

CRPA predictions of neutrino-nucleus

<u>interactions</u>

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Independent-particle shell-model from self-consistent mean field with effective Skyrme interaction.



Mean field nucleus

- Mean field potential
- Single-particle wavefunctions with (I,j,E,s)
- Binding energies
- Orthogonal states (→ Pauli-blocking)

Independent-particle shell-model from self-consistent mean field with effective Skyrme interaction.

$$-\nabla \left[\frac{\hbar^2}{2m_q^*(\mathbf{r})}\nabla\phi_{\alpha_q}(\mathbf{r})\right] + \left[U_q(\mathbf{r}) - iW_q(\mathbf{r})\cdot(\nabla\times\sigma)\right]\phi_{\alpha_q}(\mathbf{r}) = \varepsilon_{\alpha_q}^{\mathrm{HF}}\phi_{\alpha_q}(\mathbf{r}) . \quad (2.9)$$

Density dependent effective mass:

$$\frac{\hbar^2}{2m_q^*}(\mathbf{r}) = \frac{\hbar^2}{2m_q} + \frac{1}{4}(t_1 + t_2)\rho_{\text{tot}}(\mathbf{r}) + \frac{1}{8}(t_2 - t_1)\rho_q(\mathbf{r}) + \frac{1}{24}t_4(\rho_{\text{tot}}^2(\mathbf{r}) - \rho_q^2(\mathbf{r})) \,.$$

Density dependent potential:

$$\begin{split} J_q(\mathbf{r}) &= t_0 [(1 + \frac{1}{2}x_0)\rho_{\text{tot}} - (\frac{1}{2} + x_0)\rho_q] + \frac{1}{4}(t_1 + t_2)\tau_{\text{tot}} + \frac{1}{8}(t_2 - t_1)\tau_q \\ &\quad + \frac{1}{8}(t_2 - 3t_1)\nabla^2\rho_{\text{tot}} + \frac{1}{16}(3t_1 + t_2)\nabla^2\rho_q + \frac{1}{4}t_3(\rho_{\text{tot}}^2 - \rho_q^2) \\ &\quad - \frac{1}{2}W'_0(\nabla \cdot \mathbf{J}_{\text{tot}} + \nabla \cdot \mathbf{J}_q) + \delta_{qp}V^C(\mathbf{r}) + \frac{1}{24}t_4[2\rho_{\text{tot}}\tau_{\text{tot}} - 2\rho_q\tau_q \\ &\quad + \frac{5}{2}\rho_q\nabla^2\rho_q - \frac{5}{2}\rho_{\text{tot}}\nabla^2\rho_{\text{tot}} + \frac{5}{4}(\nabla\rho_q)^2 - \frac{5}{4}(\nabla\rho_{\text{tot}})^2 + \frac{1}{2}J_{q'}^2], \end{split}$$

Self-referential equation:

$$\rho_q(\mathbf{r}) = \sum_{\alpha_q \gamma_q} \rho_{\alpha \gamma}^{(q)} \phi_{\alpha_q}^*(\mathbf{r}) \phi_{\gamma_q}(\mathbf{r}) ,$$

 \rightarrow Solve the system iteratively

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Independent-particle shell-model from **self-consistent mean field** with **effective Skyrme interaction**. Parameters in nucleon-nucleon interaction fit to nuclear matter and ground-state properties of nuclei

K $(E/A)_{n.m.}$ $k_{\mathbf{F}}$ t₄ a_{τ} m^*/m $(MeV \cdot fm^8)$ (fm^{-1}) (MeV) (MeV) (MeV) -16.0 1.33 0.72 SkE2 -15808.79200 29.7 250 -16.01.310.75 30.0 SkE4 -12258.970.76SkIII 0.0356 -15.871.29 28.2E/AE/Ar_n $r_{\rm c}$ rp r_n r_{c} r_p ⁴⁰Ca 16O SkE2 2.602.68-8.563.31 3.42 -7.922.633.37 2.62 SkE4 -7.962.65 2.70-8.593.40 3.35 3.46 2.61 2.703.36 3.46 SkIII -8.032.64-8.573.41 3.48^b) 2.71 ª) 3.36°) -7.98-8.55 exp 90Zr ¹³²Sn SkE2 4.24 4.21 -8.364.84 4.66 -8.674.174.624.22 4.26 SkE4 -8.714.29 -8.36 4.68 4.89 4.71 4.31 4.30 4.78 SkIII -8.694.26 -8.364.73 4.904.27°) -8.71-8.36 exp

