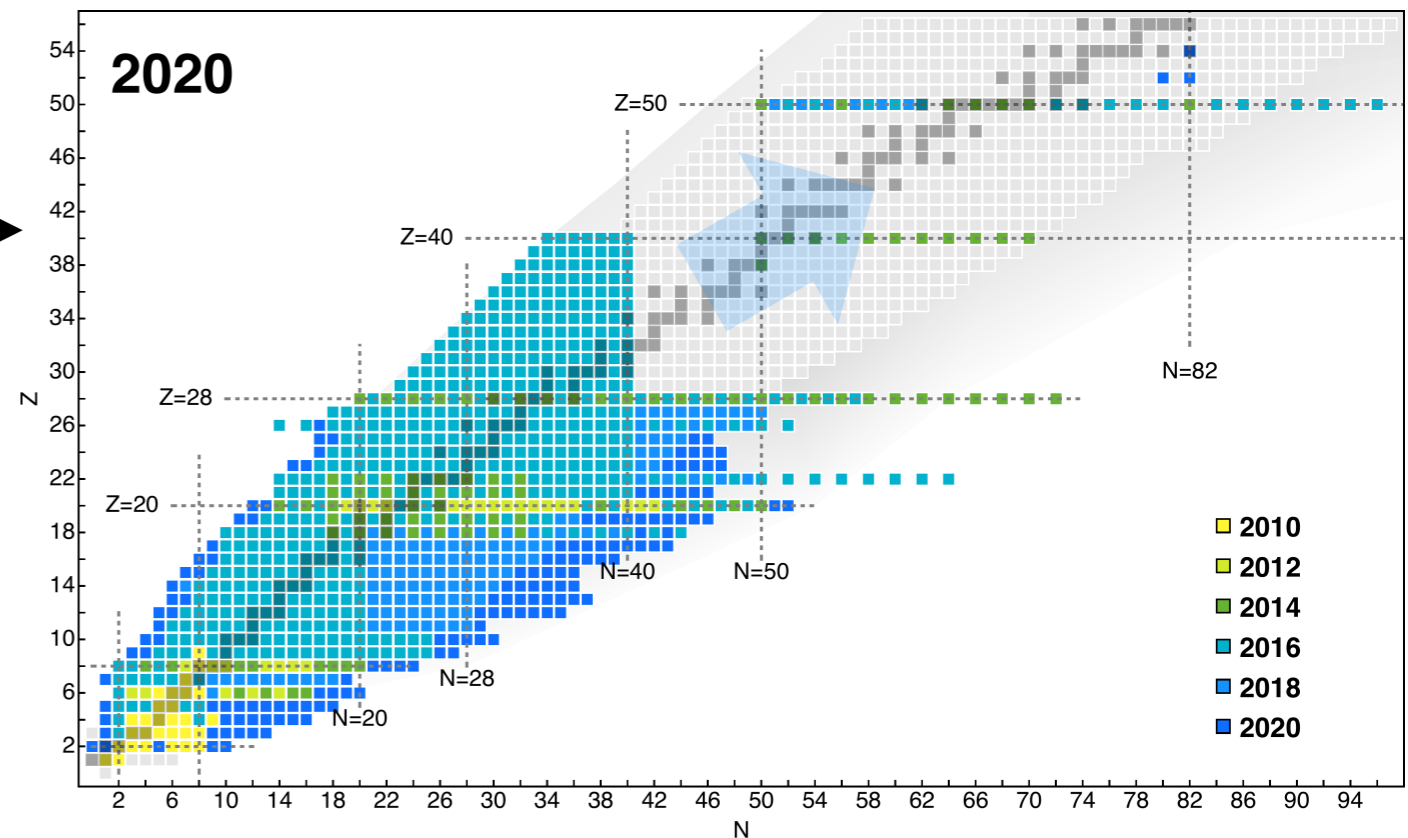
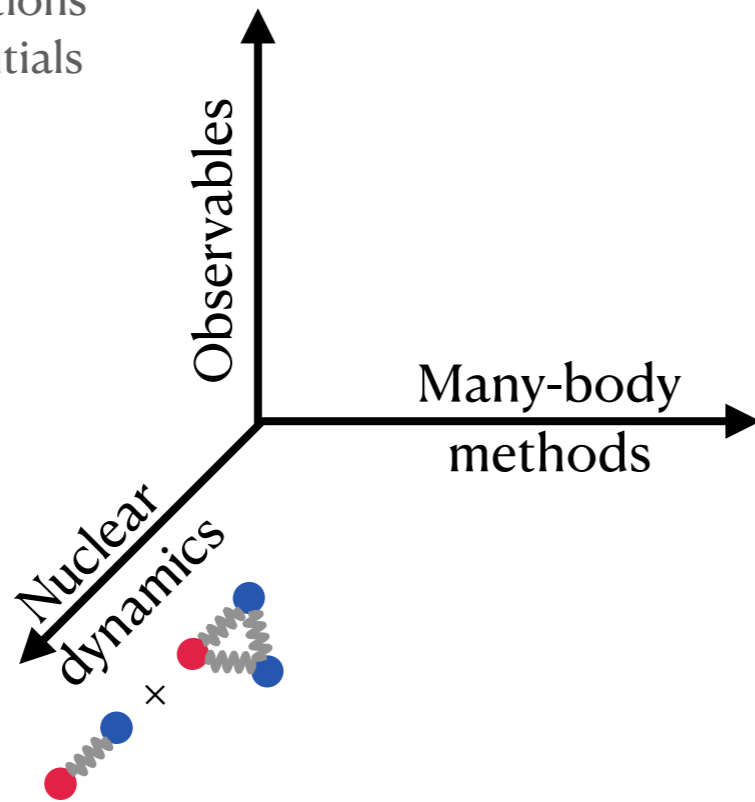


# Motivation



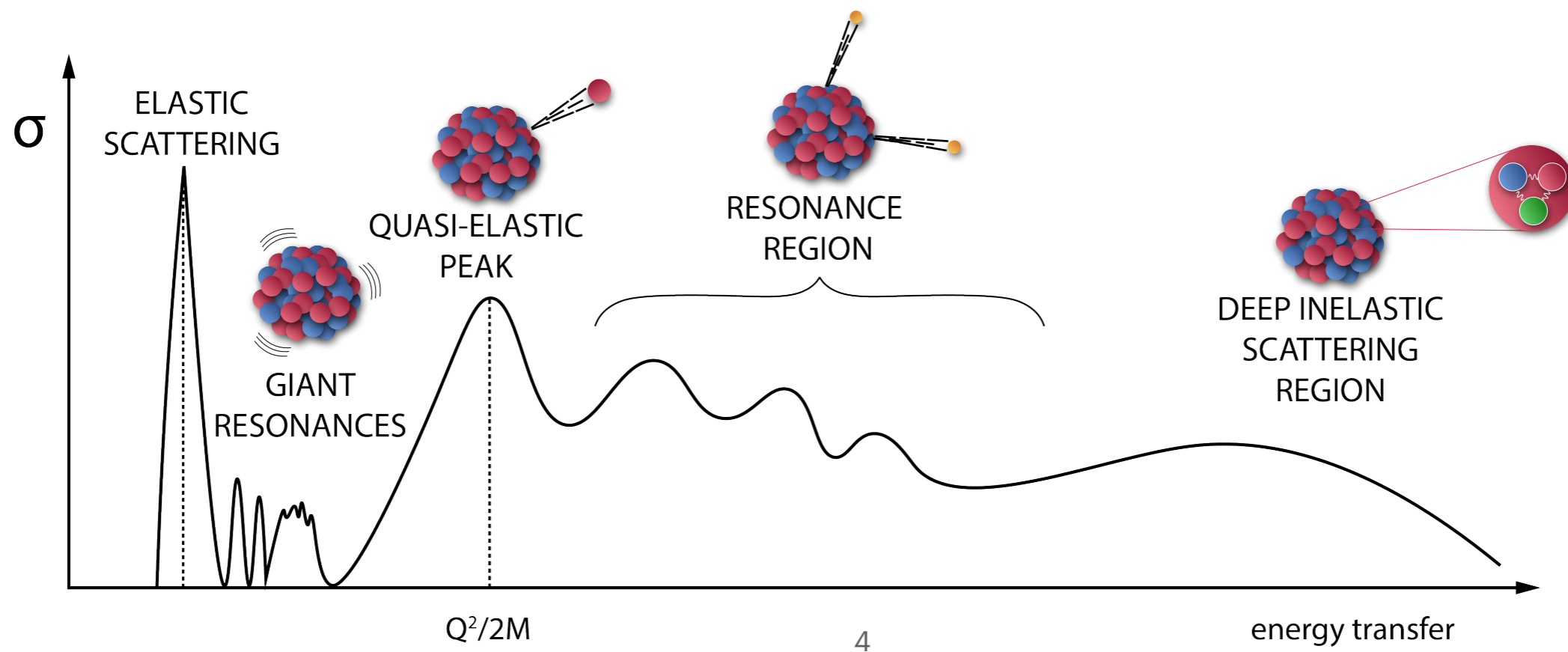
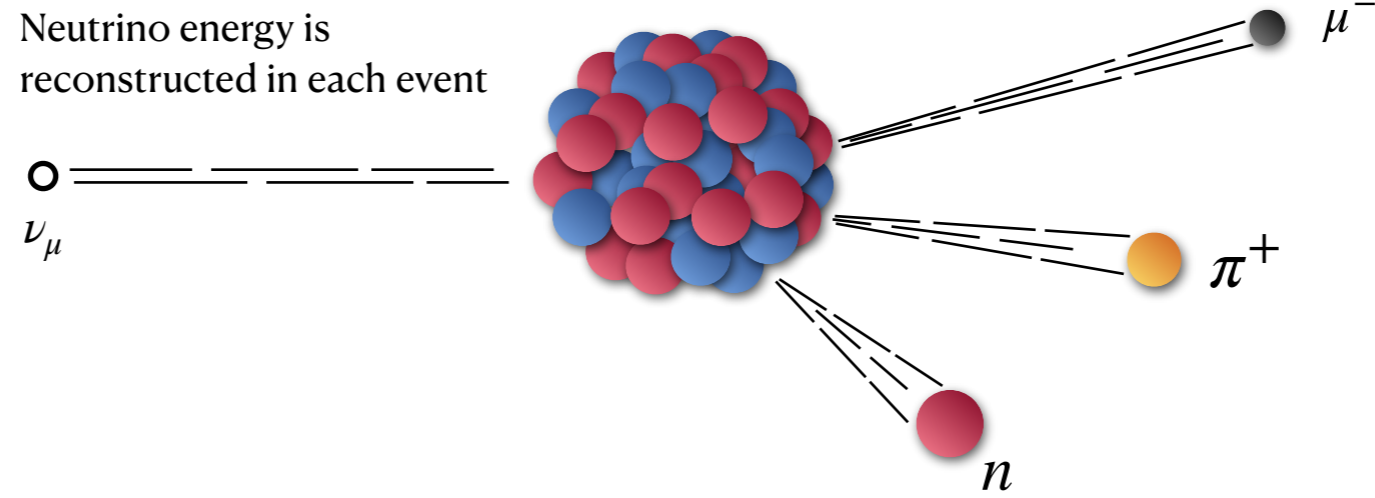
# Motivation

- ➔ Nuclear responses
- ➔ Spectral functions
- ➔ Optical potentials
- ...

