Non-equilibrium theory of non-relativistic pairs inside an environment

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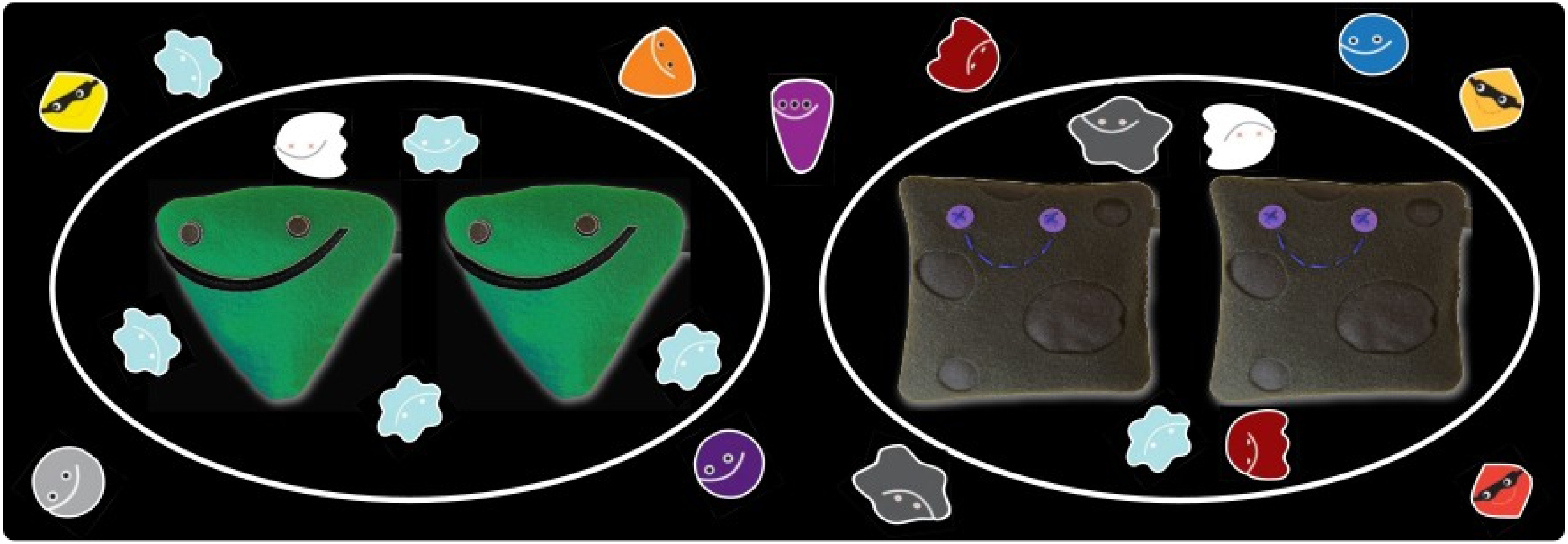
@ CERN (online)
15th July 2021

1808.06472 (PRD), 1910.11288 (PRL),
2002.07145 (JHEP),
2106.03629, 2107.03945, ...

System



Consider heavy non-relativistic pairs inside a plasma environment:

- **Qarkonia in a quark-gluon plasma (QGP)**
- **Heavy WIMPs in the early Universe plasma**
- ...

See also “Quarkonia meet Dark Matter”:
https://indico.ipmu.jp/event/389/overview

Quantum mechanical effects: Positronium

- **Bound-state decay** and **Sommerfeld-enhanced annihilation:**

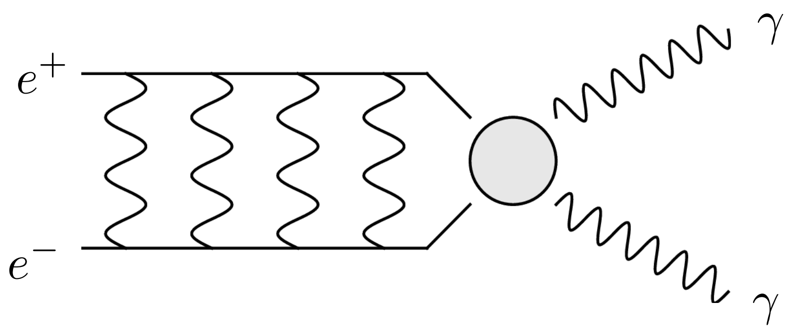
(for s-wave)

$$\Gamma_n = (\sigma v)_0 \times |\psi_n(r=0)|^2$$

J. Pirenne 1946, J. Wheeler 1946

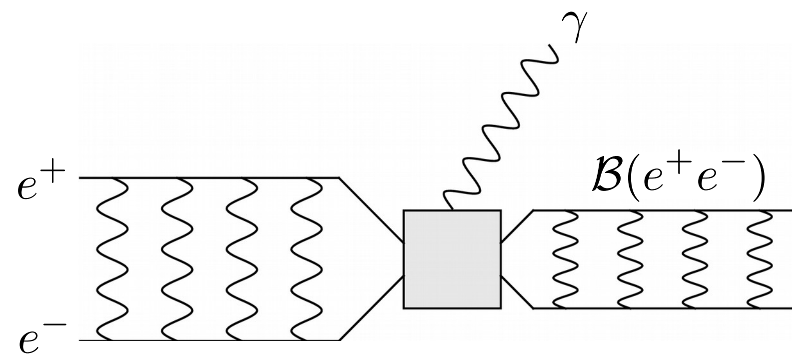
$$(\sigma v)^{\rm ann} = (\sigma v)_0 \times |\psi(r=0)|^2 \propto (\sigma v)_0 \, (\alpha/v) \,, \text{ for } v \lesssim \alpha.$$

A. Sommerfeld 1931, A. Sakharov 1948



- **Bound-state formation:**

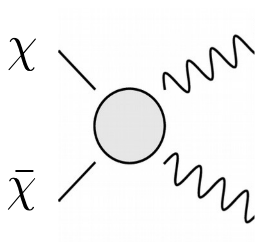
$$(\sigma v)^{\rm bsf}_{100} \sim 3 \times (\sigma v)^{\rm ann}, \text{ for } v \ll \alpha$$



Massive mediator

For massive mediators, QM effects play a role if the range of the attractive potential is larger than the Bohr radius, or, equivalently: $\mathrm{m}_{\mathrm{mediator}} \lesssim \alpha m_\chi$

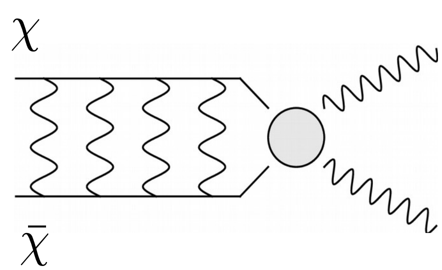
WIMP



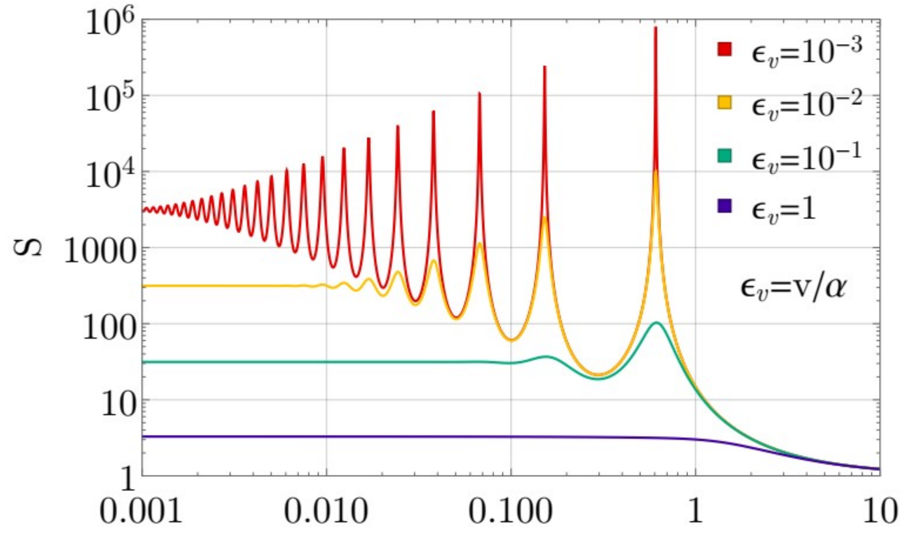
Plane-wave initial state

$m_{\mathrm{mediator}} \gg \alpha m_\chi$

“Heavy WIMP”



$m_{\mathrm{mediator}} \lesssim \alpha m_\chi$



Modification of annihilation cross section has consequences for:

- Prediction of DM relic abundance (Overclosure bound)
- Predicted flux of SM particles from DM annihilation in, e.g., the galactic center (Indirect detection)

[Seminal works by J. Hisano et al. 2006+]