M. Waroquier et al. / Effective Skyrme-type interaction (I) Nuclear Physics A404 (1983) 269-297

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Independent-particle shell-model from **self-consistent mean field** with **effective Skyrme interaction**. Parameters in nucleon-nucleon interaction fit to nuclear matter and **ground-state properties** of nuclei



Charge form factors (FT of charge-density) [N. Van Dessel et al. Arxiv:2007.03658] (→ also weak FF)

Independent-particle shell-model from self-consistent mean field with effective Skyrme interaction.



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- Effective interaction projects complexity on Mean field



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Single-nucleon excitation of the nucleus

Independent-particle shell-model from self-consistent mean field with effective Skyrme interaction.



Non-relativistic reduction of the current

$$\begin{split} \vec{J}_{V}^{\alpha}\left(\vec{x}\right) &= \vec{J}_{convection}^{\alpha}\left(\vec{x}\right) + \vec{J}_{magnetization}^{\alpha}\left(\vec{x}\right) \\ \text{with} & \vec{J}_{c}^{\alpha}\left(\vec{x}\right) = \frac{1}{2Mi} \sum_{i=1}^{A} G_{E}^{i,\alpha} \left[\delta\left(\vec{x} - \vec{x}_{i}\right) \overrightarrow{\nabla}_{i} - \overleftarrow{\nabla}_{i} \delta\left(\vec{x} - \vec{x}_{i}\right)\right], \\ & \vec{J}_{m}^{\alpha}\left(\vec{x}\right) = \frac{1}{2M} \sum_{i=1}^{A} G_{M}^{i,\alpha} \overrightarrow{\nabla} \times \vec{\sigma}_{i} \delta\left(\vec{x} - \vec{x}_{i}\right), \\ & \vec{J}_{A}^{\alpha}\left(\vec{x}\right) &= \sum_{i=1}^{A} G_{A}^{i,\alpha} \vec{\sigma}_{i} \delta\left(\vec{x} - \vec{x}_{i}\right), \\ & J_{V}^{0,\alpha}\left(\vec{x}\right) = \rho_{V}^{\alpha}\left(\vec{x}\right) &= \sum_{i=1}^{A} G_{E}^{i,\alpha} \delta\left(\vec{x} - \vec{x}_{i}\right), \\ & J_{A}^{0,\alpha}\left(\vec{x}\right) = \rho_{A}^{\alpha}\left(\vec{x}\right) &= \frac{1}{2Mi} \sum_{i=1}^{A} G_{A}^{i,\alpha} \vec{\sigma}_{i} \cdot \left[\delta\left(\vec{x} - \vec{x}_{i}\right) \overrightarrow{\nabla}_{i} - \overleftarrow{\nabla}_{i} \delta\left(\vec{x} - \vec{x}_{i}\right)\right] \end{split}$$

$$J_P^{0,\alpha}\left(\vec{x}\right) = \rho_P^{\alpha}\left(\vec{x}\right) = \frac{m_{\mu}}{2M} \sum_{i=1}^A G_P^{i,\alpha} \vec{\nabla} \cdot \vec{\sigma}_i \,\delta\left(\vec{x} - \vec{x}_i\right)$$

Natalie Jachowicz Neutrino Inte

The Random Phase approximation



- Long-range correlations are correlations over the whole size of the nucleus
- They can redistribute the incoming energy transfer to the nucleus over all the nuclear constituents.
- They manifest themselves in collective excitations such as giant resonances

