Some thoughts on theory uncertainties

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On Z resonance (leading pole term):

$$A_4 = \frac{\sum_q X_q \, 8 \frac{v_\ell}{a_\ell} \frac{v_q}{a_q}}{\sum_q X_q \left(1 + \frac{v_\ell^2}{a_\ell^2}\right)\left(1 + \frac{v_q^2}{a_q^2}\right)} \qquad X_q = f_q(x_1) f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1) f_q(x_2)$$

$$\frac{v_\ell}{a_\ell} = 1 - 4 s_\ell^2, \qquad s_\ell^2 \equiv \sin^2 \theta^\ell_{\text{eff}}$$

$$\frac{v_q}{a_q} = 1 - 4|e_q|(s_\ell^2 + \Delta_q) \qquad \Delta_q = \underbrace{\Delta_{q(1)}}_{\text{implemented}} + \underbrace{\Delta_{q(2)}}_{\text{missing}}$$

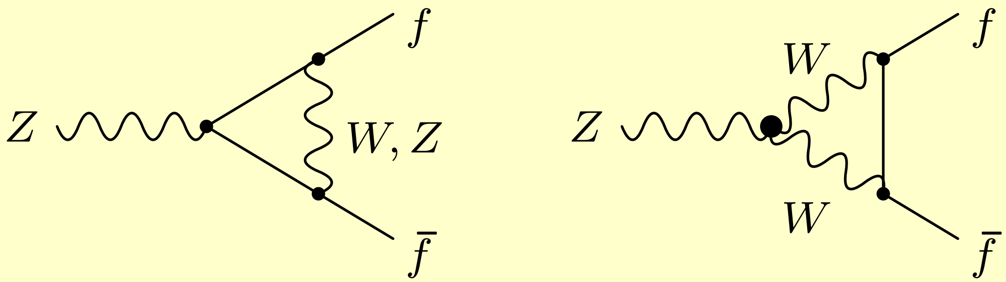
$$\frac{\delta A_4}{A_4} \approx \frac{\sum_q X_q \, (-4|e_q|\Delta_{q(2)})}{\sum_q X_q \, (1 - 4|e_q|s_\ell^2)} + \frac{\sum_q X_q \, 8|e_q| \, (1 - 4|e_q|s_\ell^2)\Delta_{q(2)}}{\sum_q X_q \, [1 + (1 - 4|e_q|s_\ell^2)^2]}$$

$\Delta_{q(2)}$ is known (in SM) for leading Z pole term

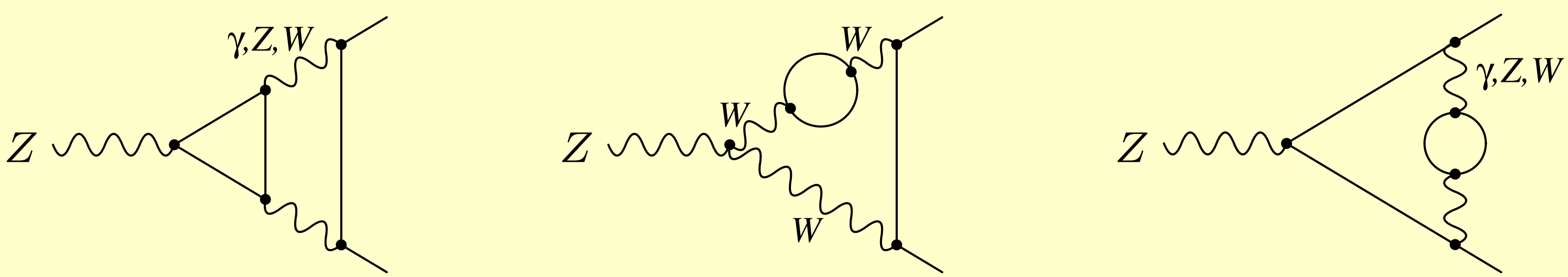
Freitas '14

Dubovyuk, Freitas, Gluza, Riemann, Usovitsch '19

Example contributions to $\Delta_{q(1)}$:



Example contributions to $\Delta_{q(2)}$:



Z-pole 2-loop flavor dependence:

Assume: no EW 2-loop corrections included in analysis (i.e. they are theory unc.)

