A collection of results for level 3

Valerio Bertone[1] and Giuseppe Bozzi[2]

[1] IRFU, CEA, Université Paris-Saclay,

[2] INFN and Università di Pavia

LHC EW WG Subgroup Meeting

June 17, 2021

Matching with fixed order

- We implement the *additive* matching, i.e., :

$$\frac{d\sigma}{dq_T} = \left[\frac{d\sigma^{res.}}{dq_T}\right] + \left[\frac{d\sigma}{dq_T}\right]_{f.o.} - \left[\frac{d\sigma^{res}}{dq_T}\right]_{f.o.}$$

- Expansion of resummed calculation:

$$\left[\frac{d\sigma^{\mathrm{res.}}}{dq_T}\right]_{\mathrm{f.o.}} = \sum_{n=0}^{\mathrm{f.o.}} \alpha_s^n \sum_{l=0}^{2n} B^{(n,l)} I_l\left(q_T\right)$$

$$I_l(q_T) = \int_0^\infty db\, b J_0(b q_T) \ln^l\left(\frac{b^2 Q^2}{b_0^2} (+1)\right)$$

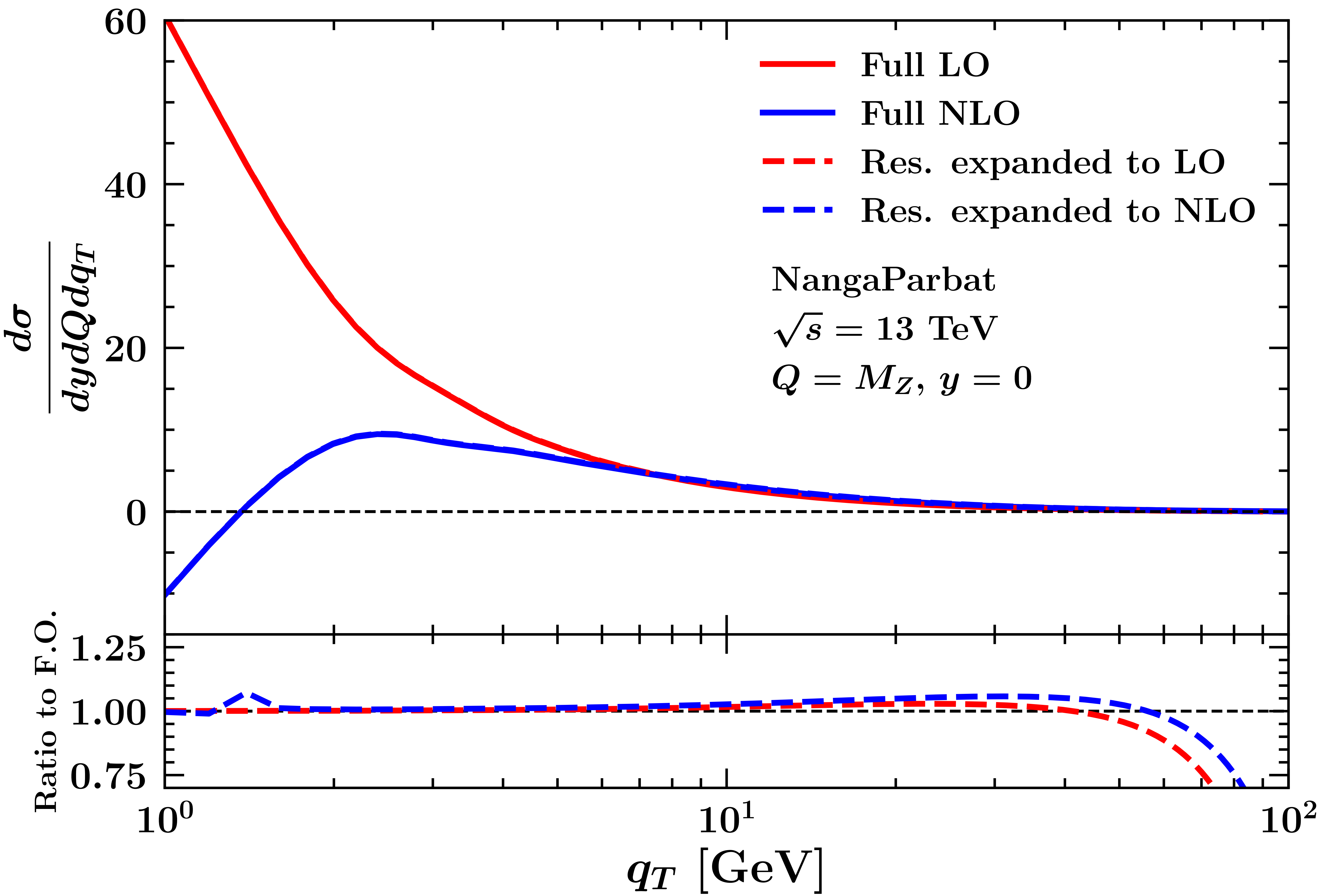
- $I_0 \propto \delta(q_T)$ irrelevant here

