KKMC-hh: Results from a Recent Run, Description of the IFI Implementation

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June 17, 2021

KKMC-hh is a collaboration with S. Jadach, B.F.L. Ward and Z. Wąs.

Computational resources provided by IFJ-PAN, Kraków.

Contents

I will show the results of a new 9 billion event run, which is largely similar to the previous runs, but fixes a bug in the script used to combine the farm data that had a small effect on normalizations. The new run also uses a smaller soft-photon cutoff, $v_{\mathrm{min}} = 10^{-7}$ rather than $v_{\mathrm{min}} = 10^{-3}$ as a fraction of the CM energy of the quarks or leptons, which avoids the possibility of boosting it to something significant in collisons far from the lab CM.

I will also describe the structure of KKMC's IFI calculation in a simplified context of the semi-analytical program KKhhFoam, which gives integrated results in a semi-soft approximation and is useful for cross-checks.

This discussion of the IFI calculation will help to clarify why it doesn't make sense to ask what any particular fixed-order contribution is to the calculation – in particular, why there is no "$O(\alpha)$" IFI result that can somehow be extracted from it.

Results Presented in the Following Tables

- All results are for muon pair final states with proton collisions with $\sqrt{s} = 8000$ GeV. Comparisons are made for two runs: Nov. 2020 and May 2021. The new run fixed a bug in the farming script that could have had a small effect on normalizations. Also, the soft photon cutoff was dropped by a factor of 100.
- Our tabulated results all include a dilepton mass cut in all cases:

$$60 \text{ GeV} < M_{ll} < 150 \text{ GeV}.$$

- The following table shows only A_4 calculated from A_{FB} in the full phase space without fermion cuts.
- We also calculated A_{FB} with lepton cuts $P_T > 25$ GeV, $|\eta| < 2.5$ on both muons. The corresponding table is on the following page. All results use NNPDF3.1 NLO and include FSR corrections.
- After the tables, I have included 9 billion event distributions for A_4 calculated as $\frac{8}{3} A_{FB}$ with 1 GeV binning for the full range 60 GeV < M_{ll} < 150 GeV. These are not new.

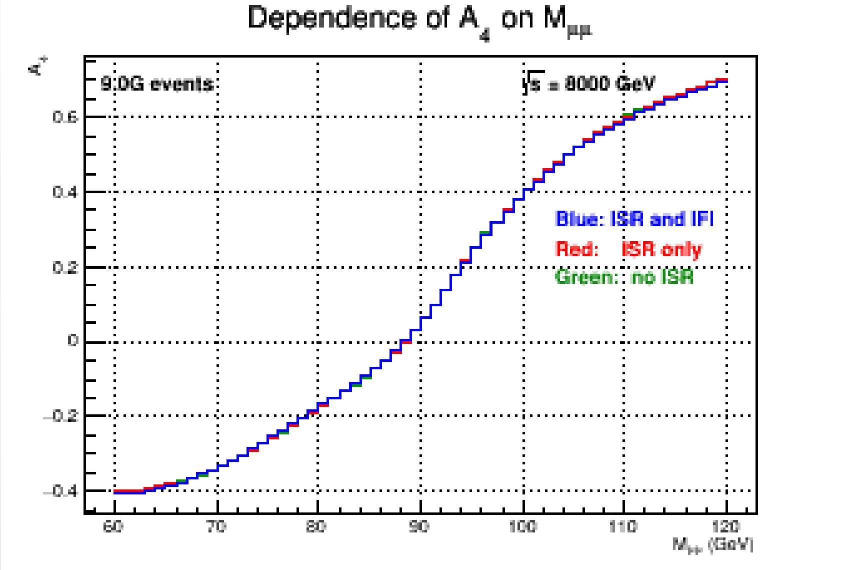
Comparisons of A_4 for New vs Old Run

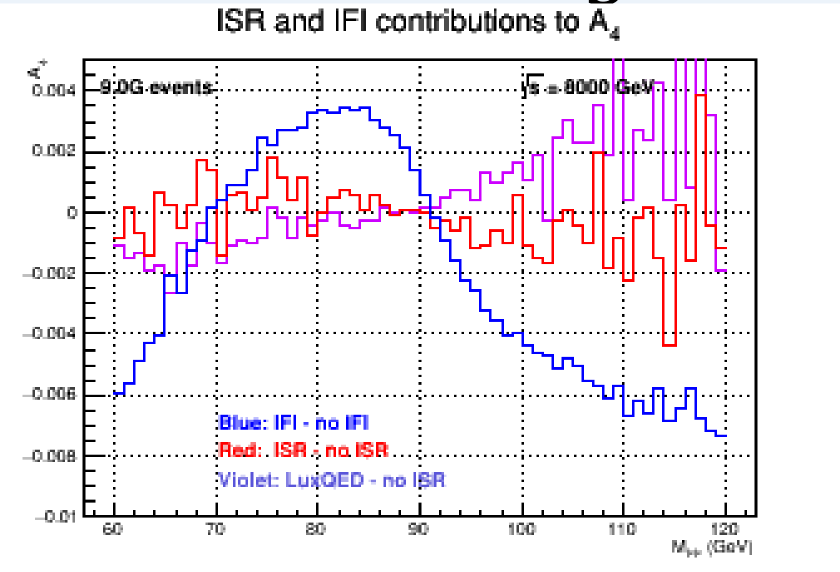
$A_4 = \frac{8}{3} A_{\mathrm{FB}}$ is calculated in the full phases space with complete (KKMC best) photonic corrections.
ΔISR is the difference in A_4 with ISR on minus ISR off, with IFI off in both cases. ΔIFI is the difference in A_4 with IFI on minus IFI off. The numbers are based on a sample of 9G – 10G muon events.

