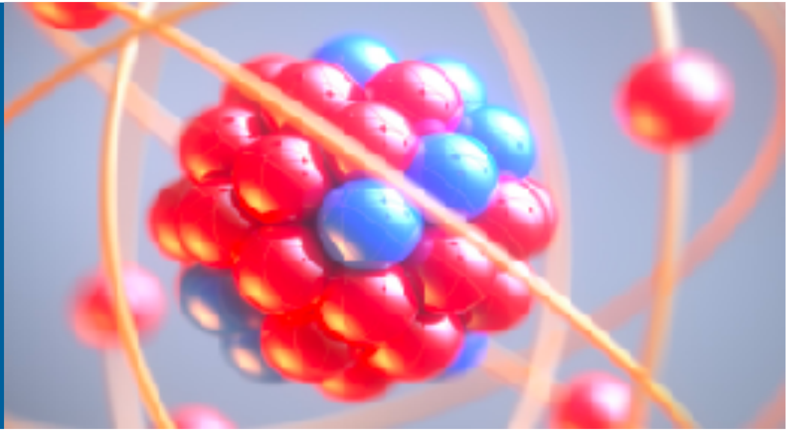
GREEN'S FUNCTION MONTE CARLO PREDICTIONS OF NEUTRINO-NUCLEUS CROSS SECTION



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19 January 2022

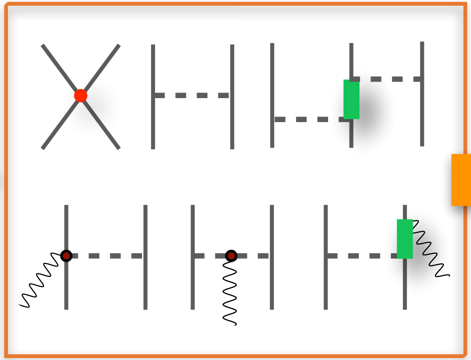
Neutrino–Nucleus Interactions in the Standard Model and Beyond

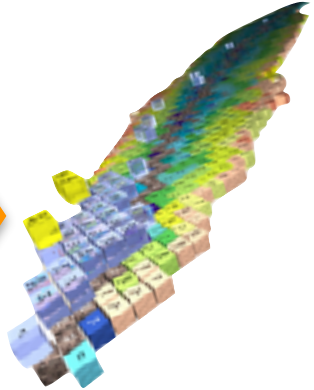
MICROSCOPIC MODEL OF NUCLEAR THEORY

- Nuclear effective field theories allow to systematically derive Hamiltonians and consistent electroweak currents

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots \qquad J = \sum_i j_i + \sum_{i<j} j_{ij} + \dots$$

- They connect the underlaying theory of strong interactions, QCD, with nuclear observables





QUANTUM MONTE CARLO

Nuclear many-body methods aim at solving the Schrödinger equation with controlled approximations

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

The variational Moment Carlo method approximates the ground-state with a variational ansatz that explicitly includes two and three-body correlations

$$|\Psi_T\rangle = \Big(1 + \sum_{ijk} F_{ijk}\Big)\Big(\mathcal{S}\prod_{i<j} F_{ij}\Big)|\Phi_{J,T_z}\rangle$$

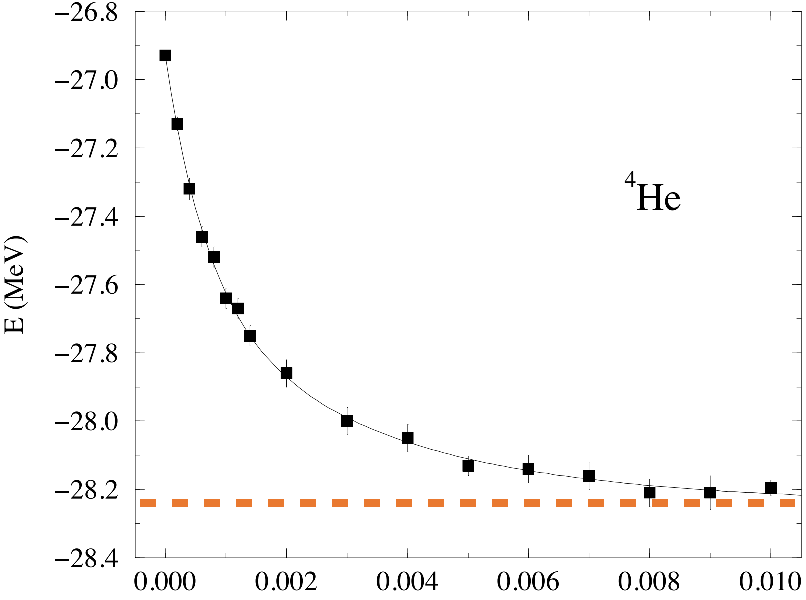
The optimal set of variational parameters is found by minimizing the variational energy

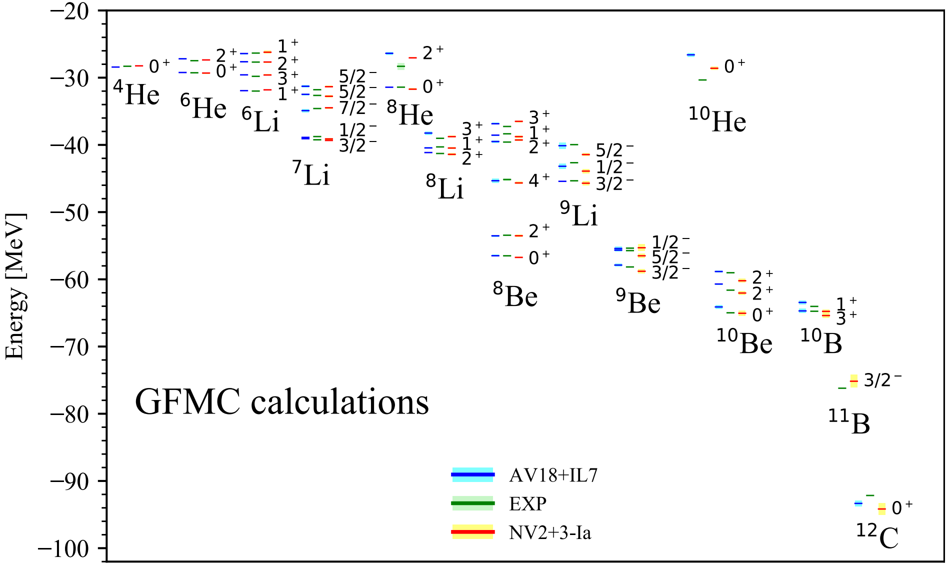
$$E_T = \frac{\langle\Psi_T|H|\Psi_T\rangle}{\langle\Psi_T|\Psi_T\rangle} \geq E_0$$

QUANTUM MONTE CARLO

The Green's function Monte Carlo uses imaginary-time projection techniques to extract the ground-state of the system from the trial wave function

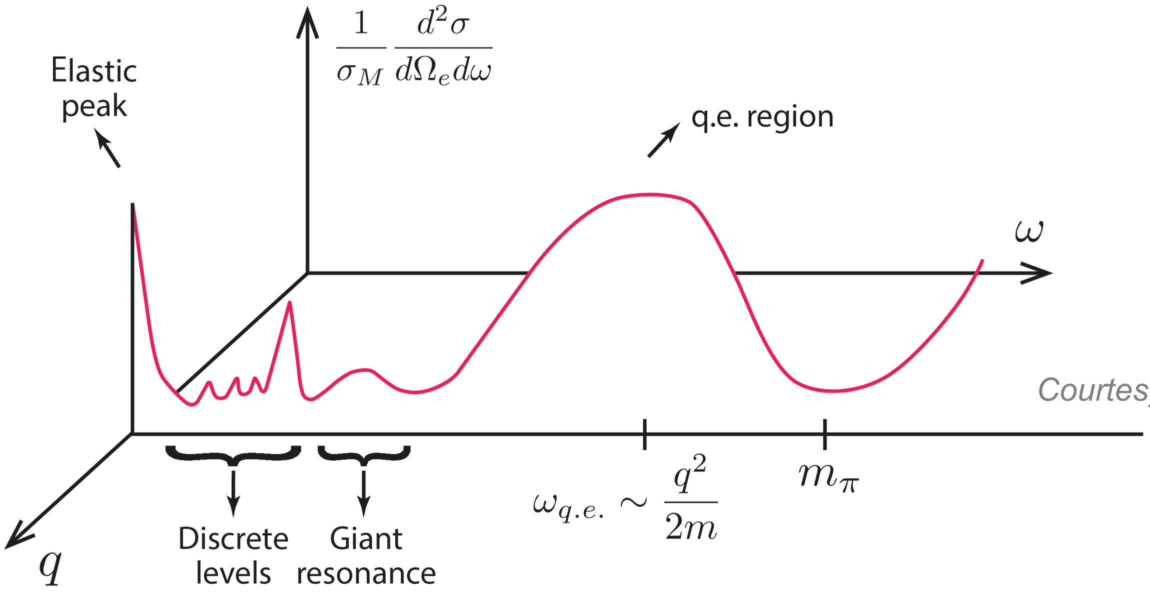
$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = \lim_{\tau\to\infty}\sum_n c_n e^{-(E_n-E_0)\tau}|\Psi_n\rangle = c_0|\Psi_0\rangle$$