- The **+1** defines the modified logarithms:

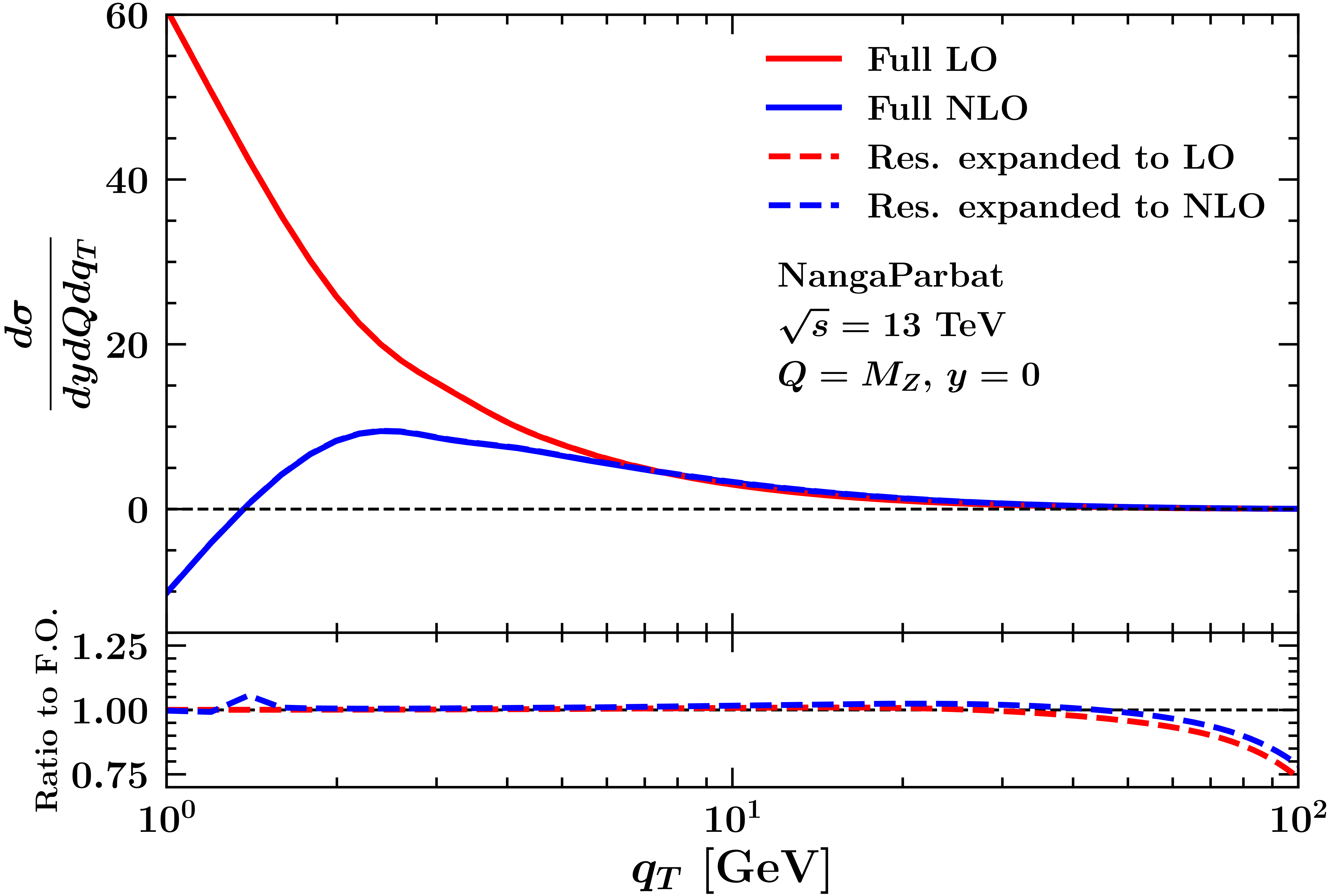
    - originally introduced to guarantee "unitarity"

    - gives raise to power corrections $O(q_T/Q)$ that do not spoil the cancellation at low q_T between fixed order and expansion of the resummation.