Prime example: Wino neutralino

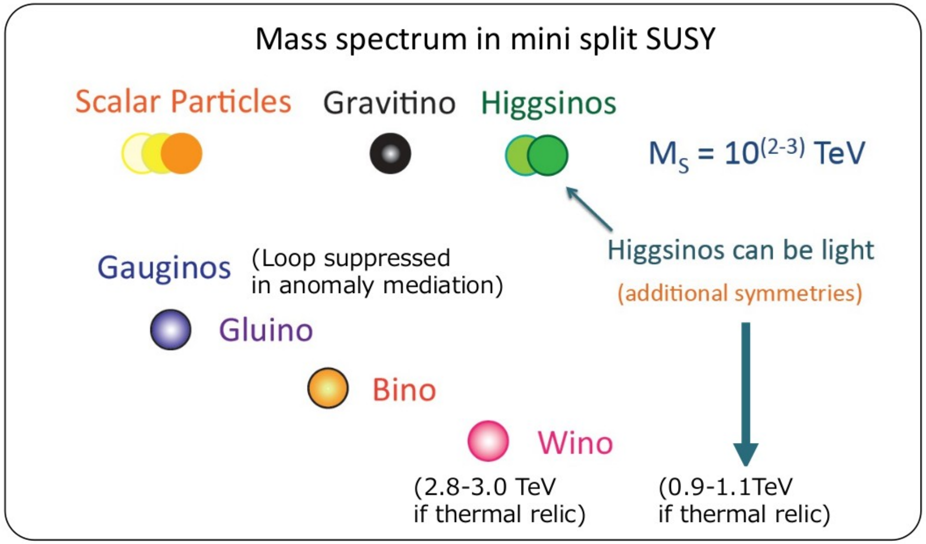
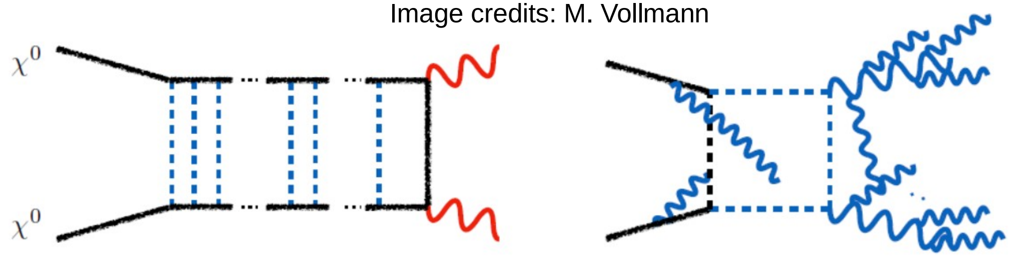


Image credits: M. Vollmann



Wino
(I=1, Y=0, S=1/2)
Partners of weak bosons

$$\chi^0,\ \chi^\pm$$

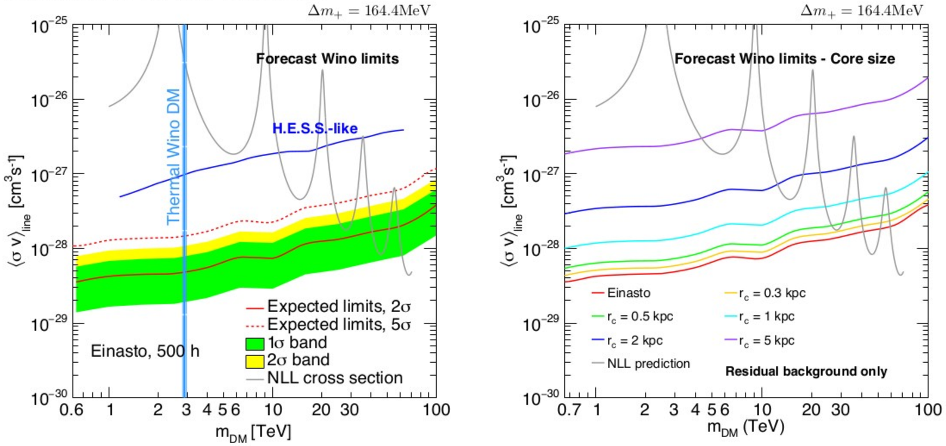
[see talk by J. Hisano, M. Beneke]

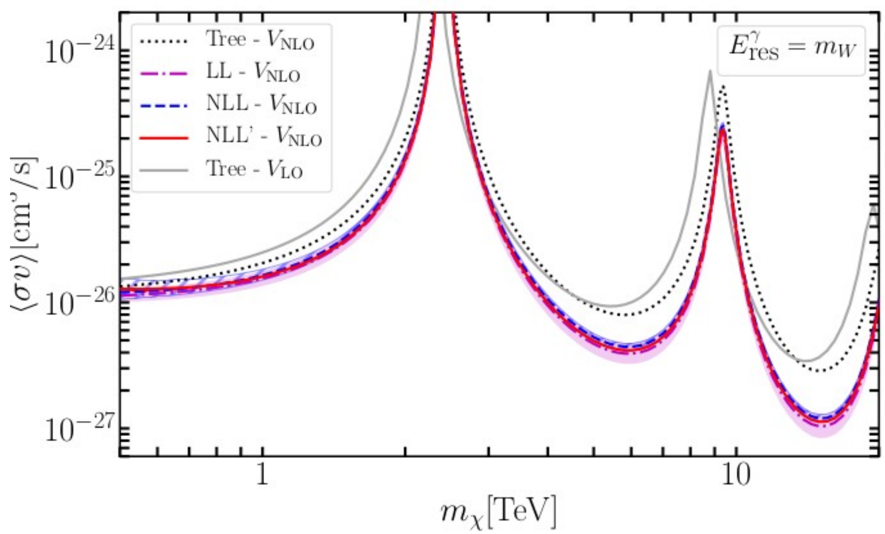
$$\Delta m_+ = 166\text{MeV} + O\left(\frac{v^4}{M^3}\right)$$

(EW radiative correction)

Line gamma rays from GC of Wino/Higgsino

CTA Prospect for Wino DM (Rinchiuso, Slatyer, et al (20))
(Resummation of Sudalov double log, continuum emission, endpoint photons, energy resolution of CTA are included.)

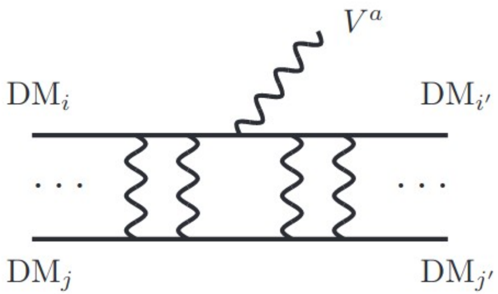


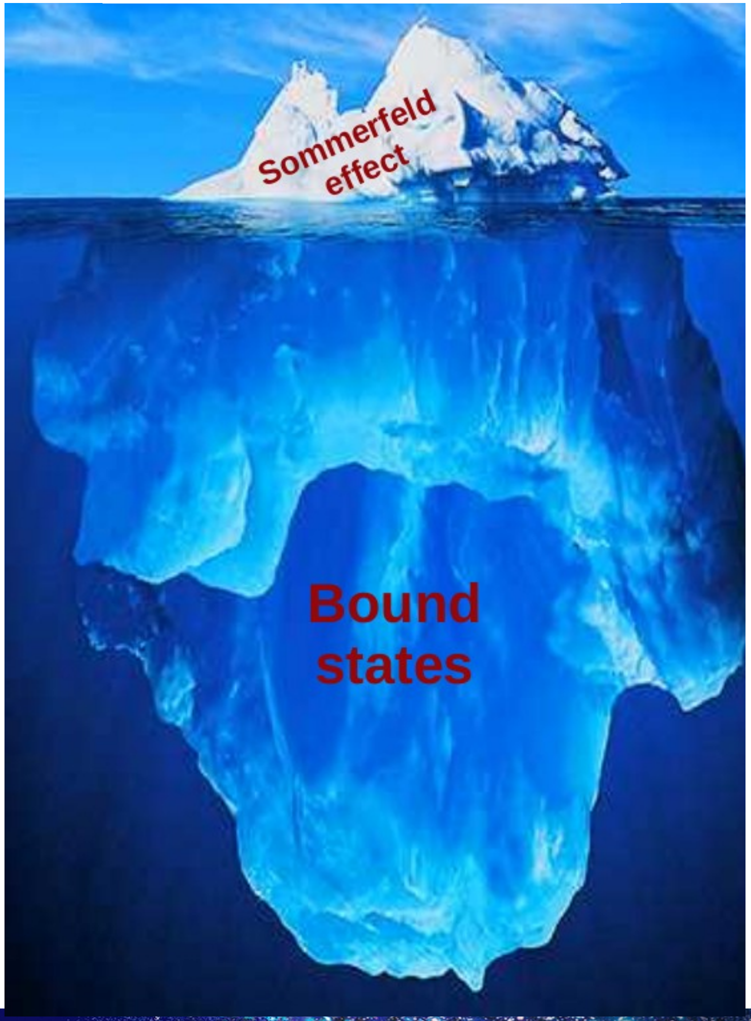


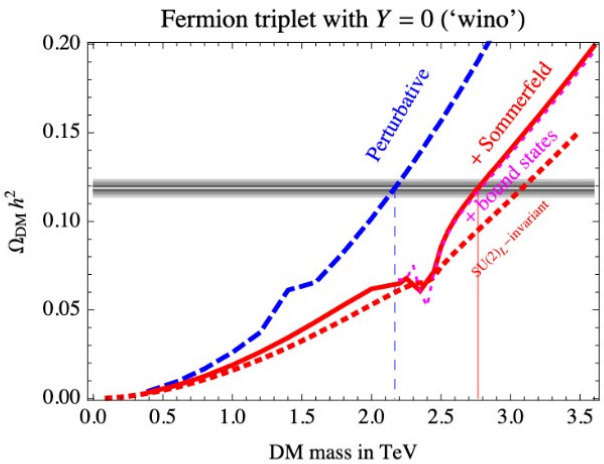
The precise evaluation of thermal relic abondance is important since the x section to line gamma is quite sensitive to wino mass.

