Fiducial Higgs and Drell-Yan distributions at N^3LL'+NNLO with RadISH

Luca Rottoli

University of Zurich[UZH]

Based on: Re, LR, Torrielli 2104.07509

RadISH formalism: Monni, Re, Torrielli 1604.02191, Bizon, Monni, Re, LR, Torrielli 1705.09127

Resummation of transverse observables in colour-singlet production

Transverse observables (independent on the rapidity) $V(k) = g(\phi)\left(\frac{k_t}{M}\right)^a \qquad a > 0$

In this talk: focus on **inclusive observables** $V(k_1, \ldots k_n) = V(k_1 + \ldots + k_n)$

n.b.: RadISH can be formulated also for non-inclusive observables

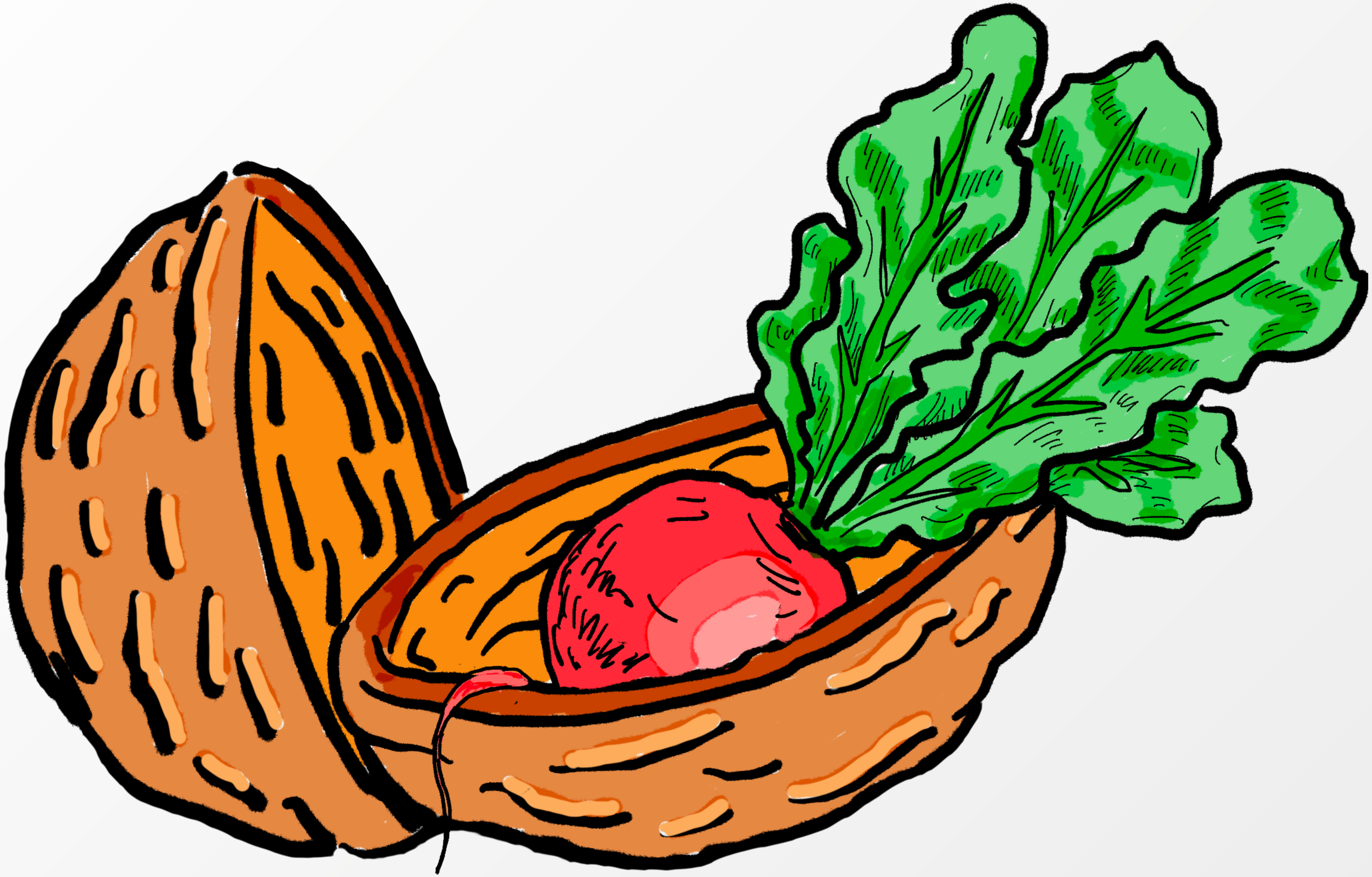
[Monni, LR, Torrielli '19]

$$\frac{p_t}{M}: \qquad V(k_1, \ldots, k_n) = \frac{1}{M}\left|\sum_i \vec{k}_{ti}\right| \qquad\qquad \phi^*: \qquad V(k_1, \ldots, k_n) = \frac{1}{M}\left|\sum_i k_{ti}\sin\phi_i\right| + \mathcal{O}(k_{ti}^2/M^2)$$

In regions dominated by soft/collinear radiation, large logarithms $L = \ln(v)$

Logarithmic counting defined at the **cumulative level**

$$\ln\Sigma(v) = \ln\int_0^v dV\frac{d\sigma}{dV} \sim \underbrace{\mathcal{O}(\alpha_s^n L^{n+1})}_{\text{LL}} + \underbrace{\mathcal{O}(\alpha_s^n L^{n})}_{\text{NLL}} + \underbrace{\mathcal{O}(\alpha_s^n L^{n-1})}_{\text{NNLL}} + \underbrace{\mathcal{O}(\alpha_s^n L^{n-2})}_{\text{N}^3\text{LL}} + \underbrace{\mathcal{O}(\alpha_s^n L^{n-3})}_{\text{N}^4\text{LL}} + \ldots$$



RadISH in a nutshell

All-order resummation: CAESAR/ARES approach

Translate the resummability into properties of the observable in the presence of multiple radiation: **recursive infrared and collinear (rIRC) safety**

[Banfi, Salam, Zanderighi '01, '03, '04]
[Banfi, McAslan, Monni, Zanderighi, El-Menoufi '14, '18]

Simple observable easy to calculate

$$\tilde{\sigma} \sim \int \frac{dv_1}{v_1} \Sigma_s(v_1) \mathcal{F}(v, v_1)$$

Transfer function relates the resummation of the full observable to the one of the simple observable.

i.e. conditional probability

All-order resummation: CAESAR/ARES approach

Translate the resummability into properties of the observable in the presence of multiple radiation: **recursive infrared and collinear (rIRC) safety**

[Banfi, Salam, Zanderighi ’01, ‘03, ’04]
[Banfi, McAslan, Monni, Zanderighi, El-Menoufi ’14, ’18]

Simple observable easy to calculate

$$\tilde{\sigma} \sim \int \frac{dv_1}{v_1} \Sigma_s(v_1) \mathcal{F}(v, v_1)$$

Transfer function relates the resummation of the full observable to the one of the simple observable.

i.e. conditional probability

Separation obtained by introducing a **resolution scale** $q_0 = \epsilon k_{t,1}$

$$\tilde{\sigma} \sim \int [dk_1] e^{-R(q_0)}$$

Unresolved emission can be treated as **totally unconstrained**
⟶ **exponentiation**

$$\times |\mathcal{M}(k_1)|^2 \left(\sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathcal{M}(k_i)|^2 \Theta(V(k_i) - q_0) \Theta\left(v - V(k_1, \dots, k_{m+1})\right) \right)$$

Resolved emission treated exclusively with **Monte Carlo methods.** Integral is finite, can be integrated in d=4 with a computer

k_t-ordering

All-order resummation: CAESAR/ARES approach

Translate the resummability into properties of the observable in the presence of multiple radiation: **recursive infrared and collinear (rIRC) safety**

[Banfi, Salam, Zanderighi '01, '03, '04]
[Banfi, McAslan, Monni, Zanderighi, El-Menoufi '14, '18]

Simple observable easy to calculate

$$\tilde{\sigma} \sim \int \frac{dv_1}{v_1} \Sigma_s(v_1) \mathscr{F}(v, v_1)$$

Transfer function relates the resummation of the full observable to the one of the simple observable.

i.e. conditional probability

Separation obtained by introducing a **resolution scale** $q_0 = \epsilon k_{t,1}$

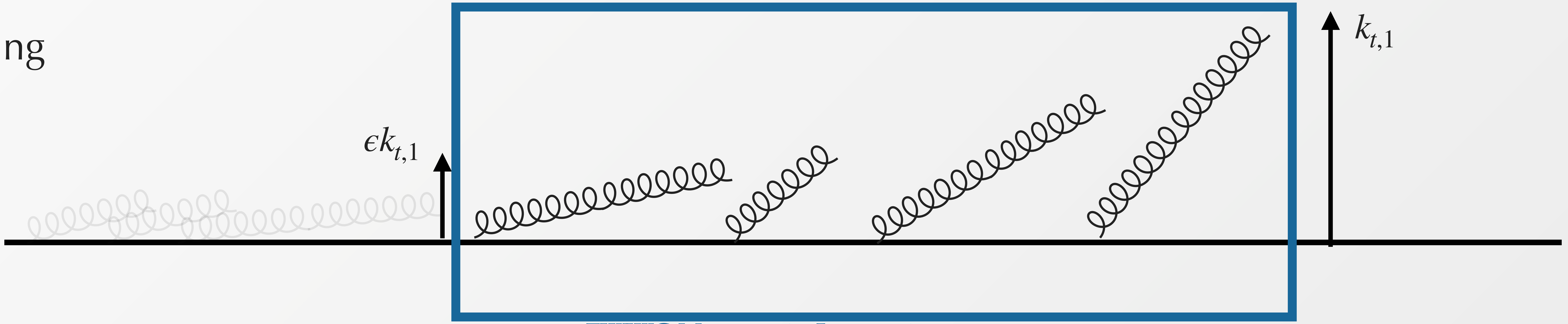
$$\tilde{\sigma} \sim \int [dk_1] e^{-R(q_0)}$$

Unresolved emission can be treated as **totally unconstrained** $\longrightarrow$ **exponentiation**

$$\times |\mathscr{M}(k_1)|^2 \left(\sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathscr{M}(k_i)|^2 \Theta(V(k_i) - q_0) \Theta\left(v - V(k_1, \ldots, k_{m+1})\right) \right)$$

Resolved emission treated exclusively with **Monte Carlo methods.** Integral is finite, can be integrated in d=4 with a computer

k_t-ordering



All-order resummation: CAESAR/ARES approach

Translate the resummability into properties of the observable in the presence of multiple radiation: **recursive infrared and collinear (rIRC) safety**

[Banfi, Salam, Zanderighi '01, '03, '04]
[Banfi, McAslan, Monni, Zanderighi, El-Menoufi '14, '18]

Simple observable easy to calculate

$$\tilde{\sigma} \sim \int \frac{dv_1}{v_1} \Sigma_s(v_1) \mathcal{F}(v, v_1)$$

Transfer function relates the resummation of the full observable to the one of the simple observable.

i.e. conditional probability

Separation obtained by introducing a **resolution scale** $q_0 = \epsilon k_{t,1}$

$$\tilde{\sigma} \sim \int [dk_1] e^{-R(q_0)}$$

Unresolved emission can be treated as **totally unconstrained**
$\longrightarrow$ **exponentiation**

$$\times |\mathcal{M}(k_1)|^2 \left(\sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathcal{M}(k_i)|^2 \Theta(V(k_i) - q_0) \Theta\left(v - V(k_1, \ldots, k_{m+1})\right) \right)$$

Resolved emission treated exclusively with **Monte Carlo methods.** Integral is finite, can be integrated in d=4 with a computer

Method entirely formulated in **direct space**

Approach recently formulated within SCET language [Bauer, Monni '18, '19 + ongoing work]

Resummation of the transverse momentum spectrum in direct space

Result at NLL accuracy (with fixed PDFs) can be written as

$$\tilde{\sigma} \sim \int \frac{dv_1}{v_1} \Sigma_s(v_1) \mathcal{F}(v, v_1)$$

$$\sigma(p_\perp) = \sigma_0 \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(v_1)}$$

Unresolved

$$v_i = k_{t,i}/m_H, \quad \zeta_i = v_i/v_1$$

$$\times\, \epsilon^{R'(v_1)} R'\left(v_1\right) \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i v_1\right)\, \Theta\left(p_\perp - |\vec{k}_{t,i} + \cdots \vec{k}_{t,n+1}|\right)$$

Resolved

Resummation of the transverse momentum spectrum in direct space

Result at NLL accuracy (with fixed PDFs) can be written as

$$\tilde{\sigma} \sim \int \frac{dv_1}{v_1} \Sigma_s(v_1) \mathcal{F}(v, v_1)$$

$$v_i = k_{t,i}/m_H, \quad \zeta_i = v_i/v_1$$

$$\sigma(p_\perp) = \sigma_0 \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(v_1)}$$

Unresolved

$$\times\, \epsilon^{R'(v_1)} R'\left(v_1\right) \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i v_1\right)\, \Theta\left(p_\perp - |\vec{k}_{t,i} + \cdots \vec{k}_{t,n+1}|\right)$$

Resolved

Sudakov and **azimuthal mechanisms** accounted for, **no assumption** on $k_{t,i}$ vs $p_\perp$ hierarchy.
Subleading effects retained: no divergence at small $p_\perp$, Parisi-Petronzio power-like behaviour respected

Logarithmic accuracy defined in terms of $\ln(m_H/k_{t1})$

Result formally equivalent to the b-space formulation [Bizon, Monni, Re, LR, Torrielli '17]

All-order formula in Mellin space [Bizon, Monni, Re, LR, Torrielli '17]

Now include effect of **collinear radiation** and terms beyond NLL accuracy

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathscr{C}_1} \frac{dN_1}{2\pi i} \int_{\mathscr{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1 c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) \mathbf{f}_{N_2}(\mu_0),$$