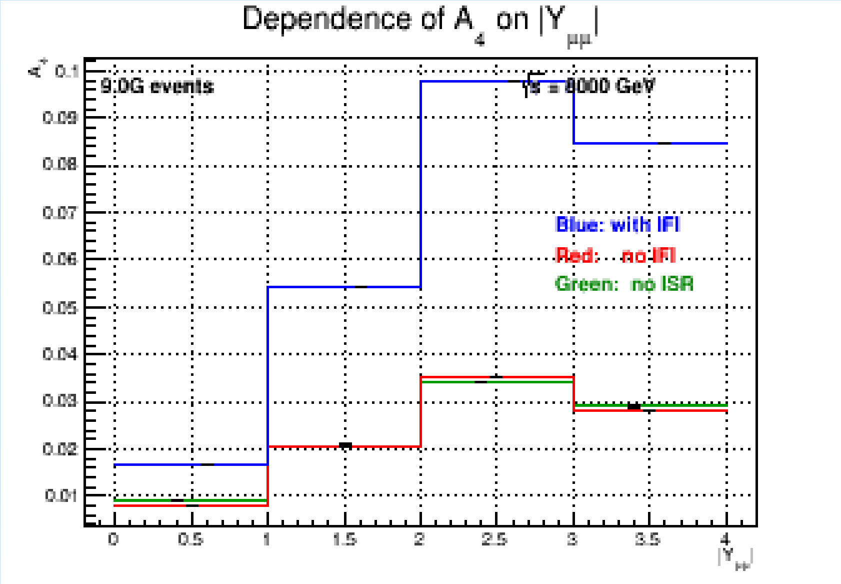
	version	$60 < M_{ll} < 81$	$81 < M_{ll} < 101$	$101 < M_{ll} < 150$	$60 < M_{ll} < 120$	$89 < M_{ll} < 93$
A_4 (best)	old	$-0.28892(8)$	$0.07785(3)$	$0.5836(1)$	$0.05606(3)$	$0.08329(2)$
	new	$-0.28871(7)$	$0.07787(3)$	$0.53649(1)$	0.05614(3)	$0.08332(6)$
Δ ISR	old	$(0.2 \pm 1.1) \times 10^{-4}$	$-(0.5 \pm 0.5\ ) \times 10^{-4}$	$-(8.1 \pm 1.9) \times 10^{-4}$	$-(0.7\ \ \pm 0.4) \times 10^{-4}$	$-(1.0 \pm 0.6) \times 10^{-4}$
	new	$(2.9 \pm 0.1) \times 10^{-4}$	$-(1.8 \pm 0.5) \times 10^{-4}$	$-(7.7 \pm 2.0) \times 10^{-4}$	$-(1.7 \pm 0.4) \times 10^{-4}$	$-(2.0 \pm 0.7) \times 10^{-4}$
Δ IFI	old	$(3.4 \pm 0.9) \times 10^{-4}$	$(\ 3.1 \pm 0.2) \times 10^{-4}$	$-(6.2 \pm 0.1) \times 10^{-3}$	$(1.3 \pm 0.4) \times 10^{-4}$	$(2.0 \pm 0.3) \times 10^{-4}$
	new	$(4.5 \pm 4.8) \times 10^{-4}$	$(3.2 \pm 1.7) \times 10^{-4}$	$-(5.6 \pm 0.4) \times 10^{-3}$	$(1.5 \pm 1.5) \times 10^{-4}$	$(2.0 \pm 2.8) \times 10^{-4}$

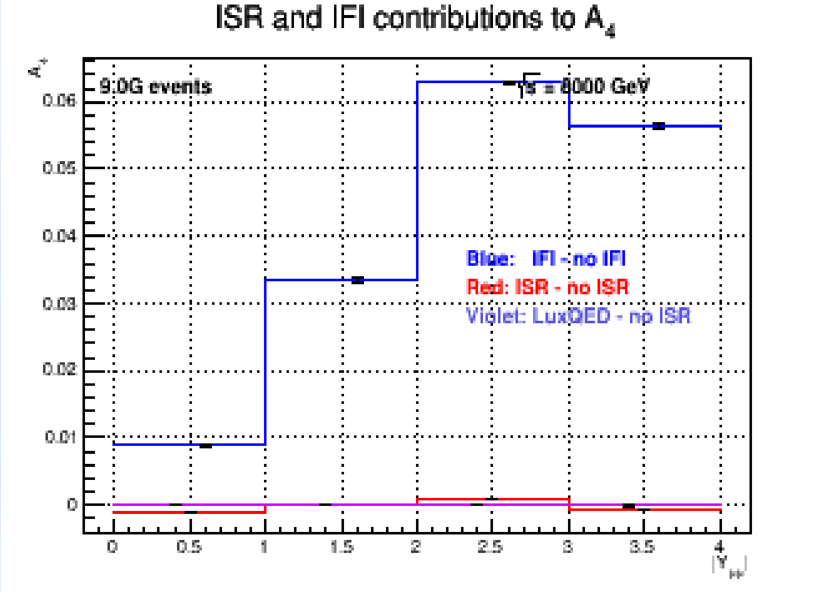
Old: Nov. 2020 New: May 2021

ISR and IFI contributions to A_4 ($\frac{8}{3}A_{\mathrm{FB}}$, no lepton cuts)