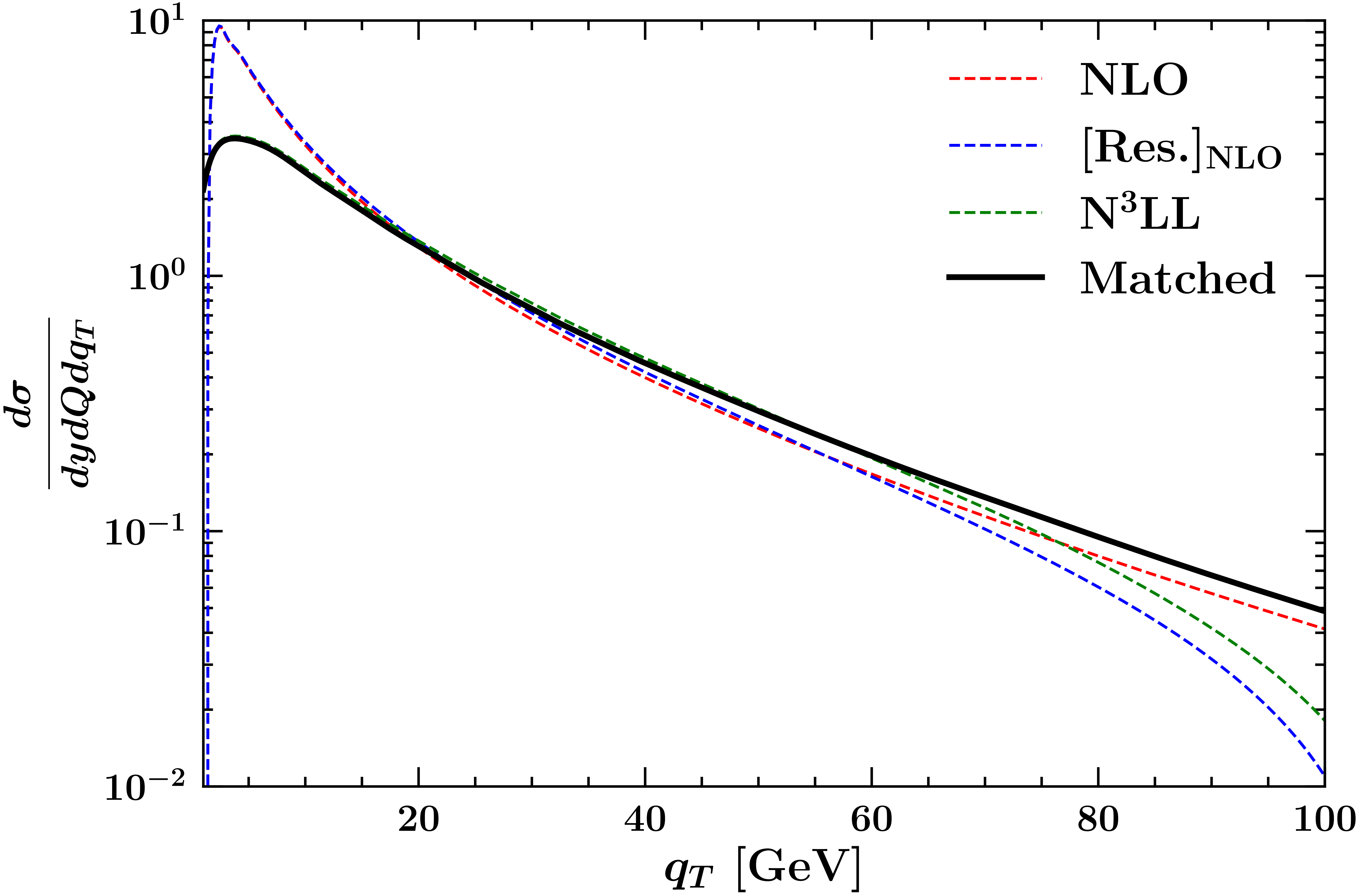
Nominal logs



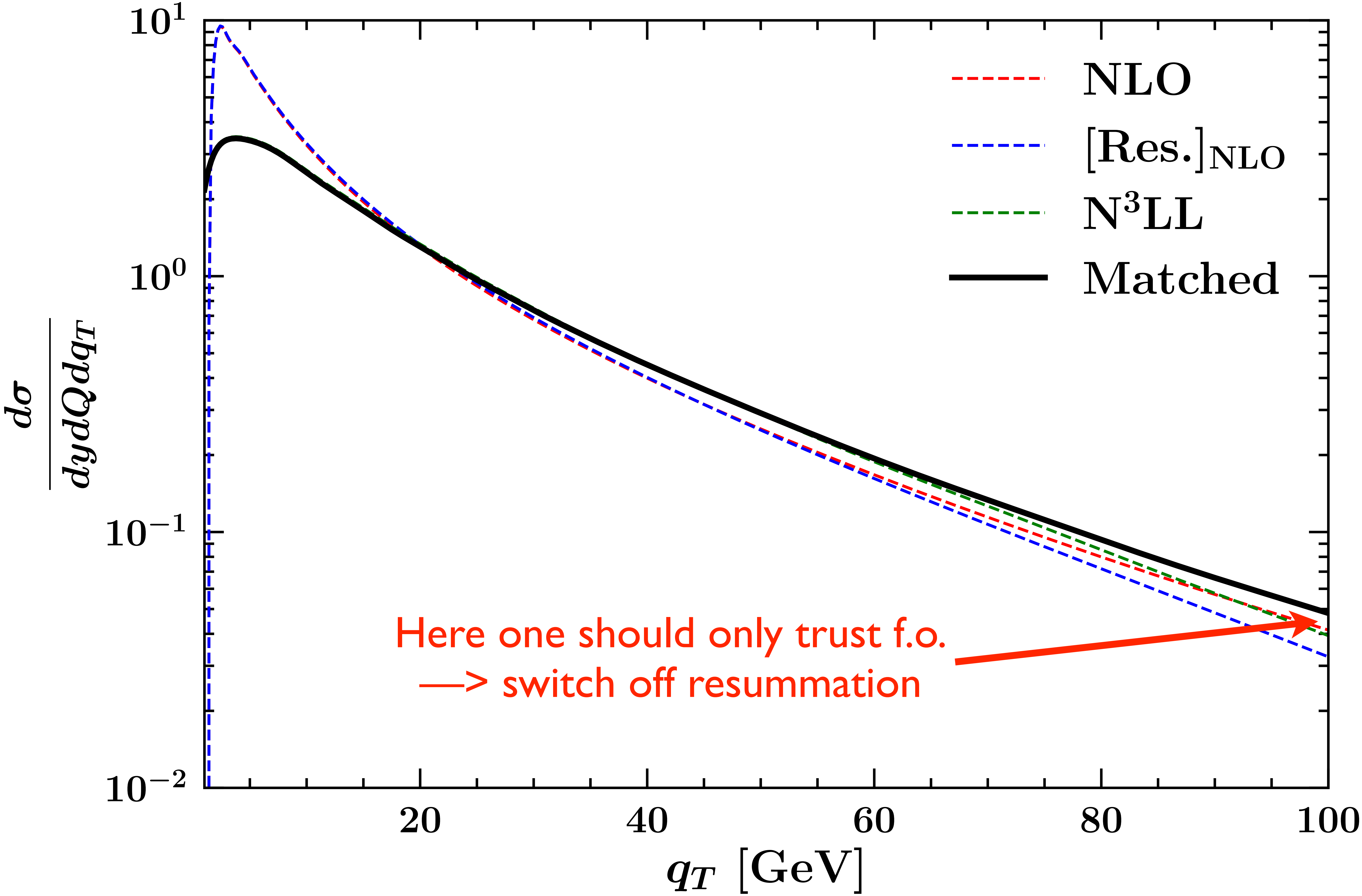
Modified logs



Nominal logs



Modified logs



Matching uncertainties

- Eventually, resummation has to be switched off
- Damping function —> $\left[\frac{d\sigma^{res.}}{dq_T}\right] \longrightarrow \left[\frac{d\sigma^{res.}}{dq_T}\right] f(q_T, Q)$

  with $f(q_T, Q) \rightarrow 0$ at high q_T to recover the pure f.o. result
- matching uncertainties (currently working on that and on N3LL')

Estimate of uncertainties

$$\frac{d\sigma}{dq_T} \propto H(M_{\ell\ell}, \mu_R) \qquad : \text{Hard factor}$$

$$\times \exp\left[S_{\mathrm{PT+NP}}(\mu_R, \mu_b)\right] \qquad : \text{Evolution}$$

$$\times C_{1,2}(\mu_b) f_{\mathrm{NP}}^{(1,2)}(\mu_b) \qquad : \text{Matching onto collinear}$$

$$\times \underbrace{\Gamma_{1,2}^{\mathrm{DGLAP}}(\mu_b, \mu_F) f_1(\mu_F) f_2(\mu_F)}_{f_1(\mu_b) f_2(\mu_b)} \qquad : \text{Collinear PDFs}$$

- Theoretical uncertainty estimate on **N³LL**:
  - variations of μ_R by a factor 2 up and down w.r.t. M_{ll},
  - variations of μ_F by a factor 2 up and down w.r.t. M_{ll},
  - estimate of subleading logarithmic corrections by including N⁴LL corrections in the Sudakov (mimicking **resummation scale variations**),
    [G. Das, S.-O. Moch, A. Vogt, arXiv:1912.12920]
  - inclusion of non-perturbative effects as determined in the **PV19** fit.
    [A. Bacchetta *et al.*, arXiv:1912.07550]

Estimate of uncertainties

- N^4LL corrections to the Sudakov: [G. Das, S.-O. Moch, A. Vogt, arXiv:1912.12920]

$$A_5 = (1.7 \pm 0.5\,,\ 1.1 \pm 0.5\,,\ 0.7 \pm 0.5) \cdot 10^5 \quad \text{for} \quad n_f = 3\,,\ 4\,,\ 5\,.$$

$$B_4^{\rm DIS}\Big|_{\rm appr} = (10.68 \pm 0.01) \cdot 10^4 + (-2.025 \pm 0.032) \cdot 10^4\, n_f + 798.0698\, n_f^2\ -12.08488\, n_f^3$$

  - we used the configuration that gave the largest difference w.r.t. N^3LL (and finally multiplied it by two both in the plus and minus directions).