Unresolved

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{ -\sum_{\ell=1}^{2} \left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t)) \right) \right\}$$

Resolved

$$\times \sum_{\ell_1=1}^{2} \left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1})) \right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),$$

$$\times \sum_{\ell_i=1}^{2} \left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti})) \right)$$

All-order formula in Mellin space [Bizon, Monni, Re, LR, Torrielli '17]

Now include effect of **collinear radiation** and terms beyond NLL accuracy

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathcal{C}_1} \frac{dN_1}{2\pi i} \int_{\mathcal{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) \mathbf{f}_{N_2}(\mu_0),$$

Unresolved

Hard-virtual coefficient

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

Resolved

$$\times \sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),$$

$$\times \sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

All-order formula in Mellin space [Bizon, Monni, Re, LR, Torrielli '17]

Now include effect of **collinear radiation** and terms beyond NLL accuracy

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathscr{C}_1} \frac{dN_1}{2\pi i} \int_{\mathscr{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \hat{\boldsymbol{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) \mathbf{f}_{N_2}(\mu_0),$$

Collinear coefficient functions and their RGE

Unresolved

$$\hat{\boldsymbol{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \boldsymbol{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \boldsymbol{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

Resolved

$$\times \sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \boldsymbol{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \boldsymbol{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),$$

$$\times \sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \boldsymbol{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \boldsymbol{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

All-order formula in Mellin space [Bizon, Monni, Re, LR, Torrielli '17]

Now include effect of **collinear radiation** and terms beyond NLL accuracy

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathscr{C}_1} \frac{dN_1}{2\pi i} \int_{\mathscr{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) \mathbf{f}_{N_2}(\mu_0),$$

Unresolved

DGLAP evolution

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{ -\sum_{\ell=1}^{2} \left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t)) \right) \right\}$$

Resolved

$$\times \sum_{\ell_1=1}^{2} \left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1})) \right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),$$

$$\times \sum_{\ell_i=1}^{2} \left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti})) \right)$$

Inclusion of N^{3}LL$'$ effects in RadISH [Re, LR, Torrielli '21]

Capture **all constant terms** of relative order $\mathcal{O}(\alpha_s^3)$

- α_s^3 is N^{4}LL (since $\alpha_s^n L^{n-3}$) but sufficient to get all $\alpha_s^n L^{2n-6}$ in the cumulant
- Allows for the computation of **N^{3}LO cross section** for H, DY production based on $q_\perp$-subtraction methods

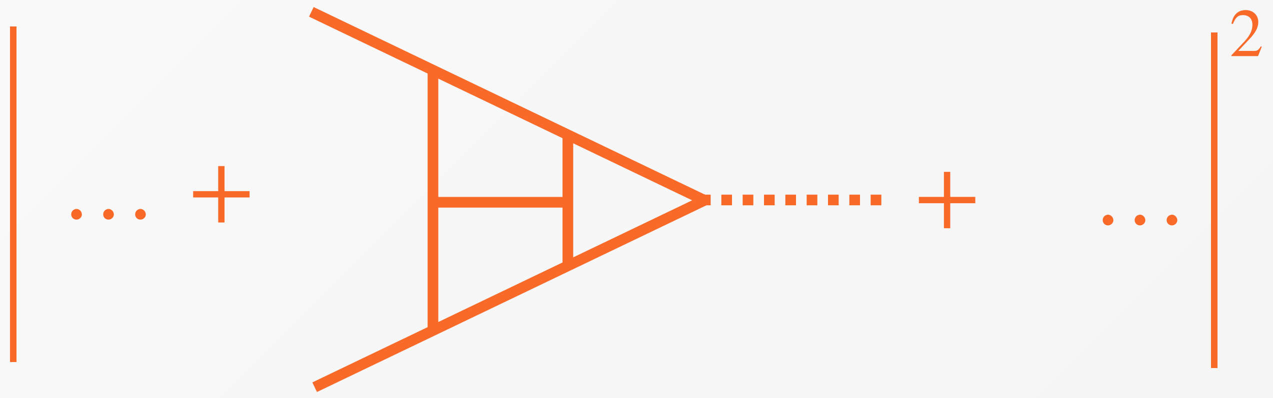
[Billis et al. '21][Cieri et al. '21]

Inclusion of N³LL′ effects in RadISH [Re, LR, Torrielli '21]

Sources of N³LL′ correction, neglected in previous RadISH implementation

Three-loop hard-virtual coefficient

$$H(\alpha_s) = 1 + \left(\frac{\alpha_s}{2\pi}\right) H_1 + \left(\frac{\alpha_s}{2\pi}\right)^2 H_2 + \left(\frac{\alpha_s}{2\pi}\right)^3 H_3$$



[Gehrmann et al. '10]

Three-loop Wilson coefficient for Higgs EFT

[Schroder, Steinhauser '05]

$$\hat{\Sigma}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0))\, H(\mu_R)\, \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi}\frac{d\phi_1}{2\pi}\, e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{-\sum_{\ell=1}^{2} \left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\boldsymbol{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\boldsymbol{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

$$\times \sum_{\ell_1=1}^{2} \left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\boldsymbol{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \boldsymbol{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!}\prod_{i=2}^{n+1}\int_\epsilon^1 \frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\,\Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right),$$

$$\times \sum_{\ell_i=1}^{2} \left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi}\boldsymbol{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \boldsymbol{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

Inclusion of N3LL′ effects in RadISH [Re, LR, Torrielli '21]

Sources of N3LL′ correction, neglected in previous RadISH implementation

$$\hat{\Sigma}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0))H(\mu_R)\mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right]\int_0^M \frac{dk_{t1}}{k_{t1}}\int_0^{2\pi}\frac{d\phi_1}{2\pi}e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

$$\times \sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=2}^{n+1}\int_{\epsilon}^{1}\frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right),$$

$$\times \sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi}\mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

Three-loop coefficient functions

$$C(\alpha_s, z) = \delta(1-z) + \left(\frac{\alpha_s}{2\pi}\right)C_1(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 C_2(z) + \left(\frac{\alpha_s}{2\pi}\right)^3 C_3(z)$$

[[Li, Zhu '16][Vladimirov '16]Luo et al. '19][Ebert et al. '20]

For Higgs production: **two-loop G coefficient functions**

[Catani, Grazzini '11]

$$G(\alpha_s, z) = \left(\frac{\alpha_s}{2\pi}\right)G_1(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 G_2(z)$$

[Luo et al. '19]

Inclusion of N3LL′ effects in RadISH [Re, LR, Torrielli '21]

Sources of N3LL′ correction, neglected in previous RadISH implementation

Constants terms coming from the Sudakov

$$R(k_{t1}) = -\log\frac{M}{k_{t1}} g_1 - g_2 - \left(\frac{\alpha_s}{\pi}\right) g_3 - \left(\frac{\alpha_s}{\pi}\right)^2 g_4 - \left(\frac{\alpha_s}{\pi}\right)^3 g_5$$

Resummation scale $Q \sim M$

$$\ln\frac{M}{k_{t1}} \to \ln\frac{Q}{k_{t1}} + \ln\frac{M}{Q}$$

Constant terms expanded in α_s and included in H

$$\hat{\Sigma}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0))H(\mu_R)\mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right]\int_0^M\frac{dk_{t1}}{k_{t1}}\int_0^{2\pi}\frac{d\phi_1}{2\pi}e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times\exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

$$\times\sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=2}^{n+1}\int_\epsilon^1\frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right),$$

$$\times\sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi}\mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

Inclusion of N3LL′ effects in RadISH [Re, LR, Torrielli '21]

Sources of N3LL′ correction, neglected in previous RadISH implementation

Constants terms coming resolved contributions

$$\Gamma(\alpha_s) = \Gamma^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)\Gamma^{(1)}$$

$$\Gamma^{(C)}(\alpha_s) = \left(\frac{\alpha_s}{2\pi}\right)^2 \Gamma^{(C,1)}$$

$$\hat{\Sigma}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0))H(\mu_R)\mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right]\int_0^M \frac{dk_{t1}}{k_{t1}}\int_0^{2\pi}\frac{d\phi_1}{2\pi}e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

$$\times \sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=2}^{n+1}\int_\epsilon^1\frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right),$$

$$\times \sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi}\mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

Momentum-space formula at N³LL

$$
\begin{aligned}
\frac{d\Sigma(v)}{d\Phi_B} = & \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}\partial_L\left(-e^{-R(k_{t1})}\mathcal{L}_{\mathrm{N^3LL}}(k_{t1})\right)\int d\mathcal{Z}\,\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right) \\
& + \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}\int_0^1 \frac{d\zeta_s}{\zeta_s}\frac{d\phi_s}{2\pi}\Bigg\{\Big(R'(k_{t1})\mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1})\Big)\times\left(R''(k_{t1})\ln\frac{1}{\zeta_s} + \frac{1}{2}R'''(k_{t1})\ln^2\frac{1}{\zeta_s}\right) \\
& - R'(k_{t1})\left(\partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - 2\frac{\beta_0}{\pi}\alpha_s^2(k_{t1})\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\ln\frac{1}{\zeta_s}\right) \\
& + \frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\} \\
& \times\left\{\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1},k_s)\right) - \Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right)\right\} \\
& + \frac{1}{2}\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}\int_0^1 \frac{d\zeta_{s1}}{\zeta_{s1}}\frac{d\phi_{s1}}{2\pi}\int_0^1 \frac{d\zeta_{s2}}{\zeta_{s2}}\frac{d\phi_{s2}}{2\pi}R'(k_{t1})\Bigg\{\mathcal{L}_{\mathrm{NLL}}(k_{t1})\big(R''(k_{t1})\big)^2\ln\frac{1}{\zeta_{s1}}\ln\frac{1}{\zeta_{s2}} - \partial_L\mathcal{L}_{\mathrm{NLL}}(k_{t1})R''(k_{t1})\left(\ln\frac{1}{\zeta_{s1}} + \ln\frac{1}{\zeta_{s2}}\right) \\
& + \frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\} \\
& \times\Big\{\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1},k_{s1},k_{s2})\right) - \Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1},k_{s1})\right) - \\
& \quad \Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1},k_{s2})\right) + \Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right)\Big\} + \mathcal{O}\left(\alpha_s^n\ln^{2n-6}\frac{1}{v}\right).
\end{aligned}
$$

Momentum-space formula at N³LL′

Luminosity factors now contains H_3 and C_3

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathscr{L}_{\mathrm{N^3LL'}}(k_{t1})\right) \int d\mathcal{Z} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right)$$

$$+ \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z} \int_0^1 \frac{d\zeta_s}{\zeta_s} \frac{d\phi_s}{2\pi} \Bigg\{ \left(R'(k_{t1}) \mathscr{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L \mathscr{L}_{\mathrm{NNLL}}(k_{t1}) \right) \times \left(R''(k_{t1}) \ln \frac{1}{\zeta_s} + \frac{1}{2} R'''(k_{t1}) \ln^2 \frac{1}{\zeta_s} \right)$$

$$- R'(k_{t1}) \left(\partial_L \mathscr{L}_{\mathrm{NNLL}}(k_{t1}) - 2 \frac{\beta_0}{\pi} \alpha_s^2(k_{t1}) \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL}}(k_{t1}) \ln \frac{1}{\zeta_s} \right)$$

$$+ \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL'}}(k_{t1}) - \beta_0 \frac{\alpha_s^3(k_{t1})}{\pi^2} \left(\hat{P}^{(0)} \otimes \hat{C}^{(1)} + \hat{C}^{(1)} \otimes \hat{P}^{(0)} \right) \otimes \mathscr{L}_{\mathrm{NLL}}(k_{(t1)}) + \frac{\alpha_s^3(k_{t1})}{\pi^2} 2\beta_0 \ln \frac{1}{\zeta_s} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL}}(k_{(t1)})$$

New structures appearing at α_s^3

$$+ \frac{\alpha_s^3(k_{t1})}{2\pi^2} \left(\hat{P}^{(0)} \otimes \hat{P}^{(1)} + \hat{P}^{(1)} \otimes \hat{P}^{(0)} \right) \otimes \mathscr{L}_{\mathrm{NLL}}(k_{(t1)}) \Bigg\} \times \left\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_s)\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \right\}$$

Convolution structure obtained after Mellin inversion

$$+ \frac{1}{2} \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z} \int_0^1 \frac{d\zeta_{s1}}{\zeta_{s1}} \frac{d\phi_{s1}}{2\pi} \int_0^1 \frac{d\zeta_{s2}}{\zeta_{s2}} \frac{d\phi_{s2}}{2\pi} R'(k_{t1}) \Bigg\{ \mathscr{L}_{\mathrm{NLL}}(k_{t1}) \left(R''(k_{t1})\right)^2 \ln \frac{1}{\zeta_{s1}} \ln \frac{1}{\zeta_{s2}} - \partial_L \mathscr{L}_{\mathrm{NLL}}(k_{t1}) R''(k_{t1}) \left(\ln \frac{1}{\zeta_{s1}} + \ln \frac{1}{\zeta_{s2}} \right)$$

$$+ \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL}}(k_{t1}) + \frac{\alpha_s^2(k_{t1})}{\pi^2} \left(\ln \frac{1}{\zeta_{s1}} + \ln \frac{1}{\zeta_{s2}} \right) R''(k_{t1}) \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL}}(k_{(t1)}) - \ln \frac{1}{\zeta_{s1}} \ln \frac{1}{\zeta_{s2}} (R''(k_{t1})^2 \partial_L \mathscr{L}_{\mathrm{NLL}}(k_{(t1)})$$

$$+ \frac{\alpha_s^2(k_{t1})}{\pi^3} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL}}(k_{t1}) \Bigg\} \times \Big\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s1}, k_{s2})\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s1})\right) -$$

$$\Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s2})\right) + \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \Big\} + \mathcal{O}\left(\alpha_s^n \ln^{2n-7} \frac{1}{v}\right)$$