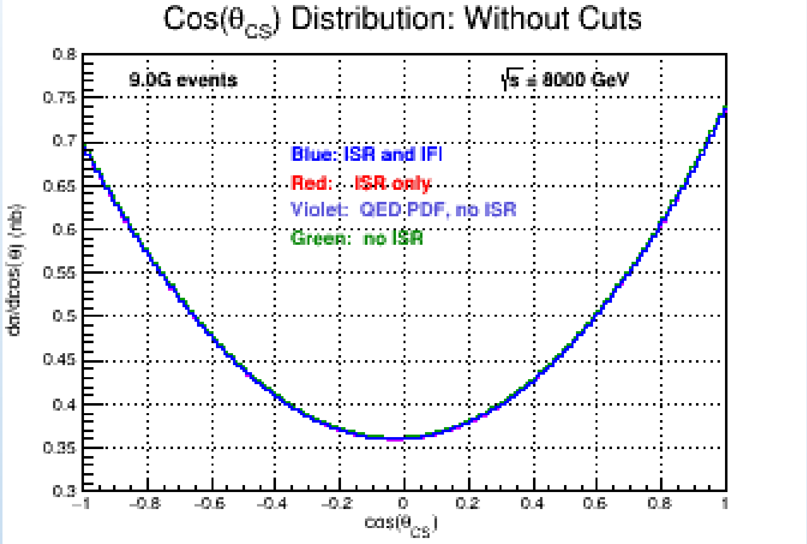


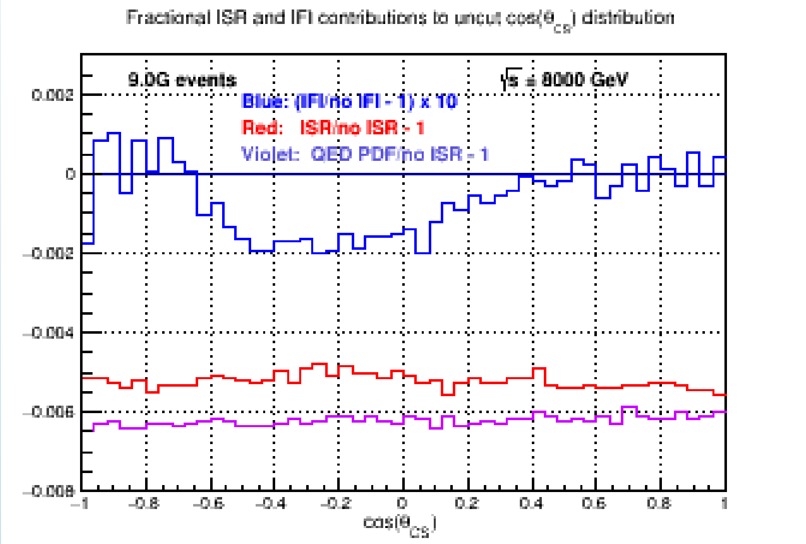
The IFI contribution has a clear structure, but the ISR contribution is near zero and flat up to statistical fluctuations.

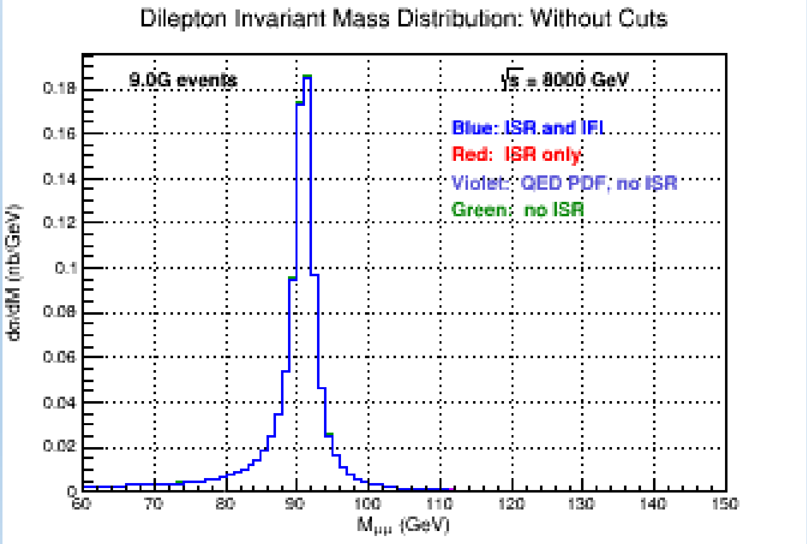
The effect of using a Lux-QED PDF set with ISR off is shown in purple.

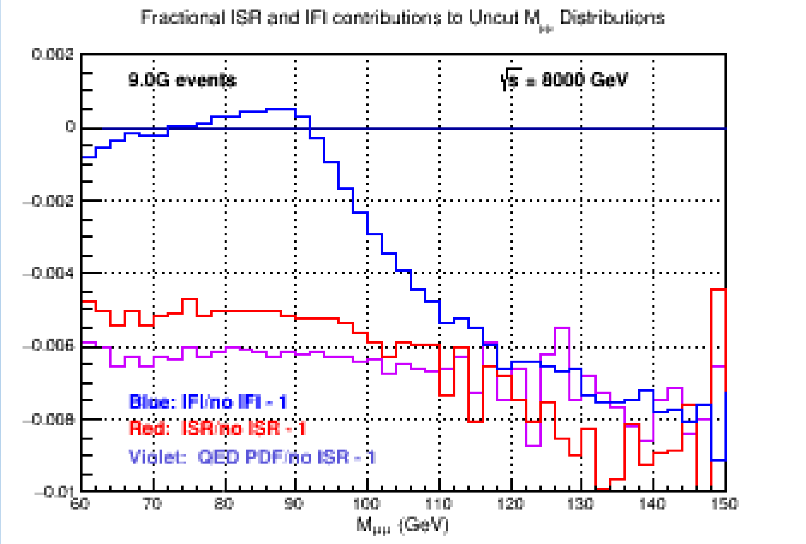
When binned in Y, the ISR contribution is much smaller than IFI.

ISR and IFI contributions to $\cos\theta_{CS}$, M_{ll} Distributions









The IFI contribution to the $\cos\theta_{CS}$ distribution shows a clear asymmetry. The ISR contribution is larger but flatter

The lower figures show the effect ISR and IFI on the M_{ll} distribution.

Using a Lux-QED PDF with ISR off has an effect similar to KKMC-hh's ISR, to within $\sim 10^{-4}$ in each case.

KKhhFoam: Semi-Analytical Implementation

KK-Foam for $e^+e^- \to Z/\gamma^* \to l^+l^- + n\gamma$ is an update of an earlier program KKsem to implement the soft photon exponentiation in a compact, relatively easy-to-understand package that can be used for cross-checks of the more versatile but much more complex KKMC generator.

This update was intended to provide a semi-independent cross-check of the Initial-Final Interference (IFI) calculation of KKMC for FCC physics.

Recently, we ported KK-foam to the hadronic environment as **KKhhFoam**.

This talk will focus primarily on KKhhFoam and cross-checks between it and KKMC-hh.

