

# **CRPA predictions of neutrino-nucleus**

# <u>interactions</u>

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Independent-particle shell-model from self-consistent mean field with effective Skyrme interaction.



### Mean field nucleus

- Mean field potential
- Single-particle wavefunctions with (I,j,E,s)
- Binding energies
- Orthogonal states (→ Pauli-blocking)

Independent-particle shell-model from self-consistent mean field with effective Skyrme interaction.

$$-\nabla \left[\frac{\hbar^2}{2m_q^*(\mathbf{r})}\nabla\phi_{\alpha_q}(\mathbf{r})\right] + \left[U_q(\mathbf{r}) - iW_q(\mathbf{r})\cdot(\nabla\times\sigma)\right]\phi_{\alpha_q}(\mathbf{r}) = \varepsilon_{\alpha_q}^{\mathrm{HF}}\phi_{\alpha_q}(\mathbf{r}) . \quad (2.9)$$

Density dependent effective mass:

$$\frac{\hbar^2}{2m_q^*}(\mathbf{r}) = \frac{\hbar^2}{2m_q} + \frac{1}{4}(t_1 + t_2)\rho_{\text{tot}}(\mathbf{r}) + \frac{1}{8}(t_2 - t_1)\rho_q(\mathbf{r}) + \frac{1}{24}t_4(\rho_{\text{tot}}^2(\mathbf{r}) - \rho_q^2(\mathbf{r})) \,.$$

Density dependent potential:

$$\begin{split} J_q(\mathbf{r}) &= t_0 [(1 + \frac{1}{2}x_0)\rho_{\text{tot}} - (\frac{1}{2} + x_0)\rho_q] + \frac{1}{4}(t_1 + t_2)\tau_{\text{tot}} + \frac{1}{8}(t_2 - t_1)\tau_q \\ &\quad + \frac{1}{8}(t_2 - 3t_1)\nabla^2\rho_{\text{tot}} + \frac{1}{16}(3t_1 + t_2)\nabla^2\rho_q + \frac{1}{4}t_3(\rho_{\text{tot}}^2 - \rho_q^2) \\ &\quad - \frac{1}{2}W'_0(\nabla \cdot \mathbf{J}_{\text{tot}} + \nabla \cdot \mathbf{J}_q) + \delta_{qp}V^C(\mathbf{r}) + \frac{1}{24}t_4[2\rho_{\text{tot}}\tau_{\text{tot}} - 2\rho_q\tau_q \\ &\quad + \frac{5}{2}\rho_q\nabla^2\rho_q - \frac{5}{2}\rho_{\text{tot}}\nabla^2\rho_{\text{tot}} + \frac{5}{4}(\nabla\rho_q)^2 - \frac{5}{4}(\nabla\rho_{\text{tot}})^2 + \frac{1}{2}J_{q'}^2], \end{split}$$

Self-referential equation:

$$\rho_q(\mathbf{r}) = \sum_{\alpha_q \gamma_q} \rho_{\alpha \gamma}^{(q)} \phi_{\alpha_q}^*(\mathbf{r}) \phi_{\gamma_q}(\mathbf{r}) ,$$

 $\rightarrow$  Solve the system iteratively

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Independent-particle shell-model from **self-consistent mean field** with **effective Skyrme interaction**. Parameters in nucleon-nucleon interaction fit to nuclear matter and ground-state properties of nuclei

K  $(E/A)_{n.m.}$  $k_{\mathbf{F}}$ t<sub>4</sub>  $a_{\tau}$  $m^*/m$  $(MeV \cdot fm^8)$  $(fm^{-1})$ (MeV) (MeV) (MeV) -16.0 1.33 0.72 SkE2 -15808.79200 29.7 250 -16.01.310.75 30.0 SkE4 -12258.970.76SkIII 0.0356 -15.871.29 28.2E/AE/Ar<sub>n</sub>  $r_{\rm c}$ rp r<sub>n</sub>  $r_{c}$ r<sub>p</sub> <sup>40</sup>Ca 16O SkE2 2.602.68-8.563.31 3.42 -7.922.633.37 2.62 SkE4 -7.962.65 2.70-8.593.40 3.35 3.46 2.61 2.703.36 3.46 SkIII -8.032.64-8.573.41 3.48<sup>b</sup>) 2.71 ª) 3.36°) -7.98-8.55 exp 90Zr <sup>132</sup>Sn SkE2 4.24 4.21 -8.364.84 4.66 -8.674.174.624.22 4.26 SkE4 -8.714.29 -8.36 4.68 4.89 4.71 4.31 4.30 4.78 SkIII -8.694.26 -8.364.73 4.904.27°) -8.71-8.36 exp