Long-range correlations = probing collective effects



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Solving the RPA equations in coordinate space

One gets coupled self-consistent integral equation for the radial transition densities :

$$\begin{aligned} \langle \Psi_0 || X_{\eta J} || \Psi_C(J; E) \rangle_r &= - \langle h || X_{\eta J} || p(\varepsilon_{ph}) \rangle_r \\ &+ \sum_{\mu, \nu} \int dr_1 \int dr_2 \ U^J_{\mu\nu}(r_1, r_2) \ \mathcal{R} \left(R^{(0)}_{\eta\mu; J}(r, r_1; E) \right) \ \langle \Psi_0 || X_{\nu J} || \Psi_C(J; E) \rangle_{r_2} \end{aligned}$$

Solved numerically by discretizing on a mesh in coordinate space Translates into a matrix inversion for the transition densities:

$$\rho_C^{RPA} = -\frac{1}{1-R U} \rho_C^{HF}$$

$$\Psi_{RPA} = \sum_c \left\{ X_{(\Psi,C)} \left| ph^{-1} \right\rangle - Y_{(\Psi,C)} \left| hp^{-1} \right\rangle \right\}$$

The 'bare' transition densities are already dressed at HF level

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Additional effects in lepton scattering



[S. Jeschonnek and T. Donnelly, PRC57, 2438 (1998)] Shift :

$$\lambda \rightarrow \lambda(\lambda + 1)$$
 $\lambda = \omega/2M_N$

$$\begin{split} \mathsf{Boost} : R^{\mathsf{V}}_{\mathsf{CC}}(q,\omega) &\to \frac{q^2}{q^2 - \omega^2} R^{\mathsf{V}}_{\mathsf{CC}}(q,\omega) \,, \\ R^{\mathsf{A}}_{\mathsf{LL}}(q,\omega) &\to \left(1 + \frac{q^2 - \omega^2}{4m^2}\right) R^{\mathsf{A}}_{\mathsf{LL}}(q,\omega) \,, \\ R^{\mathsf{V}}_{\mathsf{T}}(q,\omega) &\to \frac{q^2 - \omega^2}{q^2} R^{\mathsf{V}}_{\mathsf{T}}(q,\omega) \,, \\ R^{\mathsf{A}}_{\mathsf{T}}(q,\omega) &\to \left(1 + \frac{q^2 - \omega^2}{4m^2}\right) R^{\mathsf{A}}_{\mathsf{T}}(q,\omega) \,, \\ R^{\mathsf{VA}}_{\mathsf{T}'}(q,\omega) &\to \sqrt{\frac{q^2 - \omega^2}{q^2}} \sqrt{1 + \frac{q^2 - \omega^2}{4m^2}} R^{\mathsf{VA}}_{\mathsf{T}'}(q,\omega) \,. \end{split}$$

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Coulomb correction for outgoing/incoming lepton



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low energies: Fermi function

= Ratio of plane wave to coulomb-distorted s-wave gives Multiplicative factor $2(1 + \gamma_0)(2k_f R)^{-2(1-\gamma_0)} \frac{|\Gamma(\gamma_0 + i\eta)|^2}{(\Gamma(2\gamma_0 + 1))^2}$

high energies: Modified Effective Momentum Approximation

(MEMA)

$$e^{i\vec{k}\cdot\vec{r}} \to \sqrt{\frac{E_{eff}k_{eff}}{Ek}} e^{i\vec{k}_{eff}\cdot\vec{r}}$$
$$k_{eff} = k - V(0)$$
$$q_{eff} = q + 1.5 \left(\frac{Z'\alpha\hbar c}{R}\right),$$

Coulomb correction for outgoing/incoming lepton



Coulomb correction for outgoing/incoming lepton

Mostly relevant at low energies and heavy targets

CC interactions with Argon at low incoming energy

[Van Dessel et al. PRC100 055503 (2019)]

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HF-CRPA : comparison with electron scattering data