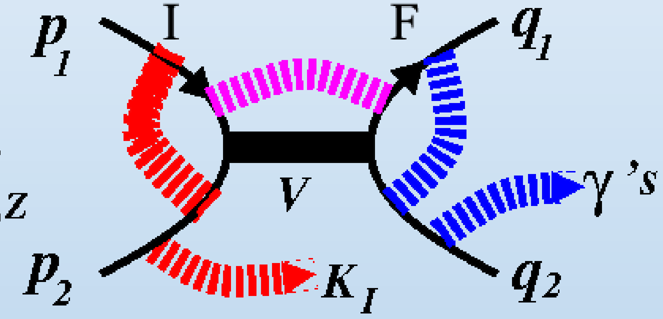
We will also look at how KKMC-hh or KKhhFoam adds photons to quarks and compare this numerically to the effect of using a QED-corrected PDF for collinear photonic ISR. [KKMC-hh also includes non-collinear ISR.]

KKhhFoam: Semi-Soft Approximation for CEEX

The structure of the CEEX matrix element, neglecting non-soft parts, is

$$\sigma(s) = \frac{1}{\text{flux}(s)} \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau_{2+n} \frac{1}{4} \sum_{\text{spins}} \mathfrak{M}^{\mu_1,\cdots,\mu_n}(k_1,\cdots,k_n)\left[\mathfrak{M}_{\mu_1,\cdots,\mu_n}(k_1,\cdots,k_n)\right]^*$$

$$\mathfrak{M}^{\mu_1,\cdots,\mu_n}(k_1,\cdots,k_n) = \sum_{V=\gamma,Z}$$



$$J_I^\mu(k) = \frac{Q_I e}{4\pi^{3/2}}\left(\frac{p_1^\mu}{p_1\cdot k} - \frac{p_2^\mu}{p_2\cdot k}\right)$$

$$J_F^\mu(k) = \frac{Q_F e}{4\pi^{3/2}}\left(\frac{q_1^\mu}{q_1\cdot k} - \frac{q_2^\mu}{q_2\cdot k}\right)$$

Resonant virtual form factor

YFS virtual form factor

$$= \sum_{V=\gamma,Z} e^{\alpha B_4 + \alpha \Delta B_4^V} \sum_{\{I,F\}} \prod_{i\in I} J_I^{\mu_i}(k_i) \prod_{f\in F} J_F^{\mu_f}(k_f)\, \mathcal{M}_V^{(0)}\left(p_1 + p_2 - \sum_{j\in I} k_j\right)$$

$$B_4 = Q_I^2 B_2(p_1,q_1) + Q_F^2 B_2(q_1,q_2) + Q_I Q_F[B_2(p_1,q_1) + B_2(p_2,q_4) - B_2(p_1,q_2) - B_2(p_2,q_3)]$$

$$B_2(p,q) \equiv \frac{i}{(2\pi)^3} \int \frac{d^4k}{k^2 - m_\gamma^2 + i\epsilon}\left(\frac{2p+k}{k^2+2p\cdot k+i\epsilon} + \frac{2q+k}{k^2-2q\cdot k+i\epsilon}\right)^2$$

IFI Near a Narrow Resonance

For a very narrow resonance, the space-time separation of ISR and FSR is significant, and IFI is correspondingly suppressed by factors $\sim \Gamma/M$. For real photons, the resonant effects are handled numerically through the MC generation in KKMC. In a semi-soft approximation, the photon sums can be done analytically leading to the results on the next slide.

The corresponding virtual interference terms were summed by Greco et al* and lead to a resonant form factor ΔB_4^V:

$$\Delta B_4^Z = -2Q_I Q_F \frac{\alpha}{\pi} \ln\left(\frac{t}{u}\right) \ln\left(\frac{M_Z^2 - iM_Z\Gamma_Z - s}{M_Z^2 - iM_Z\Gamma_Z}\right), \qquad \Delta B_4^\gamma = 0.$$

While not strictly a soft contribution, this is a numerically significant correction:

$$\frac{\alpha}{\pi} \ln\left(\frac{\Gamma_Z}{M_Z}\right) \approx 0.008.$$

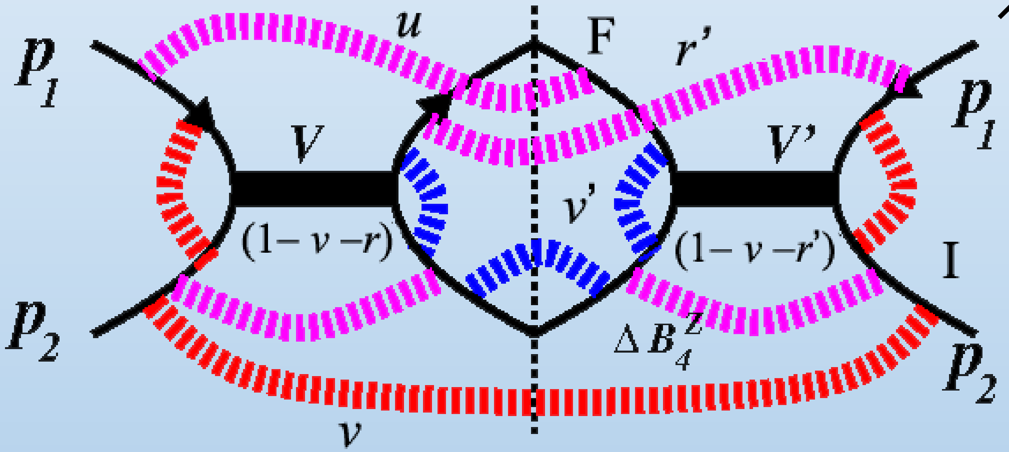
This is essential for obtaining the correct suppression of IFI at the Z pole, when combined with the other CEEX contributions.