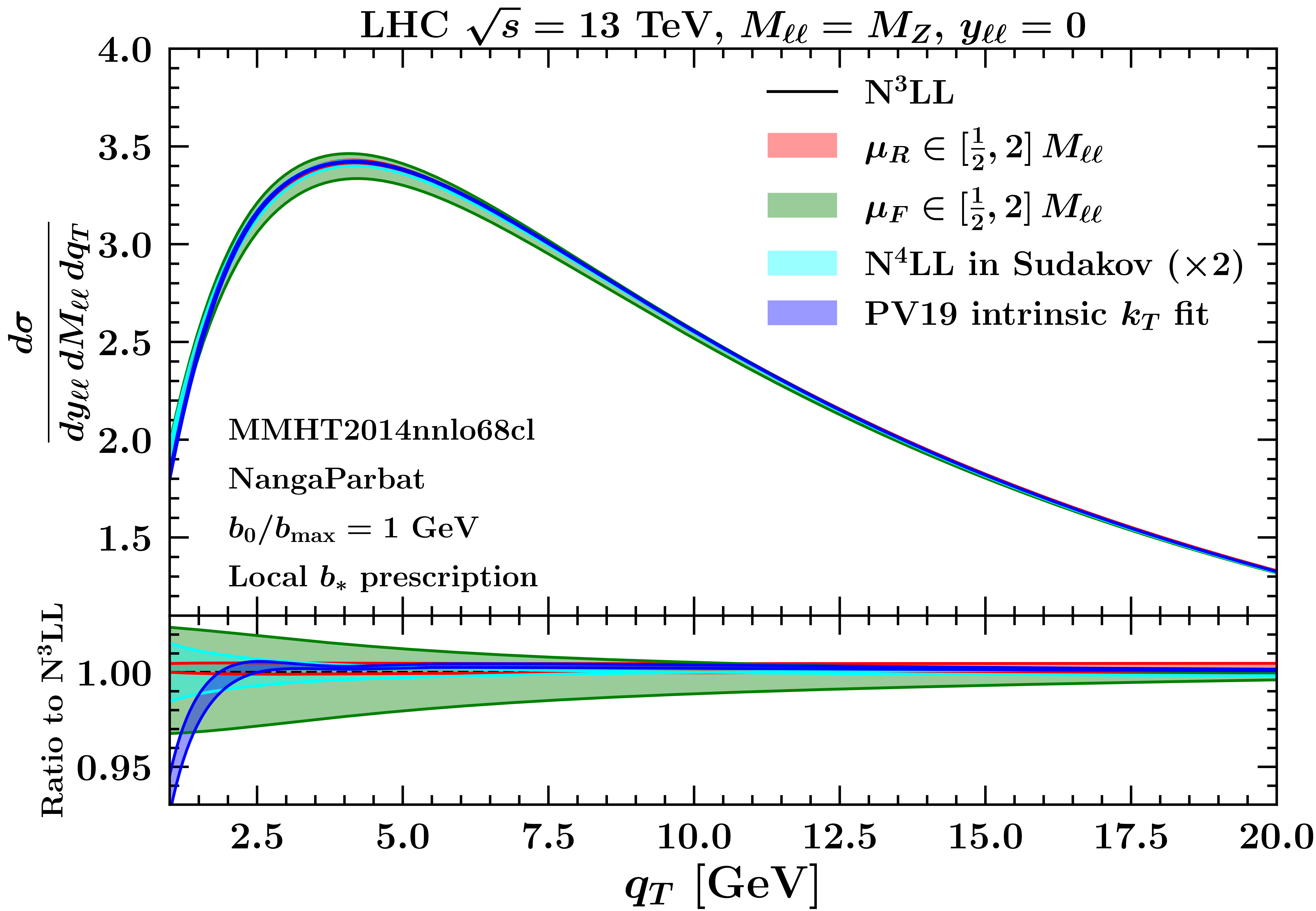
- Non-perturbative corrections determined by a fit to data at N^3LL (PV19): [A. Bacchetta *et al.*, arXiv:1912.07550]
  - Parameterisation used:

$$f_{\rm NP}(x,\mu_b)\exp\left[S_{\rm NP}(Q,\mu_b)\right] = \left[\frac{1-\lambda}{1+g_1(x)\frac{b^2}{4}} + \lambda \exp\left(-g_{1B}(x)\frac{b^2}{4}\right)\right]$$
$$\times \quad \exp\left[-\left(g_2 + g_{2B}b^2\right)\ln\left(\frac{Q^2}{Q_0^2}\right)\frac{b^2}{4}\right]$$