M. Waroquier et al. / Effective Skyrme-type interaction (I) Nuclear Physics A404 (1983) 269-297

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Independent-particle shell-model from **self-consistent mean field** with **effective Skyrme interaction**. Parameters in nucleon-nucleon interaction fit to nuclear matter and **ground-state properties** of nuclei



Charge form factors (FT of charge-density) [N. Van Dessel et al. Arxiv:2007.03658] (→ also weak FF)

Independent-particle shell-model from self-consistent mean field with effective Skyrme interaction.



### Mean field nucleus

- Mean field potential
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- Binding energies
- Orthogonal states (→ Pauli-blocking)
- Effective interaction projects complexity on Mean field



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### Single-nucleon excitation of the nucleus

Independent-particle shell-model from self-consistent mean field with effective Skyrme interaction.



### Non-relativistic reduction of the current

$$\begin{split} \vec{J}_{V}^{\alpha}\left(\vec{x}\right) &= \vec{J}_{convection}^{\alpha}\left(\vec{x}\right) + \vec{J}_{magnetization}^{\alpha}\left(\vec{x}\right) \\ \text{with} & \vec{J}_{c}^{\alpha}\left(\vec{x}\right) = \frac{1}{2Mi} \sum_{i=1}^{A} G_{E}^{i,\alpha} \left[\delta\left(\vec{x} - \vec{x}_{i}\right) \overrightarrow{\nabla}_{i} - \overleftarrow{\nabla}_{i} \delta\left(\vec{x} - \vec{x}_{i}\right)\right], \\ & \vec{J}_{m}^{\alpha}\left(\vec{x}\right) = \frac{1}{2M} \sum_{i=1}^{A} G_{M}^{i,\alpha} \overrightarrow{\nabla} \times \vec{\sigma}_{i} \delta\left(\vec{x} - \vec{x}_{i}\right), \\ & \vec{J}_{A}^{\alpha}\left(\vec{x}\right) &= \sum_{i=1}^{A} G_{A}^{i,\alpha} \vec{\sigma}_{i} \delta\left(\vec{x} - \vec{x}_{i}\right), \\ & J_{V}^{0,\alpha}\left(\vec{x}\right) = \rho_{V}^{\alpha}\left(\vec{x}\right) &= \sum_{i=1}^{A} G_{E}^{i,\alpha} \delta\left(\vec{x} - \vec{x}_{i}\right), \\ & J_{A}^{0,\alpha}\left(\vec{x}\right) = \rho_{A}^{\alpha}\left(\vec{x}\right) &= \frac{1}{2Mi} \sum_{i=1}^{A} G_{A}^{i,\alpha} \vec{\sigma}_{i} \cdot \left[\delta\left(\vec{x} - \vec{x}_{i}\right) \overrightarrow{\nabla}_{i} - \overleftarrow{\nabla}_{i} \delta\left(\vec{x} - \vec{x}_{i}\right)\right] \end{split}$$

$$J_P^{0,\alpha}\left(\vec{x}\right) = \rho_P^{\alpha}\left(\vec{x}\right) = \frac{m_{\mu}}{2M} \sum_{i=1}^A G_P^{i,\alpha} \vec{\nabla} \cdot \vec{\sigma}_i \,\delta\left(\vec{x} - \vec{x}_i\right)$$

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## The Random Phase approximation



- Long-range correlations are correlations over the whole size of the nucleus
- They can redistribute the incoming energy transfer to the nucleus over all the nuclear constituents.
- They manifest themselves in collective excitations such as giant resonances