Extra column of logs predicted

Ambiguity in the definition of primed accuracy

$$\mathscr{L}_{\mathrm{NNLL}}(k_{t1}) = \sum_{c,c'} \frac{d|\mathscr{M}_B|^2_{cc'}}{d\Phi_B} \sum_{i,j} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} f_i\left(k_{t1}, \frac{x_1}{z_1}\right) f_j\left(k_{t1}, \frac{x_2}{z_2}\right)$$
$$\times \left\{ \delta_{ci}\,\delta_{c'j}\,\delta(1-z_1)\,\delta(1-z_2)\left(1 + \frac{\alpha_s(\mu_R)}{2\pi} H^{(1)}(\mu_R)\right) + \frac{\alpha_s(\mu_R)/(2\pi)}{1 - 2\alpha_s(\mu_R)\beta_0 \ln(\mu_R/k_{t1})}\left(C^{(1)}_{ci}(z_1)\delta(1-z_2)\delta_{c'j} + \{z_1, c, i \leftrightarrow z_2, c', j\}\right)\right\}$$

Scale at which the α_s^k term is evaluated is subleading at N^{k}LL′ accuracy

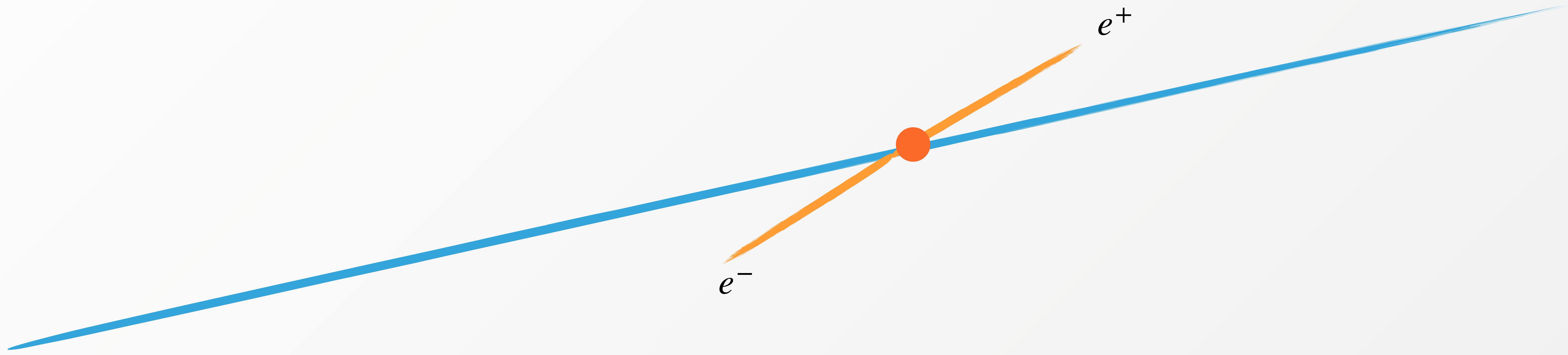
One can evaluate this contribution with $\alpha_s(\mu_R)$ rather → difference reflects **ambiguity** of these subleading effects

NLL′ **with running**: $\mathscr{L}_{\mathrm{NLL}'} = \mathscr{L}_{\mathrm{NNLL}}$ **Our default choice**

NLL′ **without running**: $\mathscr{L}_{\mathrm{NLL}'} = \mathscr{L}_{\mathrm{NNLL}}$ with $\alpha_s(\mu_R)$ in the C_1 component

Inclusion of transverse recoil effects

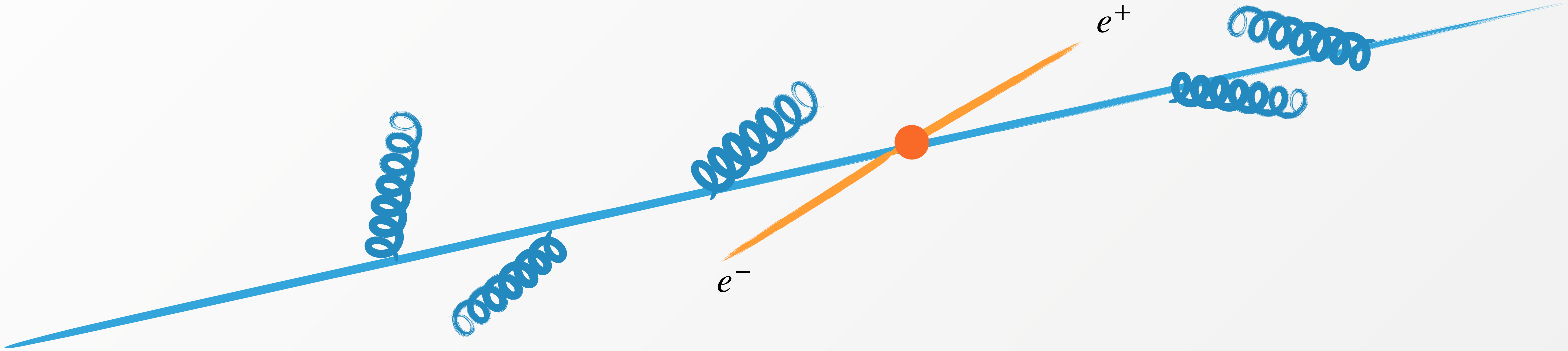
[Catani et al '15]



Born matrix element evaluated at $p_t = 0$

Inclusion of transverse recoil effects

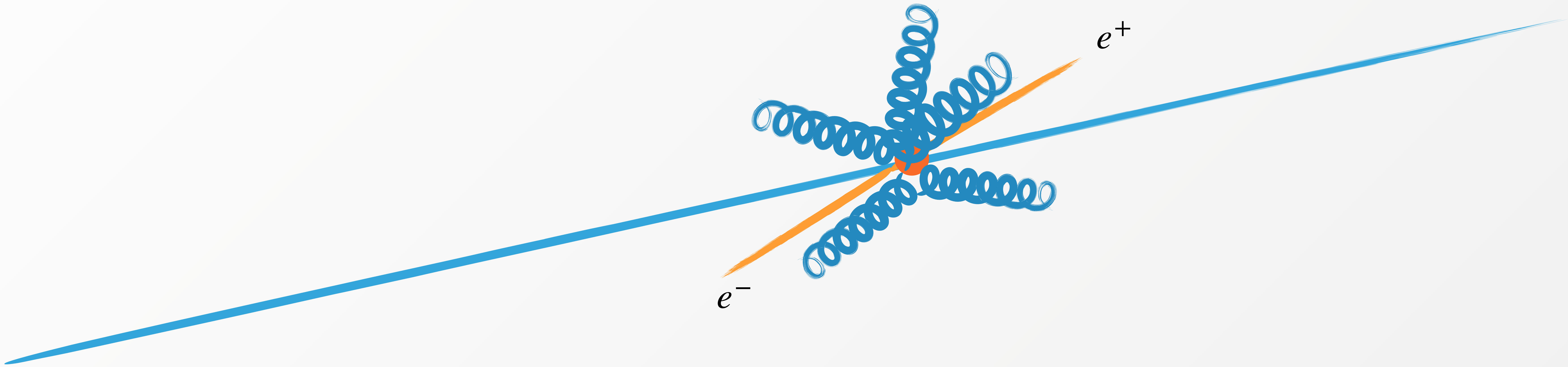
[Catani et al '15]



Generate singlet p_t by QCD radiation

Inclusion of transverse recoil effects

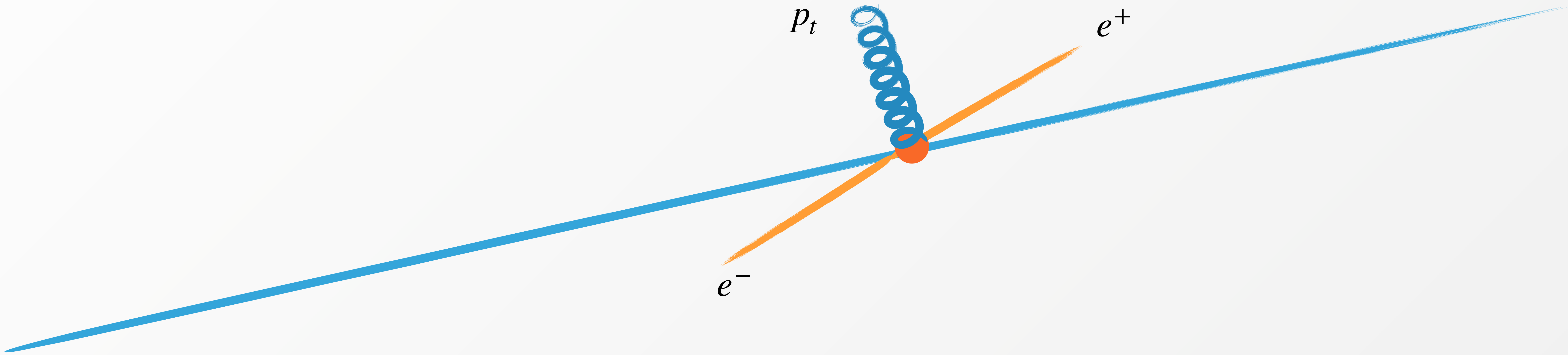
[Catani et al '15]



Generate singlet p_t by QCD radiation

Inclusion of transverse recoil effects

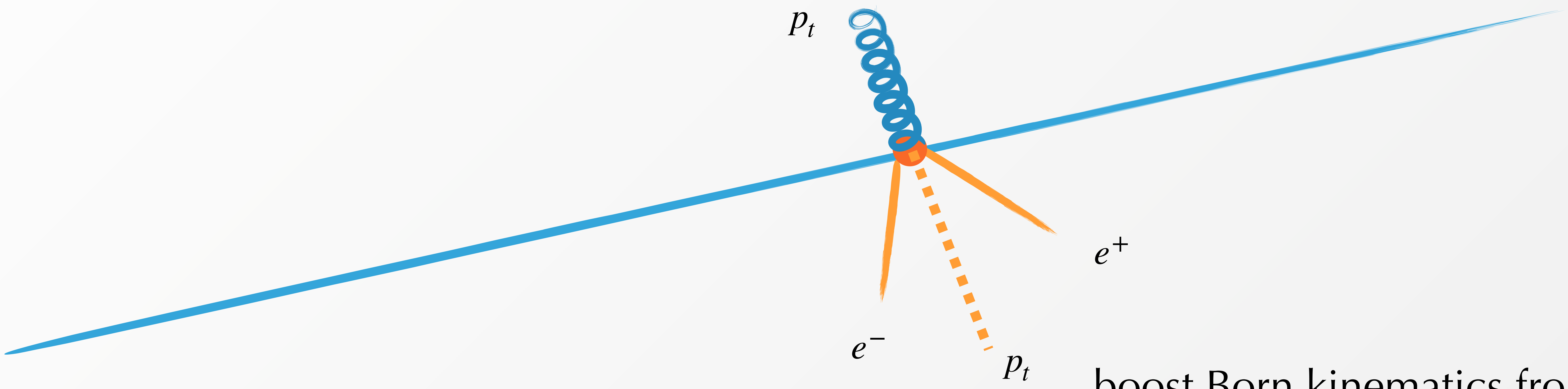
[Catani et al '15]



Generate singlet p_t by QCD radiation

Inclusion of transverse recoil effects

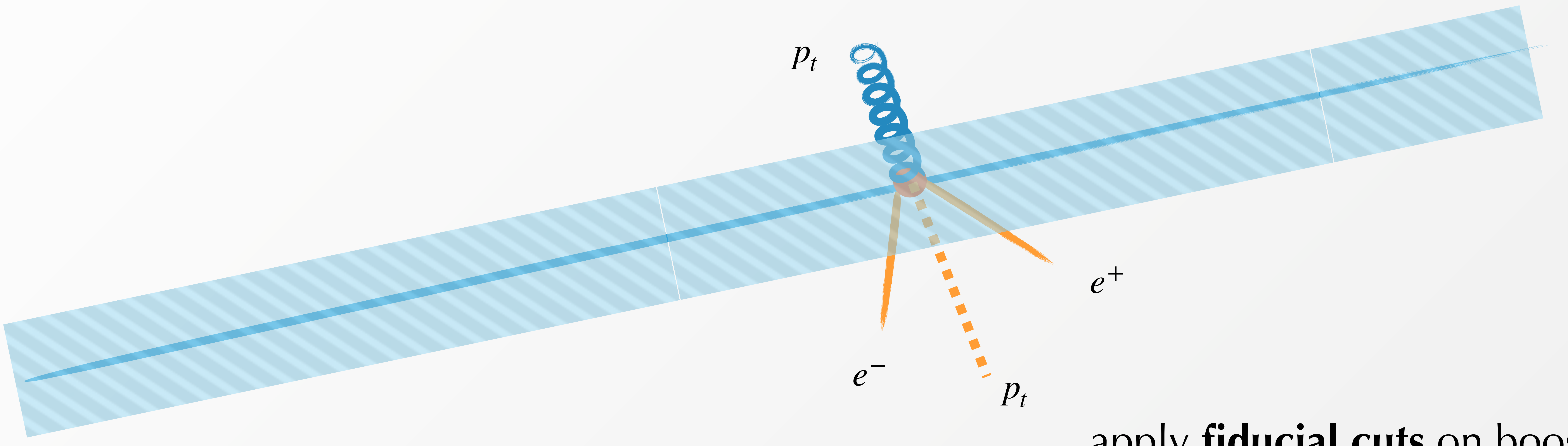
[Catani et al '15]



boost Born kinematics from boson rest frame (e.g. CS) to lab frame with that p_t

Inclusion of transverse recoil effects

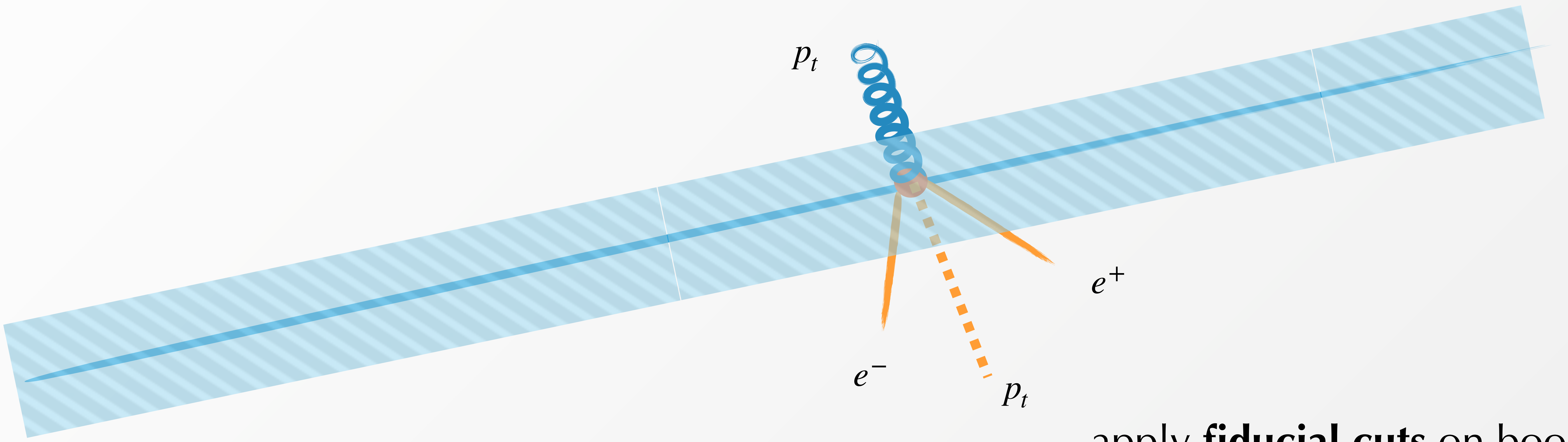
[Catani et al '15]



apply **fiducial cuts** on boosted Born kinematics

Inclusion of transverse recoil effects

[Catani et al '15]

