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Standard Model Lectures

CERN SSLP 2021 **Homework** **Date: 14.07.2021**

Feel free to send me your solutions and I will correct them and send you feedback.

Exercise 1: Phase change and latent heat

a) A small puddle is frozen and its temperature is 0 ^{o}C. How much solar energy is needed to melt all the ice? Assume that $m_{\rm ice} = 10.0\,$kg. Imagine now that the ice is originally at -20 ^{o}C. How much does the amount of energy required increase? If the solar heating rate is constant at 191W/m^2 and that the area of the puddle is 2.09m^2. How long does it take the sun to raise the temperature of the ice and then melt it? Consider now that the source of heating is not the sun but instead is warm air passing over the puddle. If the temperature of the air is 20 ^{o}C and we assume that its temperature drops to 0 ^{o}C after contacting the ice, what is the mass of air that is required to warm the ice and then melt it?

b) Climate change is predicted to cause increases in sea surface temperature, as well as decreases in sea-ice cover, wind and current velocities. These changes will have a marked effect on iceberg melting in waters off Newfoundland (Canada). Indeed in August of 2020, a massive chunk of Canada's last fully intact ice shelf, some 4000 years old, broke off, reducing the shelf by more than half. Let us consider this hulking iceberg is approximately rectangular prism 160 km long, 40 km wide, and 250 m thick. Compute its mass, given the density of ice is 917 kg/m^3. Determine also how much heat transfer (in joules) is need to melt it. Discover how many years it would take sunlight alone to melt ice this thick, if the ice absorbs an average of 100W/m^2, 12 hours per day.

Exercise 2: Natural units

a) Show that $[\hbar] = M\cdot L^2\cdot T^{-1}$ and $[c] = L\cdot T^{-1}$.

b) Check the consistency of the classical/quantum correspondence at the dimensional level:

$$E \to i\hbar\frac{\partial}{\partial t} \quad \& \quad p \to i\hbar\frac{\partial}{\partial x}$$

c) Show that

$$1\,\text{s} = 1.52\cdot 10^{27}\,\hbar/\text{TeV}, \quad 1\,\text{m} = 5.1\cdot 10^{18}\,\hbar c/\text{TeV}, \quad 1\,\text{kg} = 5.61\cdot 10^{23}\,\text{TeV}/c^2$$

d) Using the Newton constant, $\hbar$ and c, construct a mass scale, a length scale and a time scale. They are defining the Planck scales. Compute the matter density of the universe today (10^{-29} g/cm^3) in Planck units.

e) The Schwarzschild radius of an object of mass m is the measure of its mass in Planck units. The Compton wavelength is defined as $\hbar/(mc)$. Compute the Schwarzschild radius of the Earth, the Sun, a neutron star, a stellar black-hole, a super-massive BH, a micro-BH (you'll check on Wikipedia the characteristic mass of these objects). What do you conclude? Compute the Schwarzschild radius of a micro-BH assuming that the Planck scale has been reduced to 1 TeV. What do you conclude?
f) Using e, m_e and c, construct a length scale. This is the classical radius of the electron.
Using e, m_e and $\hbar$, construct a length scale. This is the Bohr radius of the electron.
g)The pion Compton wavelength in natural units is $\lambda_\pi = \hbar/(M_\pi c) \to 1/M_\pi \simeq (140 \text{ MeV})^{-1}$. Convert this to conventional units by multiplying with a combination of $\hbar$ and c to get a distance unit.
h) A typical hadronic cross section is of order $\sigma \simeq \lambda_\pi^2 \simeq 1/M_\pi^2 \simeq 1/(140)^2\text{MeV}^{-2}$. Express this quantity in units of barns (1 barn = $10^{-28}\,\text{m}^2$).

Exercise 3: Value of e in HEP units

The electromagnetic fine-structure constant was defined by A. Sommerfeld in 1916. It is given by

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c},$$

where $e = 1.6\times10^{-19}\,\text{C}$ is the unit electric charge, $\epsilon_0 = 8.8\times10^{-12}\,\text{F}\cdot\text{m}^{-1}$ is the vacuum permittivity.
a) Compute the value of α. Check that it is a dimensionless quantity (we remind that $1\,\text{F} = 1\,\text{C}^2.\text{J}^{-1}$)
b) Deduce the value of the electric charge e in the HEP units ($\hbar = c = \epsilon_0 = 1$).

Exercise 4: Wave equations

a) Derive the Schrödinger equation from the classical expression of the energy.
b) Derive the Klein–Gordon equation from the relativistic energy expression.