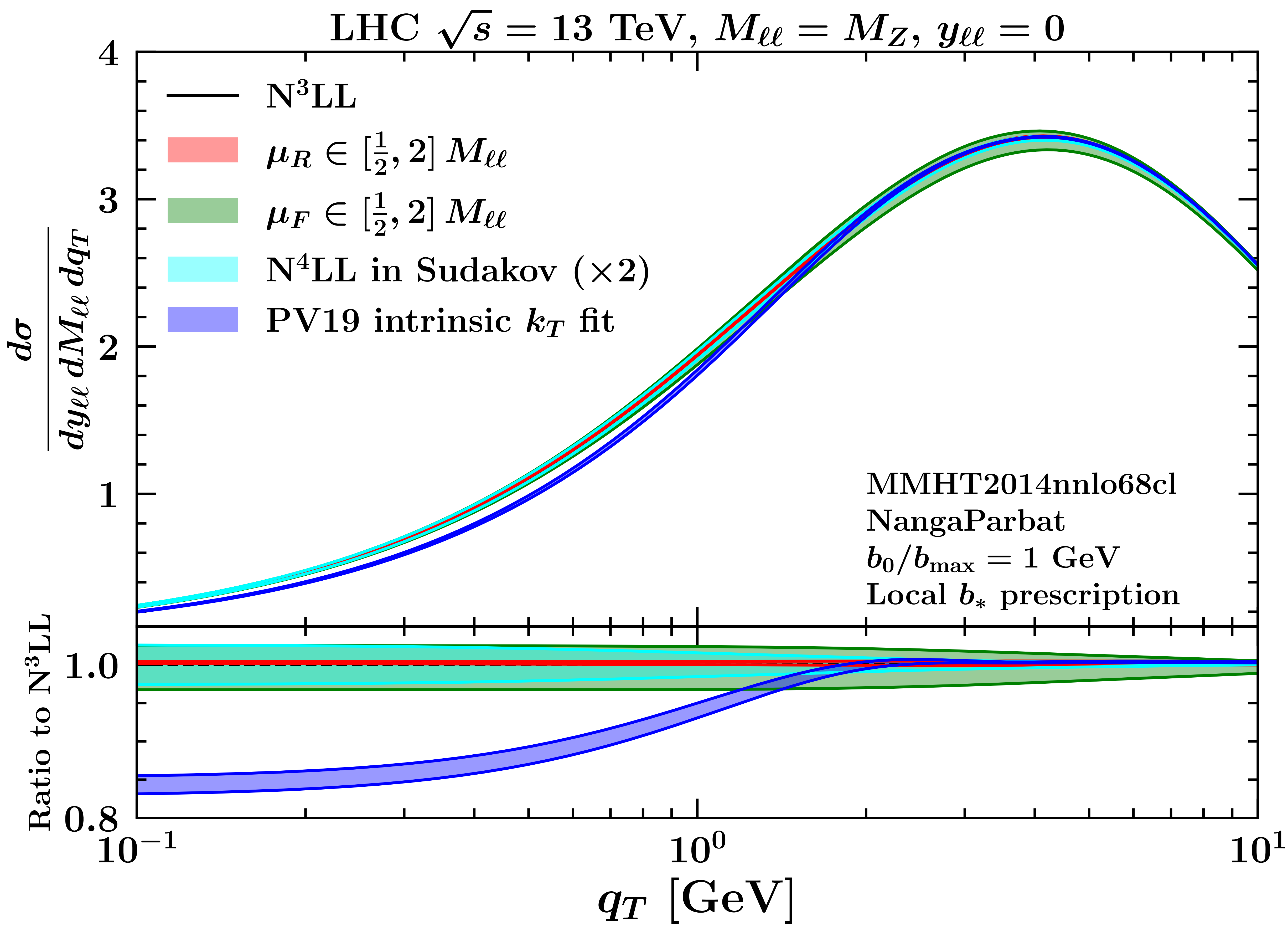
  - with:

$$g_1(x) = \frac{N_1}{x\sigma}\exp\left[-\frac{1}{2\sigma^2}\ln^2\left(\frac{x}{\alpha}\right)\right] \qquad g_{1B}(x) = \frac{N_{1B}}{x\sigma_B}\exp\left[-\frac{1}{2\sigma_B^2}\ln^2\left(\frac{x}{\alpha_B}\right)\right]$$