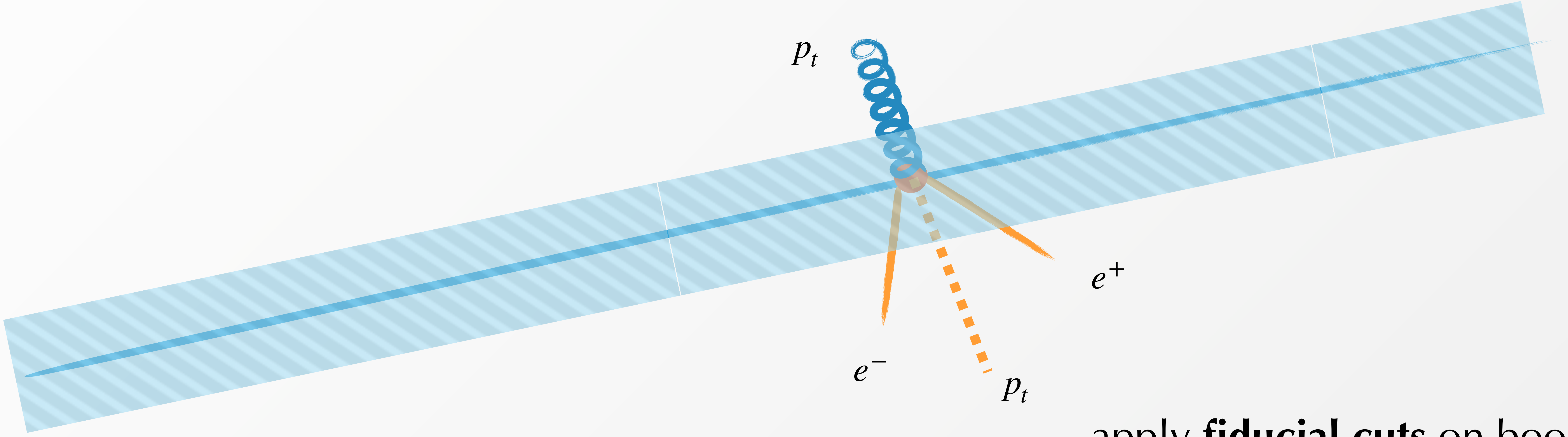


apply **fiducial cuts** on boosted Born kinematics

Sufficient to capture the **full linear fiducial power correction** for p_t [Ebert et al. '20]

Inclusion of transverse recoil effects

[Catani et al '15]



apply **fiducial cuts** on boosted Born kinematics

Implementation in RadISH:

- Each contribution in the resummation formula boosted in the corresponding frame
- Derivative of the expansion computed on-the-fly, boost computed according to the value of v

Results

Matching to fixed order

Two different families of **matching schemes**, defined at the **differential** level (due to the inclusion of **recoil effects**)

Additive matching

$$\frac{d\Sigma_{\rm add}^{{\rm N}^k{\rm LL}^{(\prime)}}(v)}{dv} = \left(\frac{d\Sigma^{{\rm N}^k{\rm LL}^{(\prime)}}(v)}{dv} - \frac{d\Sigma_{\rm exp}^{{\rm N}^k{\rm LL}^{(\prime)}}(v)}{dv}\right) Z(v) + \frac{d\Sigma^{{\rm N}^{k-1}{\rm LO}}(v)}{dv}$$

Multiplicative matching

$$\frac{d\Sigma_{\rm mult}^{{\rm N}^k{\rm LL}^{(\prime)}}(v)}{dv} = \left(\frac{d\Sigma^{{\rm N}^k{\rm LL}^{(\prime)}}(v)/dv}{d\Sigma_{\rm exp}^{{\rm N}^k{\rm LL}^{(\prime)}}(v)/dv}\right)^{Z(v)} \frac{d\Sigma^{{\rm N}^{k-1}{\rm LO}}(v)}{dv}$$

At NNLO+N^{3}LL$'$ the two matching schemes are on equal footing, differences starts at α_s^4

Damping function (does not act on linear power corrections)

$$Z(v) = \left[1 - (v/v_0)^2\right]^3 \Theta(v_0 - v)$$

v_0 varied in the interval [2/3, 3/2] around central value to **estimate matching uncertainty**

Central value $v_0 = 1$ for $p_\perp$ and $v_0 = 1/2$ for ϕ_η^*

Drell-Yan production: setup

Drell-Yan fiducial region defined as [ATLAS 2019]

$$p_t^{\ell^\pm} > 27\,\mathrm{GeV}\,, \qquad |\eta^{\ell^\pm}| < 2.5\,, \qquad 66\,\mathrm{GeV} < m_{\ell\ell} < 116\,\mathrm{GeV}$$

Central scales chosen as

$$\mu_R = \kappa_R\, M_t \qquad \mu_F = \kappa_F\, M_t, \qquad Q = \kappa_Q\, M_{\ell\ell} \qquad M_t = \sqrt{M_{\ell\ell}^2 + p_t^{\ell\ell\,2}}$$

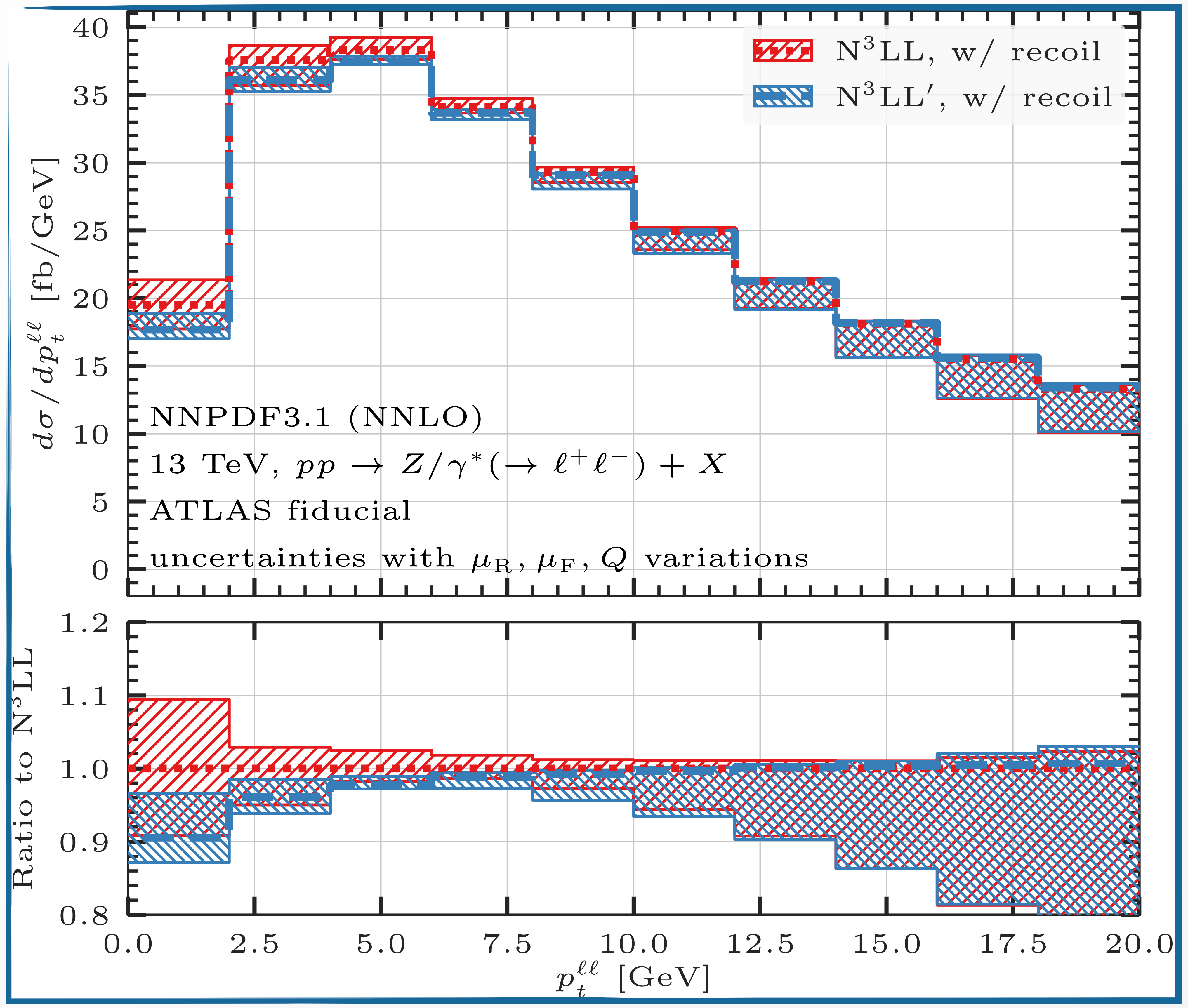
In resummed predictions $M_t \to M_{\ell\ell} = M_t + \mathcal{O}\left(\frac{p_t^{\ell\ell}}{M_{\ell\ell}}\right)^2$

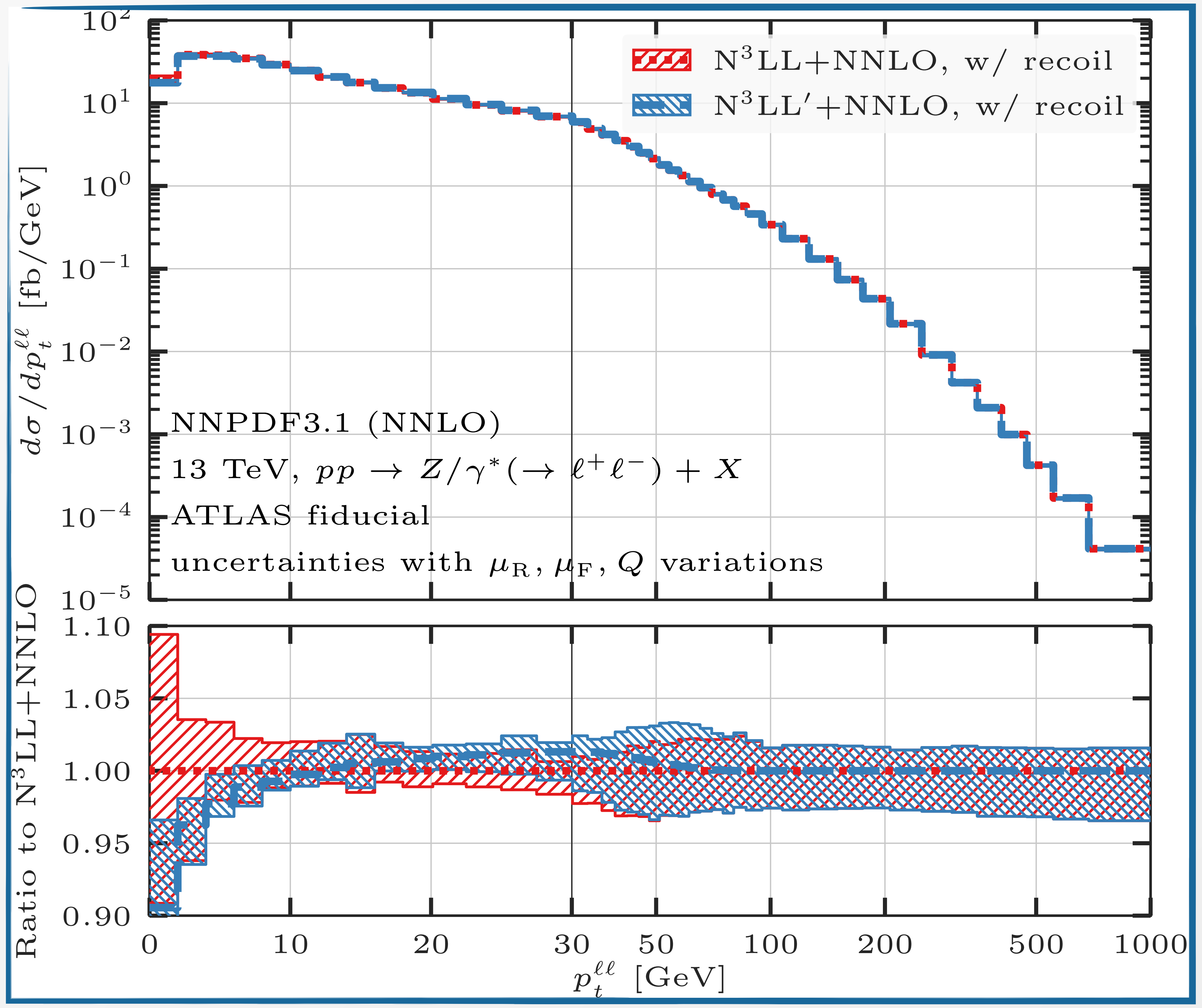
Scale uncertainty:

[canonical 7 scale variation + variation of κ_Q by a factor of 2 for central μ_R, μ_F] $\times$ 3 values of v_0 $\to$ **27 variations**

NNPDF31 NNLO parton densities with $\alpha_s = 0.118$

Drell-Yan production: N^{3}LL$'$ effects





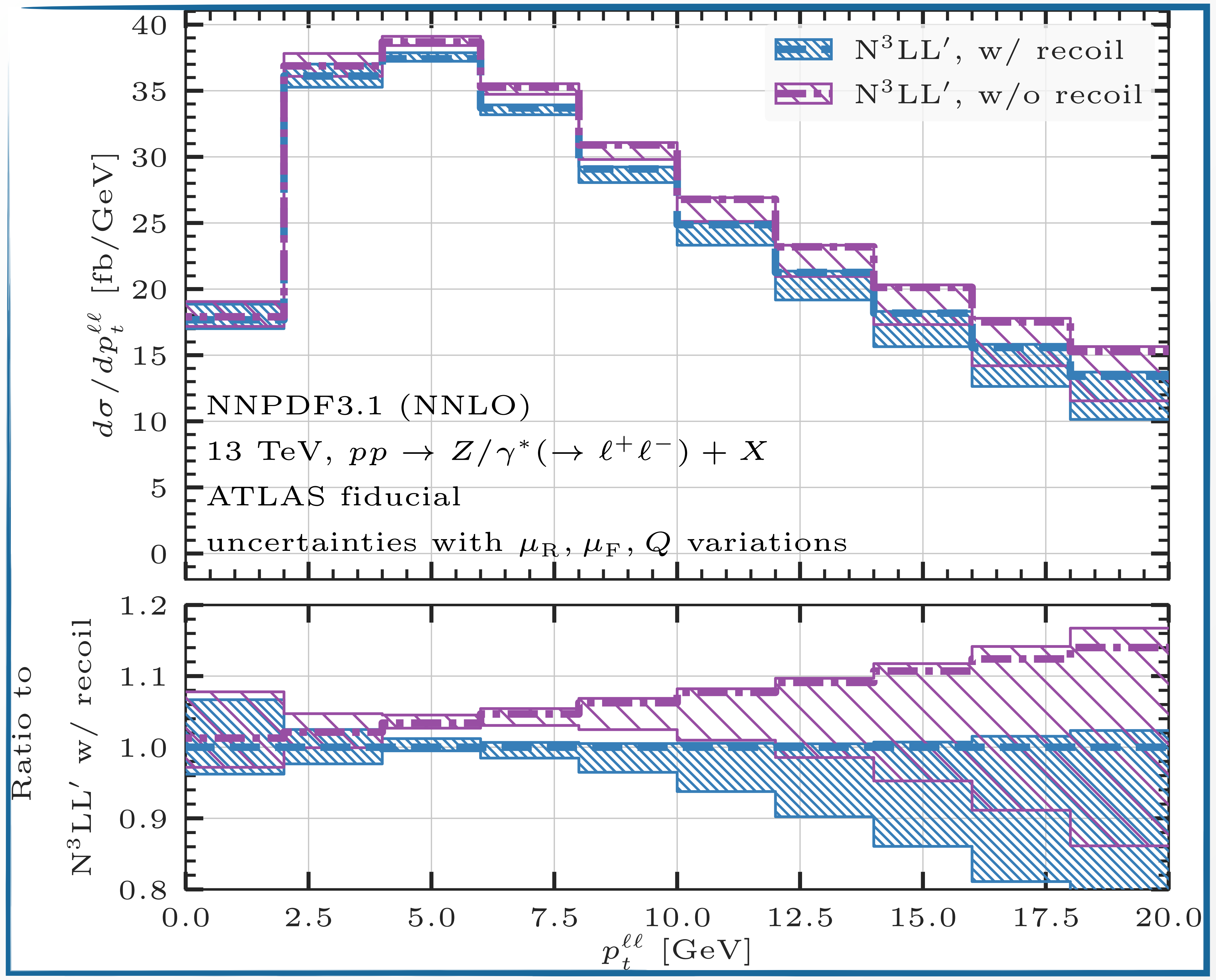
$\kappa_R = \kappa_F = 1, \qquad \kappa_Q = 1/2$

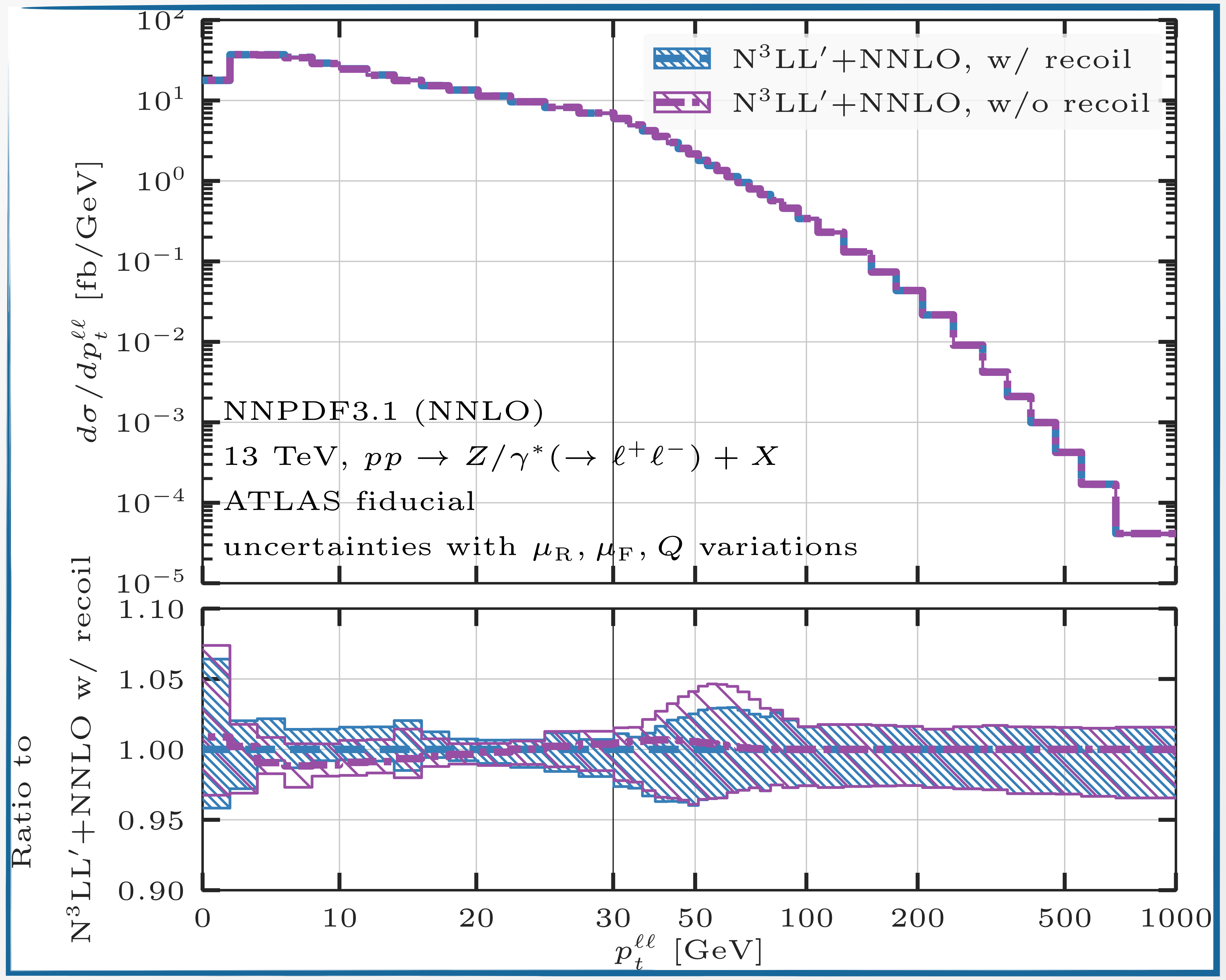
Reduction in theoretical uncertainty below 10 GeV

Modification at the **5-10% level** below 10 GeV (similar effect, but larger, present at NNLL vs NNLL$'$)

Minor differences with respect to N^{3}LL for value of p_t larger than 5 GeV

Transverse recoil effects in fiducial DY setup





At the pure resummed level recoil prescription captures whole linear power corrections from fiducial cuts

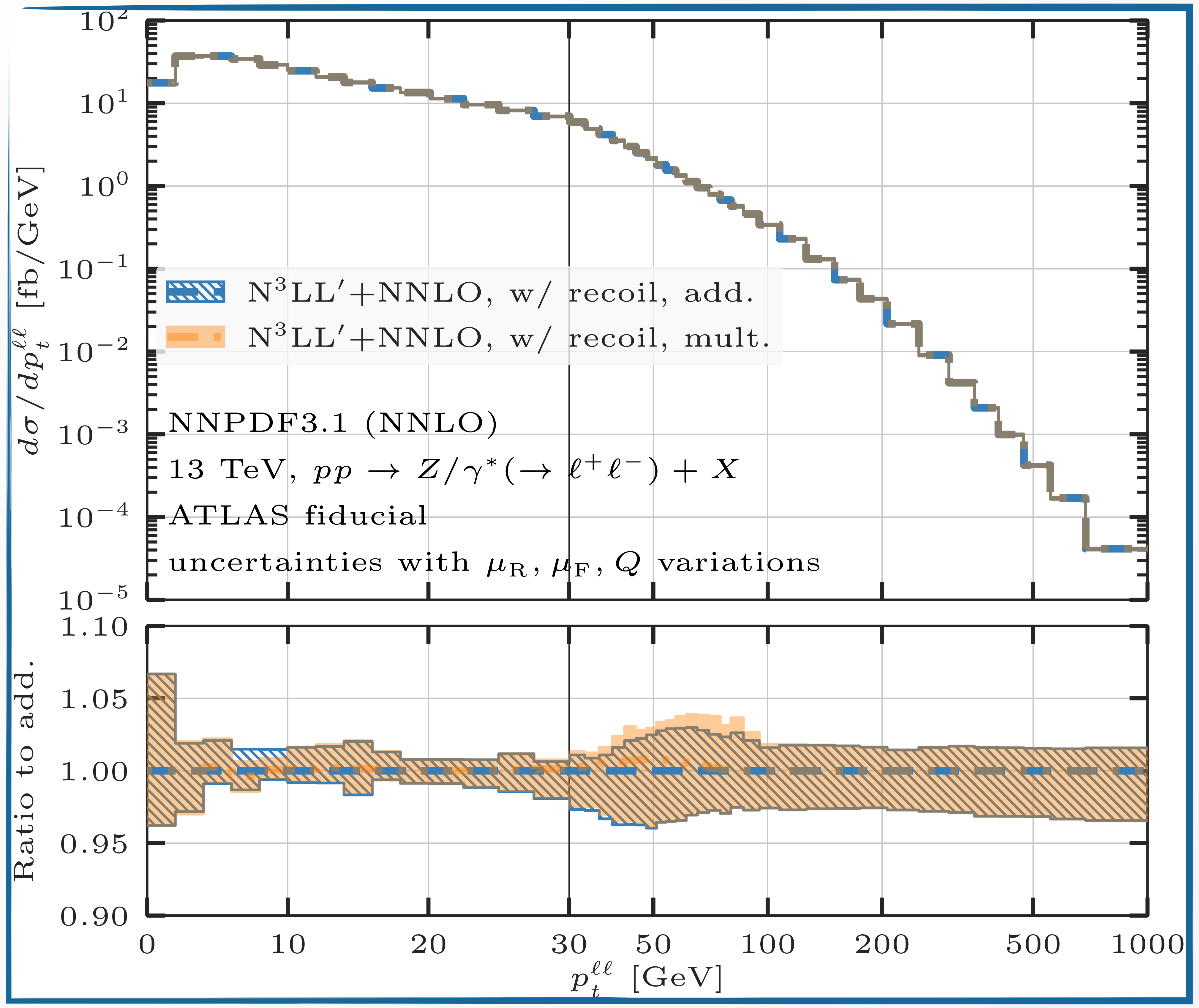
Effect reduce at 1-2% level after matching to fixed order (effect becomes $\mathcal{O}(\alpha_s^4)$)