Long-range correlations = probing collective effects



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### Solving the RPA equations in coordinate space

One gets coupled self-consistent integral equation for the radial transition densities :

$$\begin{aligned} \langle \Psi_0 || X_{\eta J} || \Psi_C(J; E) \rangle_r &= - \langle h || X_{\eta J} || p(\varepsilon_{ph}) \rangle_r \\ &+ \sum_{\mu, \nu} \int dr_1 \int dr_2 \ U^J_{\mu\nu}(r_1, r_2) \ \mathcal{R} \left( R^{(0)}_{\eta\mu; J}(r, r_1; E) \right) \ \langle \Psi_0 || X_{\nu J} || \Psi_C(J; E) \rangle_{r_2} \end{aligned}$$

Solved numerically by discretizing on a mesh in coordinate space Translates into a matrix inversion for the transition densities:

$$\rho_C^{RPA} = -\frac{1}{1-R U} \rho_C^{HF}$$

$$\Psi_{RPA} = \sum_c \left\{ X_{(\Psi,C)} \left| ph^{-1} \right\rangle - Y_{(\Psi,C)} \left| hp^{-1} \right\rangle \right\}$$

The 'bare' transition densities are already dressed at HF level

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# Additional effects in lepton scattering



[S. Jeschonnek and T. Donnelly, PRC57, 2438 (1998)] Shift :

$$\lambda \rightarrow \lambda(\lambda + 1)$$
  $\lambda = \omega/2M_N$ 

$$\begin{split} \mathsf{Boost} : R^{\mathsf{V}}_{\mathsf{CC}}(q,\omega) &\to \frac{q^2}{q^2 - \omega^2} R^{\mathsf{V}}_{\mathsf{CC}}(q,\omega) \,, \\ R^{\mathsf{A}}_{\mathsf{LL}}(q,\omega) &\to \left(1 + \frac{q^2 - \omega^2}{4m^2}\right) R^{\mathsf{A}}_{\mathsf{LL}}(q,\omega) \,, \\ R^{\mathsf{V}}_{\mathsf{T}}(q,\omega) &\to \frac{q^2 - \omega^2}{q^2} R^{\mathsf{V}}_{\mathsf{T}}(q,\omega) \,, \\ R^{\mathsf{A}}_{\mathsf{T}}(q,\omega) &\to \left(1 + \frac{q^2 - \omega^2}{4m^2}\right) R^{\mathsf{A}}_{\mathsf{T}}(q,\omega) \,, \\ R^{\mathsf{VA}}_{\mathsf{T}'}(q,\omega) &\to \sqrt{\frac{q^2 - \omega^2}{q^2}} \sqrt{1 + \frac{q^2 - \omega^2}{4m^2}} R^{\mathsf{VA}}_{\mathsf{T}'}(q,\omega) \,. \end{split}$$

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### **Coulomb correction for outgoing/incoming lepton**



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### low energies: Fermi function

= Ratio of plane wave to coulomb-distorted s-wave gives Multiplicative factor  $2(1 + \gamma_0)(2k_f R)^{-2(1-\gamma_0)} \frac{|\Gamma(\gamma_0 + i\eta)|^2}{(\Gamma(2\gamma_0 + 1))^2}$ 

### high energies: Modified Effective Momentum Approximation

(MEMA)

$$e^{i\vec{k}\cdot\vec{r}} \to \sqrt{\frac{E_{eff}k_{eff}}{Ek}} e^{i\vec{k}_{eff}\cdot\vec{r}}$$
$$k_{eff} = k - V(0)$$
$$q_{eff} = q + 1.5 \left(\frac{Z'\alpha\hbar c}{R}\right),$$

**Coulomb correction for outgoing/incoming lepton** 



**Coulomb correction for outgoing/incoming lepton** 



Mostly relevant at low energies and heavy targets

CC interactions with Argon at low incoming energy

[Van Dessel et al. PRC100 055503 (2019)]

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### HF-CRPA : comparison with electron scattering data

<sup>12</sup>C(*e*, *e*')