NEUTRINO-NUCLEUS SCATTERING

The inclusive cross section is characterized by a variety of reaction mechanisms



Courtesy of S. Pastore

The response functions contain all nuclear-dynamics information

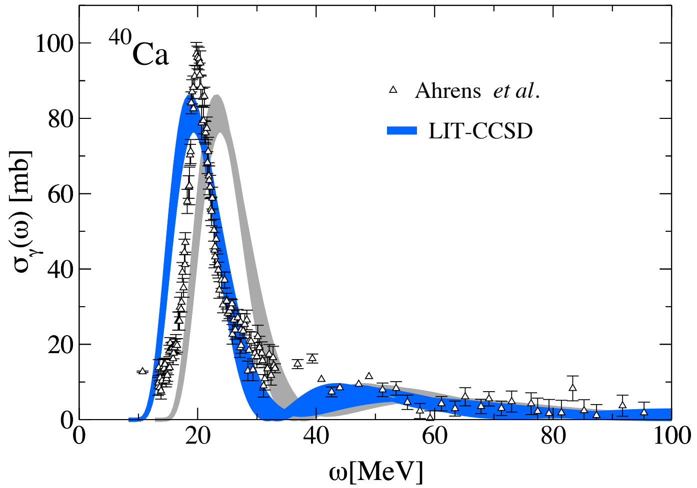
$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle\Psi_0|J_\alpha^\dagger(\mathbf{q})|\Psi_f\rangle\langle\Psi_f|J_\beta(\mathbf{q})|\Psi_0\rangle\delta(\omega - E_f + E_0)$$

EUCLIDEAN RESPONSES

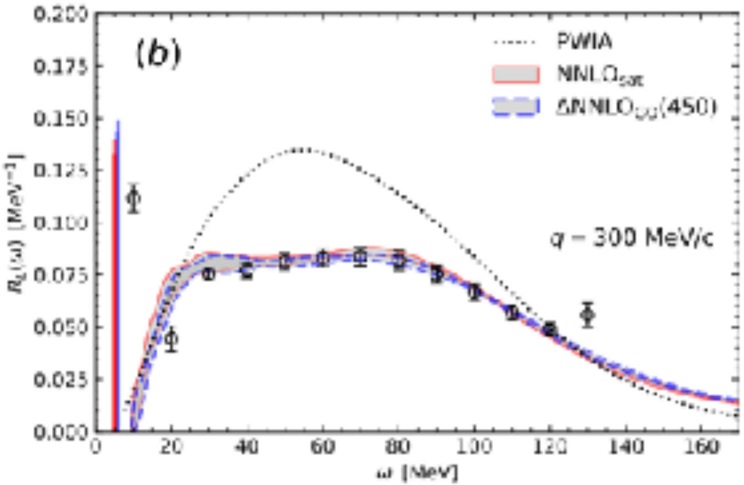
The integral transform of the response function is expressed as a ground-state expectation value

$$E_{\alpha\beta}(\sigma,\mathbf{q}) \equiv \int d\omega K(\sigma,\omega) R_{\alpha\beta}(\omega,\mathbf{q}) = \langle \Psi_0 | J^\dagger_\alpha(\mathbf{q}) K(\sigma, H - E_0) J_\beta(\mathbf{q}) | \Psi_0 \rangle$$

The Lorentz and the Gaussian kernels are typically used in many-body calculations



*S. Bacca et al., PRC **6**, 064619 (2014)*



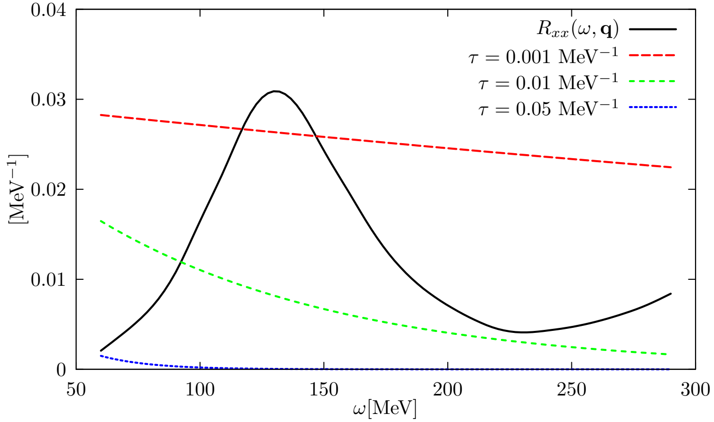
*J. Sobczyk et al., PRL **127**, 072501 (2021)*

EUCLIDEAN RESPONSES

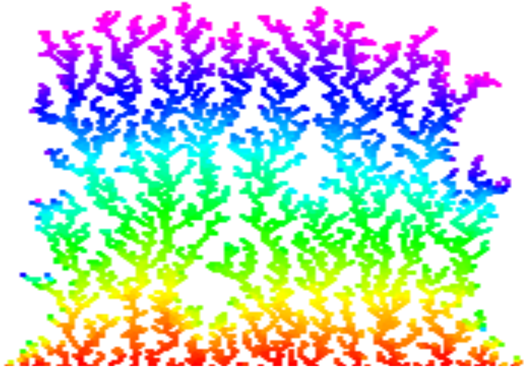
Our GFMC calculations rely on the Laplace kernel

$$E_{\alpha\beta}(\tau,\mathbf{q}) \equiv \int d\omega e^{-\omega\tau} R_{\alpha\beta}(\omega,\mathbf{q})$$

At finite imaginary time the contributions from large energy transfer are quickly suppressed



The system is first heated up by the transition operator. Its cooling determines the Euclidean response of the system



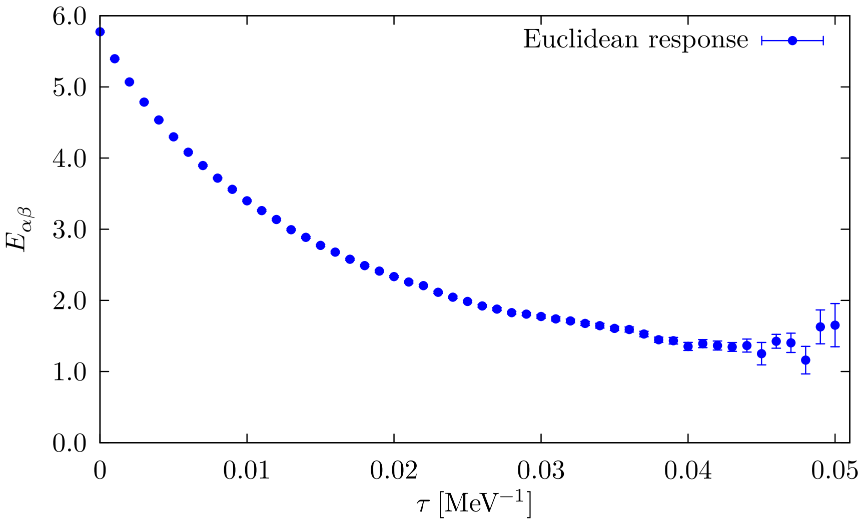
$$E_{\alpha\beta}(\tau,\mathbf{q}) = \langle\Psi_0|J_\alpha^\dagger(\mathbf{q})e^{-(H-E_0)\tau}J_\beta(\mathbf{q})|\Psi_0\rangle$$