Schemes:

- α' : Use $\alpha, M_\mathrm{W}, M_\mathrm{Z}$ as inputs, perturb. exp. in α
- α : Use $\alpha, G_\mu, M_\mathrm{Z}$ as inputs, perturb. exp. in α
- G_μ : Use $G_\mu, M_\mathrm{W}, M_\mathrm{Z}$ as inputs, perturb. exp. in G_μ

Scheme:	α'	α	G_μ
$\Delta_{u(\alpha^2)}$ $[10^{-5}]$	-1.74	-1.82	-1.45
$\Delta_{d(\alpha^2)}$ $[10^{-5}]$	-1.49	-1.67	-0.88

Inputs: $M_\mathrm{Z} = 91.1876$ GeV, $M_\mathrm{W} = 80.385$ GeV, $M_\mathrm{H} = 125.7$ GeV

$m_\mathrm{t} = 173.5$ GeV, $\Delta\alpha = 0.059$, $\alpha_\mathrm{s} = 0.1184$, $G_\mu = 1.16638 \times 10^{-5}$ GeV^{-2}

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$\Delta_{u(\alpha^2)}$ [10^{-5}]	-1.74	-1.82	-1.45
$\Delta_{d(\alpha^2)}$ [10^{-5}]	-1.49	-1.67	-0.88
including non-factorizable EW×QCD corrections:			
$\Delta_{u(\alpha^2+\alpha\alpha_\mathrm{s})}$ [10^{-5}]	$+1.47$	$+1.38$	$+1.74$
$\Delta_{d(\alpha^2+\alpha\alpha_\mathrm{s})}$ [10^{-5}]	$+2.34$	$+2.15$	$+2.95$

Czarnecki, Kühn '96

Harlander, Seidensticker, Steinhauser '97

Inputs: $M_\mathrm{Z} = 91.1876$ GeV, $M_\mathrm{W} = 80.385$ GeV, $M_\mathrm{H} = 125.7$ GeV

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Z-pole 2-loop flavor dependence:

Impact of EW 2-loop contributions (without EW$\times$QCD):

$\delta A_4/A_4$: $[10^{-4}]$

$m_{\ell\ell}$ [GeV]	Scheme:	α'	α	G_μ
60		0.07	0.08	0.06
70		0.17	0.19	0.13
80		0.33	0.38	0.26
$M_Z{-}2$		6.03	3.80	3.11
$M_Z{-}1$		0.93	0.92	0.69
M_Z		0.41	0.44	0.33
$M_Z{+}2$		0.22	0.24	0.18
$M_Z{+}1$		0.11	0.13	0.10
100		0.09	0.09	0.06
110		0.13	0.14	0.09
130		0.12	0.13	0.09
150		0.11	0.12	0.09

Including photon exchange and photon form factor estimate:

(neglecting boxes and s-dependence of Z form factors)

$$A_4 = \frac{\sum_q X_q \, 4\left(\frac{v_\ell}{a_\ell}\frac{v_q}{a_q} + \frac{v_{\ell q}(s)}{a_\ell a_q}\right)}{\sum_q X_q \left(1 + \frac{v_\ell^2}{a_\ell^2} + \frac{v_q^2}{a_q^2} + \frac{v_{\ell q}^2(s)}{a_\ell^2 a_q^2}\right)} \qquad X_q = f_q(x_1) f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1) f_q(x_2)$$

$$v_{\ell q}(s) = v_\ell v_q + \frac{s - M_Z^2 - i M_Z \Gamma_Z}{s} e^2 e_q (1 + \overline{\Delta}_q)$$

$$\frac{v_\ell}{a_\ell} = 1 - 4 s_\ell^2, \qquad s_\ell^2 \equiv \sin^2\theta_{\text{eff}}^\ell$$