## Electron scattering off medium mass nuclei



### (e,e') off Calcium

Blue band uncertainty due to residual interaction in CRPA

$$V \to \frac{V}{(1+Q^2/\Lambda)^2}$$

• Cut off determined in

[V. Pandey, et al Phys. Rev. C 92, 024606 (2015)]

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# Electron scattering off medium mass nuclei



### (e,e') off Iron

 Blue band uncertainty due to residual interaction in CRPA

$$V 
ightarrow rac{V}{(1+Q^2/\Lambda)^2}$$

• Cut off determined in

[V. Pandey, et al Phys. Rev. C 92, 024606 (2015)]

Jachowicz, N., Nikolakopoulos, A. Nuclear medium effects in neutrino- and antineutrino-nucleus scattering. Eur. Phys. J. Spec. Top. (2021)

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### Asymmetry

Free nucleon (isospin symmetric):

$$\frac{d\sigma_{\nu} - d\sigma_{\overline{\nu}}}{d\sigma_{\nu} + d\sigma_{\overline{\nu}}} = \frac{d\sigma_{VA}}{d\sigma_{VV} + d\sigma_{AA}}$$

Flux-folded (general L1) + isospin symmetric (L2)

$$\begin{split} A &= \frac{\int \varPhi_{\nu}(E_{\nu})\sigma_{\nu}(E_{\nu})dE_{\nu} - \int \varPhi_{\overline{\nu}}(E_{\overline{\nu}})\sigma_{\overline{\nu}}(E_{\overline{\nu}})dE_{\overline{\nu}}}{\int \varPhi_{\nu}(E_{\nu})\sigma_{\nu}(E_{\nu})dE_{\nu} + \int \varPhi_{\overline{\nu}}(E_{\overline{\nu}})\sigma_{\overline{\nu}}(E_{\overline{\nu}})dE_{\overline{\nu}}} \\ &= \frac{\int dE\left(\varPhi_{\nu} - \varPhi_{\overline{\nu}}\right)\sigma_{VV,AA} + \left(\varPhi_{\nu} + \varPhi_{\overline{\nu}}\right)\sigma_{VA}}{\int dE\left(\varPhi_{\nu} + \varPhi_{\overline{\nu}}\right)\sigma_{VV,AA} + \left(\varPhi_{\nu} - \varPhi_{\overline{\nu}}\right)\sigma_{VA}}. \end{split}$$





# **Asymmetry** T2K measurement

Add SuSAv2 collaboration MEC (RFG calculation) [G. D. Megias et al. PRD91, 073004 (2015)]

Add Hydrogen in anti-neutrino reactions

Dashed lines: assumption of isospin symmetry (neglect Coulomb effects)

Asymmetry quite model-independent

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### **Asymmetry** T2K measurement

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### **GFMC** results

Consistent treatment of one- and two-body Currents!

CRPA & GFMC 1b are similar

CRPA + SuSAv2 MEC & GFMC 1+2b Similar in backward bins  $\rightarrow$  Discrepancies in forward low P region

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SRC and MEC in Skyrme-Hartree Fock mean field (T. Van Cuyck 2017)

[T. Van Cuyck, N. Jachowicz, R. González-Jiménez, J. Ryckebusch, and N. Van Dessel Phys. Rev. C 95, 054611]

#### Lacks Delta currents!

MEC + Delta currents in axial sector in HF mean field [K. Niewczas et al. (in preparation)]

In RMF mean field [talk of T. Franco]

Is consistency the key ?

LFG+RPA with associated MEC model (talk J. Nieves)

GFMC 1+2b (talk A. Lovato)

Martini LFG+RPA with associated MEC [Phys.Rev.C 81 (2010) 045502]

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A 'precision era' because of

Detector technology (LarTPCs, Gd doping) Huge detectors, intense fluxes, good statistics

 $\rightarrow$  Uncertainties projected to come from cross section modeling

An open question: What do neutrino experiments need?



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### An open question: What do neutrino experiments need?