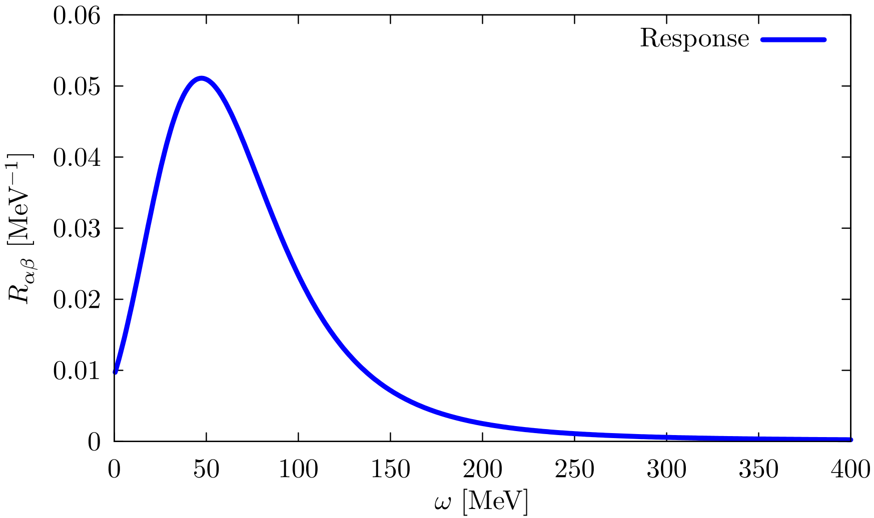
$$\sum_f |\Psi_f\rangle\langle\Psi_f|$$

EUCLIDEAN RESPONSES

Inverting the Euclidean response is an ill posed problem: any set of observations is limited and noisy and the situation is even worse since the kernel is a smoothing operator.

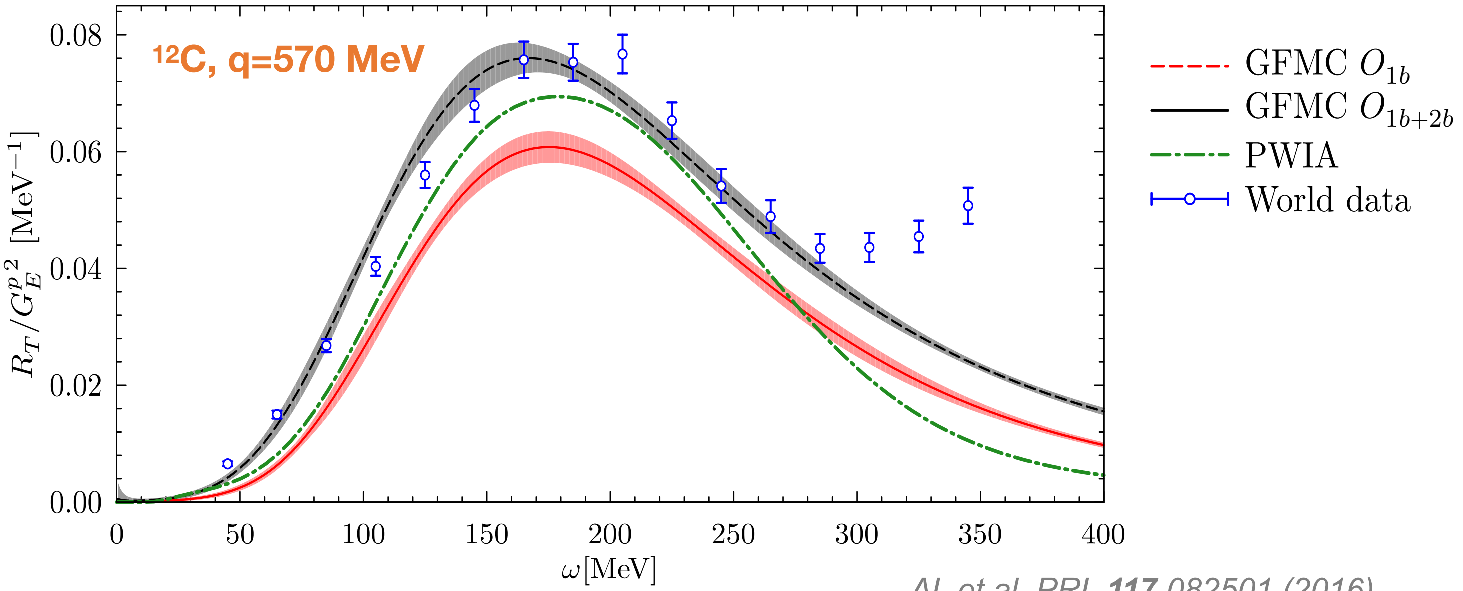
$$E_{\alpha\beta}(\tau, \mathbf{q}) \longrightarrow R_{\alpha\beta}(\omega, \mathbf{q})$$





We find Maximum-entropy techniques to be reliable enough for quasi-elastic responses

VALIDATION WITH ELECTRON SCATTERING



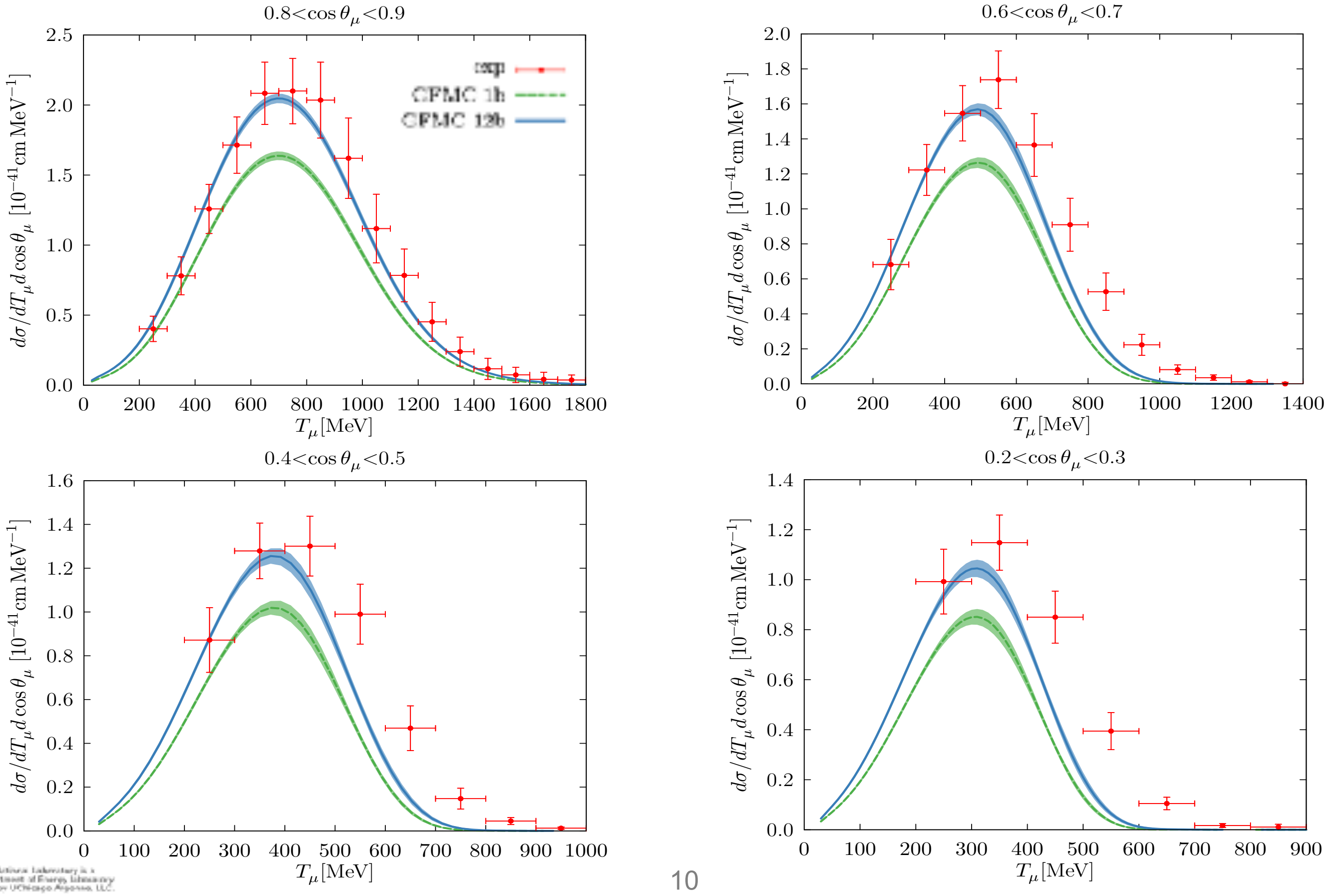
AL et al. PRL ***117*** 082501 (2016)