¹²C(*e*, *e*')

Electron scattering off medium mass nuclei

(e,e') off Calcium

Blue band uncertainty due to residual interaction in CRPA

$$V \to \frac{V}{(1+Q^2/\Lambda)^2}$$

• Cut off determined in

[V. Pandey, et al Phys. Rev. C 92, 024606 (2015)]

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Electron scattering off medium mass nuclei

(e,e') off Iron

 Blue band uncertainty due to residual interaction in CRPA

$$V
ightarrow rac{V}{(1+Q^2/\Lambda)^2}$$

• Cut off determined in

[V. Pandey, et al Phys. Rev. C 92, 024606 (2015)]

Jachowicz, N., Nikolakopoulos, A. Nuclear medium effects in neutrino- and antineutrino-nucleus scattering. Eur. Phys. J. Spec. Top. (2021)

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Asymmetry

Free nucleon (isospin symmetric):

$$\frac{d\sigma_{\nu} - d\sigma_{\overline{\nu}}}{d\sigma_{\nu} + d\sigma_{\overline{\nu}}} = \frac{d\sigma_{VA}}{d\sigma_{VV} + d\sigma_{AA}}$$

Flux-folded (general L1) + isospin symmetric (L2)

$$\begin{split} A &= \frac{\int \varPhi_{\nu}(E_{\nu})\sigma_{\nu}(E_{\nu})dE_{\nu} - \int \varPhi_{\overline{\nu}}(E_{\overline{\nu}})\sigma_{\overline{\nu}}(E_{\overline{\nu}})dE_{\overline{\nu}}}{\int \varPhi_{\nu}(E_{\nu})\sigma_{\nu}(E_{\nu})dE_{\nu} + \int \varPhi_{\overline{\nu}}(E_{\overline{\nu}})\sigma_{\overline{\nu}}(E_{\overline{\nu}})dE_{\overline{\nu}}} \\ &= \frac{\int dE\left(\varPhi_{\nu} - \varPhi_{\overline{\nu}}\right)\sigma_{VV,AA} + \left(\varPhi_{\nu} + \varPhi_{\overline{\nu}}\right)\sigma_{VA}}{\int dE\left(\varPhi_{\nu} + \varPhi_{\overline{\nu}}\right)\sigma_{VV,AA} + \left(\varPhi_{\nu} - \varPhi_{\overline{\nu}}\right)\sigma_{VA}}. \end{split}$$

Asymmetry T2K measurement

Add SuSAv2 collaboration MEC (RFG calculation) [G. D. Megias et al. PRD91, 073004 (2015)]

Add Hydrogen in anti-neutrino reactions

Dashed lines: assumption of isospin symmetry (neglect Coulomb effects)

Asymmetry quite model-independent

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GFMC results

Consistent treatment of one- and two-body Currents!

CRPA & GFMC 1b are similar

CRPA + SuSAv2 MEC & GFMC 1+2b Similar in backward bins \rightarrow Discrepancies in forward low P region

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SRC and MEC in Skyrme-Hartree Fock mean field (T. Van Cuyck 2017)

[T. Van Cuyck, N. Jachowicz, R. González-Jiménez, J. Ryckebusch, and N. Van Dessel Phys. Rev. C 95, 054611]

Lacks Delta currents!

MEC + Delta currents in axial sector in HF mean field [K. Niewczas et al. (in preparation)]

In RMF mean field [talk of T. Franco]

Is consistency the key ?

LFG+RPA with associated MEC model (talk J. Nieves)

GFMC 1+2b (talk A. Lovato)

Martini LFG+RPA with associated MEC [Phys.Rev.C 81 (2010) 045502]

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A 'precision era' because of

Detector technology (LarTPCs, Gd doping) Huge detectors, intense fluxes, good statistics

 \rightarrow Uncertainties projected to come from cross section modeling

An open question: What do neutrino experiments need?

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An open question: What do neutrino experiments need?