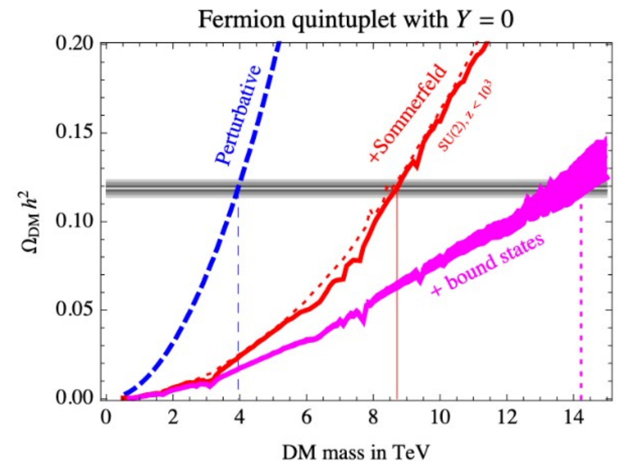
15

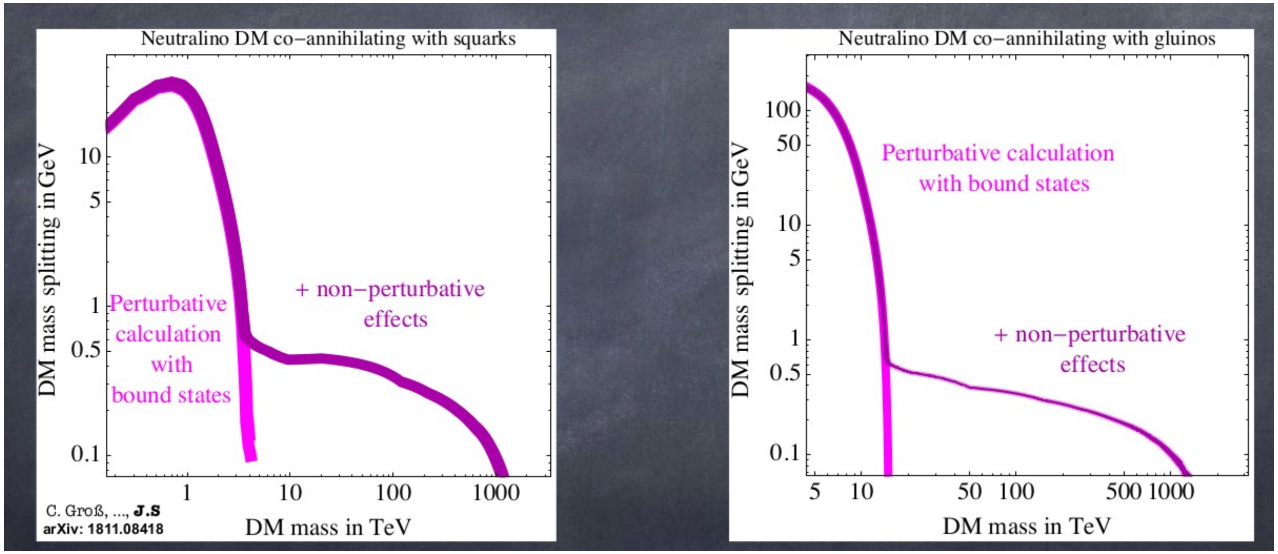
+ bound states

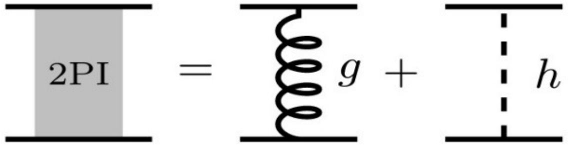






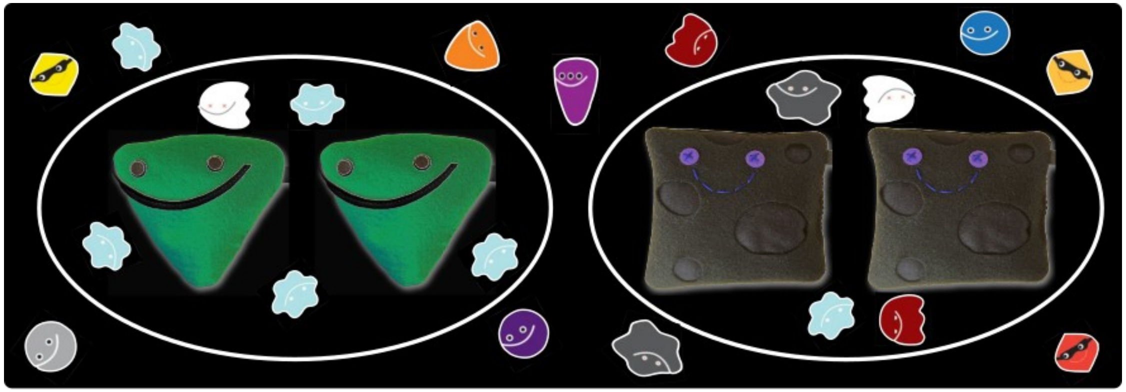




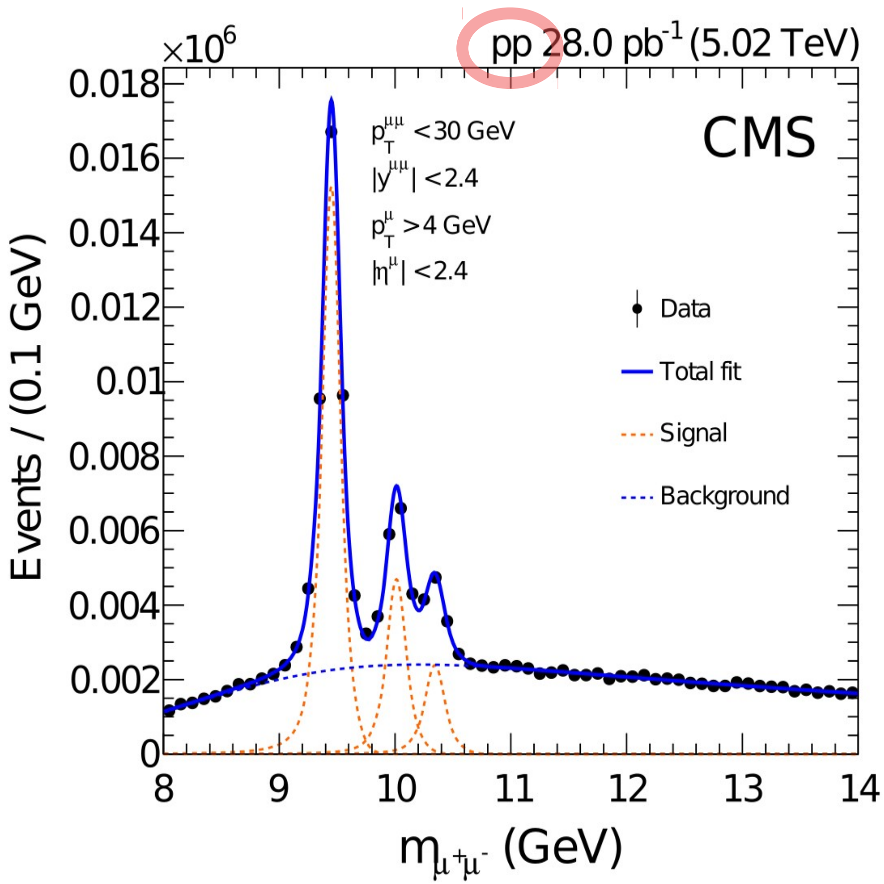


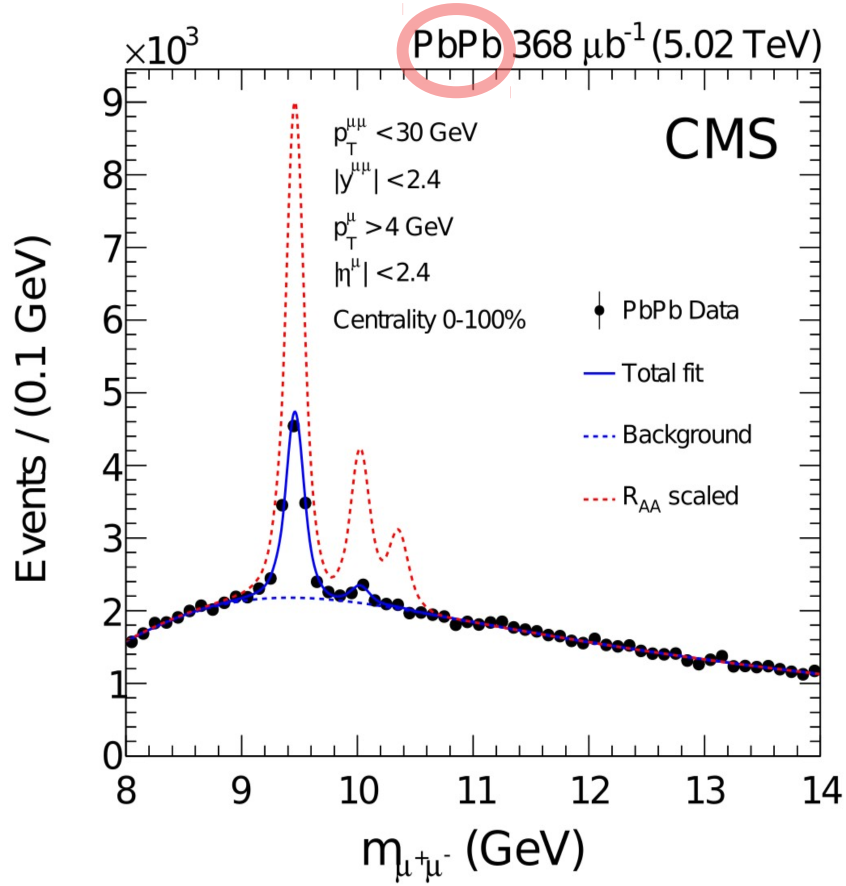
[see talks by K. Petraki, J. Smirnov, J. Harz]

Finite Temperature



Data





Sequential melting of b-bbar bound states inside QGP plasma observed.

CMS collaboration,
Phys.Lett. **B790** (2019) 270-293

Theory

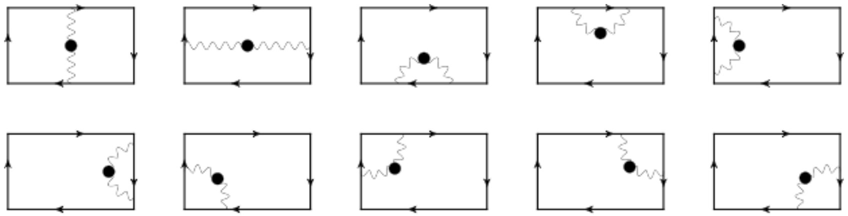
Heavy Quarkonia in QGP

- **Matsui & Satz 1986**
  - J/Psi suppression in QGP due to screening effect
  - In-medium potential from **2** Polyakov loop (Wilson line)

$$V(r) = -\frac{\alpha}{r} e^{-m_D r}, \; m_D \propto gT$$

Debye screening

- **Laine *et al.* 2007**
  - **4** Polyakov loop method



$$V(r) = -\alpha m_D - \frac{\alpha}{r} e^{-m_D r} - i\alpha T \phi(m_D r)$$

Salpeter correction

Thermal width

Heavy DM in primordial plasma

- **Cirelli *et al.* 2007** (Minimal DM)
  - Debye screened Sommerfeld enhancement
  - (Wino mass lowered)

- **Bödeker & Laine 2012**
  - First dynamical formulation based on "**linear response theory**"

[see follow-ups by Biondini & Kim & Laine]

Non-equilibrium QFT approach

In-medium potential + dynamics from Keldysh-Schwinger-Formalism

Scheme:

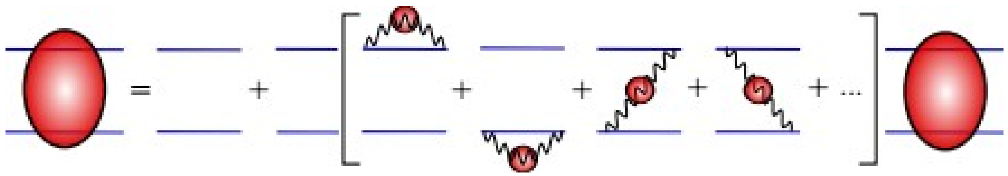
I Non-relativistic effective action

$$S_{\rm NR}[\eta,\xi]=\int_{x\in\mathcal{C}}\eta^\dagger\left[i\partial_t+\frac{\Delta}{2M}\right]\eta+\xi^\dagger\left[i\partial_t-\frac{\Delta}{2M}\right]\xi+\int_{x,y\in\mathcal{C}} i\frac{g^2}{2}\underbrace{J(x)D(x,y)J(y)}_{\text{"potential"}}+i\underbrace{O^\dagger(x)\Gamma(x,y)O(y)}_{\text{"annihilation"}},$$

$J\equiv\eta^\dagger\eta+\xi^\dagger\xi,\ O\equiv\xi^\dagger\eta$

II Number density equation from EoM of 2-point correlation function