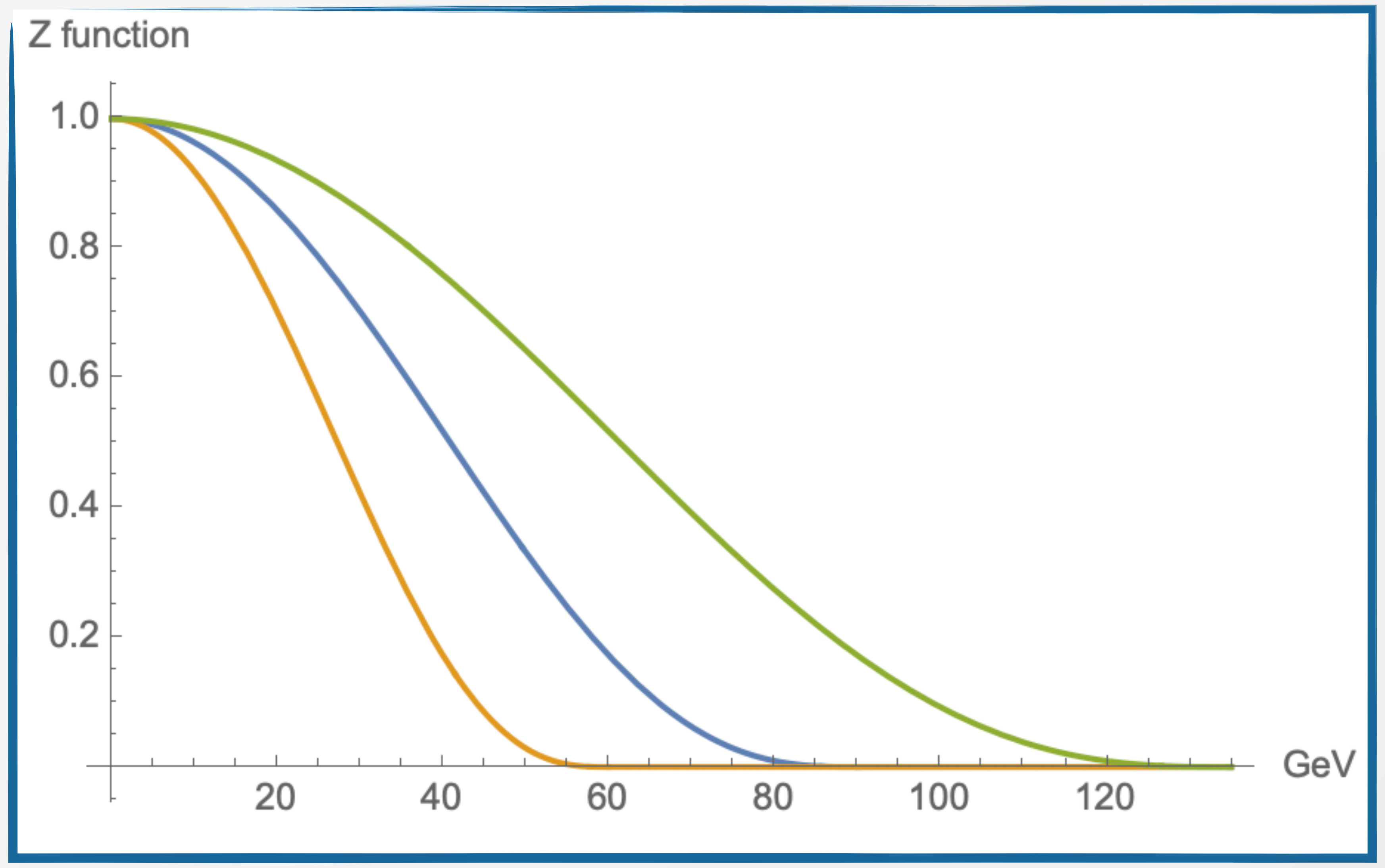
Pure resummed: band widening due to power corrections due to modified logs

$$\ln(Q/k_{t1}) \to 1/p \ln(1 + (Q/k_{t1})^p)$$

$$\int_0^M \frac{dk_{t1}}{k_{t1}} \to \int_0^\infty \frac{dk_{t1}}{k_{t1}} \frac{(Q/k_{t1})^p}{1 + (Q/k_{t1})^p}$$