Exercise 5: Schwarzschild radius

Our present understanding of the Universe and its evolution implies the existence of black holes, bodies whose masses are packed in such small volumes that not even light can escape. From a theoretical point of view, black holes are a direct consequence of the fact that we must use General Relativity to describe the late stages of gravitational collapse. The physical magnitude used to evaluate this singularity point is known as Schwarzschild radius. The aim of this exercise is to determine this important quantity for a set of different objects:
a) the entire Milky Way, with a mass of 250 billion suns ($M_{\text{Sun}} = 2\times10^{30}$ kg)
b) our moon with a mass of 7.35×10^{22} kg.
c) a black hole with a mass of an average human (M = 60 kg).
(Express the answers to two significant figures)

Exercise 6: Stephan Boltzmann Law

a) The Stefan-Boltzmann law states that a black hole loses energy at a rate $P = \sigma A T^4$. The

total energy is given by the Einstein's equation $E = mc^2$. Use these to estimate the lifetime of a black hole in terms of its mass.
b) Take the Sun to be a blackbody with a surface temperature of 6'000 K. The Sun's radius is 7.0×10^5 km. Calculate the Sun's luminosity, in watts (Joules/second).
c) Mars is 1.5 AU from the Sun. Calculate the brightness of the Sun at Mars' distance (i.e., the solar flux on Mars' surface) expressed in watts per square meter.

Exercise 7: Star Wars Death Star and Hawking Black Hole radiation

In the movie, we learn that the Death Star has a radius of about one-tenth of the Endor planet. Endor is very comparable to the Earth (life develops, humans experience gravitational potential as on Earth, there is a breathable atmosphere).
a) Estimate the size and the mass of the Death Star.
b) The Death Star is a weapon that can destroy planets similar to the Earth, like Alderande. Using dimensional arguments, estimate the energy needed to destroy Alderande, i.e. compute the gravitational potential energy of Alderande. For comparison, the total amount of energy produced on Earth in one year is of the order of 10^{20} J. Given that the Death Star needs about 3 days to produce/store this energy, compute the power of the source of energy.
c) In practise, the Death Star is doing more than destroying Alderande: each little fragment of the planet is expelled with a velocity of about 10^4 km/s. Compute the energy needed by the Death Star to achieve such a destructive action. What is the power of the source of energy? Assuming that this energy is produced by burning oil, compute the volume of oil needed. How many power-plants are needed to reach the same power?
d) Assuming that the energy of the Death Star is produced by the annihilation of matter and antimatter into energy, what would be the amount of anti-matter needed?
e) S. Hawking understood that the laws of quantum mechanics imply that a BH is actually radiating particles, hence energy. Based on dimensional arguments, find the Hawking temperature of a BH of mass M. There is a priori a 1D infinite family of solution, you'll retain the solution that scales with a single power of $\hbar$. The exact formula derived by Hawking is smaller by a factor $1/(8\pi)$ compared to the naive estimate.
f) Assuming that a BH is a perfect black-body, use the Stefan–Boltzmann law ($P \propto T^4$) to derive the luminosity of a BH of mass M. Numerically, compute the power of a BH of solar mass (we recall that the Stefan–Boltzmann constant is equal to $\pi^2 k_B^4/(60\hbar^3 c^2)$).
g) Using the conservation of energy, derive the differential equation controlling the time evolution of the BH mass. Integrate this equation to obtain the BH life time.
h) What is the lower bound on the mass of a BH to be as old as the Universe?

Exercise 8: Solar neutrino flux

The main source of energy of the Sun is the thermonuclear reaction:

$$4\,{}^1_1H + 4e^- \rightarrow {}^4_2He + 2\,e^- + 2\,\nu_e$$

a) The mass of Helium is 4.002602 atomic unit, the mass the proton is 1.007276466879 atomic unit (or 938.2720813 MeV) and the mass of the neutron is 1.00866491588 atomic unit.

Compute the amount of energy emitted by the reaction above.
b) From the luminosity of the Sun computed in the previous exercise, compute the amount of matter lost and transformed into energy every second in the Sun. Is it now compatible with the age of the Sun?
c) From the value of the luminosity of the Sun, estimate the number of neutrinos produced by the Sun every second.
d) Compute the flux of neutrinos emitted by the Sun and received on Earth.