III Resummation of 4-point correlator



Recover in-medium potential.

[TB, Covi, Mukaida 18]

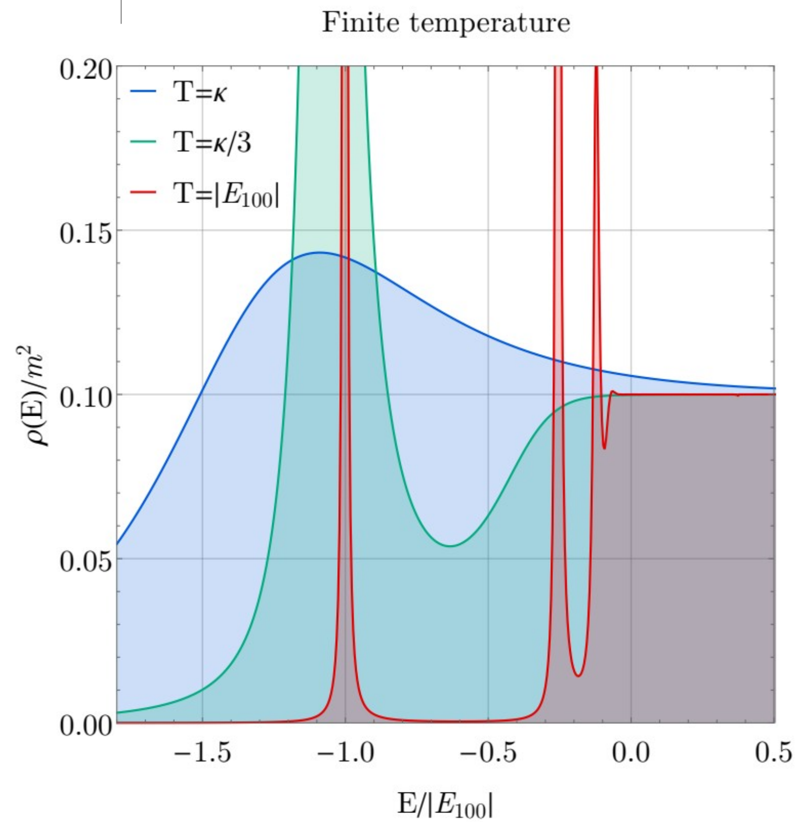
Annihilation/decay at finite temperature

- **Consistent with Boltzmann equations in zero temperature limit**
- **Finite temperature corrections modify two-particle spectrum**
- **Formalism advantages**
  - Sommerfeld enhanced annihilation
  - **ALL** bound states included
  - Finite temperature corrections
  - Simple number density equation
- **Limitation**
  - Hard-Thermal-Loop approximation (ok for |E|<T)
  - Ionization equilibrium

$$\dot{n} + 3Hn = -2(\sigma v)_0 G_4^{++--}\big|_{\rm eq}\left[e^{\beta 2\mu} - 1\right],$$

$$G_4^{++--}\big|_{\rm eq} \propto \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{(2\pi)} e^{-\beta E}\rho(E).$$

$$V(r) = -\alpha m_D - \frac{\alpha}{r}e^{-m_D r} - i\alpha T\phi(m_D r)$$



Open Quantum system treatment

Lindblad eqaution:

[see talk by A. Vairo]

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_i (C_i \rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\})$$

For quarkonia (QCD), these are coupled singlet/octet equations:

$$\frac{d\rho_s(t;t)}{dt} = -i[h_s, \rho_s(t;t)] - \Sigma_s(t)\rho_s(t;t) - \rho_s(t;t)\Sigma_s^\dagger(t) + \Xi_{so}(\rho_o(t;t), t)$$

$$\frac{d\rho_o(t;t)}{dt} = -i[h_o, \rho_o(t;t)] - \Sigma_o(t)\rho_o(t;t) - \rho_o(t;t)\Sigma_o^\dagger(t) + \Xi_{os}(\rho_s(t;t), t) + \Xi_{oo}(\rho_o(t;t), t)$$

Projection of Lindblad equation, gives bound state evolution

$$\frac{\partial}{\partial t} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) + \frac{\boldsymbol{k}}{2M} \cdot \nabla_{\boldsymbol{x}} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) = \underbrace{\mathcal{C}_{nl}^{+}(\boldsymbol{x}, \boldsymbol{k}, t)}_{\text{Recombination}} - \underbrace{\mathcal{C}_{nl}^{-}(\boldsymbol{x}, \boldsymbol{k}, t)}_{\text{Dissociation}}$$

Ok for T<< Bohr momentum.