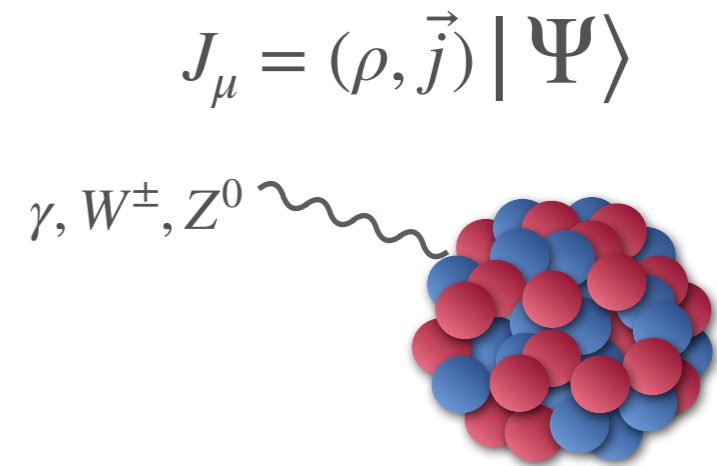
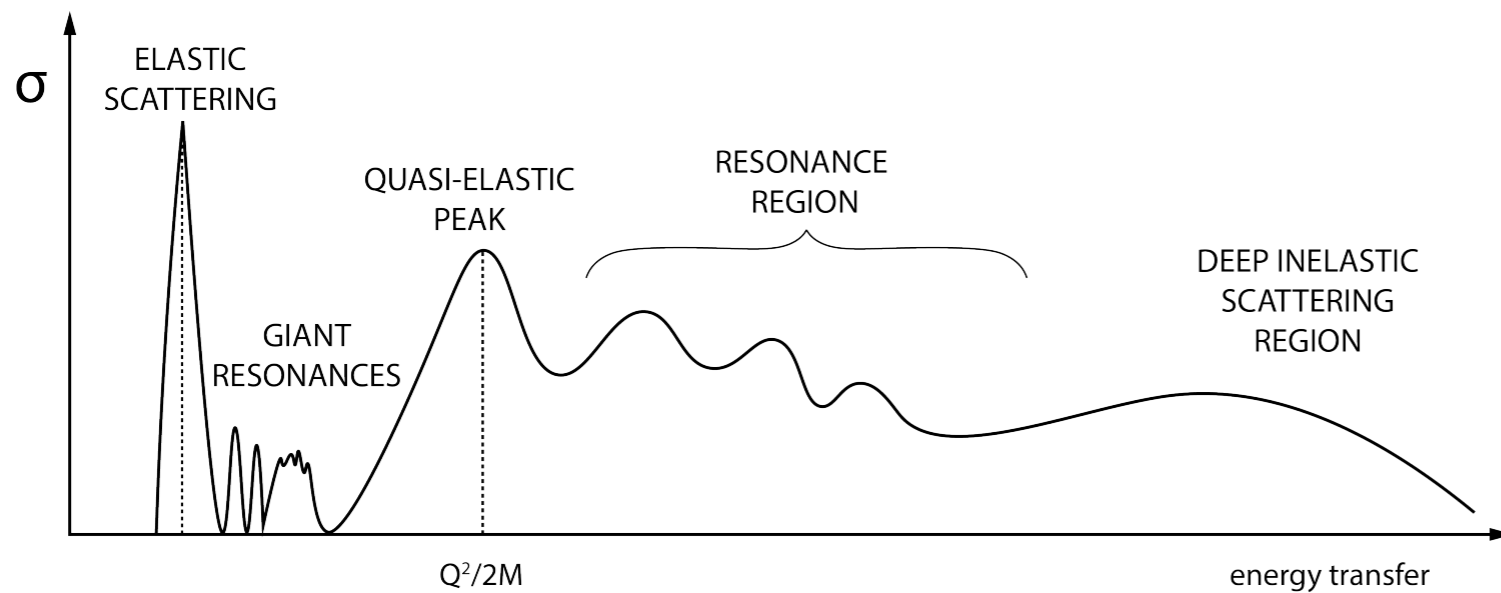


- ➔ Neutrinos challenge ab initio nuclear theory
- ➔ Controllable approximations within ab initial nuclear theory

# Motivation



# Nuclear response



$$\sigma \propto L^{\mu\nu} R_{\mu\nu}$$

lepton tensor    nuclear responses

$$R_{\mu\nu}(\omega, q) = \sum_f \langle \Psi | J_\mu^\dagger(q) | \Psi_f \rangle \langle \Psi_f | J_\nu(q) | \Psi \rangle \delta(E_0 + \omega - E_f)$$

# Ab initio nuclear theory for neutrinos

Nuclear Hamiltonian

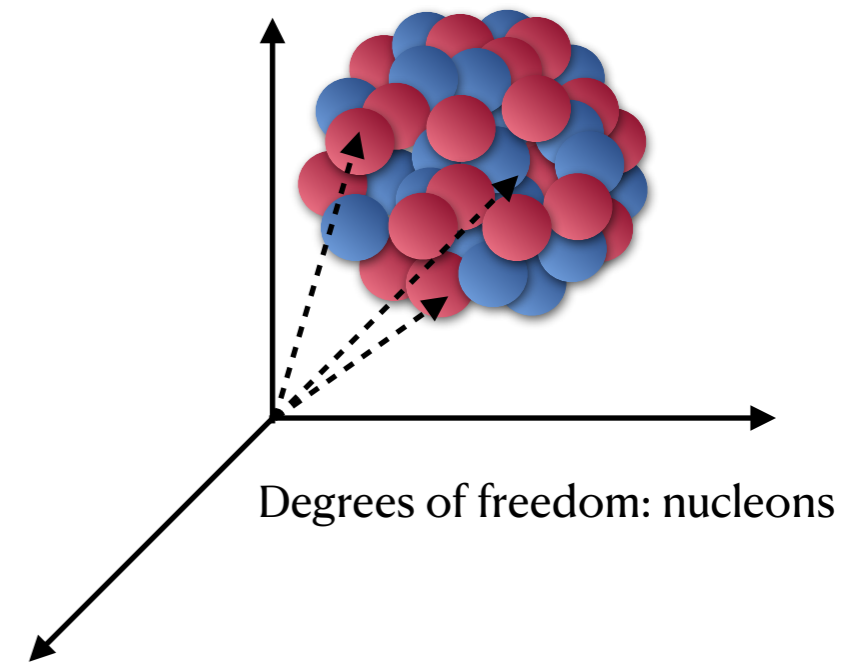
$$\mathcal{H} |\Psi\rangle = E |\Psi\rangle$$

Electroweak currents

$$J^\mu = (\rho, \vec{j})$$

Many-body method

$$\mathcal{A} = \langle \Psi_m | J_\mu | \Psi_n \rangle$$



# Nuclear hamiltonian

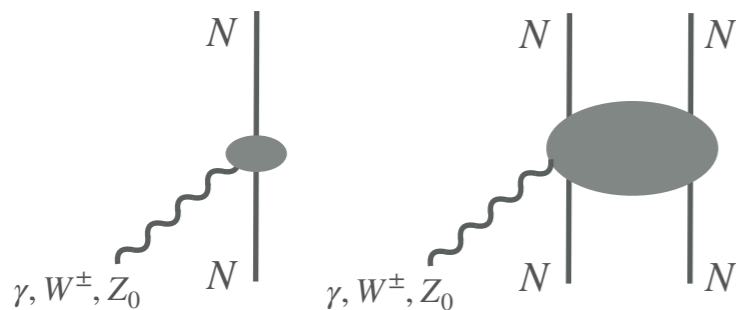
$$\mathcal{H} = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

		2N force	3N force	4N force
$n = 0$	LO			
$n = 2$	NLO			
$n = 3$	N2LO			
$n = 4$	N3LO			

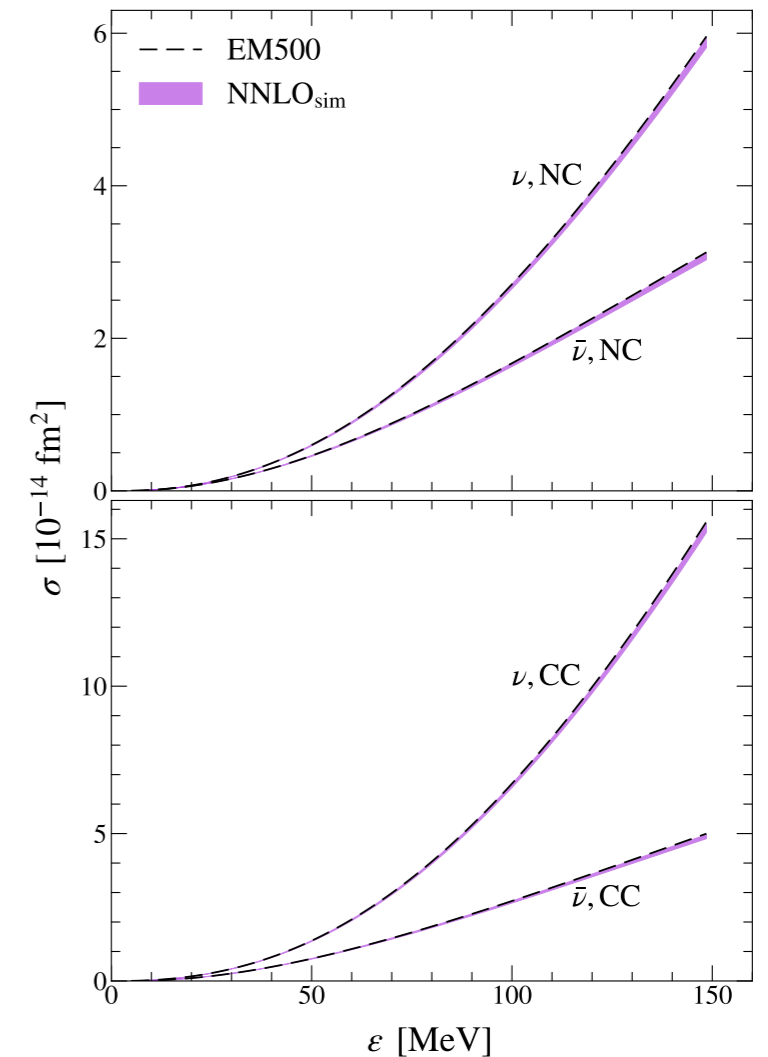
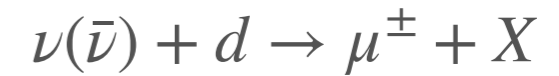
- ✓ Chiral Hamiltonians exploiting chiral symmetry (QCD);  
 $\pi, N, (\Delta)$  degrees of freedom
- ✓ counting scheme in  $\left(\frac{Q}{\Lambda}\right)^n$
- ✓ low energy constants (LEC) fit to data
- ✓ uncertainty assessment