Matching systematics

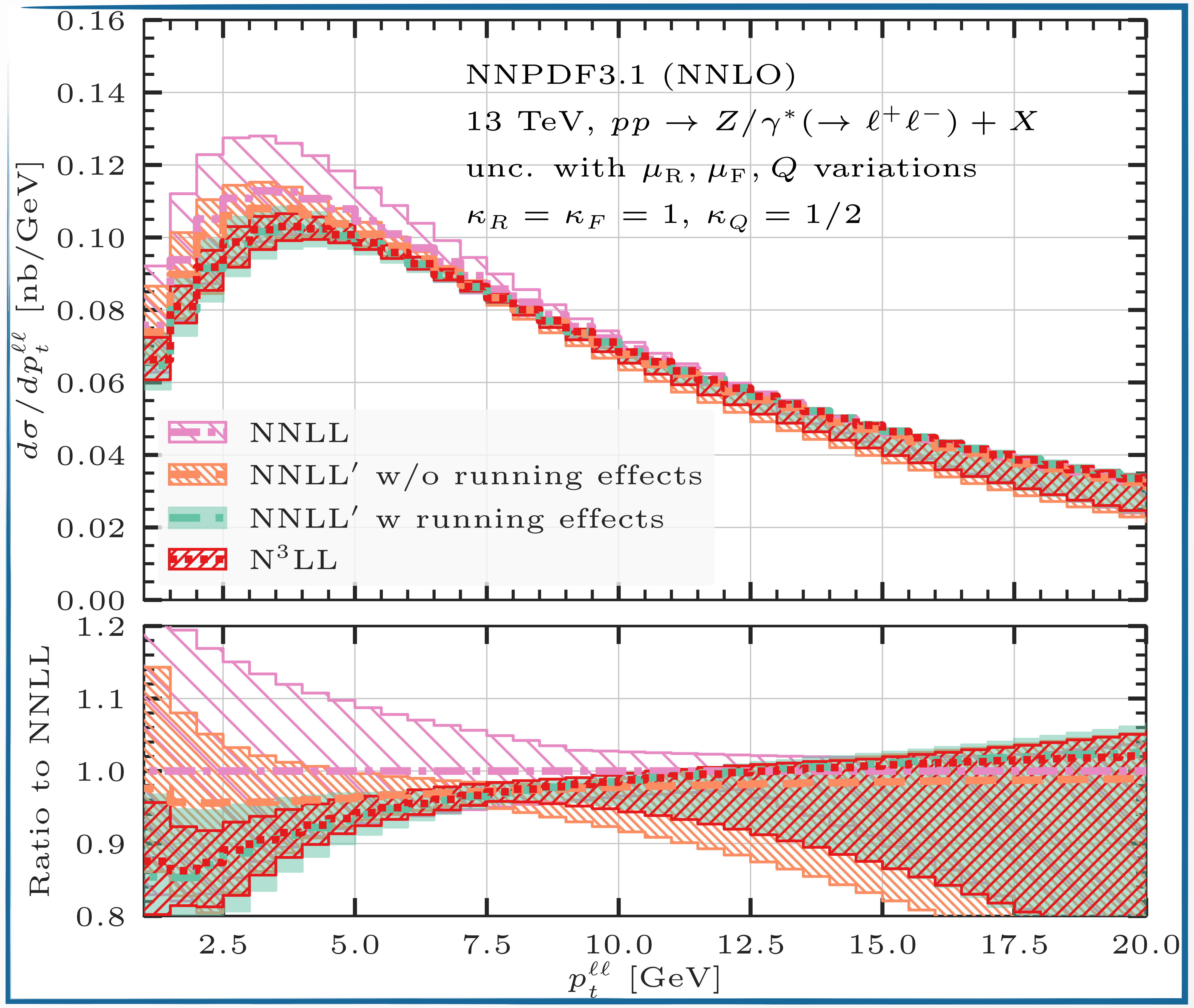


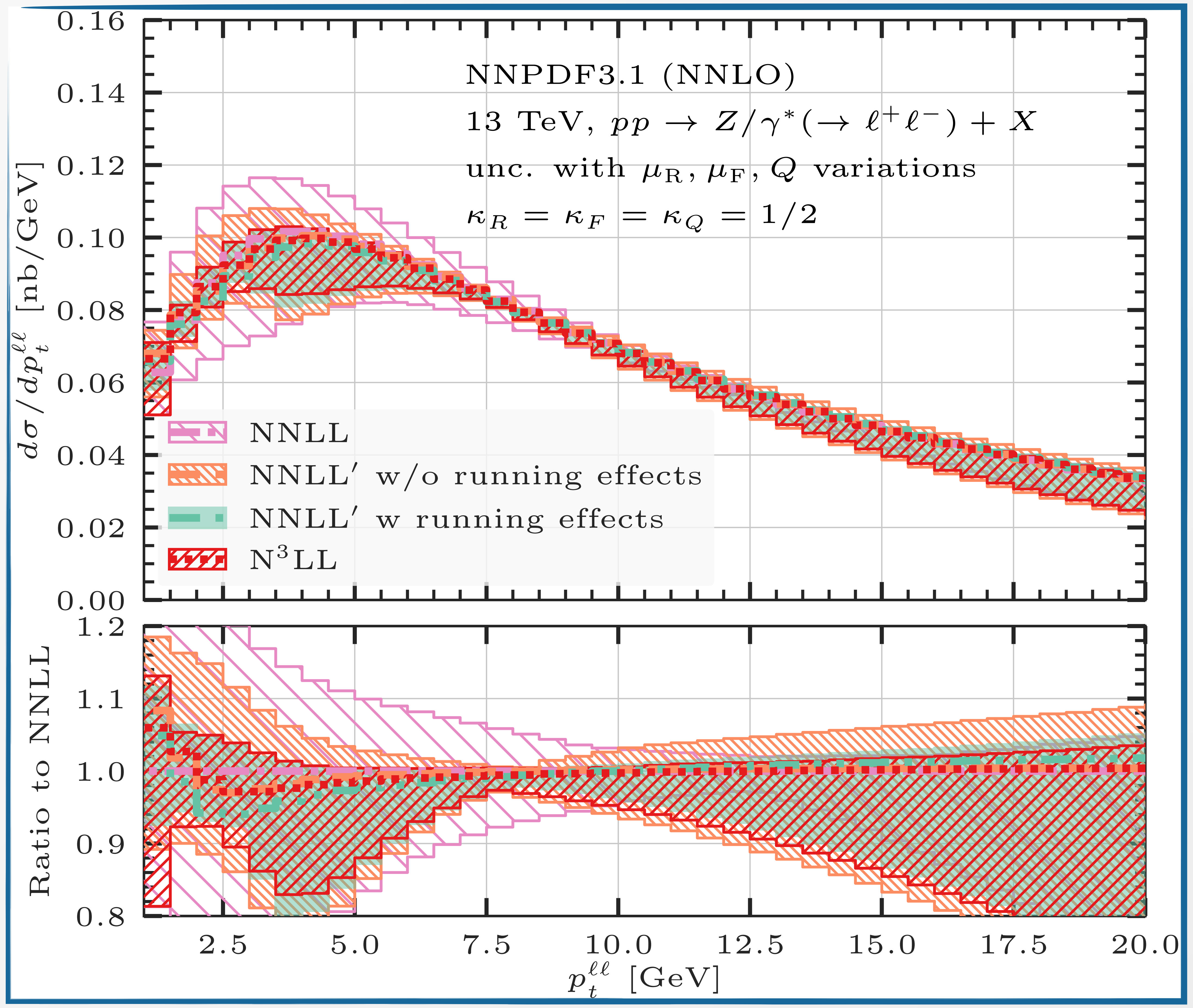


Very mild matching scheme dependence both for central results and uncertainties

Additive matching uncertainty band reliably estimate matching ambiguities

Ambiguity in the definition of primed accuracy



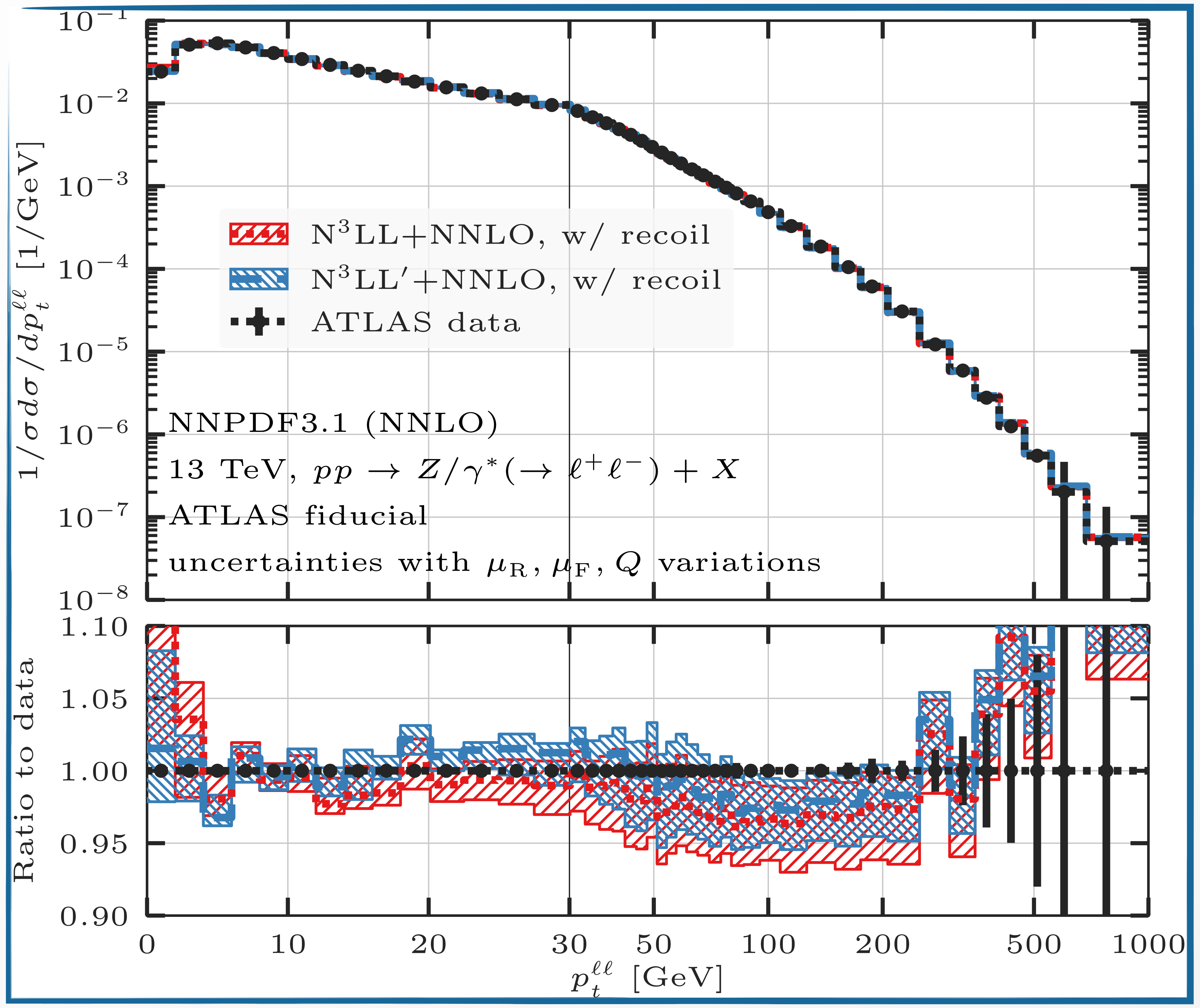


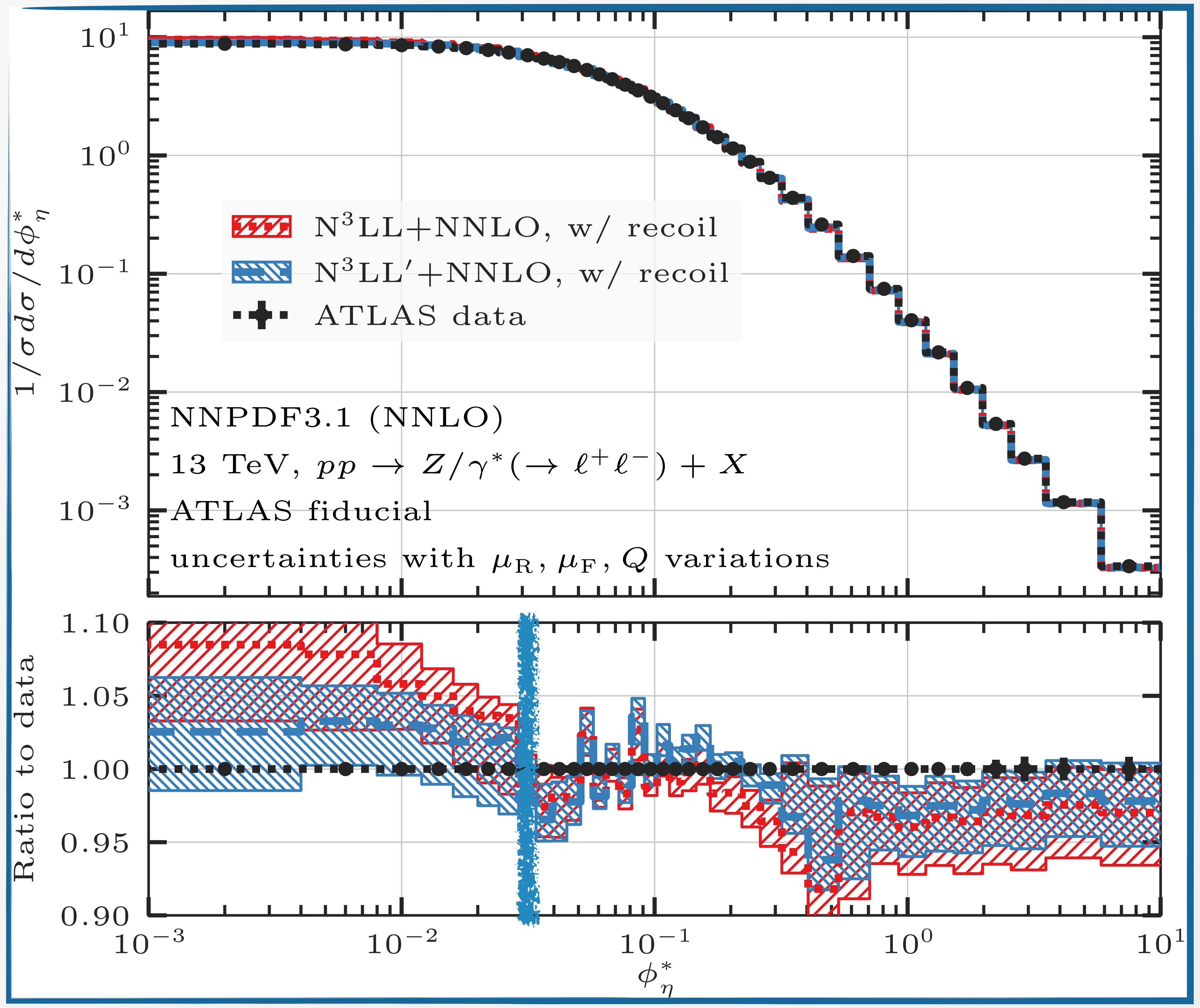
NNLL′ with and without running closer to N^{3}LL than NNLL is

NNLL′ with running band in better agreement to N^{3}LL: N^{3}LL contained within NNLL′ with running uncertainty

Band for NNLL′ with running covers difference between two NNLL′ → reliable estimate of prime ambiguity

Drell-Yan production: comparison with ATLAS data





N^{3}LL′+NNLO **improves the description of data** w.r.t. N^{3}LL+NNLO

Theoretical uncertainties at the **few percent level** across the whole range

High statistic runs needed for the description of ϕ^*_η in the singular region (fixed-order component set to 0)

Marginal effect of recoil after matching (1-2% effect)

Higgs production: setup

Higgs fiducial region defined as [ATLAS 2018]

$$\min(p_t^{\gamma_1}, p_t^{\gamma_2}) > 31.25\ \mathrm{GeV}\,, \qquad \max(p_t^{\gamma_1}, p_t^{\gamma_2}) > 43.75\ \mathrm{GeV}$$

$$0 < |\eta^{\gamma_{1,2}}| < 1.37 \quad \text{or} \quad 1.52 < |\eta^{\gamma_{1,2}}| < 2.37\,, \qquad |Y_{\gamma\gamma}| < 2.37$$

Central scales chosen as

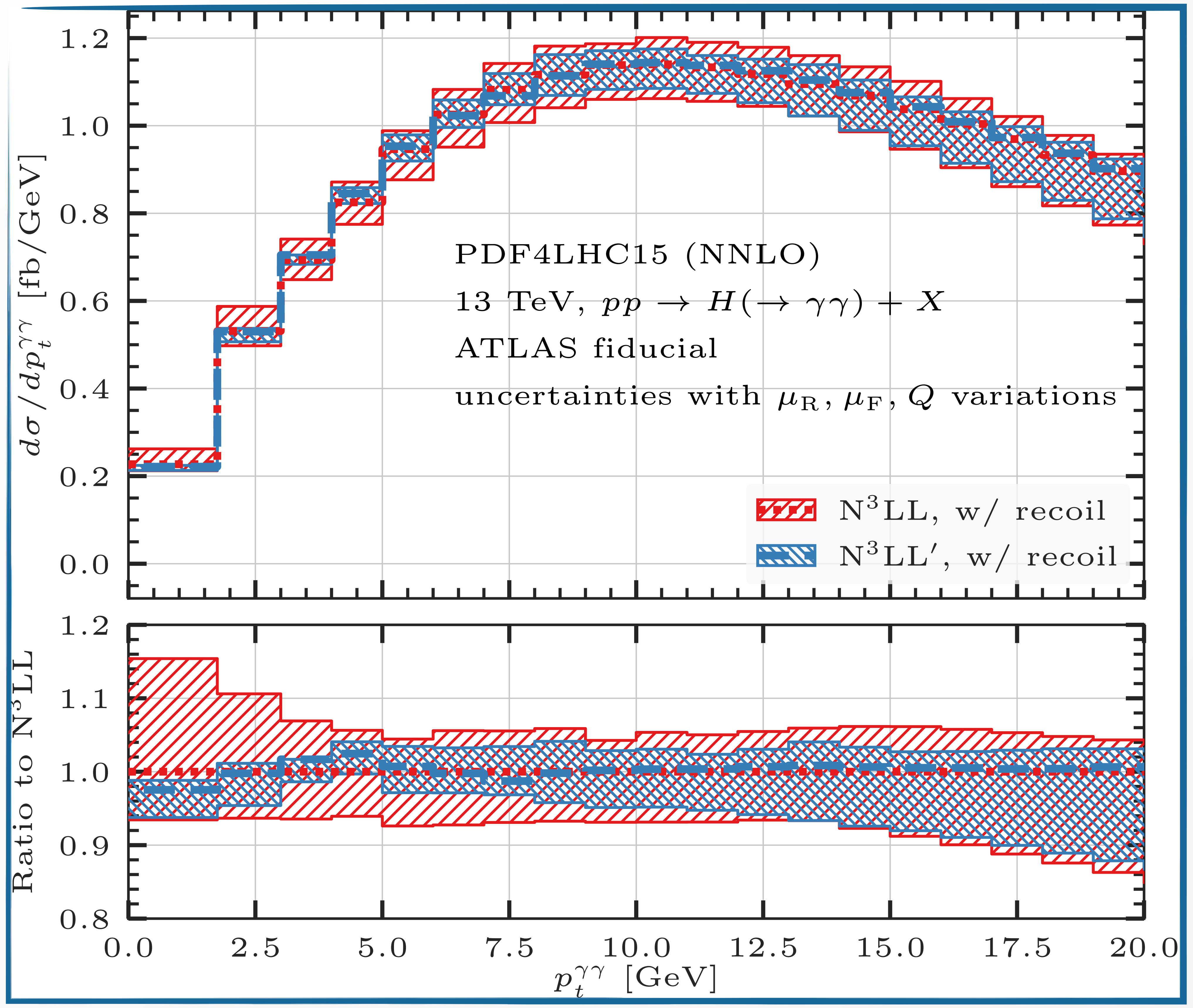
$$\mu_R = \kappa_R\, M_H \qquad \mu_F = \kappa_F\, M_H, \qquad Q = \kappa_Q\, M_H$$

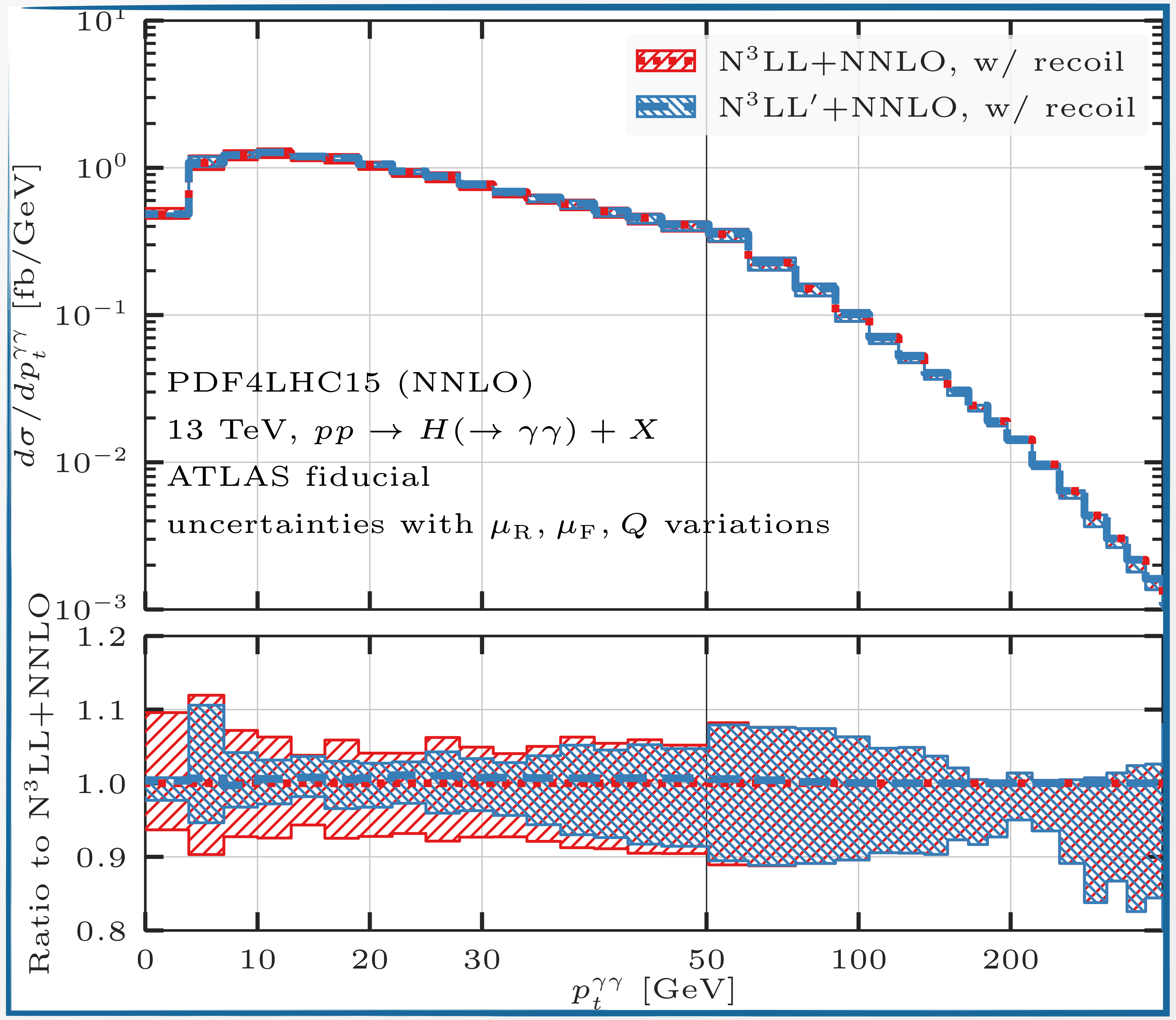
Scale uncertainty:

[canonical 7 scale variation + variation of κ_Q by a factor of 2 for central μ_R, μ_F] $\times$ 3 values of v_0 $\rightarrow$ **27 variations**