Intermediate summary

- Sommerfeld-enhanced annihilation and bound state decay at finite T,
- + Formation and dissociation of bound states at finite T,
- + e.g., fine and hyperfine splitting at finite T,
- Full out-of-equilibrium.

→ How can we theoretically describe all that at once?
Or, in other words:
i) which EFT and **ii)** which non-equilibrium quantum field theory?

Previous limitations:

- Open quantum system:
  T << Bohr momentum
- 4 Plyakov + Linear response theory:
  Linear close to chemical equilibrium
- Non-rel. EFT (w/o ultrasoft) + Keldysh-Schwinger:
  Ionization equilibrium T>>E

pNREFT

Relativistic QED:

[Dirac 1928]

$$\mathcal{L} = i\bar{\chi}\gamma^{\mu}\partial_{\mu}\chi - m\bar{\chi}\chi - g\bar{\chi}\gamma^{\mu}\chi A_{\mu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathcal{L}^{\text{env}}[A],$$

Non-relativistic QED:

[Caswell & Lepage 1986, Labelle 1992]

$$\mathcal{L}^{\text{NR}} = \eta^{\dagger}\left[iD_0 + \frac{D^2}{2m_e}\right]\eta + \xi^{\dagger}\left[iD_0 - \frac{D^2}{2m_e}\right]\xi + \int \frac{g^2}{2} \underbrace{J^0 V J^0}_{\text{“potential”}} + \ldots$$

Potential non-relativistic QED:

[Brambilla et al. 2000 +]

$$\mathcal{L}^{\text{pNR}} = \int \mathrm{d}^3 r \; \mathrm{Tr}\{O^{\dagger}(\mathbf{x},\mathbf{r},t)\left[i\partial_t - h + \mathbf{r}\cdot g\mathbf{E}(\mathbf{x},t) + \vec{\mu}\cdot g\mathbf{B}(\mathbf{x},t)\right]O(\mathbf{x},\mathbf{r},t)\} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathcal{L}^{\text{env}}[A],$$

Non-equilibrium pNRQFT

Two-body field two-point function:

$$G_{oss'o'}(x,y;\mathbf{r},\mathbf{r}') \equiv \langle T_{\mathcal{C}} O_{os}(x,\mathbf{r}) O^{\dagger}_{s'o'}(y,\mathbf{r}')\rangle$$

From path integral, we derive **two-body field two-point function** EoM:

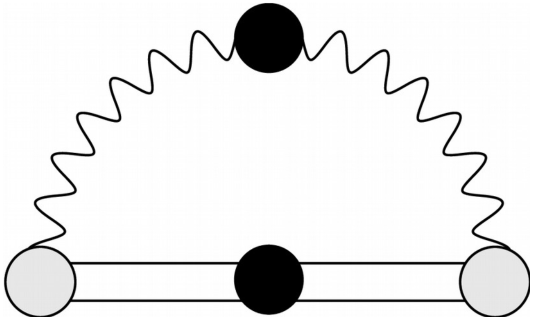
$$\begin{aligned}
(i\partial_{x^0} - h_x)_{o\bar{o}} G_{\bar{o}ss'o'}(x,y;\mathbf{r},\mathbf{r}') &= i\delta_{ss'}\delta_{oo'}\delta^4_{\mathcal{C}}(x,y)\delta^3(\mathbf{r}-\mathbf{r}') \qquad (1)\\
&- g r_i \langle E_i(x)\rangle G^0_{oss'o'}(x,y;\mathbf{r},\mathbf{r}') - g\mu_i^{o\bar{o}}\langle B_i(x)\rangle G^0_{\bar{o}ss'o'}(x,y;\mathbf{r},\mathbf{r}')\\
&- ig^2 \int_{\mathcal{C}} \mathrm{d}^4 w \mathrm{d}^3\bar{r}\, G^0_{os\bar{s}'\bar{o}'}(x,w;\mathbf{r},\bar{\mathbf{r}}) r_i \bar{r}_j \langle T_{\mathcal{C}} E_i(x) E_j(w) O_{\bar{o}'\bar{s}'}(w,\bar{\mathbf{r}}) O^{\dagger}_{s'o'}(y,\mathbf{r}')\rangle\\
&- ig^2 \int_{\mathcal{C}} \mathrm{d}^4 w \mathrm{d}^3\bar{r}\, G^0_{os\bar{s}'\bar{o}'}(x,w;\mathbf{r},\bar{\mathbf{r}}) (\mu_i)^{\bar{o}'\bar{\bar{o}}} (\bar{\mu}_j)^{\bar{\bar{o}}\bar{o}} \langle T_{\mathcal{C}} B_i(x) B_j(w) O_{\bar{o}\bar{s}'}(w,\bar{\mathbf{r}}) O^{\dagger}_{s'o'}(y,\mathbf{r}')\rangle,\\
(-i\partial_{y^0} - h'_y)_{\bar{o}o'} G_{oss'\bar{o}}(x,y;\mathbf{r},\mathbf{r}') &= i\delta_{ss'}\delta_{oo'}\delta^4_{\mathcal{C}}(x,y)\delta^3(\mathbf{r}-\mathbf{r}') \qquad (2)\\
&- g r'_i \langle E_i(y)\rangle G^0_{oss'o'}(x,y;\mathbf{r},\mathbf{r}') - g(\mu'_i)^{\bar{o}o'}\langle B_i(y)\rangle G^0_{oss'\bar{o}}(x,y;\mathbf{r},\mathbf{r}')\\
&- ig^2 \int_{\mathcal{C}} \mathrm{d}^4 w \mathrm{d}^3\bar{r} \langle T_{\mathcal{C}} O_{os}(x,\mathbf{r}) O^{\dagger}_{\bar{s}'\bar{o}'}(w,\bar{\mathbf{r}}) E_j(w) E_i(y)\rangle \bar{r}_j r'_i G^0_{\bar{o}'\bar{s}'s'o'}(w,y;\bar{\mathbf{r}},\mathbf{r}')\\
&- ig^2 \int_{\mathcal{C}} \mathrm{d}^4 w \mathrm{d}^3\bar{r} \langle T_{\mathcal{C}} O_{os}(x,\mathbf{r}) O^{\dagger}_{\bar{s}'\bar{o}}(w,\bar{\mathbf{r}}) B_j(w) B_i(y)\rangle\rangle (\bar{\mu}_j)^{\bar{o}\bar{\bar{o}}} (\mu'_i)^{\bar{\bar{o}}\bar{o}'} G^0_{\bar{o}'\bar{s}'s'o'}(w,y;\bar{\mathbf{r}},\mathbf{r}').
\end{aligned}$$

Equivalent representation of the full path integral in terms of its infinitely coupled field moments (correlation functions).

[TB 21]

Closure of correlation function hierarchy

"Mean field" **+**



$$(i\partial_{x^0} - h_x)G(x,y;\mathbf{r},\mathbf{r}') = i\delta^4_{\mathcal{C}}(x,y)\delta^3(\mathbf{r}-\mathbf{r}') - gr_i\langle E_i(x)\rangle G(x,y;\mathbf{r},\mathbf{r}') - g\mu_i\langle B_i(x)\rangle G(x,y;\mathbf{r},\mathbf{r}') - ig^2\int_{\mathcal{C}} \mathrm{d}^4w\mathrm{d}^3\bar{r}\langle T_{\mathcal{C}}E_i(x)E_j(w)\rangle G(x,w;\mathbf{r},\bar{\mathbf{r}})r_i\bar{r}_jG(w,y;\bar{\mathbf{r}},\mathbf{r}') - ig^2\int_{\mathcal{C}} \mathrm{d}^4w\mathrm{d}^3\bar{r}\langle T_{\mathcal{C}}B_i(x)B_j(w)\rangle G(x,w;\mathbf{r},\bar{\mathbf{r}})\mu_i\bar{\mu}_jG(w,y;\bar{\mathbf{r}},\mathbf{r}'), \quad (1)$$

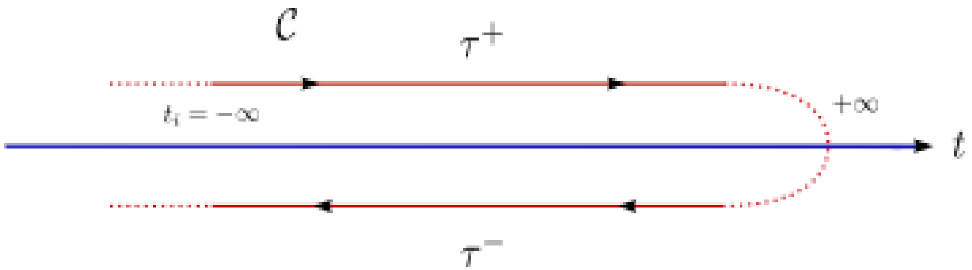
$$(-i\partial_{y^0} - h'_y)G(x,y;\mathbf{r},\mathbf{r}') = i\delta^4_{\mathcal{C}}(x,y)\delta^3(\mathbf{r}-\mathbf{r}') - gr'_i\langle E_i(y)\rangle G(x,y;\mathbf{r},\mathbf{r}') - g\mu'_i\langle B_i(y)\rangle G(x,y;\mathbf{r},\mathbf{r}') - ig^2\int_{\mathcal{C}} \mathrm{d}^4w\mathrm{d}^3\bar{r}G(x,w;\mathbf{r},\bar{\mathbf{r}})\bar{r}_jr'_iG(w,y;\bar{\mathbf{r}},\mathbf{r}')\langle T_{\mathcal{C}}E_j(w)E_i(y)\rangle - ig^2\int_{\mathcal{C}} \mathrm{d}^4w\mathrm{d}^3\bar{r}G(x,w;\mathbf{r},\bar{\mathbf{r}})\bar{\mu}_j\mu'_iG(w,y;\bar{\mathbf{r}},\mathbf{r}')\langle T_{\mathcal{C}}B_j(w)B_i(y)\rangle. \quad (2)$$

- Truncation at the two-point function level. Note bound states included!
- Dressing of free correlators takes into account some higher terms which are otherwise neglected by truncation.
- Information of environment implicitly contained in gauge field correlators.