# Electroweak currents

$$J = \sum_i J_i + \sum_{i < j} J_{ij} + \dots$$



known to give significant contribution for neutrino-nucleus scattering



Multipole decomposition for 1- and 2-body EW currents

B. Acharya, S. Bacca

Phys.Rev.C 101 (2020) 1, 015505

Current decomposition into multipoles needed for various *ab initio* methods: CC, No Core Shell Model, In-Medium Similarity Renormalization Group



# Coupled cluster method

Reference state (Hartree-Fock):  $|\Psi\rangle$

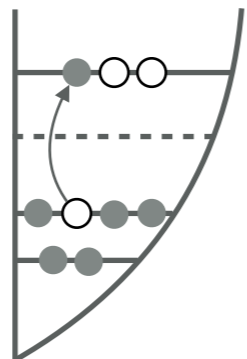
Include correlations through  $e^T$  operator

similarity transformed  
Hamiltonian (non-Hermitian)

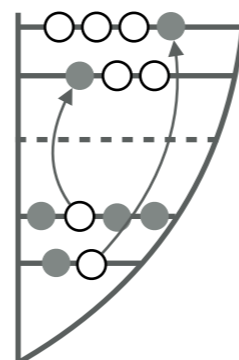
$$e^{-T} \mathcal{H} e^T |\Psi\rangle \equiv \bar{\mathcal{H}} |\Psi\rangle = E |\Psi\rangle$$

Expansion:  $T = \sum t_a^i a_a^\dagger a_i + \sum t_{ab}^{ij} a_a^\dagger a_b^\dagger a_i a_j + \dots$

singles



doubles



← coefficients obtained  
through coupled cluster  
equations

# Coupled cluster method

- ✓ Controlled approximation through truncation in  $T$
- ✓ Polynomial scaling with  $A$  (predictions for  $^{132}\text{Sn}$  and  $^{208}\text{Pb}$ )
- ✓ Works most efficiently for doubly magic nuclei

nature  
physics

LETTERS

<https://doi.org/10.1038/s41567-019-0450-7>

## Discrepancy between experimental and theoretical $\beta$ -decay rates resolved from first principles

P. Gysbers<sup>1,2</sup>, G. Hagen<sup>3,4\*</sup>, J. D. Holt<sup>5</sup>, G. R. Jansen<sup>6,5</sup>, T. D. Morris<sup>3,4,6</sup>, P. Navrátil<sup>6</sup>, T. Papenbrock<sup>6,3,4</sup>, S. Quaglioni<sup>6,7</sup>, A. Schwenk<sup>8,9,10</sup>, S. R. Stroberg<sup>1,11,12</sup> and K. A. Wendt<sup>7</sup>

## Coupled-Cluster Calculations of Neutrinoless Double- $\beta$ Decay in $^{48}\text{Ca}$

S. Novario,<sup>1,2</sup> P. Gysbers,<sup>3,4</sup> J. Engel<sup>5</sup>, G. Hagen,<sup>2,1,3</sup> G. R. Jansen<sup>6,2</sup>, T. D. Morris,<sup>2</sup> P. Navrátil<sup>6,3</sup>, T. Papenbrock<sup>6,1,2</sup> and S. Quaglioni<sup>6,7</sup>

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<sup>2</sup>Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA

<sup>3</sup>TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

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<sup>5</sup>Department of Physics, University of North Carolina, Chapel Hill, North Carolina 27514, USA

<sup>6</sup>National Center for Computational Sciences, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA

<sup>7</sup>Lawrence Livermore National Laboratory, P.O. Box 808, L-414, Livermore, California 94551, USA

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We use coupled-cluster theory and nuclear interactions from chiral effective field theory to compute the nuclear matrix element for the neutrinoless double- $\beta$  decay of  $^{48}\text{Ca}$ . Benchmarks with the no-core shell model in several light nuclei inform us about the accuracy of our approach. For  $^{48}\text{Ca}$  we find a relatively small matrix element. We also compute the nuclear matrix element for the two-neutrino double- $\beta$  decay of  $^{48}\text{Ca}$  with a quenching factor deduced from two-body currents in recent *ab initio* calculation of the Ikeda sum rule in  $^{48}\text{Ca}$  [Gysbers *et al.*, *Nat. Phys.* **15**, 428 (2019)].

## *Ab initio* predictions link the neutron skin of $^{208}\text{Pb}$ to nuclear forces

Baishan Hu,<sup>1,\*</sup> Weiguang Jiang,<sup>2,\*</sup> Takayuki Miyagi,<sup>1,3,\*</sup> Zhonghao Sun,<sup>4,5,\*</sup> Andreas Ekström,<sup>2</sup> Christian Forssén,<sup>2,†</sup> Gaute Hagen,<sup>5,4,1</sup> Jason D. Holt,<sup>1,6</sup> Thomas Papenbrock,<sup>4,5</sup> S. Ragnar Stroberg,<sup>7,8</sup> and Ian Vernon<sup>9</sup>

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<sup>2</sup>Department of Physics, Chalmers University of Technology, SE-412 96 Göteborg, Sweden

<sup>3</sup>Technische Universität Darmstadt, Department of Physics, 64289 Darmstadt, Germany

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<sup>5</sup>Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA

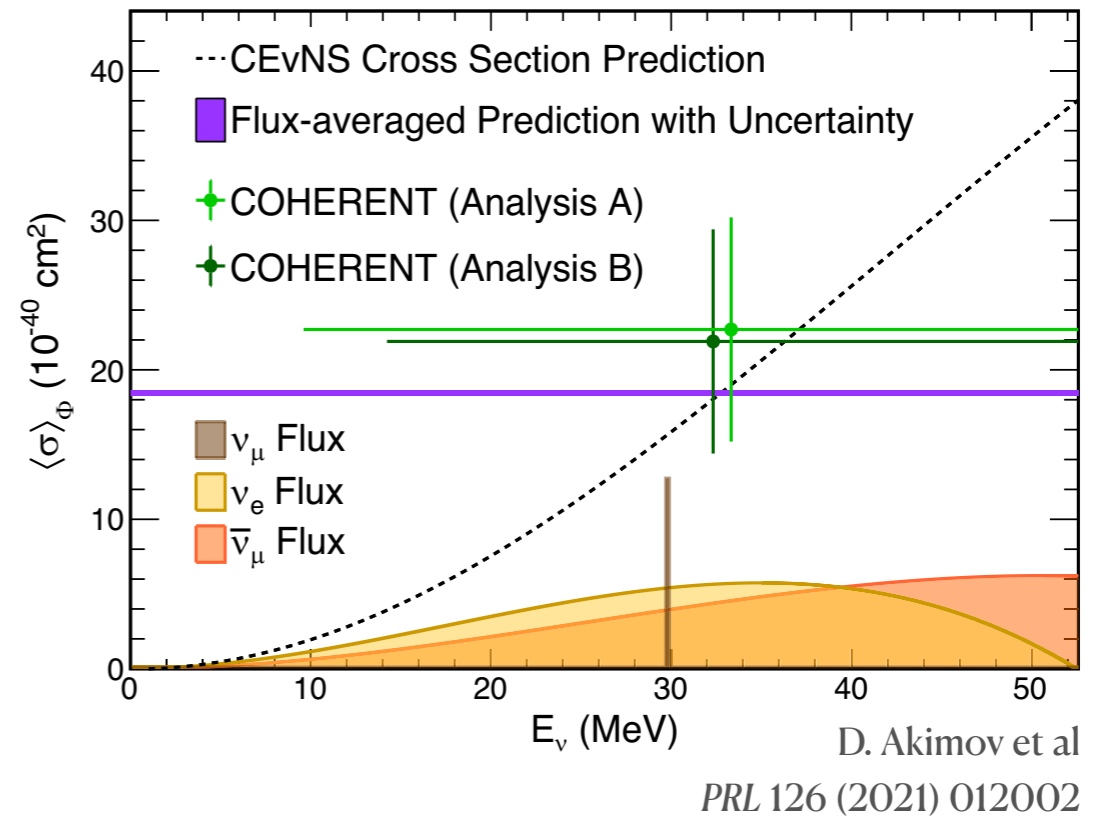
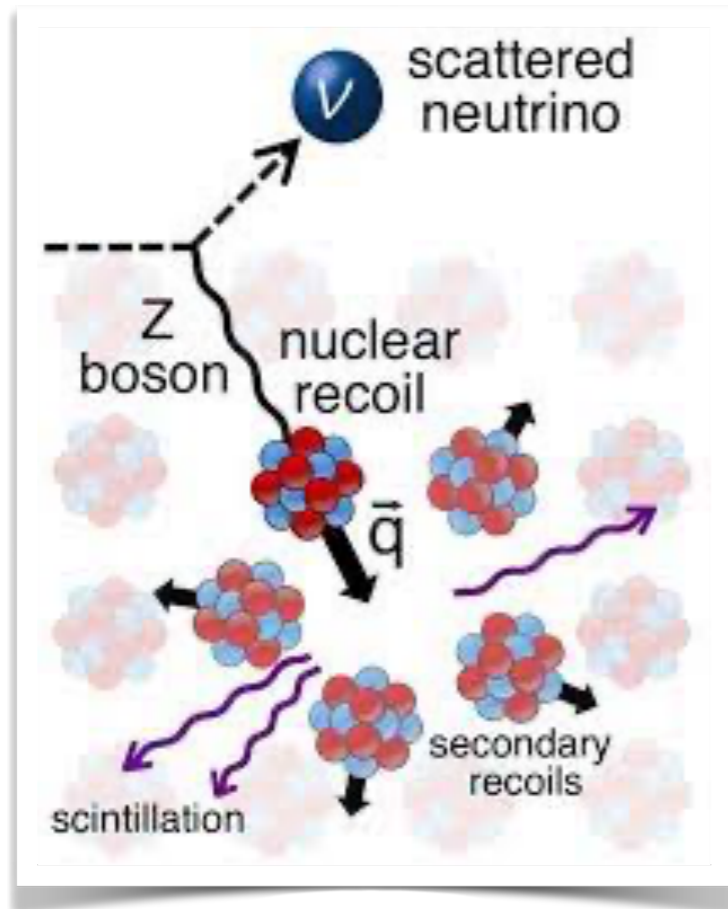
<sup>6</sup>Department of Physics, McGill University, 3600 Rue University, Montréal, QC H3A 2T8, Canada