Two-body currents generate additional strength in over the whole quasi-elastic region

Correlations redistribute strength from the quasi-elastic peak to high-energy transfer regions

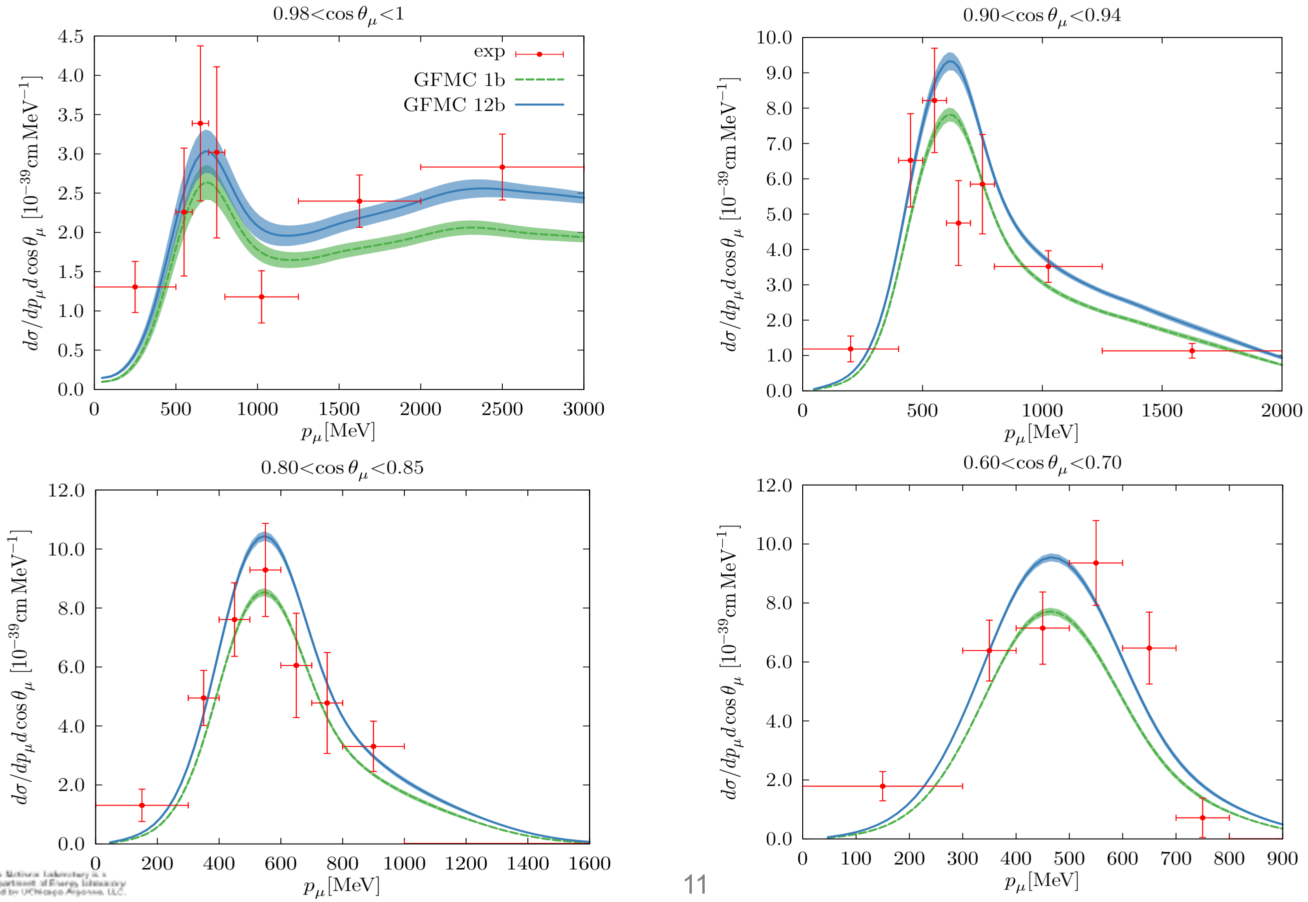
MINIBOONE CROSS SECTIONS

*AL et al., PRX **10**, 031068 (2020)*



T2K CROSS SECTIONS

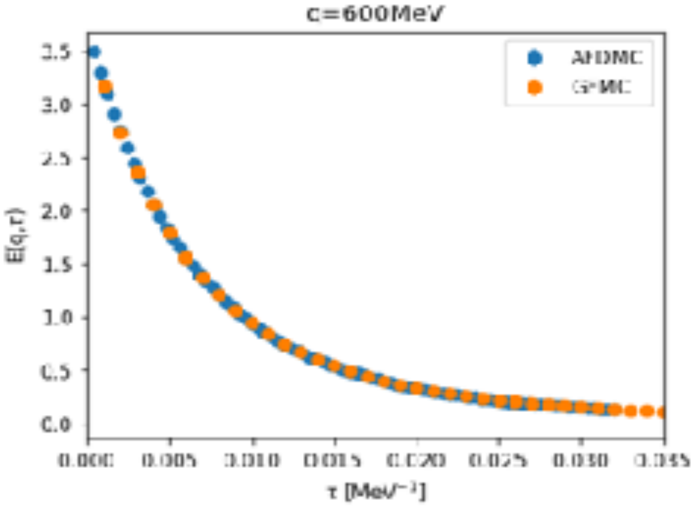
*AL et al., PRX **10**, 031068 (2020)*

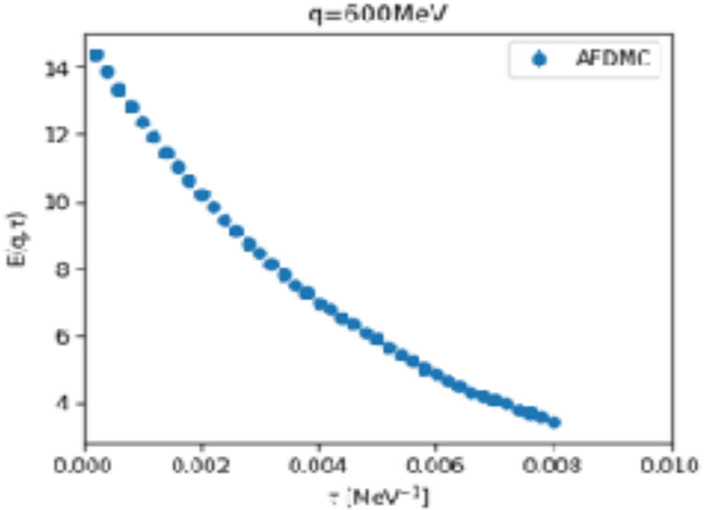


BEYOND ^{12}C: AFDMC AND MACHINE LEARNING

The auxiliary-field diffusion Monte Carlo method allows to treat ^{16}O and beyond by sampling the spin-isospin degrees of freedom

We extensively developed the AFDMC to compute the Euclidean response functions





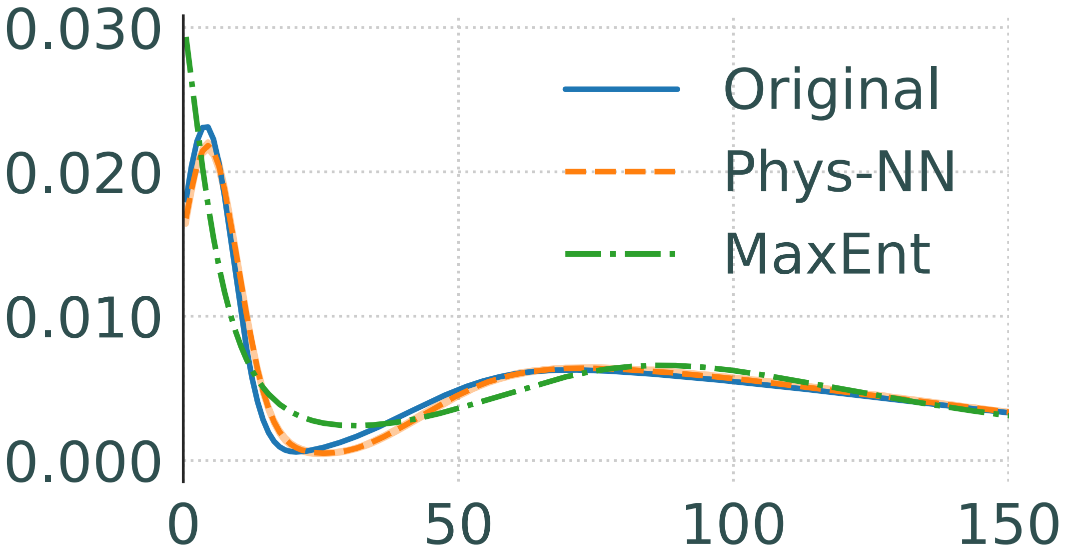
- Excellent agreement with the GFMC in the ^{4}He case;
- Somewhat noisier estimates in ^{16}O because of the fermion-sign problem;