*M. Greco et al., Nucl. Phys. B101 (1975) 234, Phys. Lett. B56 (1975) 367, Nucl. Phys. B171 (1980) 118 [Erratum Nucl. Phys. B197 (1982) 543

Result of Analytic Photon Integration

The integrals can be evaluated in the semi-soft limit giving a compact expression

$$\frac{d\sigma}{d\Omega}(s, v_{\max}) = \frac{3}{16}\sigma_0(s) \sum_{V,V'} \int_0^1 dv dv' du du' \;\theta(v_{\max} - v - v' - r - r') e^{Y(p_1,p_2,q_1,q_2)}$$



$$\times \rho(\gamma_I, 1-v)\, \rho(\gamma_F, 1-v')\, \rho(\gamma_X, 1-r)\, \rho(\gamma_X, 1-r')$$

$$\times \frac{1}{4} \mathrm{Re} \sum_{\{\lambda\}} e^{\alpha \Delta B_4^V(s(1-v-r))} \mathfrak{M}_{\{\lambda\}}^{V}(s(v+r), t)$$

$$\times \left[e^{\alpha \Delta B_4^V(s(1-v-r'))} \mathfrak{M}_{\{\lambda\}}^{V'}(s(v+r'), t) \right]^*$$

with $Y(p_1, p_2, q_1, q_2)$ = standard YFS form factor, $\rho(\gamma, z) = \frac{e^{-C_E \gamma}}{\Gamma(1+\gamma)} \gamma (1-z)^{\gamma-1}$,

$$\gamma_I = Q_I^2 \frac{\alpha}{\pi}\left[\ln\left(\frac{(p_1+p_2)^2}{m_I^2}\right) - 1\right], \quad \gamma_F = Q_F^2 \frac{\alpha}{\pi}\left[\ln\left(\frac{(q_1+q_2)^2}{m_F^2}\right) - 1\right], \quad \gamma_X = Q_I Q_F \frac{\alpha}{\pi} \ln\left(\frac{1-\cos\theta}{1+\cos\theta}\right)$$

Beyond the Semi-Soft Approximation

KKhhFoam extrapolates this calculation to the entire phase space by replacing the additive constraint $(q_1+q_2)^2 = (p_1+p_2)^2(1-v-v'-r-r')$ by a multiplicative ansatz

$$\frac{(q_1+q_2)^2}{(p_1+p_2)^2} = (1-v)(1-v')(1-r)(1-r') \equiv zz'ww'$$

and upgrading the radiative factors $\rho(\gamma_I, v_I), \rho(\gamma_F, v_F)$ to order α^2 following KKMC's expressions. The complete order α^1 virtual contributions are completed by adding the non-IR parts of the $\gamma\gamma$ and γZ box diagrams to the Born spin amplitudes, replacing $\mathfrak{M}(s,t)$ with

$$\mathfrak{M}(s,t) + \mathfrak{M}^{\gamma\gamma}(s,t,m_\gamma) + \mathfrak{M}^{\gamma Z}(s,t,m_\gamma) - 2\alpha B_4(s,t,m_\gamma) - \alpha\Delta B_4^Z(s,t).$$

EW corrections are included in the Born amplitudes via Dizet 6.45, as in KKMC.

Relation to KKMC

KKhhFoam works in a semi-soft approximation. The sum calculated analytically is closely related to the starting point for KKMC, $O(\alpha^0)_{\rm exp}$. It has the exact soft photon limit to all orders in α, but does not include the exact $O(\alpha^n)$ for any n, including $n = 1$.

KKMC adds YFS residuals at the amplitude level to correct the cross section to the desired order, which is presently $O(\alpha^2 L)$, where L is a large logarithm in the calculation: $\ln(\hat{s}/ m_q^2)$ for ISR. ($\hat{s} \equiv (\mathrm{p}_1 + \mathrm{p}_2)^2$)

To order α^2, the parton-level cross section has the structure

$$\sigma^{(2)} = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau_{2+n}\; e^{2\alpha\,\mathrm{Re}\,B_4} \frac{1}{4} \sum_{\{\lambda\}} \left| \mathfrak{M}_{n\{\lambda\}}^{(2)}(p_1, p_2, q_1, q_2, k_1, \dots, k_n) \right|^2 ,$$

$$\mathfrak{M}_n^{(2)}(\cdots) = \prod_{s=1}^{n} \mathfrak{s}(k_s) \left[\hat{\beta}_0^{(2)} + \sum_{j=1}^{n} \frac{\hat{\beta}_1^{(2)}(k_j)}{\mathfrak{s}(k_j)} + \sum_{j_1<j_2} \frac{\hat{\beta}_2^{(2)}(k_{j_1}, k_{j_2})}{\mathfrak{s}(k_{j_1})\mathfrak{s}(k_{j_2})} \right] ,$$

where the $\hat{\beta}_i^{(2)}$ are IR-finite subtracted amplitudes (“residuals”) with i photons. For example,

$$\hat{\beta}_0^{(2)} = \mathfrak{M}_0^{(2)} = \left[e^{-\alpha\,\mathrm{Re}\,B_4} \mathcal{M}_0^{(2)} \right]\Big|_{O(\alpha^2)}$$

with $\mathcal{M}_0^{(2)}(p_1, p_2, q_1, q_2)$ a Born-like spin amplitude calculated up to order α^2 via Feynman diagrams.

KKhhFoam Complete Cross Section

KKhhFoam also must generate the initial quark flavor and momentum fractions, adding adding three dimensions to the parameters v_I, v_F, v_{IF}, v_{FI} and angles θ, ϕ of the final state fermion, giving a 9-dimensional integral evaluated by the Foam adaptive MC by S. Jadach.

Including the PDFs $f_q^h(x, \hat{s})$ for quark q in hadron h with momentum fraction x and scale $\hat{s} = (p_1 + p_2)^2 = s x_1 x_2$ (with $s = E_{\text{CM}}^2$ in terms of the proton CM energy) gives a cross section

$$\sigma = \sum_q \int_0^1 dx_1 dx_2\, f_q^{h_1}(x_1, \hat{s}) f_{\bar{q}}^{h_2}(x_2, \hat{s})\, \sigma_q(\hat{s})$$

with quark-level cross section $\sigma_q(\hat{s})$ constructed as described on the previous pages.