[TB 21]

Splitting the information

Keldysh-Schwinger formalism (closed time-path contour)



1.) Statistical component:

$$G^{+-}(x, y; \mathbf{r}, \mathbf{r}')$$

2.) Retarded component:

$$G^{R}(x, y; \mathbf{r}, \mathbf{r}') = G^{++}(x, y; \mathbf{r}, \mathbf{r}') - G^{+-}(x, y; \mathbf{r}, \mathbf{r}')$$

In total there are **4 (6) coupled differential equations**, which cover the dynamics and the constraint of the statistics and the spectrum (full information). Note the difference to the standard Kadanoff-Bayam equations.

[TB 21]

Statistical component

"Kadanoff-Bayam Ansatz" for two-body field statistical two-point functions:

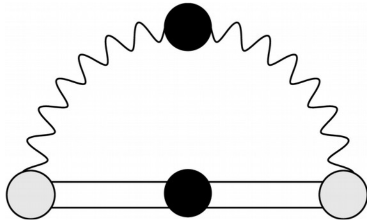
$$G^{+-}(T,R,P;\mathbf{r},\mathbf{r}') = f(T,R,M+P^0)G^{\rho}(T,R,P;\mathbf{r},\mathbf{r}'),$$
$$G^{-+}(T,R,P;\mathbf{r},\mathbf{r}') = \left[1+f(T,R,M+P^0)\right]G^{\rho}(T,R,P;\mathbf{r},\mathbf{r}').$$

Inserting this into one of the statistical equations (dynamical, Wigner transf., gradient exp...)

"Quantum Boltzmann equation" for two-body states

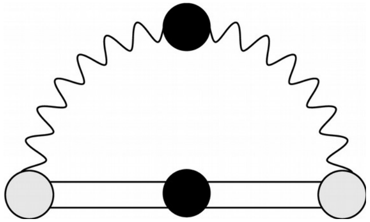
$$\left(\partial_T + \frac{\mathbf{p}}{M}\cdot\vec{\nabla}_R\right) f(T,R,M+P^0)\int \mathrm{d}^3r\, G^{\rho}(T,R,P;\mathbf{r},\mathbf{r}) =$$
$$-\int\frac{\mathrm{d}^4K}{(2\pi)^4}\int \mathrm{d}^3r\int \mathrm{d}^3\bar{r}\, G^{\rho}(T,K;\mathbf{r},\bar{\mathbf{r}})g^2 r_i\bar{r}_j G^{\rho}(T,R,P;\bar{\mathbf{r}},\mathbf{r})E^{\rho}_{ij}(P-K)\times$$
$$\left[f(M+P^0)\left[1+f(M+K^0)\right]\left[1+f_E(P^0-K^0)\right] - \left[1+f(M+P^0)\right]f(M+K^0)f_E(P^0-K^0)\right]$$
$$+\,\text{magnetic dipole contributions}$$

For spin-unpolarized medium, mean field terms canceled.
Remaining interactions:
Bound state formation, dissociation, Bremsstrahlung, higher emissions, **full finite T corrections inside spectral functions.**



[TB 21]

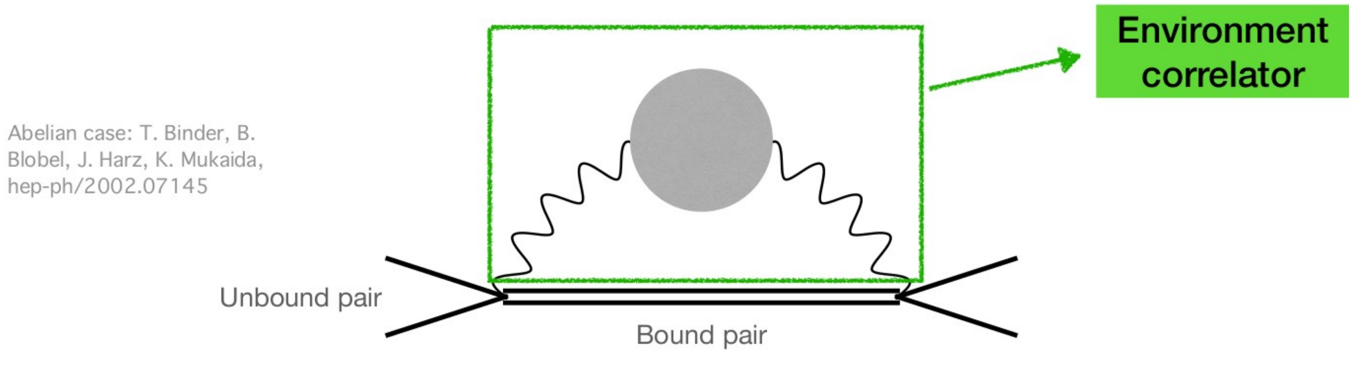
Recovery of open quantum system treatment



Assumption:

Vacuum two-body field spectral function, OK for $\mathrm{E} \gtrsim m_D$

Open quantum system (OQS) treatment



OQS assumes "Bound state propagator" can be treated as in vacuum (standard Schrödinger wave functions) + factorization of sub-system and environmental density matrix.

Non-Abelian electric field correlator at NLO

NEW: Singlet-adjoint transition in (any higher rep.) of **SU(N):**

$$\mathscr{L}_{\text{pNRQCD}} = \mathscr{L}_{\text{light quarks}} + \mathscr{L}_{\text{gluon}} + \int d^3r \text{Tr}_{\text{color}} \Big[S^\dagger (i\partial_0 - H_s) S + O^\dagger (iD_0 - H_o) O$$
$$+ V_A \left(O^\dagger \mathbf{r} \cdot g\mathbf{E} S + \text{h.c.} \right) + \frac{V_B}{2} O^\dagger \{ \mathbf{r} \cdot g\mathbf{E}, O \} + \cdots \Big]$$

Environment degrees of freedom $\Longleftrightarrow H_E$

Subsystem degrees of freedom $\Longleftrightarrow H_S$

Interactions $\Longleftrightarrow H_I$

Directly via Lindblad equation, or, vacuum limit of spectral function in statistical equation.

$$\frac{\partial}{\partial t} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) + \frac{\boldsymbol{k}}{2M} \cdot \nabla_{\boldsymbol{x}} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) = \mathcal{C}^+_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) - \mathcal{C}^-_{nl}(\boldsymbol{x}, \boldsymbol{k}, t)$$