BEYOND ^{12}C: AFDMC AND MACHINE LEARNING

We developed an artificial-neural network approach suitable to invert the Laplace transform that:

- Outperforms Maximum Entropy, especially in the low-energy region;

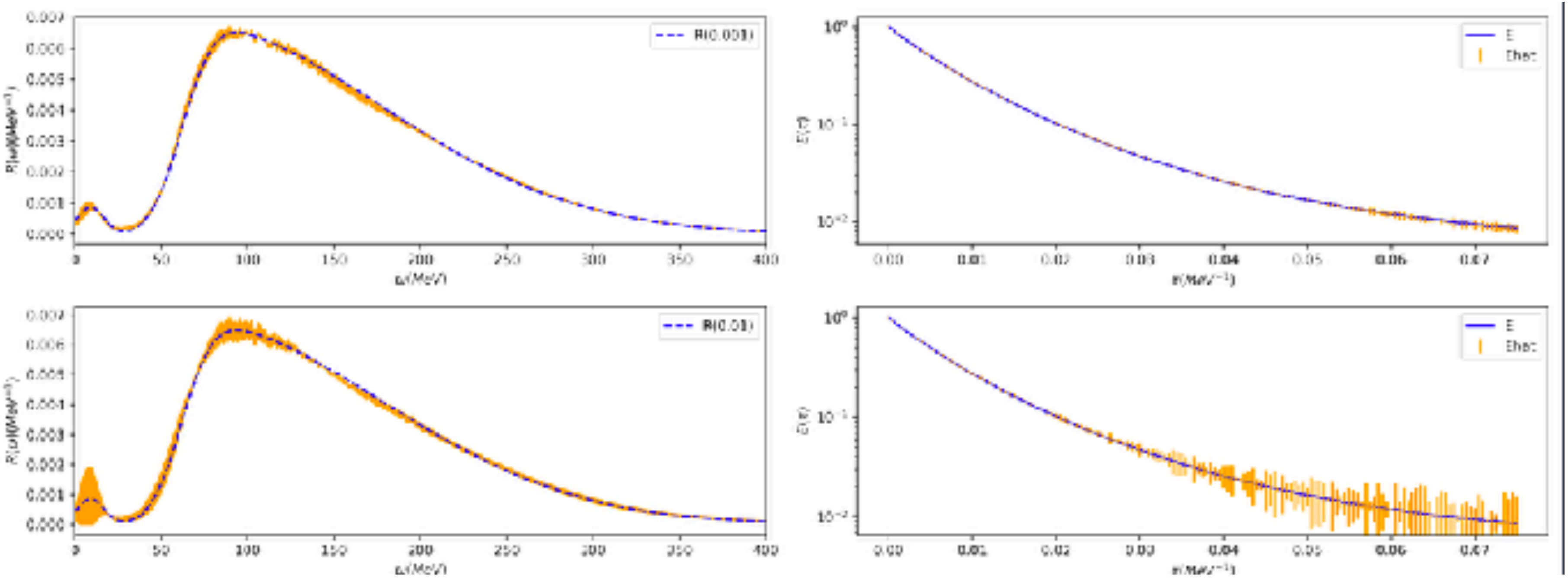
*K. Raghavan, AL, al., PRC **103**, 035502 (2021)*



BEYOND ^{12}C: AFDMC AND MACHINE LEARNING

We developed an artificial-neural network approach suitable to invert the Laplace transform that:

- Provides robust estimates of the uncertainty of the inversion;



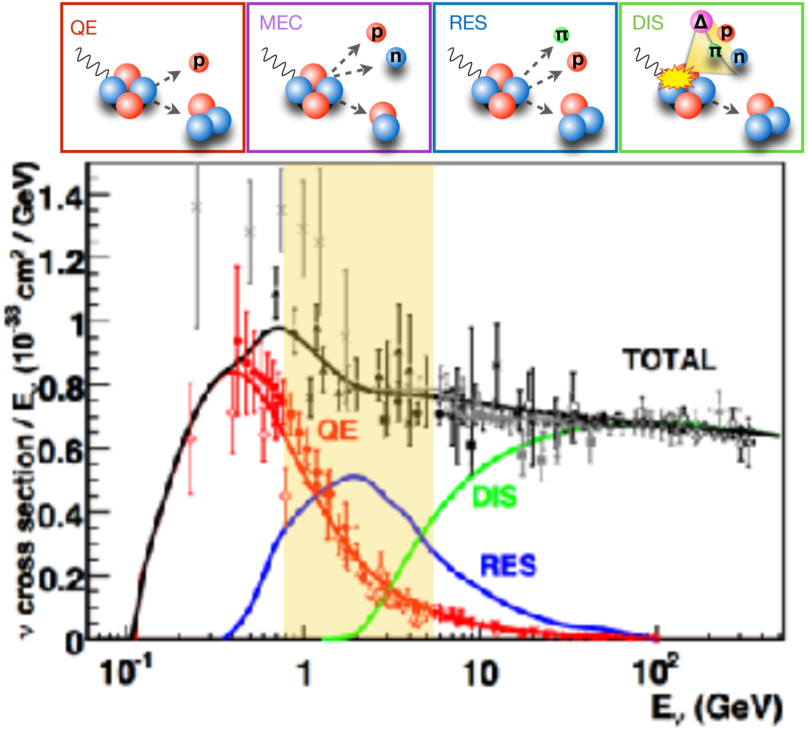
ACCELERATOR NEUTRINO EXPERIMENTS

Achieving a robust description of the reaction mechanisms at play in accelerator-neutrino experiments is a **formidable nuclear-theory challenge**

- Realistic description of nuclear correlations

- Relativistic effects in the current operators and kinematics

- Description of resonance-production and DIS region



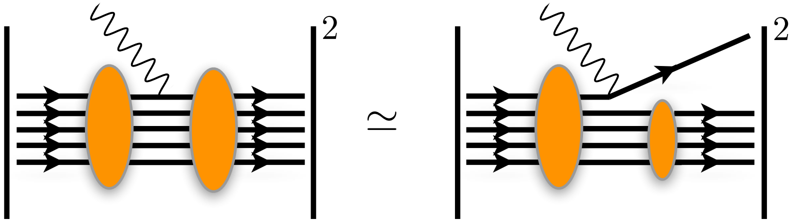
FACTORIZATION SCHEME

At large momentum transfer, the scattering reduces to the sum of individual terms

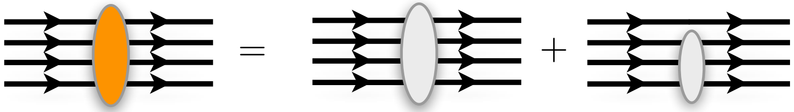
$$J^\mu \to \sum_i j_i^\mu \qquad |\psi_f^A\rangle \to |p\rangle \otimes |\psi_f^{A-1}\rangle \qquad E_f = E_f^{A-1} + e(\mathbf{p})$$

The incoherent contribution of the one-body response reads

$$R_{\alpha\beta} \simeq \int \frac{d^3k}{(2\pi)^3} dE P_h(\mathbf{k}, E) \sum_i \langle k|{j_\alpha^i}^\dagger|k+q\rangle\langle k+q|j_\beta^i|k\rangle\delta(\omega + E - e(\mathbf{k}+\mathbf{q}))$$



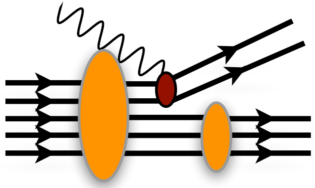
We include excitations of the A-1 final state with two nucleons in the continuum