PDF4LHC15 NNLO parton densities

Higgs production: N3LL′ effects



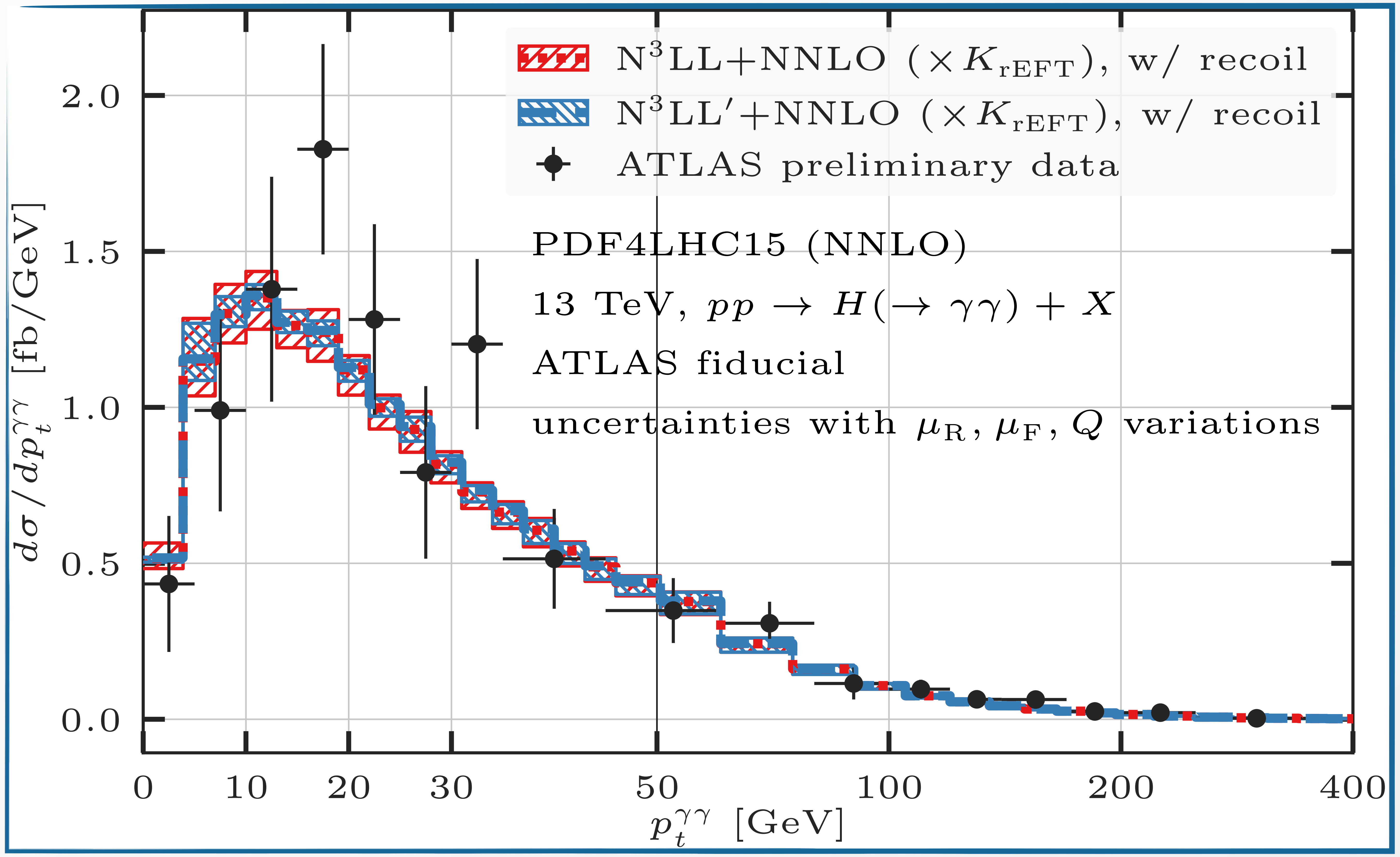


Significant reduction in theoretical uncertainty below 15 GeV, especially **below 5 GeV** $\kappa_R = \kappa_F = \kappa_Q = 1/2$

Central value almost unchanged between N3LL and N3LL′

Reduction in scale uncertainty limited at matched level (**statistical fluctuations** of the fixed order at small p_t)

Higgs production: comparison with ATLAS data



ATLAS preliminary data from https://cds.cern.ch/record/2682800

Theoretical predictions rescaled by $K_{\mathrm{rEFT}} = 1.06584$ to account for exact LO top-mass dependence

Recapitulation and outlook

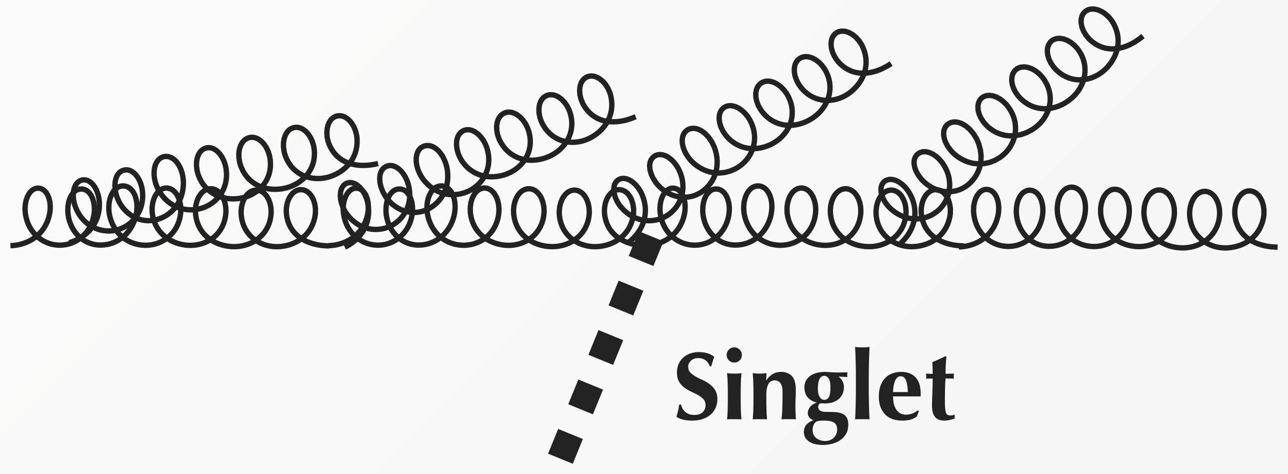
- Results for singlet p_t (H/DY production) and ϕ_η^* (DY) at N^3LL'+NNLO accuracy by including all constant terms of relative order α_s^3 in the RadISH formalism
- RadISH now includes recoil effects which improve the description of decay kinematics in the fiducial region
- Precise theoretical prediction in the fiducial region for $Z/\gamma^* \to \ell^+\ell^-$ and $H \to \gamma\gamma$
- Reduction of theoretical uncertainty at N^3LL'. Improved description of DY data
- Resummation uncertainty at the few percent level (DY), 5-10% level (Higgs)
- Marginal effect of recoil in matched results

Backup

Resummation of the transverse momentum spectrum

Resummation of transverse momentum is particularly delicate because $p_\perp$ is a **vectorial quantity**

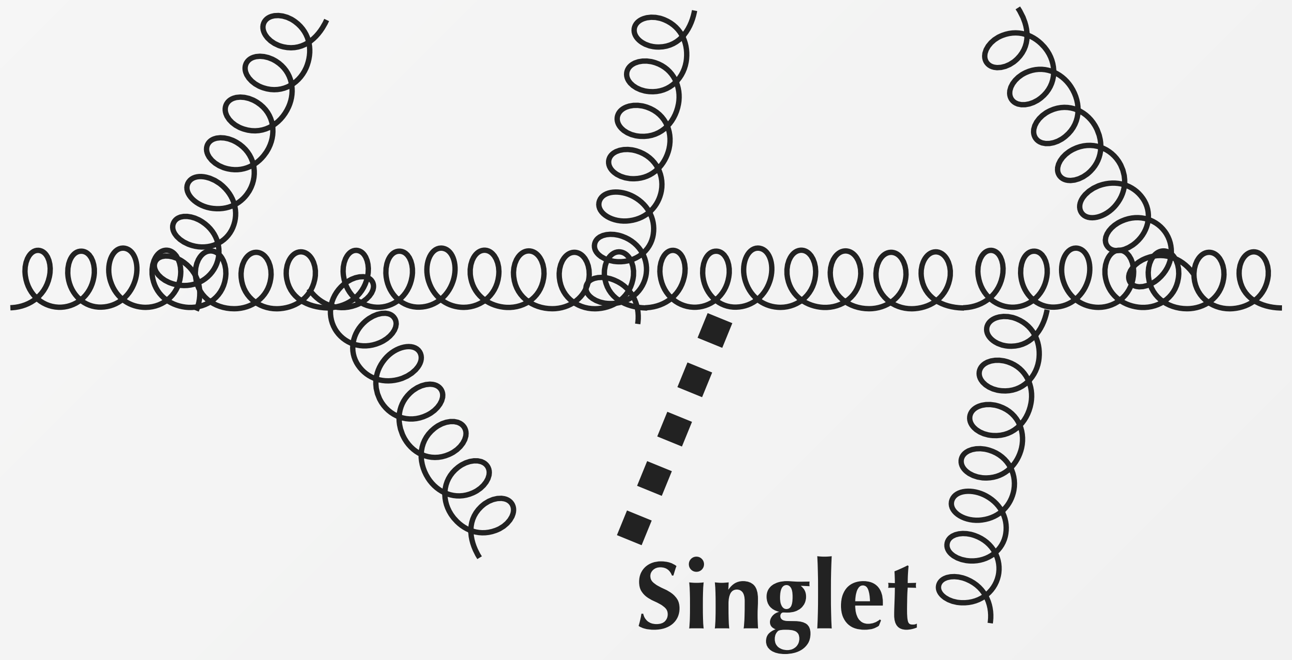
Two concurring mechanisms leading to a system with small $p_\perp$



$$p_\perp^2 \sim k_{t,i}^2 \ll m_H^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

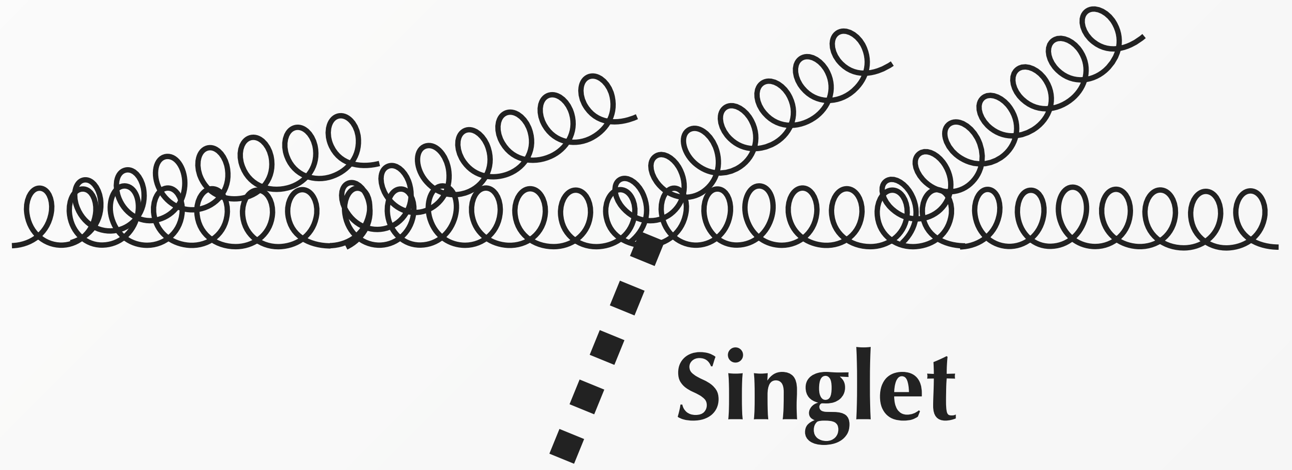
$p_\perp$ ~0 far from the Sudakov limit

Power suppression

Resummation of the transverse momentum spectrum

Resummation of transverse momentum is particularly delicate because $p_\perp$ is a **vectorial quantity**

Two concurring mechanisms leading to a system with small $p_\perp$



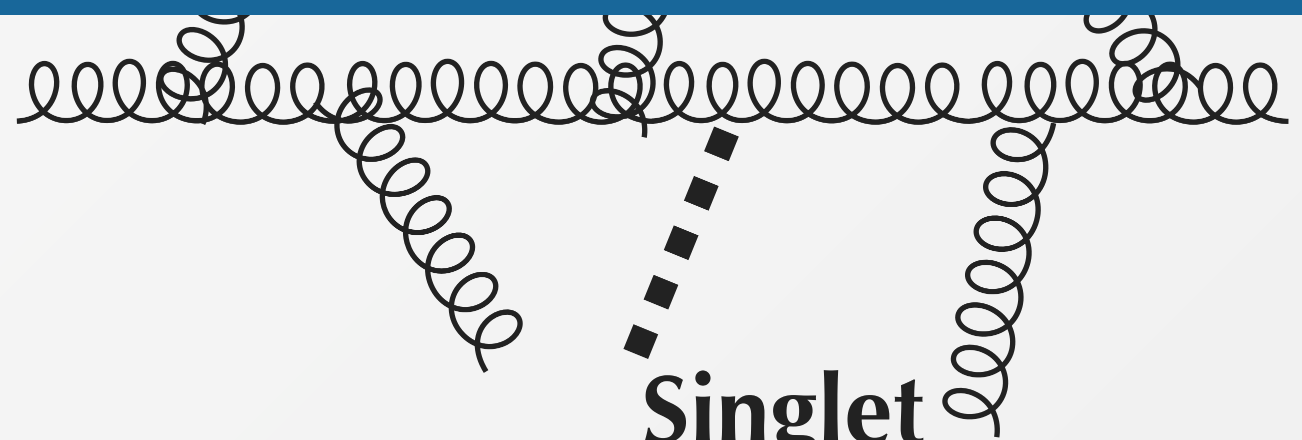
$$p_\perp^2 \sim k_{t,i}^2 \ll m_H^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression

Dominant at small $p_\perp$

[Parisi, Petronzio, 1979]



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

$p_\perp$ ~0 far from the Sudakov limit

Power suppression