<sup>7</sup>Department of Physics, University of Washington, Seattle, Washington 98195, USA

<sup>8</sup>Physics Division, Argonne National Laboratory, Lemont, Illinois 60439, USA

<sup>9</sup>Department of Mathematical Sciences, University of Durham, South Road, Durham, DH1 3LE, UK

# CEvNS on $^{40}\text{Ar}$



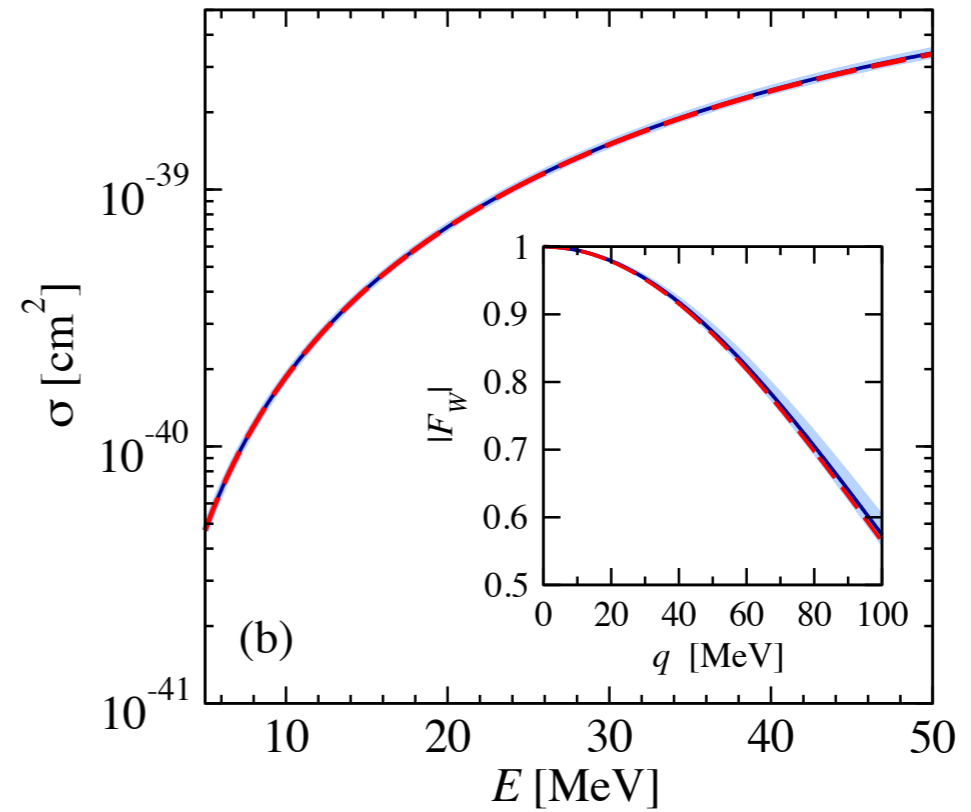
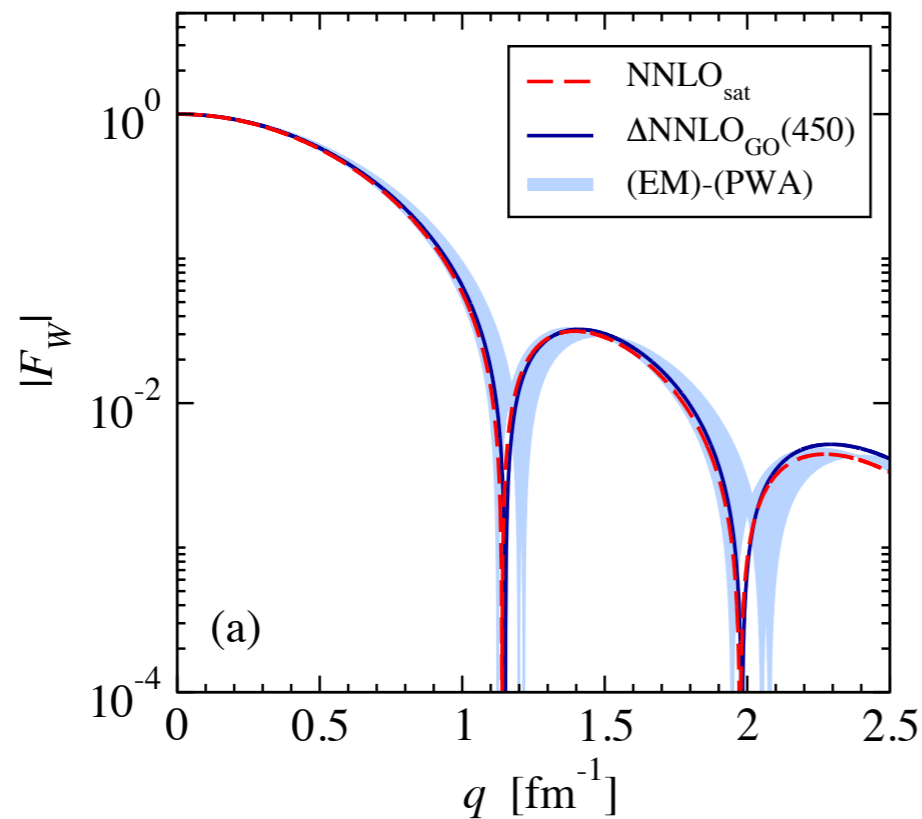
$$\frac{d\sigma}{dT}(E_\nu, T) \simeq \frac{G_F^2}{4\pi} M \left[ 1 - \frac{MT}{2E_n u^2} \right] Q_W^2 F_W^2(q^2) \propto N^2$$

$$Q_W = N - (1 - 4 \sin^2 \theta_W) Z$$

$$F_W(q^2) = \frac{1}{Q_W} \left[ N F_n(q^2) - (1 - 4 \sin^2 \theta_W) Z F_p(q^2) \right]$$

# CEvNS on $^{40}\text{Ar}$

$$F_W(q^2) = \frac{1}{Q_W} \left[ NF_n(q^2) - (1 - 4 \sin^2 \theta_W) ZF_p(q^2) \right]$$



C. Payne et al.

*Phys.Rev.C* 100 (2019) 6, 061304

- Coupled-cluster predictions for the weak form factor and cross section
- Small theoretical uncertainty

# Lorentz Integral Transform

$$R_{\mu\nu}(\omega, q) = \int_f \langle \Psi | J_\mu^\dagger | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi \rangle \delta(E_0 + \omega - E_f)$$

continuum spectrum

Instead we calculate

$$S_{\mu\nu}(\omega, q) = \int d\sigma K(\omega, \sigma) R_{\mu\nu}(\omega, q) = \int d\sigma \langle \Psi | J_\mu^\dagger K(\mathcal{H} - E_0, \sigma) J_\nu | \Psi \rangle$$

$S_{\mu\nu}$  has to be inverted to get access to  $R_{\mu\nu}$

Lorentzian kernel:

$$K_\Lambda(\omega, \sigma) = \frac{1}{\pi} \frac{\Lambda}{\Lambda^2 + (\omega - \sigma)^2}$$

→ LIT-CC used for photo-absorption

# Electrons for neutrinos

$$\left. \frac{d\sigma}{d\omega dq} \right|_{\nu/\bar{\nu}} = \sigma_0 \left( v_{CC} R_{CC} + v_{CL} R_{CL} + v_{LL} R_{LL} + v_T R_T \pm v_{T'} R_{T'} \right)$$

$$\left. \frac{d\sigma}{d\omega dq} \right|_e = \sigma_M \left( v_L R_L + v_T R_T \right)$$

- ✓ much more precise data
- ✓ we can get access to  $R_L$  and  $R_T$  separately (Rosenbluth separation)
- ✓ experimental programs of electron scattering in JLab, MESA

# Quasielastic response

- Momentum transfer  
~hundreds MeV
- Upper limit for ab initio methods
- Important mechanism for T2HK, DUNE
- Role of final state interactions
- Role of 1-body and 2-body currents