Lepton Invariant Mass^2 Distribution

With $z \equiv 1 - v, z' \equiv 1 - v', w \equiv 1 - r$ and $w' \equiv 1 - r'$ and defining scales

$$\hat{s} \equiv x_1 x_2 s \text{ (quarks before ISR)}, \quad \bar{s} \equiv z\hat{s}, \quad s' = M_{ll}^2 = zz'ww'\hat{s} \text{ (leptons after FSR)},$$

the integral over z can be swapped for one over $\bar{s}$ and the s' constraint for one on z' giving

$$\frac{d\sigma}{ds'} = \frac{3\pi}{4}\sigma_0(s)\sum_q \int_{x_1x_2 \geq s'/s}^{1} dx_1 dx_2 \;\; f_q^{h_1}(x_1,\hat{s}) f_{\bar{q}}^{h_2}(x_2,\hat{s}) \int_{s'/\hat{s}}^{1} dz \; \boxed{\rho(\gamma_I(\hat{s}), z)} \;\text{ISR}$$

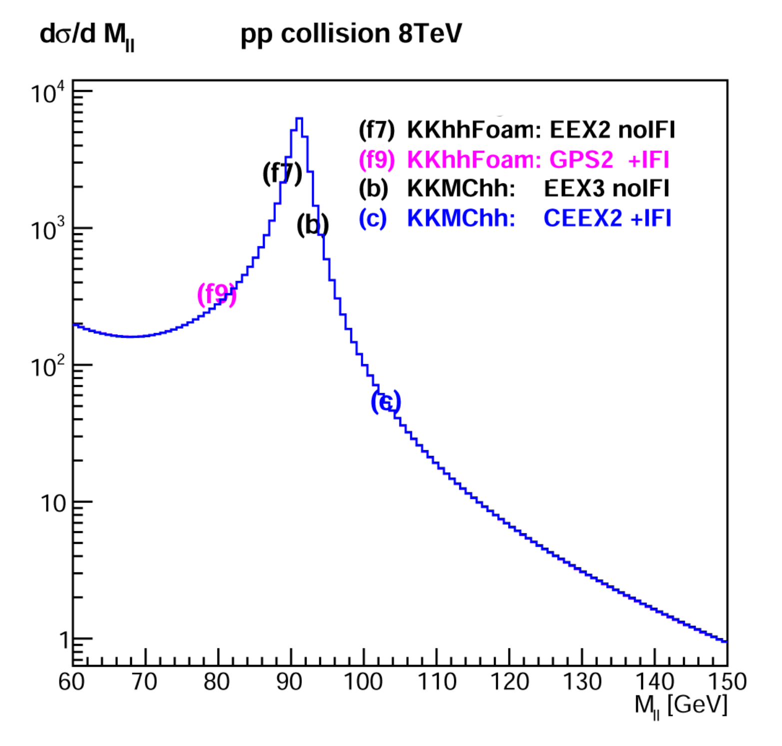
$$\times \int_{ww' \geq s'/\bar{s}}^{1} \frac{dwdw'}{ww'} \int_{-1}^{1} d\cos\theta \; \underbrace{\boxed{\rho(\gamma_X(\cos\theta), w)\, \rho(\gamma_X(\cos\theta), w')}}_{\text{IFI factors}} \; \boxed{\rho\left(\gamma_F(s'), \frac{s'}{ww'\bar{s}}\right)} \;\text{FSR}$$

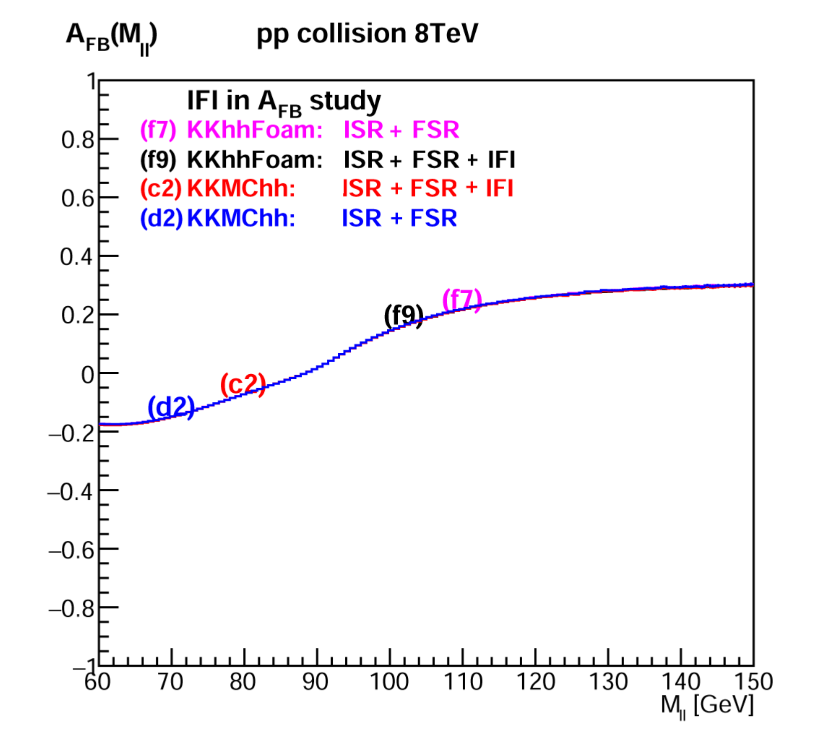
$$\times \frac{1}{4}\mathrm{Re}\sum_{\{\lambda\}} e^{\alpha\Delta B_4^V(\bar{s}w)} \mathfrak{M}_{\{\lambda\}}^{V}(\bar{s}w, \cos\theta) \left[e^{\alpha\Delta B_4^V(\bar{s}w')} \mathfrak{M}_{\{\lambda\}}^{V'}(\bar{s}w', \cos\theta)\right]^*$$

Different scales

Invariant Mass Distributions, A_{FB} Comparisons

The next slides will show comparisons of invariant mass distributions made with KKMC-hh and KKhhSem, with some comparisons to calculations without KKMC-hh ISR but with a QED-corrected PDF. The slides will show ratios of M_{ll} or A_{FB} distributions, with muon final states.