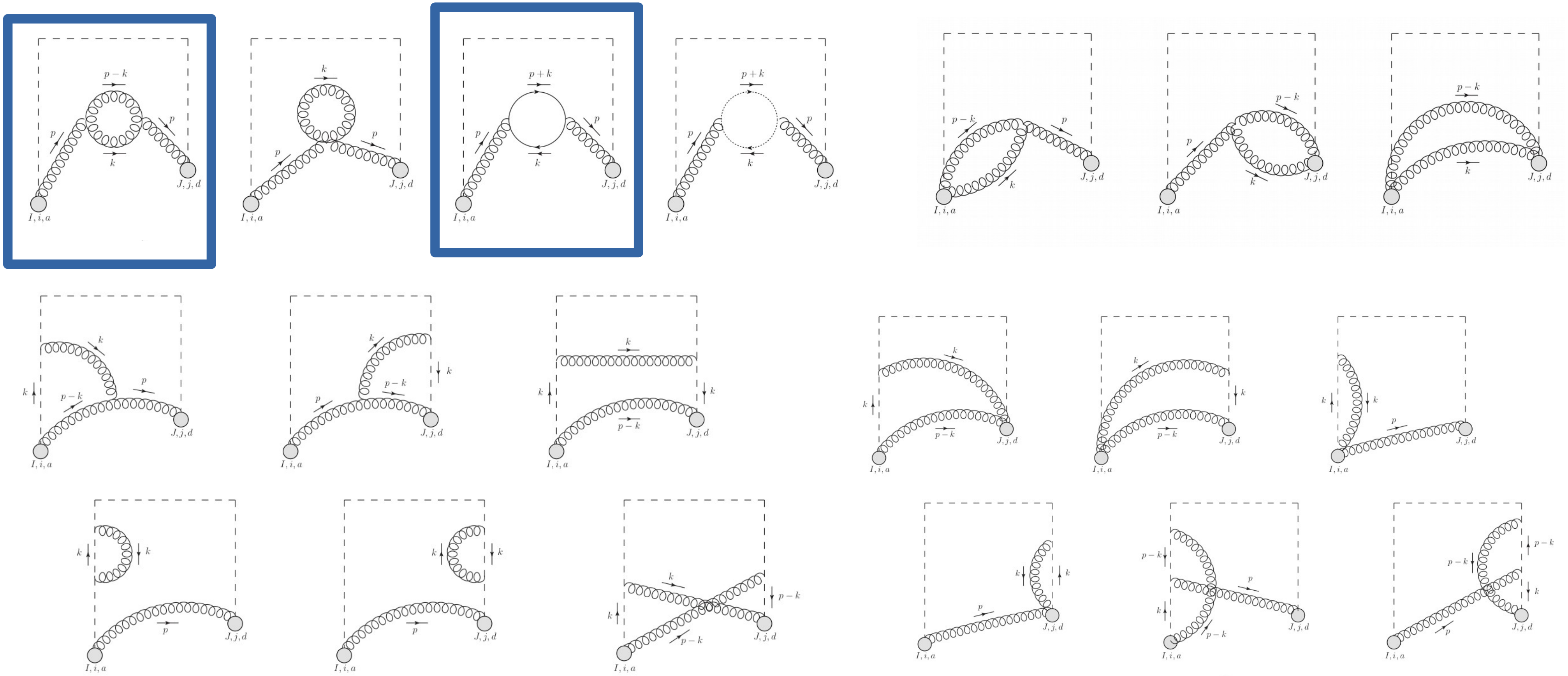
Recombination — Dissociation

[TB et al. 21, see B. Hitschfeld talk]

$$g^{E++}_{i_1 i_2}(t_1, t_2, \mathbf{R}_1, \mathbf{R}_2) = \left\langle \text{Tr}_{\text{color}} \left(E_{i_1}(\mathbf{R}_1, t_1) \mathscr{W}_{[(\mathbf{R}_1, t_1), (\mathbf{R}_1, +\infty)]} \mathscr{W}_{[(\mathbf{R}_2, +\infty), (\mathbf{R}_2, t_2)]} E_{i_2}(\mathbf{R}_2, t_2) \right) \right\rangle_T$$

SU(N) Electric dipole correlator for singlet-adjoint transition.
(Other transitions may involve different Wilson lines)

Non-Abelian Electric Field Correlator at NLO

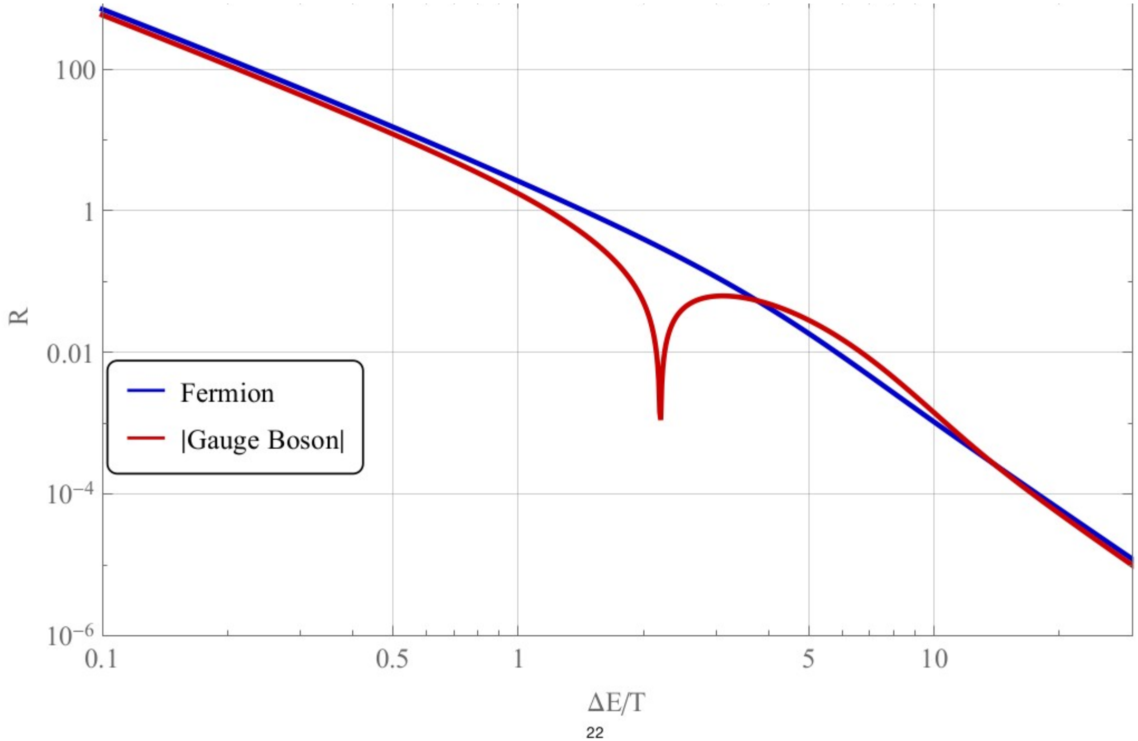


Gauge invariance, infrared and collinear safety proven.

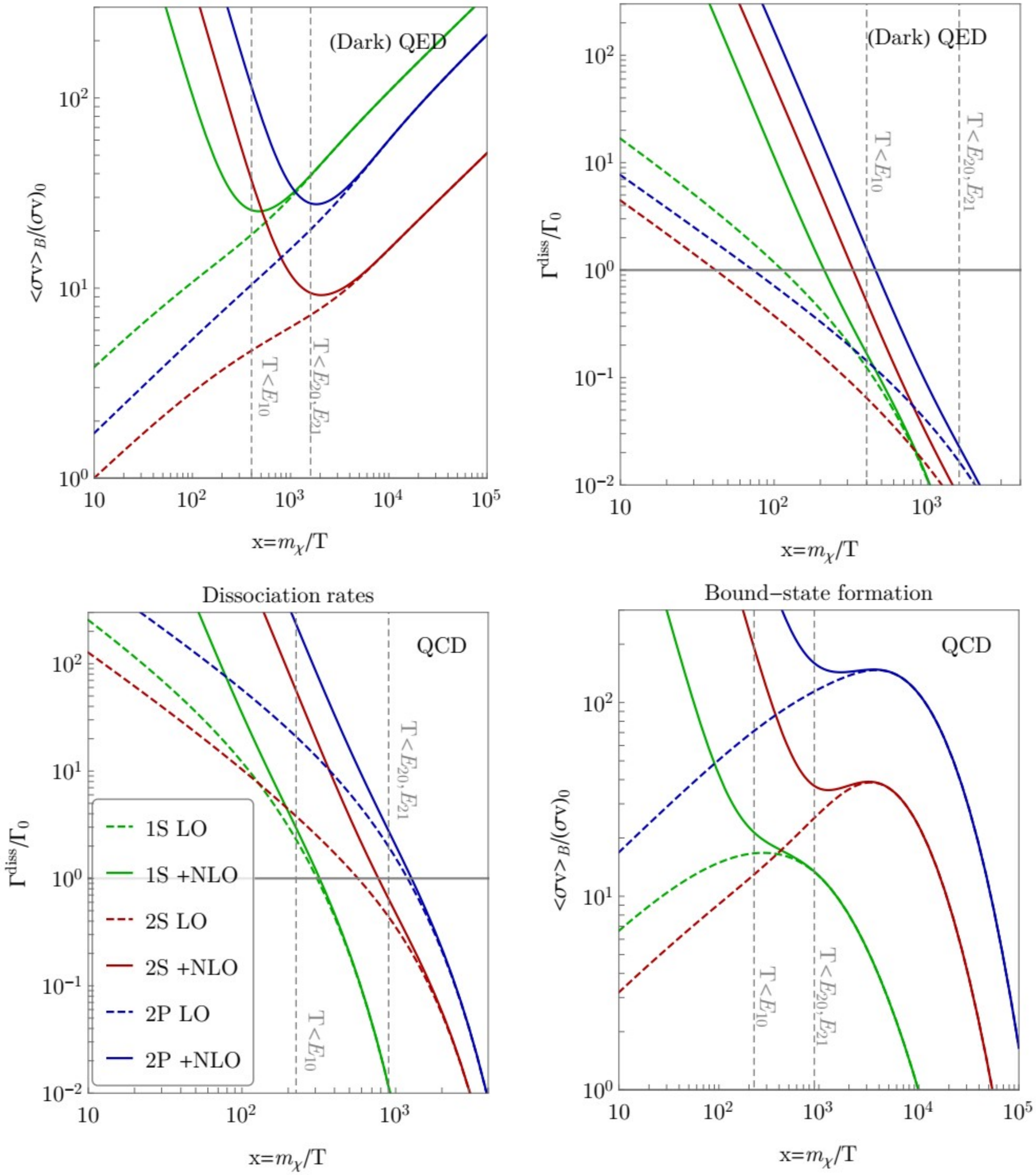
[TB et al. 21, see
B. Hitschfeld talk]