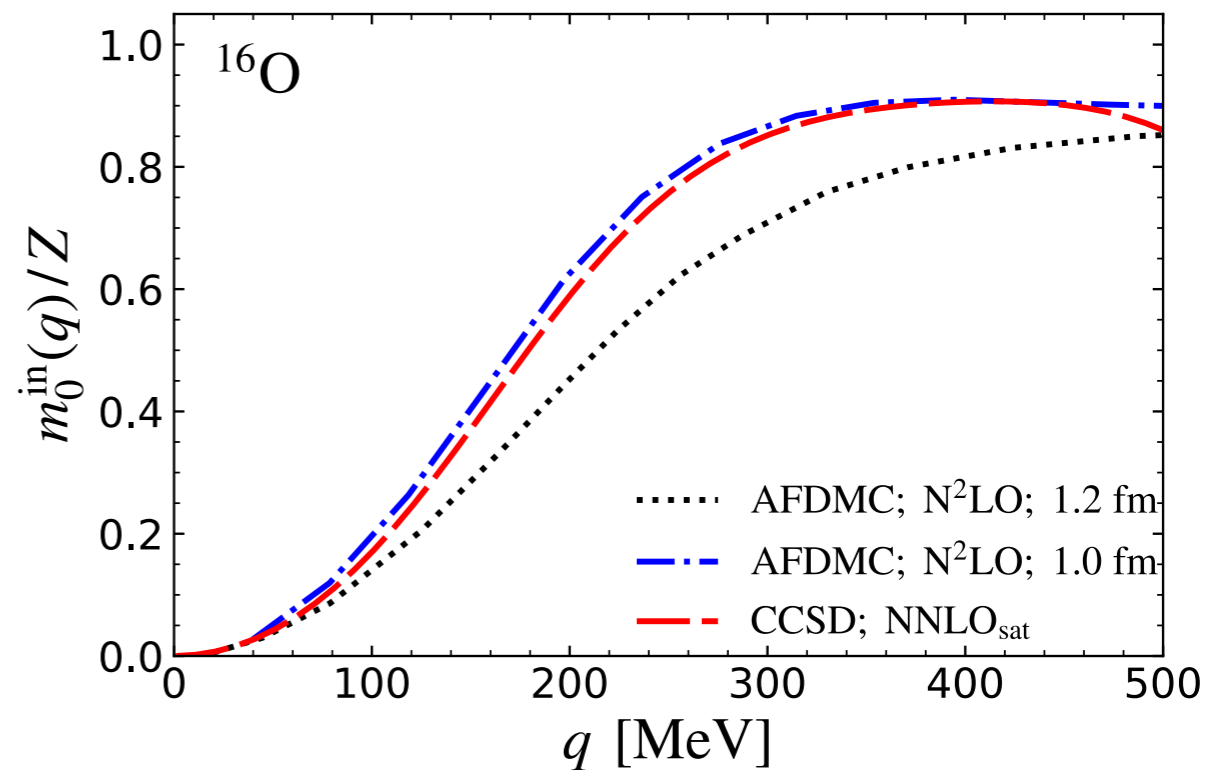
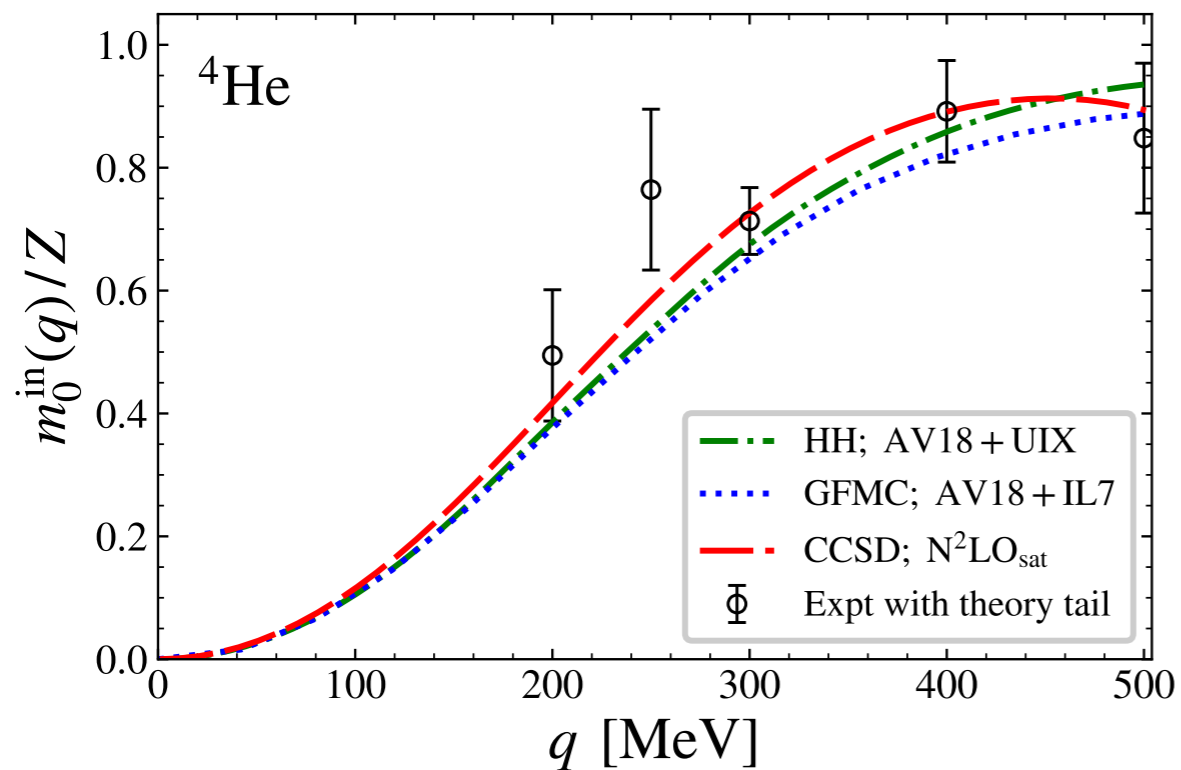
First step: analyse the longitudinal response

$$\left. \frac{d\sigma}{d\omega dq} \right|_e = \sigma_M \left( v_L R_L + v_T R_T \right)$$

$$\text{charge operator } \hat{\rho}(q) = \sum_{j=1}^Z e^{iqz'_j}$$

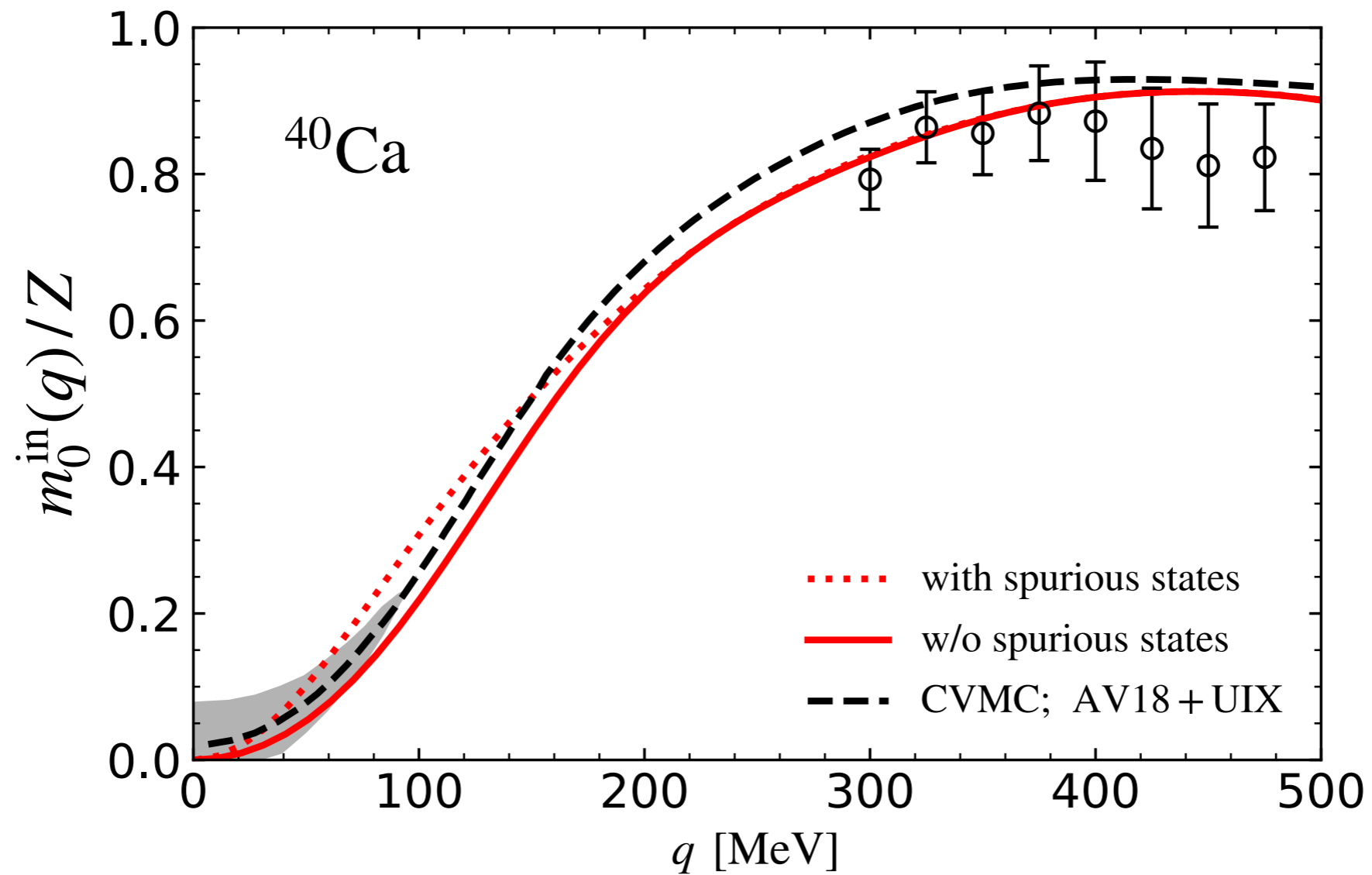
# Coulomb sum rule

$$m_0(q) = \int d\omega R_L(\omega, q) = \sum_{f \neq 0} |\langle \Psi_f | \hat{\rho} | \Psi \rangle|^2 = \langle \Psi | \hat{\rho}^\dagger \hat{\rho} | \Psi \rangle - |F_{el}(q)|^2$$