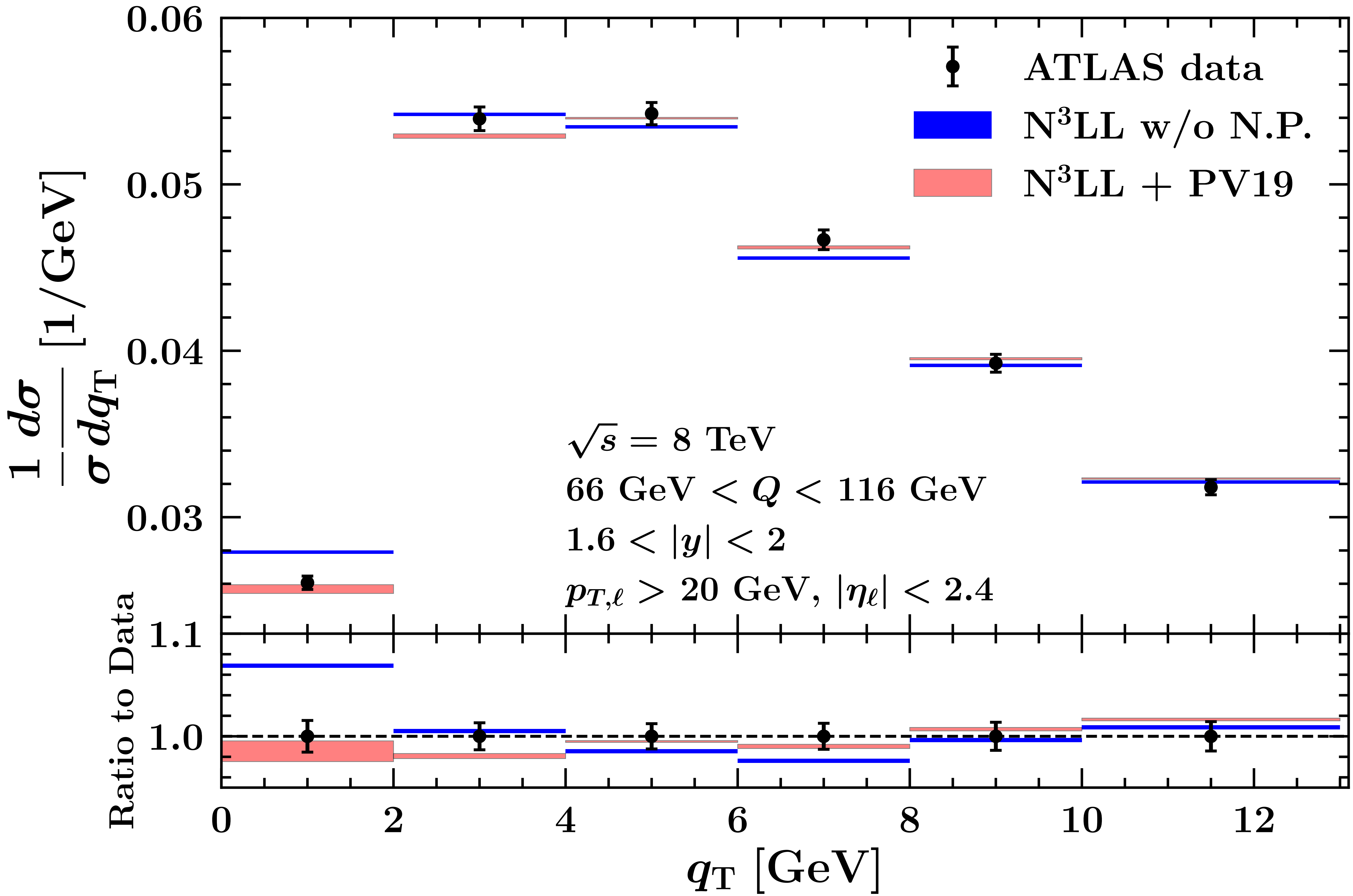
Estimate of uncertainties



Estimate of uncertainties



Estimate of uncertainties



Backup slides

TMD evolution equations

- TMD factorisation allows one to obtain the **evolution equations**:

$$\left\{\begin{array}{l} \dfrac{d\ln F}{d\ln \mu} = \gamma(\mu,\zeta) \\[2ex] \dfrac{d\ln F}{d\ln\sqrt{\zeta}} = K(\mu) \end{array}\right. , \quad \frac{d^2\ln F}{d\ln\mu\, d\ln\sqrt{\zeta}} = \left\{\begin{array}{l} \dfrac{d\gamma}{d\ln\sqrt{\zeta}} \\[2ex] \dfrac{dK}{d\ln\mu} \end{array}\right. = \gamma_K(\alpha_s(\mu))$$

- To solve these equations we need to fix **two pairs of (*i.e.* four) scales**:
  - **initial** scales: (μ_0, ζ_0)
  - **final** scales: (μ, ζ)
- The solution is **unique** and reads:

$$F(\mu,\zeta) = R\left[(\mu,\zeta) \leftarrow (\mu_0,\zeta_0)\right] F(\mu_0,\zeta_0)$$

$$R\left[(\mu,\zeta) \leftarrow (\mu_0,\zeta_0)\right] = \exp\left\{K(\mu_0)\ln\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F(\alpha_s(\mu')) - \gamma_K(\alpha_s(\mu'))\ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\}$$