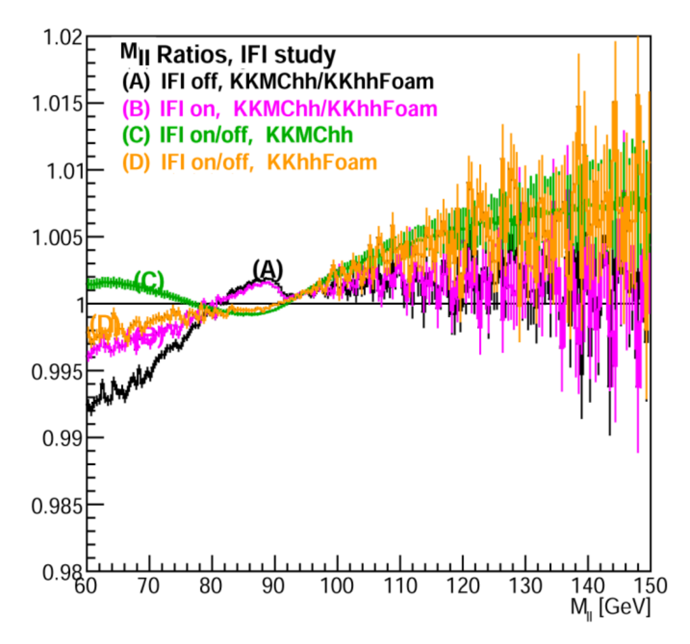


All results on the following pages were made with a “beta” version of KKhhFoam or a “beta” version of the newly created C++ implementation of KKMC-hh, and are preliminary.

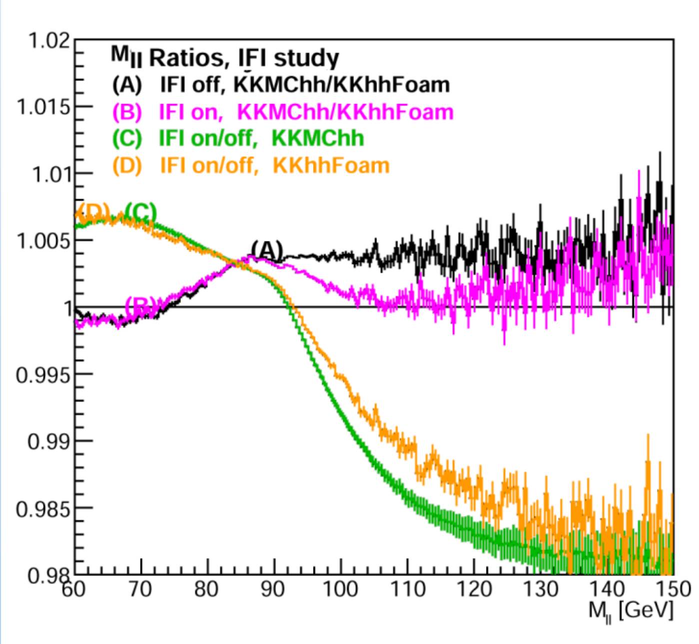
A_{FB} is calculated using the Collins-Soper angle: $\cos\theta_{CS} = \mathrm{sgn}(P_{ll}^{z}) \frac{p_l^+ p_{\bar{l}}^- - p_l^- p_{\bar{l}}^+}{\sqrt{P_{ll}^2 P_{ll}^+ P_{ll}^-}}$ for $P_{ll} = p_l + p_{\bar{l}}$, $p^{\pm} = p^0 \pm p^z$.

Ratios of M_{ll} distributions: single quarks

These graphs show ratios of M_{ll} distributions for muons in $8\ \mathrm{TeV}$ collisions. The ratio of KKMC-hh to KKhhFoam is shown for IFI off (black) or on (magenta). The IFI on/off ratio is shown for KKMChh (green) and KKhhFoam (gold). Agreement is best where hard photon contributions, which are incomplete in KKhhFoam, are less important.



Down



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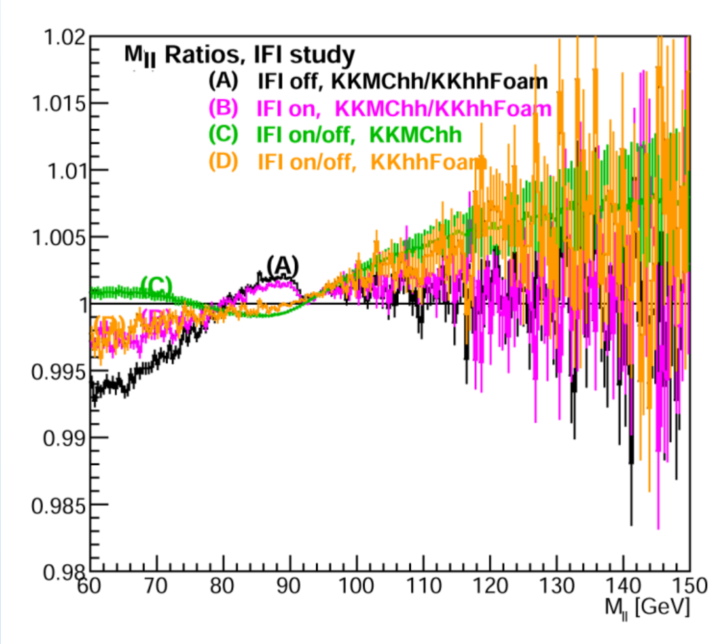
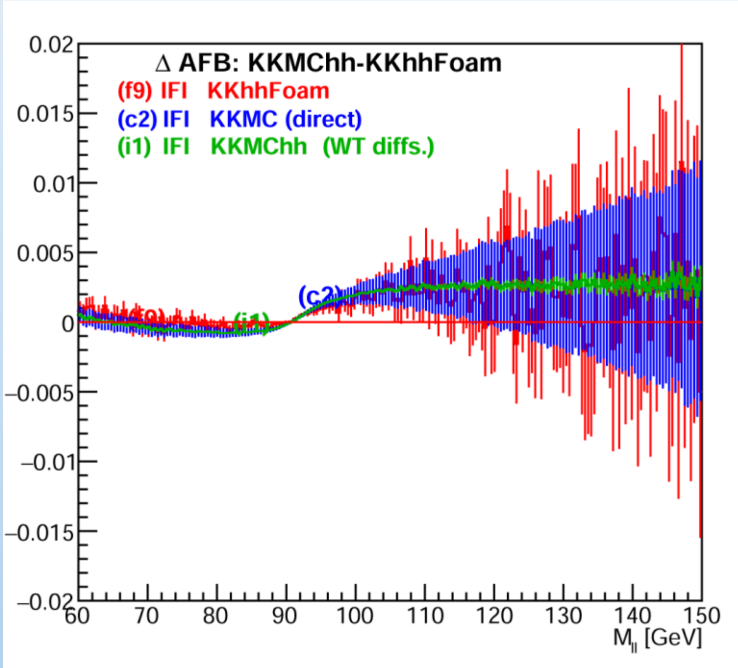