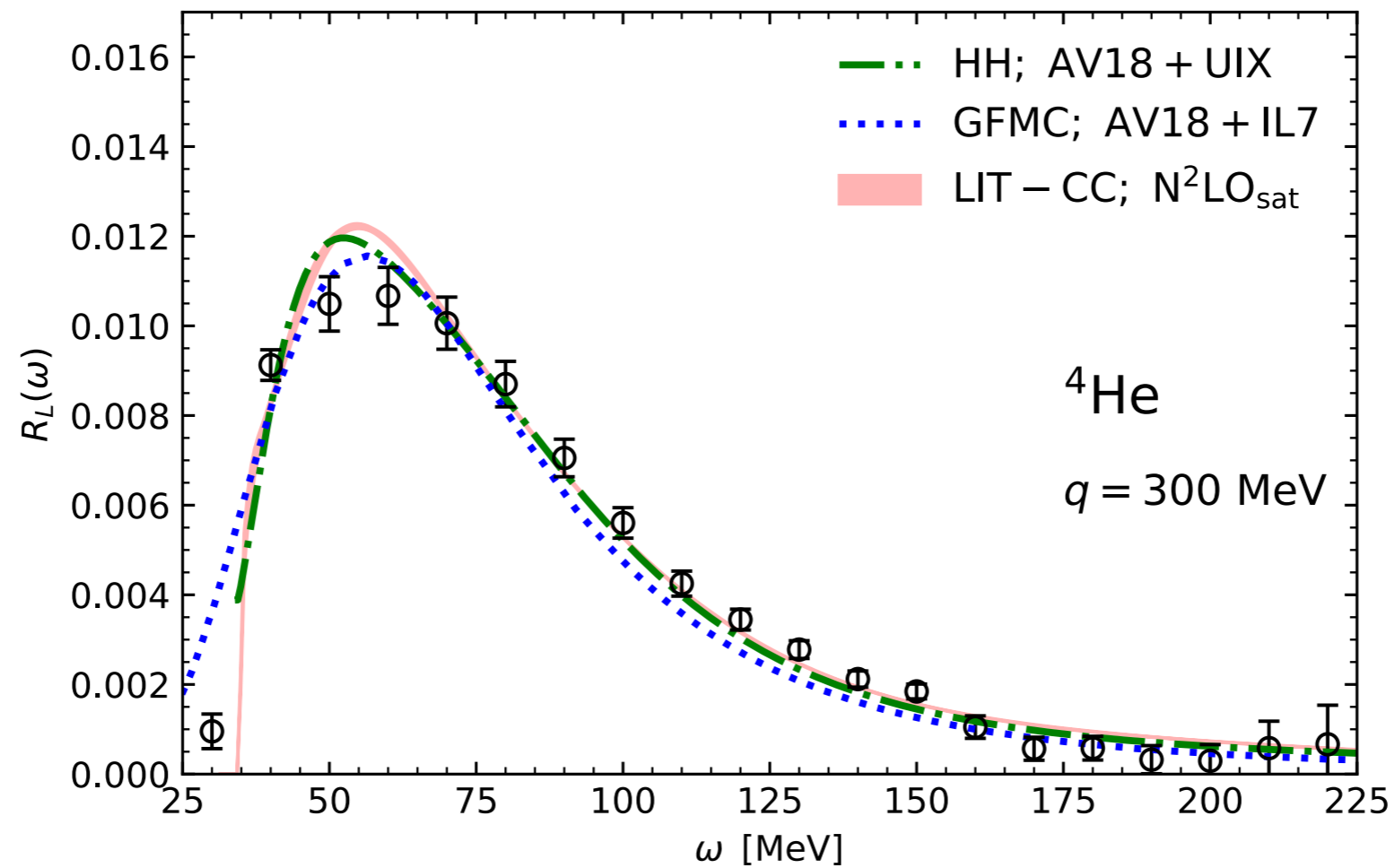




# Coulomb sum rule

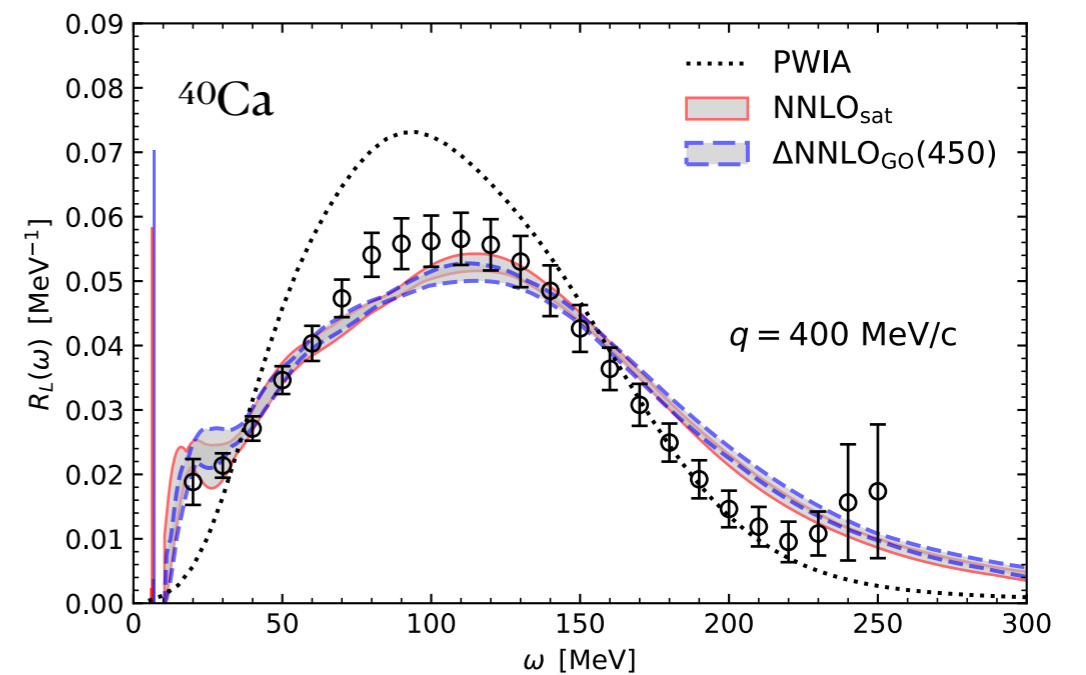
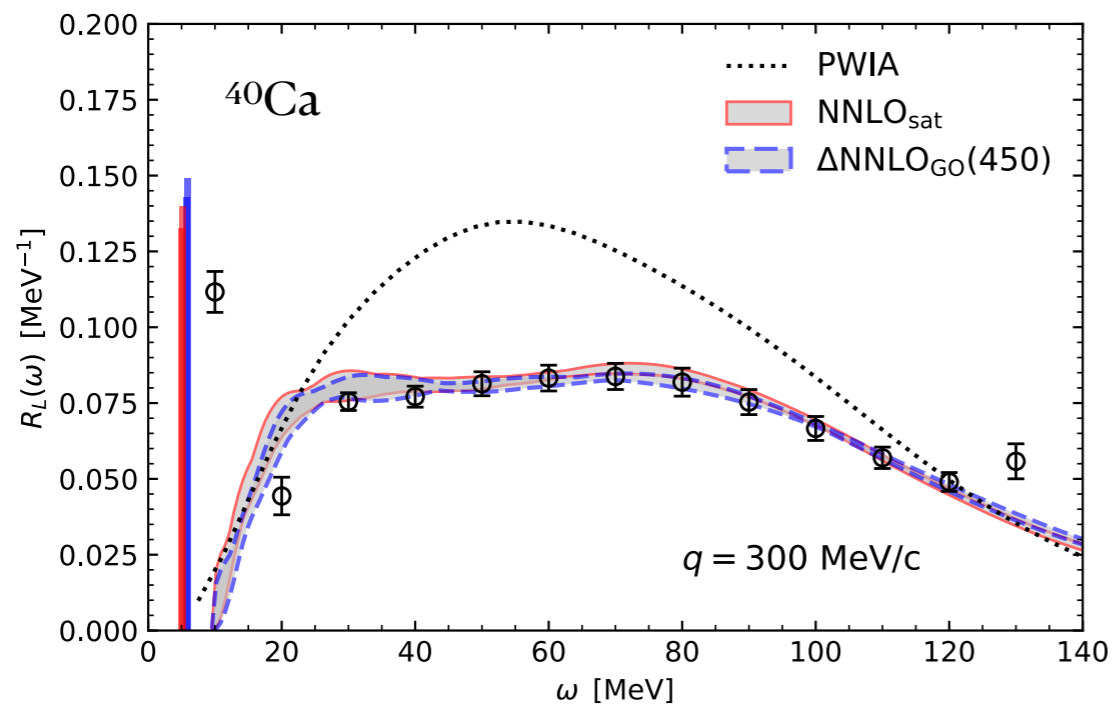