- The question is: **how do we choose these four scales?**

Scale variations

- A sensible choice of the scales is important to **allow perturbation theory to be reliable**:
  - **no large unresummed logarithms** should be introduced,
  - each scale has to be set in the **vicinity of its natural (central) value**,
  - scale variations (within a reasonable range) give an estimate of HO corrs.
- In TMD factorisation ($q_{\mathrm{T}} \ll Q$) for DY the relevant scales are $\boldsymbol{q_{\mathrm{T}}}$ and $\boldsymbol{Q}$:
  - natural to expect $\mu_0 \sim \sqrt{\zeta_0} \sim q_T \sim b_T^{-1}$ and $\mu \sim \sqrt{\zeta} \sim Q$
- In fact, it turns out that (in the $\overline{\mathrm{MS}}$ scheme) the **central scales** are:

$$\mu_0 = \sqrt{\zeta_0} = \frac{2e^{-\gamma_E}}{b_T} \equiv \mu_b \quad \text{and} \quad \mu = \sqrt{\zeta} = Q$$

- This choice **nullifies** all unresummed logs. One should thus consider:

$$\mu_0 = C_i^{(1)}\mu_b, \quad \sqrt{\zeta_0} = C_i^{(2)}\mu_b, \quad \mu = C_f^{(1)}Q, \quad \sqrt{\zeta} = C_f^{(2)}Q,$$

Scale variations

- To reason why variations of ζ have **no effect** is that:

$$\frac{d\sigma}{dq_T} \propto H\left(\frac{\mu}{Q}\right) F_1(\mu,\zeta_1) F_2(\mu,\zeta_2) \quad \text{with} \quad \boxed{\zeta_1\zeta_2 \overset{!}{=} Q^4}$$

- It is easy to see that:

$$F_1(\mu,\zeta_1)F_2(\mu,\zeta_2) = \underbrace{R\left[(\mu,\zeta_1)\leftarrow(\mu_0,\zeta_0)\right] R\left[(\mu,\zeta_2)\leftarrow(\mu_0,\zeta_0)\right]}_{f(\zeta_1\zeta_2)=f(Q^4)} F_1(\mu_0,\zeta_0)F_2(\mu_0,\zeta_0)$$

- The single dependence on ζ_1 and ζ_2 **drops** in the combination:
  - we choose $\zeta_1 = \zeta_2 = Q^2$ but any other choice such that $\zeta_1\zeta_2 = Q^4$ is **identical**.
- In addition, in NangaParbat we have chosen to set $\mu_0 = \sqrt{\zeta_0}$:
  - not strictly necessary but **probably a conservative choice**.
- At the end of the day, we have **two scales** to be varied:

$$\boxed{\mu_0 = \sqrt{\zeta_0} = C_i\mu_b \quad \text{and} \quad \mu = C_f Q}$$

Comparison to q_T resummation

- In q_T resummation, the **resummation scale** M is introduced as:

$$L = \ln\left(\frac{Q}{\mu_b}\right) = \ln\left(\frac{M}{\mu_b}\right) + \ln\left(\frac{Q}{M}\right)$$

- These logs are **exposed** by expressing integral representations of the argument of the Sudakov in terms of the **functions** g_n:

$$\int_{\mu_b}^{Q} \frac{d\mu'}{\mu'}\left[A(\alpha_s(\mu'))\ln\left(\frac{Q}{\mu'}\right) + B(\alpha_s(\mu'))\right] = Lg_0(\alpha_s L) + \sum_{n=1}^{\infty} \alpha_s^{n-1} g_n(\alpha_s L)$$

$$= Lg_0(\alpha_s L) + \sum_{n=1}^{k} \alpha_s^{n-1} g_n(\alpha_s L) + \mathcal{O}(\alpha_s^{k+n} L^n)$$

- The series in the r.h.s. is **truncated** according to the log accuracy:
    - the truncation is responsible for the **explicit dependence on** M.
- If the l.h.s. integral is computed exactly, no dependence on M appears:
    - this is what we do in NangaParbat by computing the integral numerically,
    - therefore, we have **no resummation scale dependence**.

Comparison to q_T resummation

- The **renormalisation scale** μ_R in q_T resummation is probably to be (partly) identified with the scale μ in the TMD formalism:
  - this is the large scale at which the strong coupling α_s is computed.
- The **factorisation scale** μ_F present in q_T resummation is absent in the TMD formalism:
  - in the TMD approach, PDFs are computed at the low scale μ_0:
    - μ_0 is varied around μ_{b},
  - in q_T resummation, PDFs are evolved from *exactly* μ_{b} up to μ_F:
    - μ_F is varied around Q.
  - variations of μ_0 are typically much larger than variations of μ_F because at the energies relevant to the benchmark $\alpha_s(\mu_0) \gg \alpha_s(\mu_F)$:
    - problems with NangaParbat in using a $b_{\max}$ too large with scale variations.