Strange

Ratios of A_{FB} distributions: single quarks

These graphs show ratios of M_{ll} distributions for muons in 8 TeV collisions. The difference ΔA_{FB} for IFI on – off is shown for KKhhFoam(red), for KKMChh (blue) by comparing distributions or (green) via weight differences in a single run.



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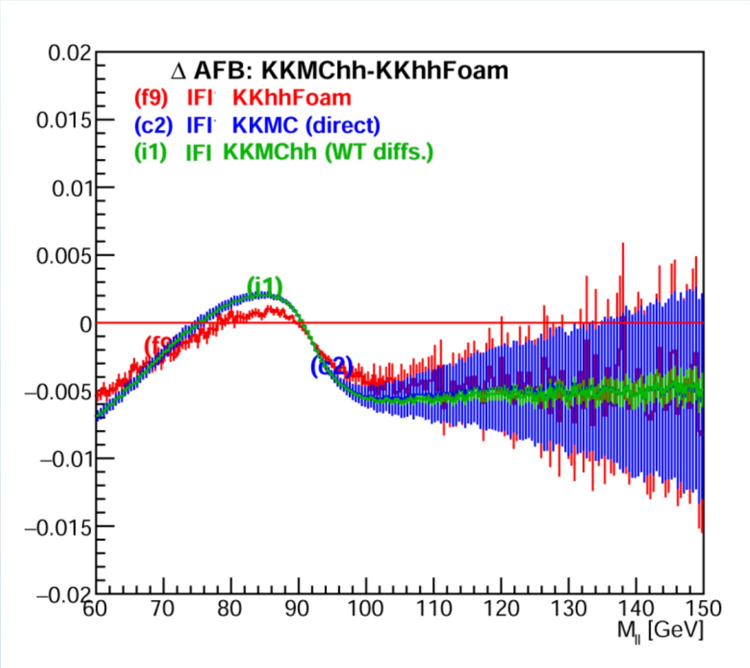

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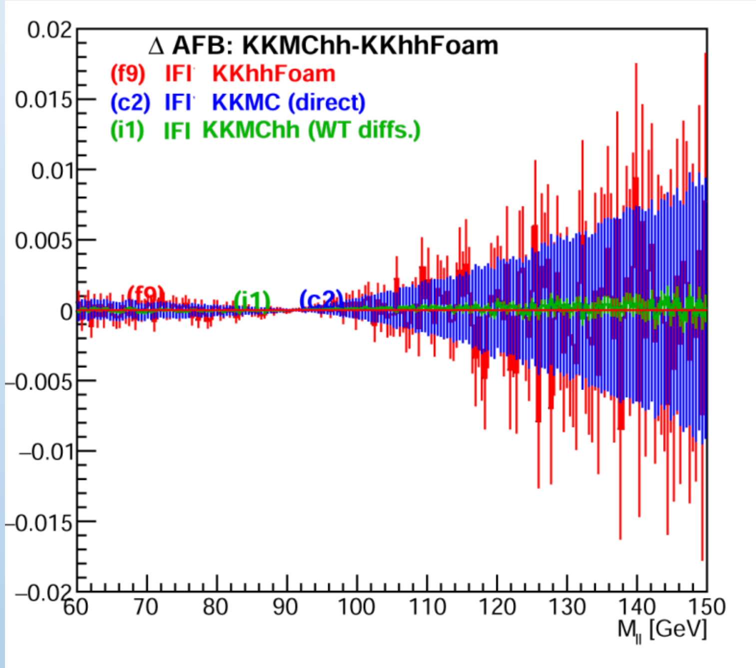

Strange

The IFI contribution is linear in the quark charge, so the up quark gives about twice the effect as the down quark, with the opposite sign. Sea quarks have equal $q, \bar{q}$ PDFs, so no asymmetry.

Quark Mass Dependence of IFI?

The dominant behavior of IFI is determined by an angle-dependent "big logarithm". If Δ is an effective cutoff on the total radiated photon energy fraction (due to cuts or kinematic constraints), it takes the form, with hard CM scale $\bar{s} \equiv z\hat{s}$

$$\int_{1-\Delta}^{1} dz\, \rho(\gamma_I(\hat{s}), z)\ \mathrm{Born}(\bar{s}, \cos\theta) \approx \left(\frac{1\ - \cos\theta}{1 + \cos\theta}\right)^{2Q_i Q_f \frac{\alpha}{\pi} \ln\Delta} \mathrm{Born}(\bar{s}, \cos\theta)$$

for a collision between *given* quarks with $\hat{s} = (p_1 + p_2)^2$. This *suggests* that IFI should be independent of the quark mass, but for hadron scattering, the quark momentum distribution is integrated over PDFs, and the quark mass enters into the relation between $\bar{s}$ and $\hat{s}$, making this less obvious.

We are in the process of working out a more detailed study of this effect, but essentially, the dependence on m_q^2 can be traded for the dependence on $\hat{s}$ in the PDFs. To the extent that this dependence is weak in the relevant kinematic region, the mass dependence of IFI should be small.

This can be tested numerically…

Quark Mass Dependence of IFI Contribution

These plots compare the IFI contribution calculated 3 ways for an up quark with a mass of 2 MeV (left) or 500 MeV (right). There is no significant change in the result.