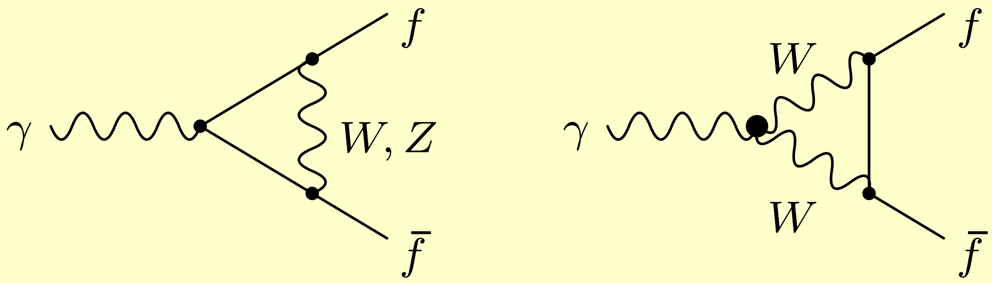
$$\frac{v_q}{a_q} = 1 - 4|e_q|(s_\ell^2 + \Delta_q) \qquad \Delta_q = \Delta_{q(1)} + \Delta_{q(2)}$$

$$\Delta_q = \underbrace{\overline{\Delta}_{q(1)}}_{\text{known}} + \underbrace{\overline{\Delta}_{q(2)}}_{\text{unknown}}$$

$\Delta_{q(2)}$ is known (in SM) for leading Z pole term

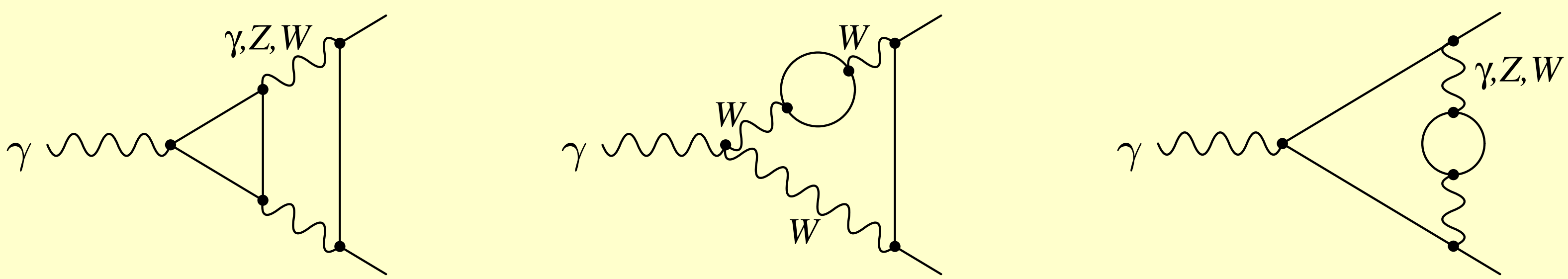
$\overline{\Delta}_{q(2)} = \pm \overline{\Delta}_{q(1)} \times \frac{g^2}{16\pi^2} n_f, \qquad n_f = 6 + 6N_c$ (maybe underestimate?)

Example contributions to $\overline{\Delta}_{q(1)}$:



Note: 1-loop boxes and s-dependence of Z vertex form factors also contribute at same order (1-loop without Z pole)

Example contributions to $\overline{\Delta}_{q(2)}$:



Including photon exchange and photon form factor estimate:

Impact of EW 2-loop contributions (without EW$\times$QCD):

$\delta A_4/A_4$: [10^{-4}]

$m_{\ell\ell}$ [GeV]	Scheme:	α'	α	G_μ
60		0.37	0.35	15.50
70		0.52	0.60	8.99
80		1.53	1.61	37.37
M_Z-2		17.54	10.27	208.5
M_Z-1		2.14	1.97	27.6
M_Z		0.58	0.59	0.57
M_Z+2		0.45	0.46	10.61
M_Z+1		0.55	0.55	16.15
100		0.84	0.83	24.85
110		0.80	0.81	21.71
130		0.53	0.56	12.34
150		0.34	0.38	6.04

- dominated by photon form factor unc. $\overline{\Delta}_q$
- artifically large corrections for G_μ scheme
  [same for (G_μ, s_ℓ, M_Z) scheme?]

Comments on mass/width scheme

- Pole expansion scheme (PS) and complex-mass scheme (CMS):
  Gauge-invariant (GI), consistent to all orders (at least conceptually)
- Factorization scheme (FS):
  Gauge-invariant (GI), not extendable beyond NLO
- Naive scheme (NS) and other gauge-dependent (GD) schemes:
  can lead to completely wrong results

- Difference GI–GD is meaningless, cannot be used for theory error estimate
- Difference PS–FS, PS–CMS, CMS–FS is of higher order (NNLO)
  $\rightarrow$ Can be used as indication for theory error, but may not fully capture it