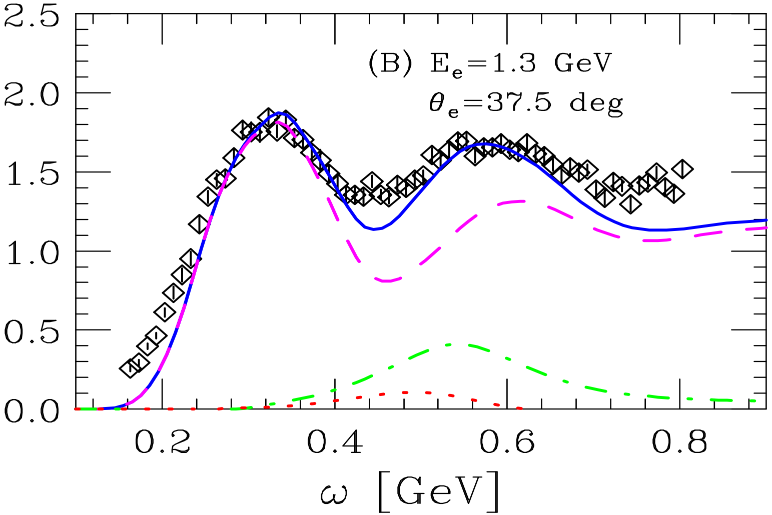
EXTENDED FACTORIZATION SCHEME

Using relativistic MEC requires extending the factorization scheme to two-nucleon emissions

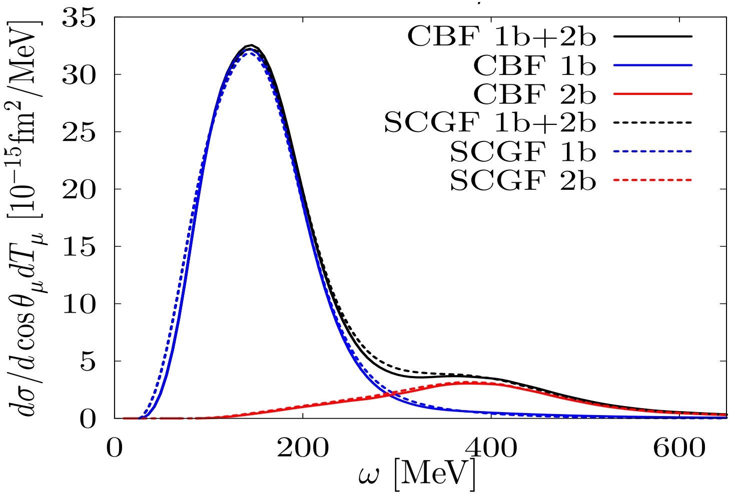
$$|\Psi_f^A\rangle \to |p_1 p_2\rangle \otimes |\Psi_f^{A-2}\rangle$$



We compute electron and neutrino inclusive cross sections using CBF and SCGF spectral functions



N. Rocco, et al. PRL. ***116*** *192501 (2016)*

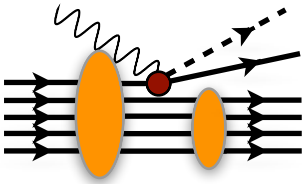


N. Rocco, et al. PRC ***99*** *025502 (2019)*

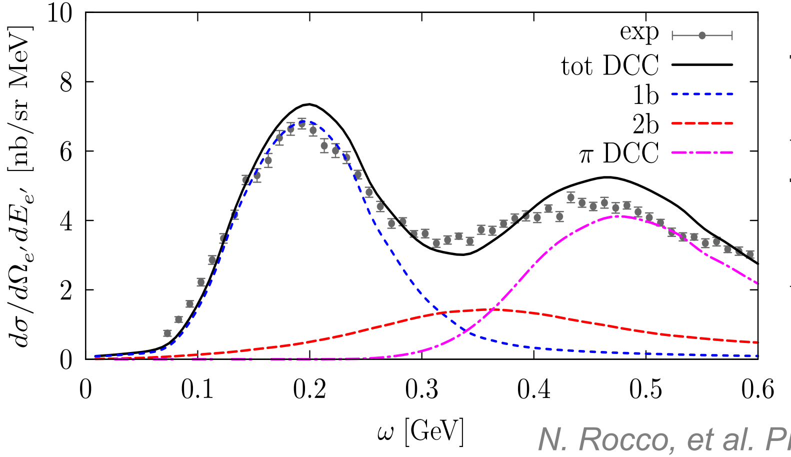
EXTENDED FACTORIZATION SCHEME

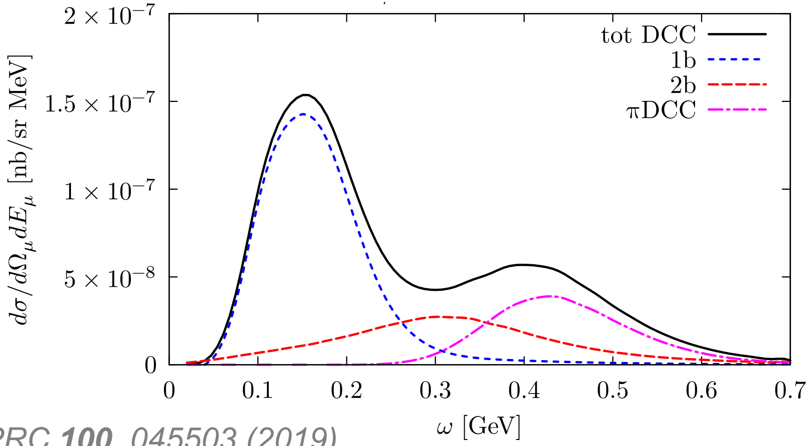
The factorization scheme can be further extended to include real pions in the final state

$$|\Psi_f^A\rangle \rightarrow |p_1, p_\pi\rangle \otimes |\Psi_f^{A-1}\rangle$$



The DCC model, suitable to accurately describe single-nucleon pion-production, is folded with a realistic spectral function





*N. Rocco, et al. PRC **100**, 045503 (2019)*

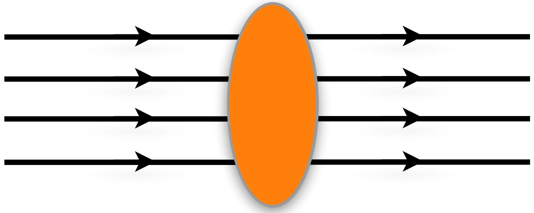
QMC-BASED SPECTRAL FUNCTION

The hole spectral function is a sum of two contributions

$$P_h(\mathbf{k},E) = \sum_n |\langle \Psi_0^A|[|k\rangle \times |\Psi_n^{A-1}\rangle|^2 \delta(E + E_0^A - E_n^{A-1}) = P_h^{MF}(\mathbf{k},E) + P_h^{corr}(\mathbf{k},E)$$

Mean-field component

$$|\bar{\Psi}_n^{A-1}\rangle =$$

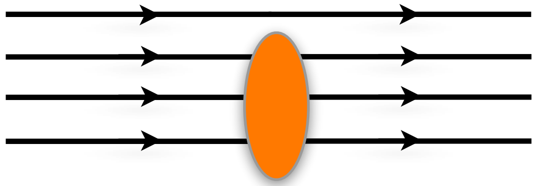


$$P_h^{MF}(\mathbf{k},E) = \sum_n |\langle \Psi_0^A|[|k\rangle \times |\bar{\Psi}_n^{A-1}\rangle|^2$$
$$\times\, \delta\left(E - B_0^A + B_n^{A-1} - \frac{k^2}{2M^{A-1}}\right)$$