# Cross section for all possible semi-inclusive final-states, multiplicities, plus FSI ? $\rightarrow$ Very difficult to combine with 'precision'

Confidence in a detailed description of a 'simple' topology, with error budget, parameters ?  $\rightarrow$  Semi-inclusive one-nucleon knockout

### Both can co-exist of course

Neutrino energy reconstruction from semi-inclusive samples

R. González-Jiménez,<sup>1</sup> M. B. Barbaro,<sup>2,3</sup> J. A. Caballero,<sup>4,5</sup> T. W. Donnelly,<sup>6</sup> N. Jachowicz,<sup>7</sup> G. D. Megias,<sup>4,8</sup> K. Niewczas,<sup>7,9</sup> A. Nikolakopoulos,<sup>7</sup> J. W. Van Orden,<sup>10</sup> and J. M. Udías<sup>1</sup>

Analysis of 111p events  $\rightarrow$  A pure signal of single-nucleon knockout minimizes uncertainy on energy reconstruction

$$\left\langle \frac{d^5 \sigma}{d|\vec{k}_f| d\cos\theta_f d|\vec{k}_N| d\Omega_N} \right\rangle$$
$$= \int dE_m \tilde{\Phi}(E_i) \frac{d^5 \sigma(E_m)}{d|\vec{k}_f| d\cos\theta_f d|\vec{k}_N| d\Omega_N},$$

$$E_m = E_i - E_f - T_N - T_B = M_B + M_N - M_A.$$

Could be experimentally restricted



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Neutrino Interactions in the Standard Model and Beyond, CERN, January 19 2022



Semi-inclusive 1 nucleon knockout in event generators



Illustration from K. Niewczas (NuSTEC Workshop on Neutrino-Nucleus Pion Production in the Resonance Region)

'Factorized' approach can be consistent for simple initial-state or in the PWIA

What if the tabularized approach is used ? No simple relation between inclusive cross section and hadron kinematics [S. Dolan et al. Phys. Rev. D 101, 033003 (2020)]

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### FSI in one-nucleon knockout



J. M. Udías, P. Sarriguren, E. Moya de Guerra, E. Garrido, and J. A. Caballero Phys. Rev. C 48, 2731 – Published 1 December 1993

# FSI in one-nucleon knockout



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### Validation of cascade models

Can we bridge between DWIA with optical potential and cascade ?



'Inelastic FSI'

Use unfactorized RDWIA with **real** potential as input to NEUT cascade

### Validation of cascade models

Can we bridge between DWIA with optical potential and cascade ?

Use unfactorized RDWIA with **real** potential (rROP) as input to NEUT cascade

Missing energy cut reduces rROP+NEUT to '1-track' results → Can be directly compared to RDWIA with ROP



Neutrino Interactions in the Standard Model and Beyond, CERN, January 19 2022

### Validation of cascade models

Can we bridge between DWIA with optical potential and cascade ?

Robust results for carbon, oxygen, calcium Elastic channel after FSI in NEUT cascade yields similar results to ROP for T > 100-150 MeV Discrepancies arise in the low kinetic energy region



# Conclusions

### Mean-field + CRPA calculations

Robust description of (e,e'), (e,e'p) for a large A-range Suitable to describe low-energy structure and collective Excitations in addition to 'direct' interactions

### **Application to neutrino-data**

Mostly succesfull, but not over all kinematics or Across different experiments

**CRPA and model-comparisons for neutrino scattering** Uncertainties for T2K kinematics even in QE scattering A-dependence can be studied Difficult to compare model-to-model, 'degeneracy' of Neutrino data  $\rightarrow$  uncertainty even in 'simplest' mechanism

### Validation of cascade models

Straightforward bridge between DWIA with optical potential and cascade can be used to validate/constrain FSI, and check the implication of kinematic factorization Excitation and decay of giant resonances in the  ${}^{40}$ Ca(e,e'x) reaction

H. Diesener, U. Helm, G. Herbert, V. Huck, P. von Neumann-Cosel, C. Rangacharyulu, A. Richter, G. Schrieder, A. Stascheck, A. Stiller, J. Ryckebusch, and J. Carter Phys. Rev. Lett. **72**, 1994 – Published 28 March 1994



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