Enhanced rates inside plasma

$$(\sigma v_{\rm rel})^{\rm LO+NLO}_{\mathscr{B}} = (\sigma v_{\rm rel})^{\rm LO}_{\mathscr{B}} \times \Big[1+\alpha N_c R_g^{T=0}(\mu/\Delta E) + \alpha N_c R_g^{T\neq0}(\Delta E/T)$$
$$+\alpha N_f R_f^{T=0}(\mu/\Delta E) + \alpha N_f R_f^{T\neq0}(\Delta E/T)\Big]$$



Flipped hierarchy of rates inside plasma



Full treatment requires dressed retarded

$$\left(\partial_T + \frac{\mathbf{p}}{M}\cdot\vec{\nabla}_R\right) f(T,R,M+P^0)\int \mathrm{d}^3r\; G^\rho(T,R,P;\mathbf{r},\mathbf{r}) =$$
$$-\int\frac{\mathrm{d}^4K}{(2\pi)^4}\int \mathrm{d}^3r\int \mathrm{d}^3\bar{r}\; G^\rho(T,K;\mathbf{r},\bar{\mathbf{r}})g^2 r_i\bar{r}_j G^\rho(T,R,P;\bar{\mathbf{r}},\mathbf{r})E^\rho_{ij}(P-K)\times$$
$$\left[f(M+P^0)\left[1+f(M+K^0)\right]\left[1+f_E(P^0-K^0)\right]-\left[1+f(M+P^0)\right]f(M+K^0)f_E(P^0-K^0)\right]$$
$$+\,\text{magnetic dipole contributions}$$

$$\mathrm{G}^\rho = 2\mathrm{Im}[iG^R]$$

$$\left(2P^0+\frac{-2\mathbf{p}^2+\partial_R^2/2}{2M}+\frac{\nabla^2_{\mathbf{r}}+\nabla^2_{\mathbf{r}'}}{2\mu_r}-[V_{\mathrm{eff}}(r)+V_{\mathrm{eff}}(r')]+i\epsilon\right)G^R(T,R,P;\mathbf{r},\mathbf{r}') = i2\delta^3(r-r') \tag{1}$$
$$-\,g\mu_i\langle B_i(x)\rangle G^R(T,R,P;\mathbf{r},\mathbf{r}') - g\mu'_i\langle B_i(y)\rangle G^R(T,R,P;\mathbf{r},\mathbf{r}')$$
$$-\,g\left(r_i\langle E_i(x)\rangle + r'_i\langle E_i(y)\rangle\right)G^R(T,R,P;\mathbf{r},\mathbf{r}')$$
$$-\,i\int \mathrm{d}^3\bar{r}\;\left[\Sigma^{R,E}_{ij}(T,R,P;\mathbf{r},\bar{\mathbf{r}})r_i\bar{r}_j G^R(T,R,P;\bar{\mathbf{r}},\mathbf{r}') + G^R(T,R,P;\mathbf{r},\bar{\mathbf{r}})\bar{r}_j r'_i\Sigma^{R,E}_{ij}(T,R,P;\bar{\mathbf{r}},\mathbf{r}')\right]$$
$$-\,i\int \mathrm{d}^3\bar{r}\;\left[\Sigma^{R,B}_{ij}(T,R,P;\mathbf{r},\bar{\mathbf{r}})\mu_i\bar{\mu}_j G^R(T,R,P;\bar{\mathbf{r}},\mathbf{r}') + G^R(T,R,P;\mathbf{r},\bar{\mathbf{r}})\bar{\mu}_j\mu'_i\Sigma^{R,B}_{ij}(T,R,P;\bar{\mathbf{r}},\mathbf{r}')\right].$$

Summary Generalizations / Consistency checks

- We performed many consistency checks with previous literature results, *under approximations/limits/neglecting terms* in our more general equations.
- DM bound state formation/dissociation at finite temperature **now** fully available. I.e., we can **for the first time** describe DM decoupling from ionization equilibrium including full finite temperature corrections.
- *In vacuum limit of the retarded correlator*, the equations reduce to the **open quantum system treatment**, as established in Quarkonia and Dark matter.
- *Vacuum limit of the retarded correlator* contains fine and hyper fine corrections to the energy levels.
- Interesting cosmological application: free electron evolution during CMB decoupling, spin-flip temperature evolution at the cosmic Dawn (remember we have **anomaly discovery** in the latter, while the **Hubble tension** in the former).

There is even more…

- *Bose Einstein condensation*
- *Critical gauge field values* can lead to spontaneous symmetry breaking of U(1) (local phases become global, spontaneously)
- Gives rise to phenomena like superconductivity, ferromagnetism, ...
- In general, the one point function of the two body field depends on

TWO LOCAL PHASES

(where one is known in quarkonia, as the ultra-soft gauge transformation)

$$\langle O(x, \mathbf{r}) \rangle \sim \rho e^{-i\phi_1(t,\mathbf{x}) - i\phi_2(t,\mathbf{r})}$$

Superconductivity (?)

Self-consistent fully quantum EoMs of electric and magnetic field:

$$\langle \mathbf{E}(x) \rangle = \int \mathrm{d}^3 r \mathrm{Tr}[\langle O^\dagger(x,\mathbf{r}) \mathbf{r} g O(x,\mathbf{r}) \rangle] + \ldots \,, \tag{1}$$

$$\langle \mathbf{B}(x) \rangle = -\int \mathrm{d}^3 r \mathrm{Tr}[\langle O^\dagger(x,\mathbf{r}) \mu g O(x,\mathbf{r}) \rangle] + \ldots \tag{2}$$

Vanishing E and B field (Meissner effect) if ϕ_2 turns (spontaneously) global (infinite correlation length, superconductivity (?)). Ferromagnetic effects may be related to ϕ_1 .

Equations may allow to study the **DYNAMICAL FORMATION and EXISTENCE** of extreme phases, such as a *FERROMAGNETIC SUPERCONDUCTING BOSE-EINSTEIN CONDENSATE*. Currently, we dont claim the co-existance or possible formation of such kind of fascinating material. Need to solve our equations!

Summary and Conclusion

- **Potential non-relativistic effective field theory** is powerful, allows to reduce the complexity of NR EFT to the essential information of non-relativistic N-body fields.
- Combined with **path-integral based non-equilibrium QFT**, our resulting equations generalize DM decoupling description, Quarkonia transport, may have interesting other cosmological application, and seem to be able to even cope with phase transitions.

  → Existence of extreme phases **+** numerical proof of dynamical formation possible
- Scheme for any (Lagrangian based) theory:

  Start from a given Lagrangian theory, derive the non-relativistic effective Lagrangian~\cite{CASWELL1986437, Labelle:1992hd}, project it to the desired N-body state and multipole expand the interaction fields which leads to potential-non-relativistic effective field theory~\cite{Pineda:1997bj,Brambilla:1999xf,Brambilla:2004jw}, and follow the procedure presented here to derive dynamical and spectral functions in the Keldysh-Schwinger~\cite{Schwinger:1960qe,Keldysh:1964ud} non-equilibrium quantum field theory formalism. The latter assumes that interactions are adiabatically switched on, however, initial memory effects can also be included by appropriately extending the time contour integration (e.g., needed for MBL theory). Note that this formalism can deal with non-hermitian potentials since it is based on the path integral.

Heavy Dark Matter candidates

[see talk by J. Hisano]

- Supersymmetric standard model (SUSY SM)
  Wino (I=1, S=1/2), Higgsino (I=1/2, S=1/2)
- Extra-dimensional models
  Kaluza-Klain weak gauge bosons (I=1, S=1)
- Neutrino mass models
  - Scotogenic models
- Extended Higgs sector models
  - Inert Higgs models
- Minimal dark matter models
  - automatically stable fermion (I=2, S=1/2)
- …………

5

SUSY SM before LHC

Energy scale

SUSY GUTs ~ 10^{16} GeV

SUSY SM @ < O(1) TeV

Standard model

Motivation of Low-scale SUSY (<~1TeV):

- Hierarchy problem
- WIMP dark matter (R parity)
- Gauge coupling unification (SUSY GUTs)

Shortcoming of SUSY :

- FCNC and CP problems
- Gravitino problem in nucleosynthesis
- D=5 proton decay in SUSY GUTs
- 125GeV Higgs mass (after 2012)

SUSY SM after LHC

Energy scale

SUSY GUTs ~ 10^{16} GeV

SUSY SM @ O($10^{2\text{-}3}$) TeV

Standard model

Motivation of mini-split SUSY (~O($10^{2\text{-}3}$) TeV).

- Solution of following problems
  - FCNC and CP problems
  - Gravitino problem in nucleosynthesis
  - D=5 proton decay in SUSY GUTs
  - 125GeV Higgs mass
- Easy model building of SUSY breaking by anomaly mediation
- WIMP dark matter (R parity)
  Wino/Higgsino
- Improved gauge coupling unification

Phenomenologically successful model ! (Except for Naturalness)

Fixed NLO vs. EFT treatment