Computed using VMC spectroscopic overlaps

Correlation component

$$|k'\rangle \times |\Psi_0^{A-2}\rangle =$$

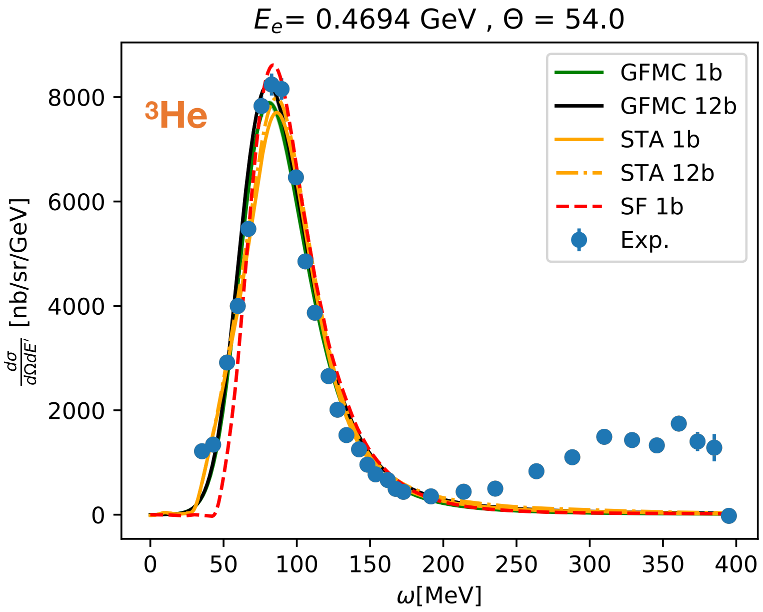


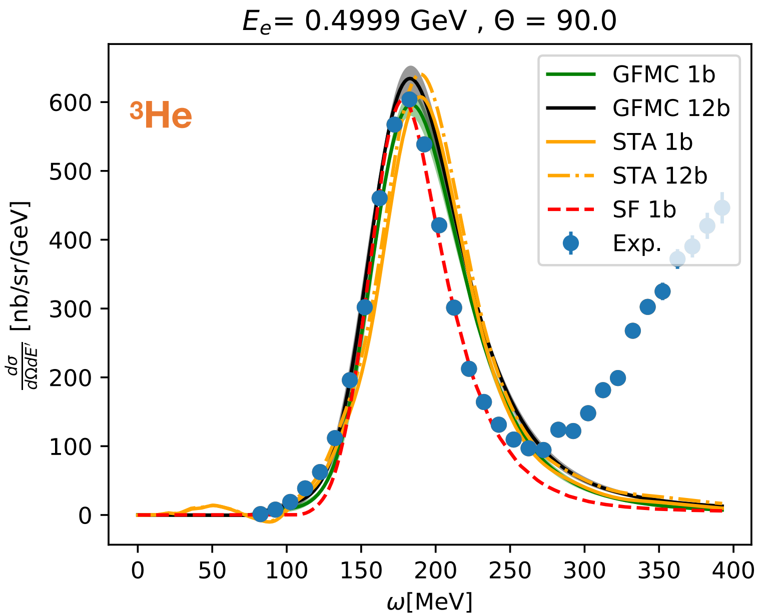
$$P_h^{corr}(\mathbf{k},E) \simeq \sum_{k'} |\langle \Psi_0^A|[|kk'\rangle \times |\bar{\Psi}_0^{A-2}\rangle|^2$$
$$\times\, \delta\left(E - B_0^A - e(k') + B_0^{A-2} - \frac{(\mathbf{k}+\mathbf{k}')^2}{2M^{A-2}}\right)$$

Computed from the short-range contributions of VMC two-body momentum distributions

QMC-BASED SPECTRAL FUNCTION

We compare the inclusive electron-^{3}He cross section obtained with the QMC spectral functions and those computed within the GFMC and STA approaches





L. Andreoli, AL, et al. PRC ***105****, 014002 (2022)*

We are using the same strategy to compute the spectral function of ^{12}C.

QMC-BASED INTRANUCLEAR CASCADE

The propagation of nucleons through the nuclear medium is crucial in the analysis of electron-nucleus scattering and neutrino oscillation experiments.

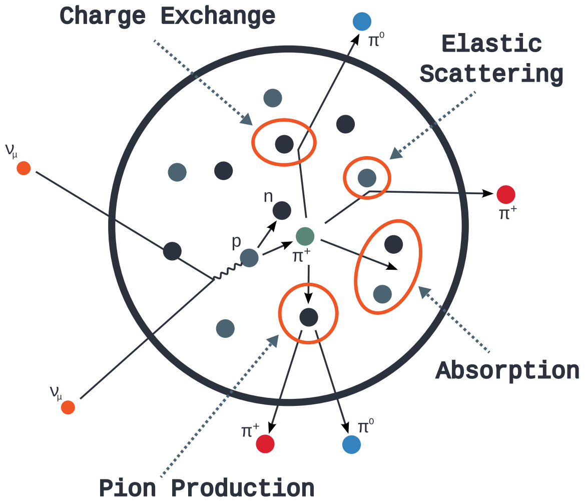


Figure from T. Golan

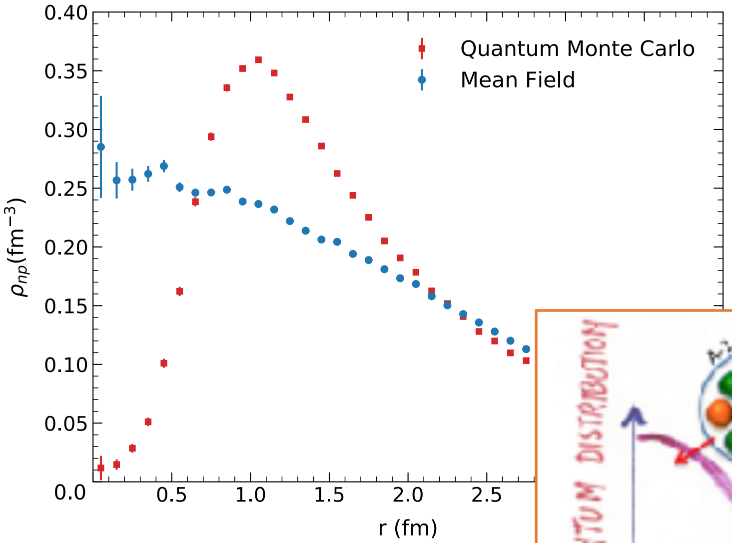
Ingredients:

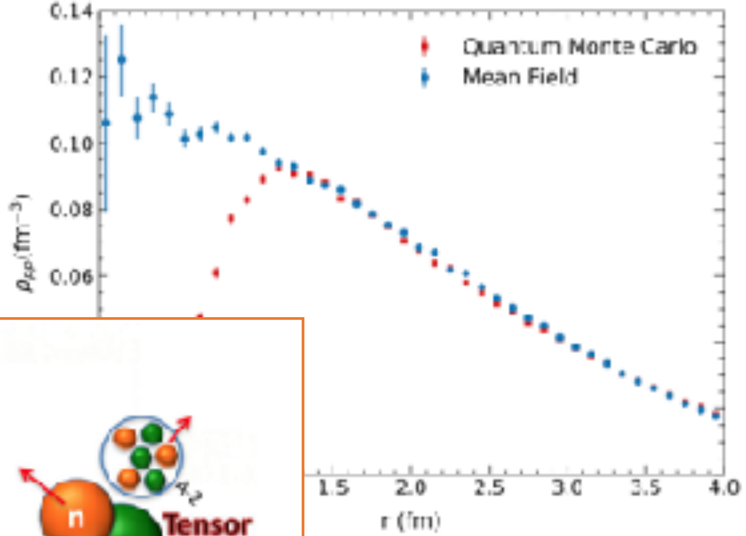
- Propagation of particles
- Elastic scattering
- Pion Production
- Pion Absorption

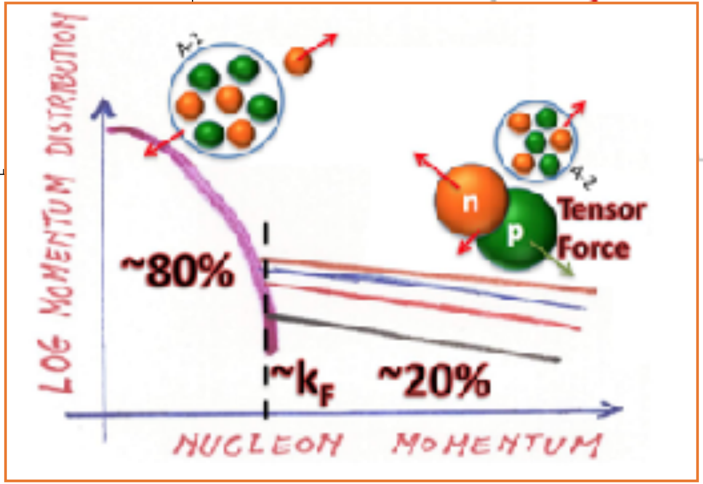
We have developed a semi-classical intra-nuclear cascade (INC) that assume classical propagation between consecutive scatterings and use QMC configurations as inputs;

QMC-BASED INTRANUCLEAR CASCADE

There is an enhancement of the neutron-proton two-body density distribution, consistent with the dominance of neutron-proton over proton-proton SRC pairs for a variety of nuclei