# Longitudinal response



Uncertainty band: inversion procedure

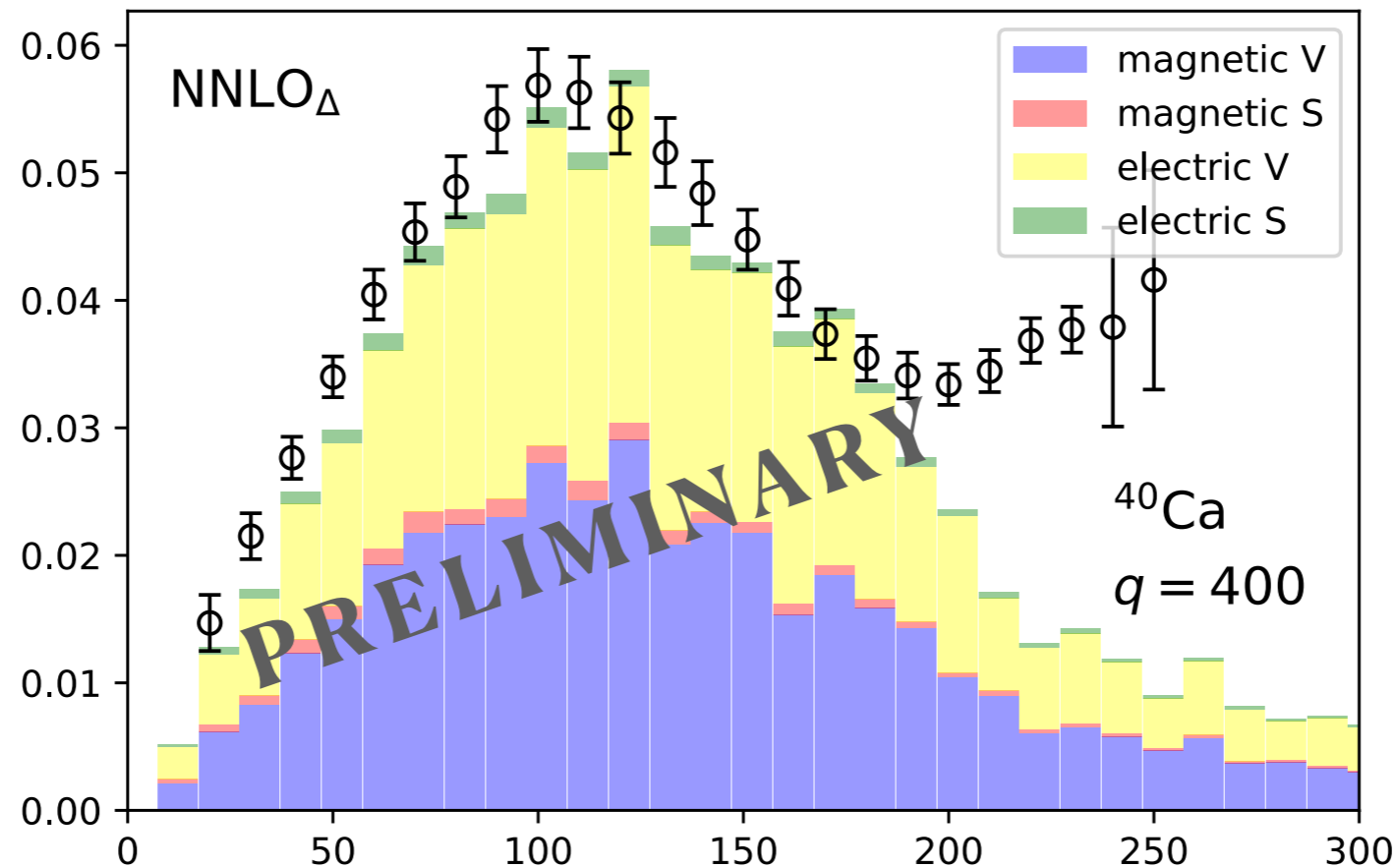
# Longitudinal response



*PRL* 127 (2021) 7, 072501 JES, B. Acharya, S. Bacca, G. Hagen

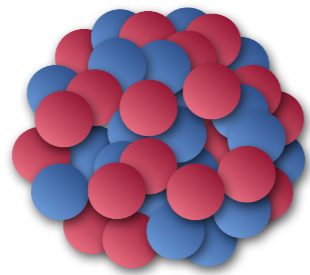
First ab-initio results for many-body system of 40 nucleons

# Transverse response



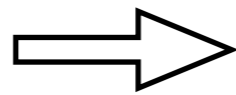
- ➔ This allows to predict electron-nucleus cross-section
- ➔ Currently only 1-body current
- ➔ Response obtained using expansion in Chebyshev polynomials instead of LIT (no inversion procedure)

# Low/high energies

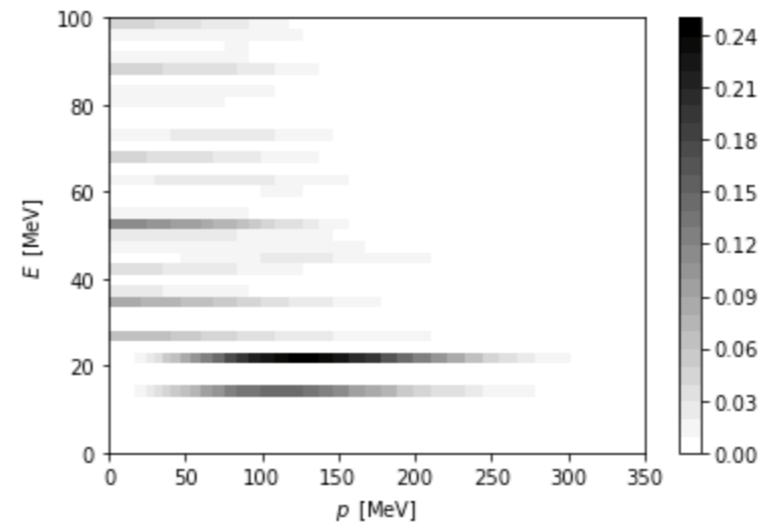


$$\hat{H}|\psi_A\rangle = E|\psi_A\rangle$$

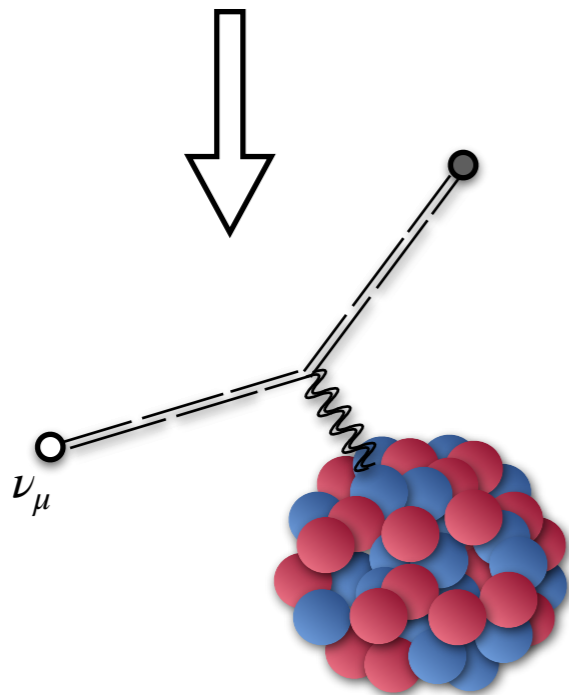
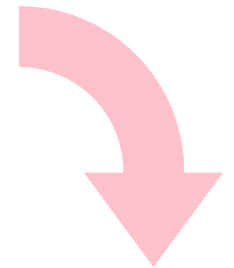
Many-body problem



Spectral function

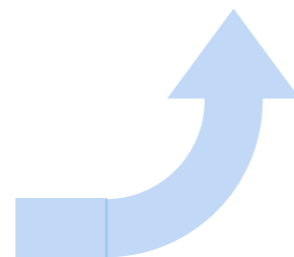
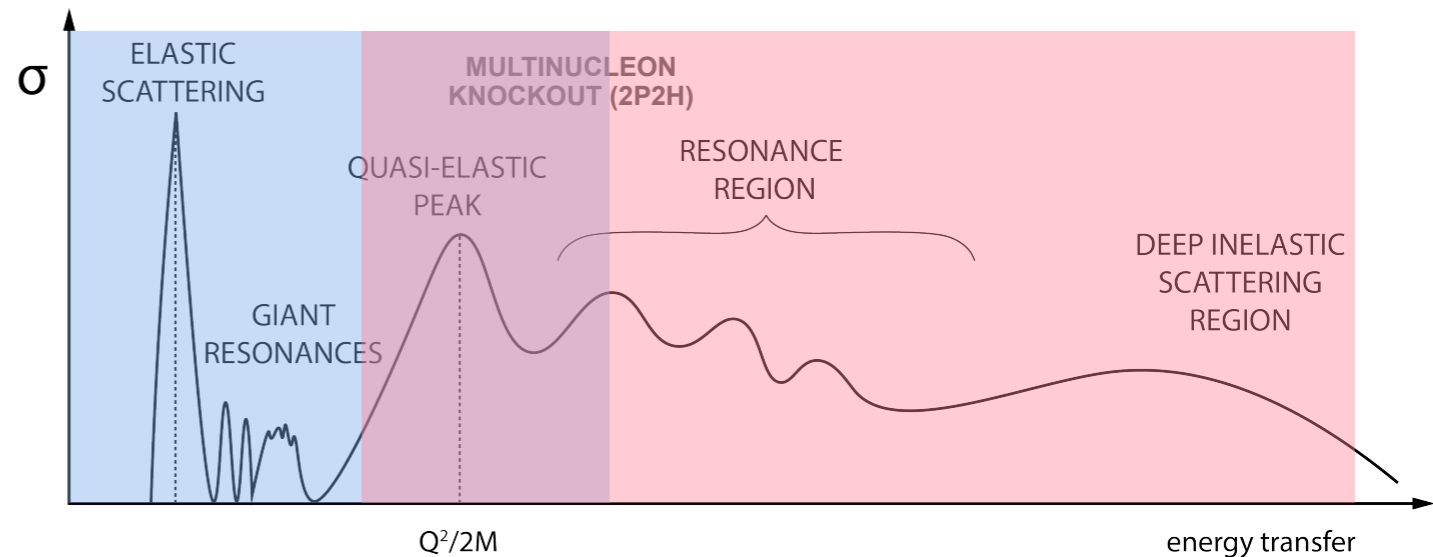


Probability density of finding nucleon  $(E, \mathbf{p})$  in ground state nucleus