Cross section for all possible semi-inclusive final-states, multiplicities, plus FSI ? \rightarrow Very difficult to combine with 'precision'

Confidence in a detailed description of a 'simple' topology, with error budget, parameters ? \rightarrow Semi-inclusive one-nucleon knockout

Both can co-exist of course

Neutrino energy reconstruction from semi-inclusive samples

R. González-Jiménez,¹ M. B. Barbaro,^{2,3} J. A. Caballero,^{4,5} T. W. Donnelly,⁶ N. Jachowicz,⁷ G. D. Megias,^{4,8} K. Niewczas,^{7,9} A. Nikolakopoulos,⁷ J. W. Van Orden,¹⁰ and J. M. Udías¹

Analysis of 111p events \rightarrow A pure signal of single-nucleon knockout minimizes uncertainy on energy reconstruction

$$\left\langle \frac{d^5 \sigma}{d|\vec{k}_f| d\cos\theta_f d|\vec{k}_N| d\Omega_N} \right\rangle$$
$$= \int dE_m \tilde{\Phi}(E_i) \frac{d^5 \sigma(E_m)}{d|\vec{k}_f| d\cos\theta_f d|\vec{k}_N| d\Omega_N},$$

$$E_m = E_i - E_f - T_N - T_B = M_B + M_N - M_A.$$

Could be experimentally restricted

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Neutrino Interactions in the Standard Model and Beyond, CERN, January 19 2022

Semi-inclusive 1 nucleon knockout in event generators

Illustration from K. Niewczas (NuSTEC Workshop on Neutrino-Nucleus Pion Production in the Resonance Region)

'Factorized' approach can be consistent for simple initial-state or in the PWIA

What if the tabularized approach is used ? No simple relation between inclusive cross section and hadron kinematics [S. Dolan et al. Phys. Rev. D 101, 033003 (2020)]

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FSI in one-nucleon knockout

J. M. Udías, P. Sarriguren, E. Moya de Guerra, E. Garrido, and J. A. Caballero Phys. Rev. C 48, 2731 – Published 1 December 1993

FSI in one-nucleon knockout

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Validation of cascade models

Can we bridge between DWIA with optical potential and cascade ?

'Inelastic FSI'

Use unfactorized RDWIA with **real** potential as input to NEUT cascade

Validation of cascade models

Can we bridge between DWIA with optical potential and cascade ?

Use unfactorized RDWIA with **real** potential (rROP) as input to NEUT cascade

Missing energy cut reduces rROP+NEUT to '1-track' results → Can be directly compared to RDWIA with ROP

Neutrino Interactions in the Standard Model and Beyond, CERN, January 19 2022

Validation of cascade models

Can we bridge between DWIA with optical potential and cascade ?

Robust results for carbon, oxygen, calcium Elastic channel after FSI in NEUT cascade yields similar results to ROP for T > 100-150 MeV Discrepancies arise in the low kinetic energy region

Conclusions

Mean-field + CRPA calculations

Robust description of (e,e'), (e,e'p) for a large A-range Suitable to describe low-energy structure and collective Excitations in addition to 'direct' interactions

Application to neutrino-data

Mostly succesfull, but not over all kinematics or Across different experiments

CRPA and model-comparisons for neutrino scattering Uncertainties for T2K kinematics even in QE scattering A-dependence can be studied Difficult to compare model-to-model, 'degeneracy' of Neutrino data \rightarrow uncertainty even in 'simplest' mechanism

Validation of cascade models

Straightforward bridge between DWIA with optical potential and cascade can be used to validate/constrain FSI, and check the implication of kinematic factorization Excitation and decay of giant resonances in the 40 Ca(e,e'x) reaction

H. Diesener, U. Helm, G. Herbert, V. Huck, P. von Neumann-Cosel, C. Rangacharyulu, A. Richter, G. Schrieder, A. Stascheck, A. Stiller, J. Ryckebusch, and J. Carter Phys. Rev. Lett. **72**, 1994 – Published 28 March 1994

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