*O. Hen, et al. RMP **89**, 4 (2017)*

QMC-BASED INTRANUCLEAR CASCADE

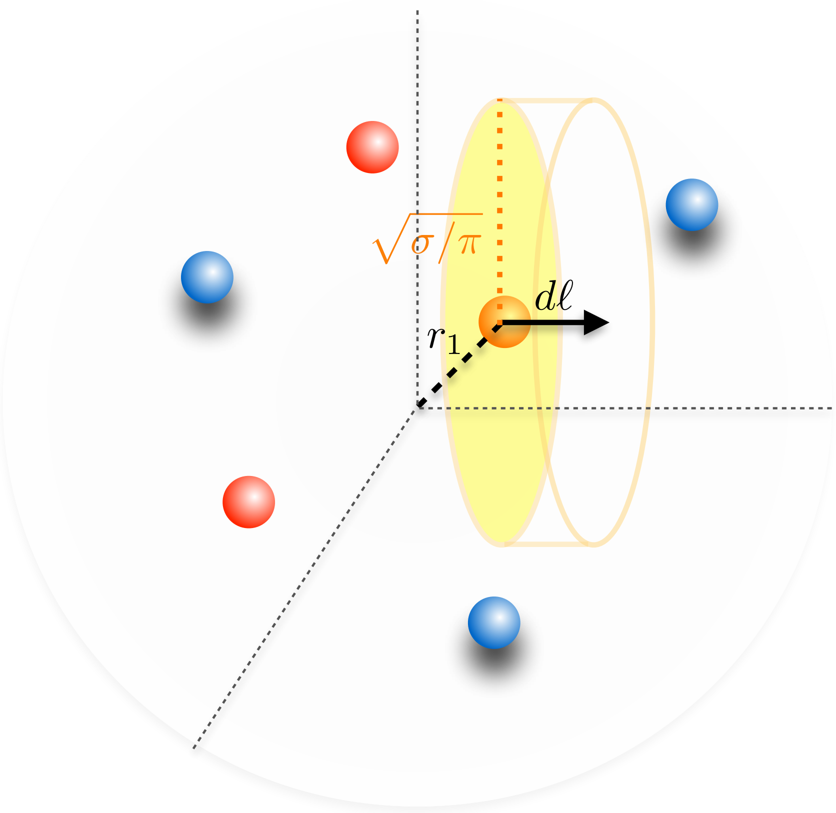
To check whether an interaction has occurred we consider an accept/reject algorithm based on a "cylinder" and a "gaussian" distributions

$$P_{\rm cyl}(b) = \theta(\sigma/\pi - b^2)$$

$$P_{\rm Gau}(b) = \exp\left(-\frac{\pi b^2}{\sigma}\right)$$

We have also implemented a standard mean free path approach

$$P_{\rm int} = (\rho_p \sigma_p + \rho_n \sigma_n) d\ell$$



PROTON-CARBON CROSS SECTION

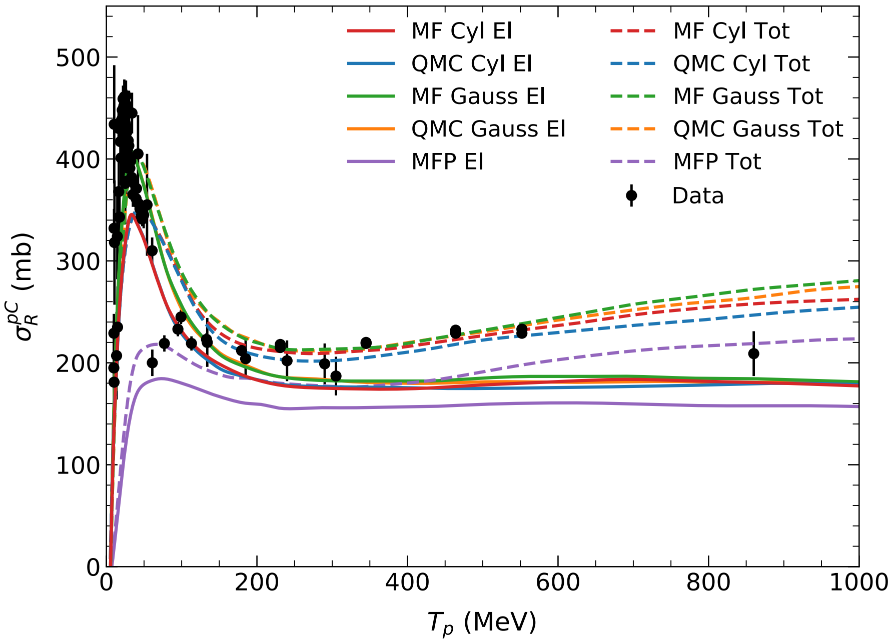
Reproducing proton-nucleus cross section measurements is an important test for INC.

- We define a beam of protons with kinetic energy T_p, uniformly distributed over an area A;
- We propagate each proton in time and check for scattering at each step;
- Monte Carlo cross section is defined as:

$$\sigma_{\mathrm{MC}} = A\frac{N_{\mathrm{scat}}}{N_{\mathrm{tot}}}$$

See also S. Dytman et al., 2103.07535

*J. Isaacson, et al., PRC **103**, 015502 (2021)*



Solid lines: elastic NN cross-section
Dashed lines: total NN cross section

NUCLEAR TRANSPARENCY

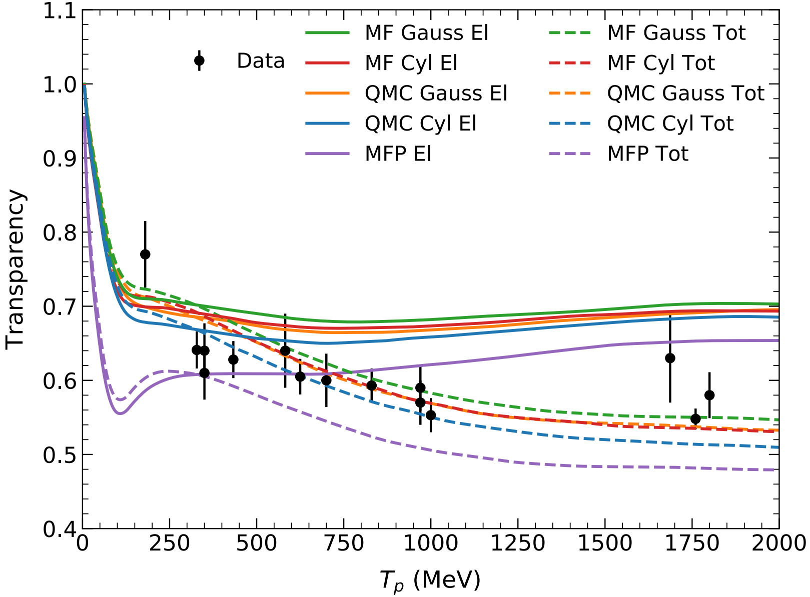
The nuclear transparency yields the average probability that a struck nucleon leaves the nucleus without interacting with the spectator particles

- The nuclear transparency is measured in (e,e'p) scattering experiments
- Simulation: we randomly sample a nucleon inside the nucleus from our configurations
- We give this nucleon a kinetic energy T_p and propagate it through the nuclear medium

$$T_{\mathrm{MC}} = 1 - \frac{N_{\mathrm{hits}}}{N_{\mathrm{tot}}}$$

See also S. Dytman et al., 2103.07535

*J. Isaacson, et al., PRC **103**, 015502 (2021)*



Solid lines: elastic NN cross-section
Dashed lines: total NN cross section

SUMMARY AND OUTLOOK

Lepton-nucleus scattering from quantum Monte Carlo

- Validated our approach on electron-^{12}C scattering
- Two-body currents enhance electromagnetic and charged-current responses
- Good agreement with MiniBooNE and T2K inclusive data → First ab-initio results!
- Use the AFDMC and ML methods to reach ^{16}O (and beyond)

Extended factorization scheme

- Two-body currents and pion-production are essential to reproduce electron-scattering data
- Need to treat two-pion production and deep-inelastic scattering regions
- Using VMC to construct an "ab initio" spectral function

Intra-nuclear cascade

- Validated the algorithm studying transparency and cross section
- The inclusion of the interaction vertex within the SF formalism is nearly completion
- Need to treat propagating pions and decaying resonances