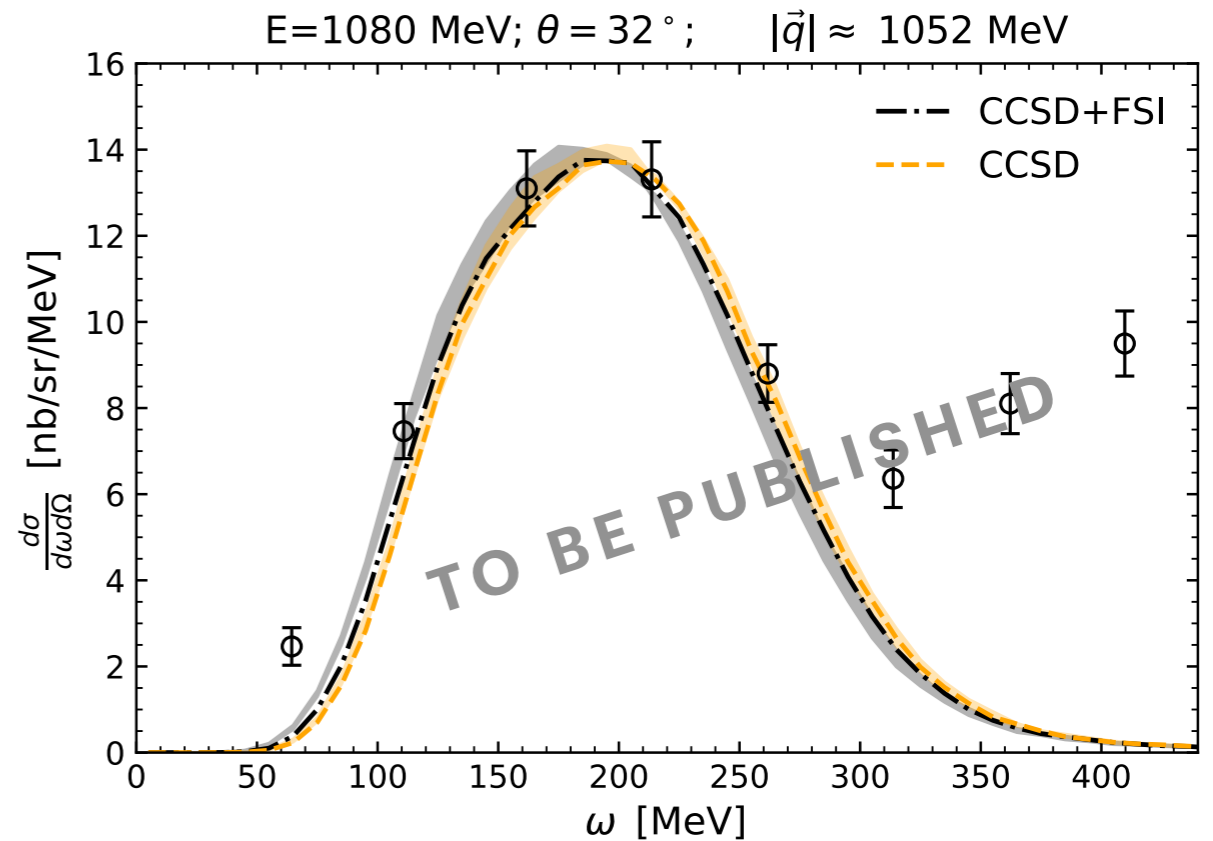
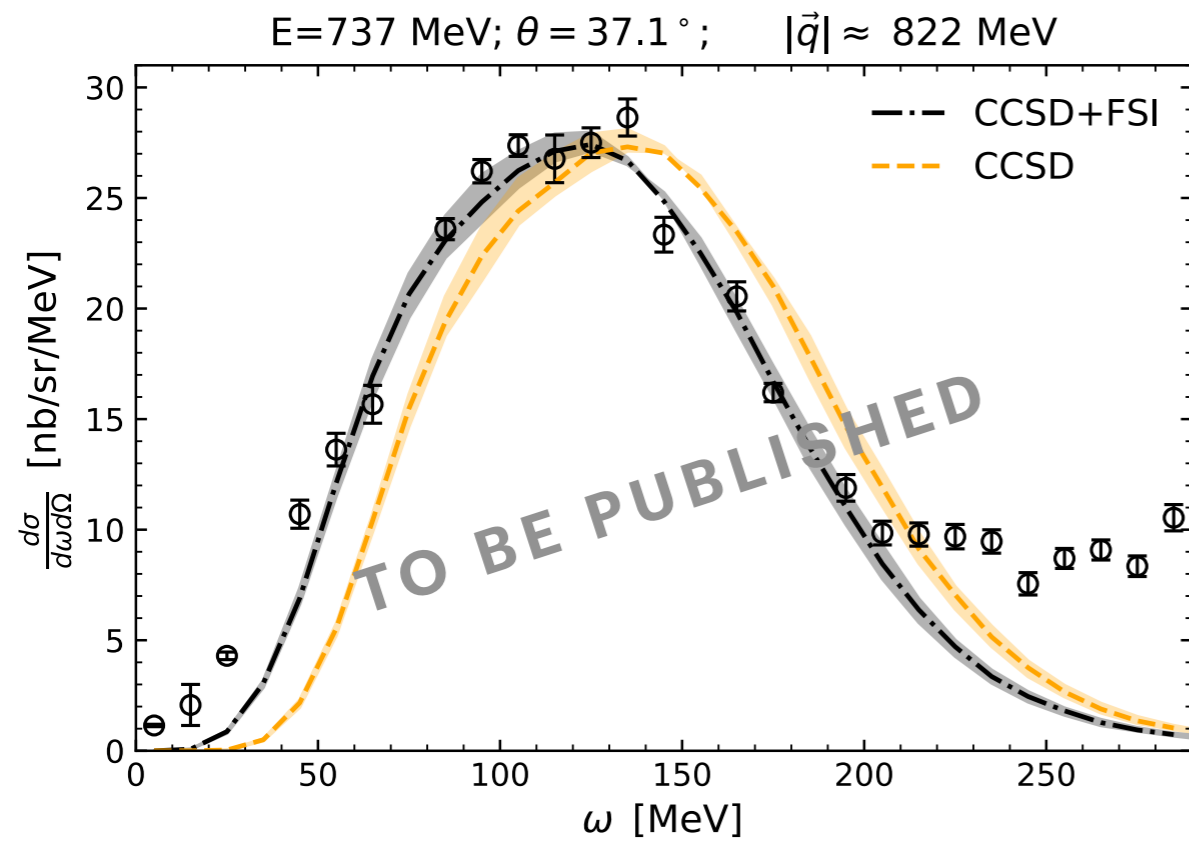


$$\langle \psi_f | \hat{j} | \psi_A \rangle$$

Electroweak responses



# Spectral function from coupled cluster



JES et al, in preparation (2022)

# Summary & Outlook

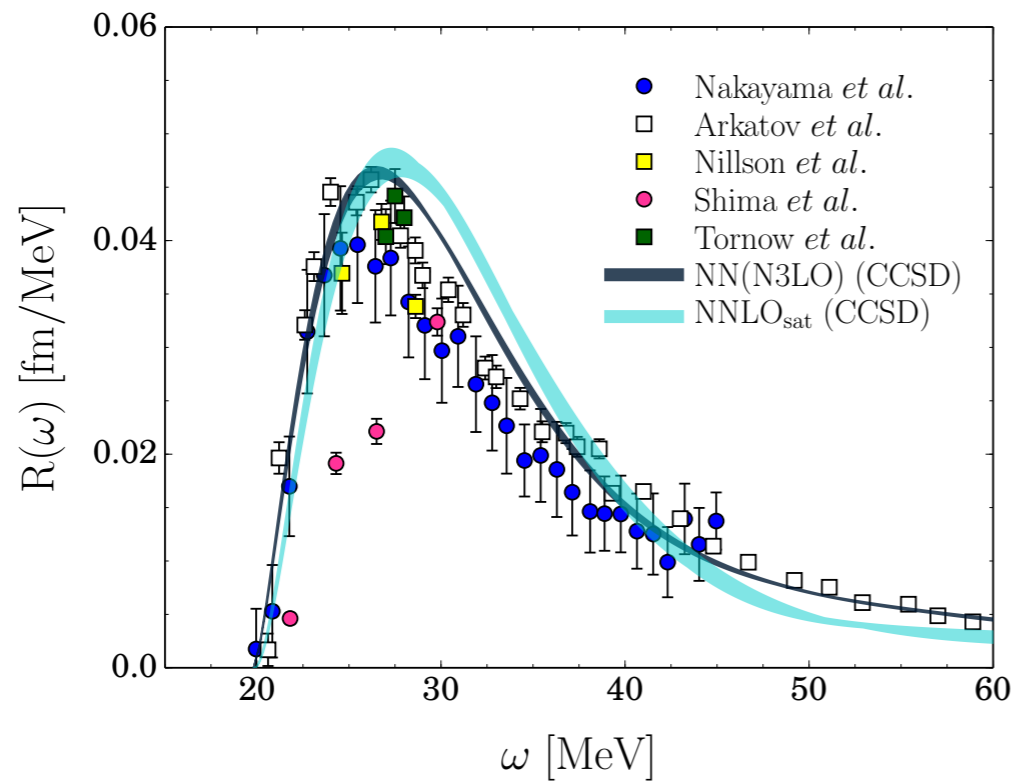
- Nuclear ab initio methods: connection to underlying QCD through chiral hamiltonians & uncertainty control
- First results from the coupled cluster theory: on the way to obtain cross-section for neutrino scattering on medium-mass nuclei
- With **spectral functions** we gain direct impact on the experimental analysis (through Monte Carlo event generators used by experimental collaboration)

**Thank you for attention!**



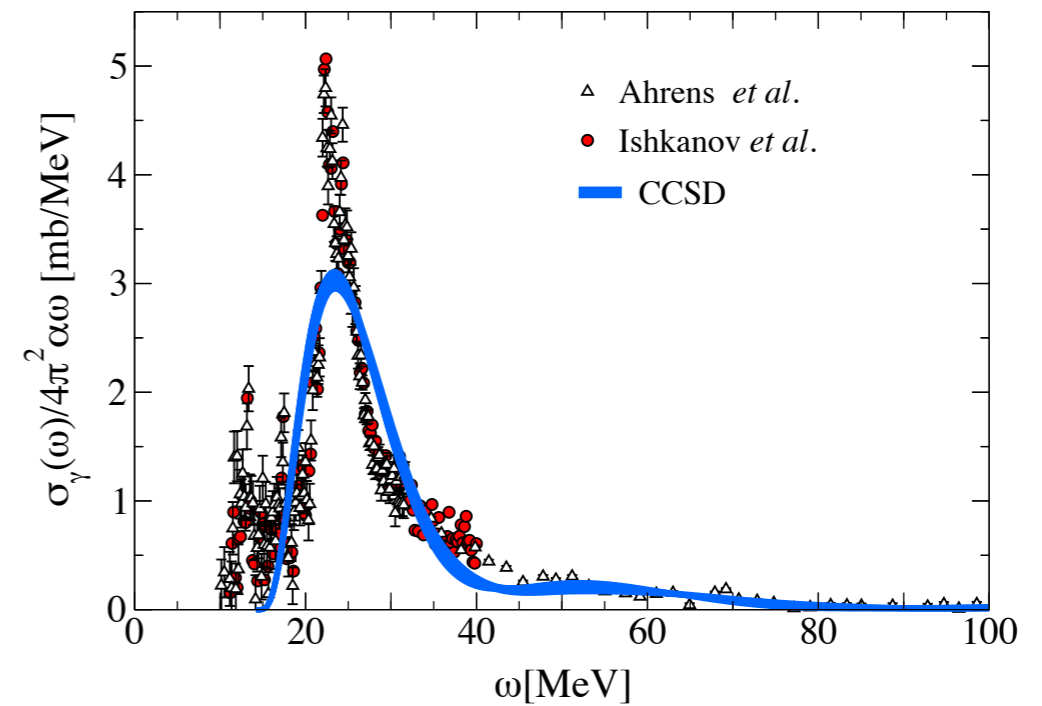
# Back up

# LIT-CC method



M. Miorelli *et al.*  
*Phys.Rev.C* 94 (2016) 3, 034317

$^4\text{He}$  photo-absorption



S. Bacca *et al.*  
*Phys.Rev.C* 90 (2014) 6, 064619

giant dipole resonance in  $^{16}\text{O}$