Exercise 9: LHC as a Higgs discovery machine

a) The scattering cross section of a neutron on ^{235}U is about 1 b (=$10^{-28}\,\mathrm{m}^2$). Assuming a simple scaling of the volume and area of a nucleus with the number of nucleons, obtain an estimate of the proton-proton cross section. Deduce the size of the proton (assuming that the proton is a hard sphere).
b) Estimate the cross-section $\sigma(pp \to W)$. Argue that $\sigma(pp \to h)$ should be about 3 orders of magnitude smaller.
c) One way to identify the Higgs boson is to look at its decay mode into 2 photons. About 1 out of 100 such events can be reconstructed and seen in the detector. Compute the collision rate (i.e., the number of proton-proton collisions) needed to observe 100 Higgs bosons in a year.
d) On disk, the recorded data associated to one proton-proton collision occupies about 1 MB. What is the fraction of collisions that one can afford to record? This is called the trigger rate.
e) Protons are grouped into bunches, each containing about 10^{11} protons. What should be the separation between two bunches to be able to observe 100 Higgs boson a year? What should be the transverse size of the bunches? And the minimum longitudinal size of the bunches?

Exercise 10: High energy physics: multiple choice questions

Circle the correct answer(s) (more than one answer can be correct).
a) The total yearly world consumption of energy is of the order of 10^{20} J. How much mass would have to be completely converted into energy to provide this amount of energy?
1) 10^{-20} kg 2) 10^{-10} kg 3) 10^{-3} kg 4) 10^{0} kg 5) 10^{3} kg 6) 10^{10} kg 7) 10^{20} kg
(reminder: 1 J=6.25×10^{18} eV and $m_{\text{proton}} = 1.67\times10^{-27}$ kg)

b) A certain radioactive element has a half-life of 20 days. The time it will take for 7/8 of the atoms originally present to disintegrate is
1) 20 days 2) 40 days 3) 60 days 4) it cannot be predicted

c) An atom moving at speed $0.30c$ emits an electron along the same direction with speed $0.60c$ in the internal rest frame of the atom. The speed of the electron in the lab frame is equal to:
1) $0.30c$ 2) $0.60c$ 3) $0.66c$ 4) $0.77c$ 5) $0.9c$
6) such a process violates energy conservation and cannot happen

d) A particle moves in such a way that its kinetic energy just equals its reste energy. The velocity of this particle is
1) $c/4$ 2) $c/2$ 3) $0.866c$ 4) c 5) it can never happen

e-f) A lump of matter whose rest frame is 4 kg is traveling at the $3/5c$ speed when it collides head-on with an identical lump of matter going in the opposite direction at the same speed. If the two lumps of matter stick together and no energy is radiated away, what is the mass of the composite lump?

1) 2 kg 2) 4 kg 3) 8 kg 4) 10 kg 5) it depends of the interactions at work

The energy of this system corresponds to which fraction of the yearly world consumption of energy?

1) 10^{-20} 2) 10^{-10} 3) 10^{-5} 4) 1% 5) 100%

g) An antiproton is a particle that has
1) the mass of a proton and the charge of an electron
2) the mass of an electron and the charge of a proton
3) the mass of a neutron and the charge of a proton
4) the mass of a proton and the charge of a neutron

h) Which of the following is not true?
1) each meson consists of a quark and an antiquark
2) each baryon consists of three quarks
3) the magnitude of the charge of every quark is 1/3
4) a particle consisting of a single quark has not been observed

i) The charge of the particle *dds* is

1) e 2) e/3 3) -2e/3 4) -e 5) such a particle does not exist

j) Electron capture corresponds to the process: $p + e^- \to X + Y$. What could be the identity of the X and Y particles?

1) $X = p, Y = K^-$ 2) $X = e^-, Y = e^+$ 3) $X = n, Y = \nu_e$ 4) $X = n, Y = \pi^0$
5) such a process cannot happen

k) Which force acts on all quarks and leptons:

1) strong interaction 2) weak interaction 3) electromagnetic interaction

l) The muon decay, $\mu^- \to e^- \bar{\nu}_e \nu_\mu$, conserves:
1) the total lepton and baryon numbers
2) the muon number but not the electron number
3) neither the muon nor the electron number
4) both the muon and the electron numbers
5) the electric charge
6) energy, 3D momentum and angular momentum

m) When the β-decay of ^{60}Co nuclei is observed at low temperatures in a magnetic field that aligns the spins of the nuclei, it is found that the electrons are emitted preferentially in a direction opposite to the ^{60}Co spin. Which of the following invariance is violated by this decay?

1) Gauge invariance
2) Time invariance
3) Translational invariance
4) Rotational invariance
5) Reflection invariance
6) Energy conservation

n) The wave function for identical fermions is anti-symmetric under particle interchange. Which of the following is a consequence of this property?

1) Pauli exclusion principle
2) Bohr correspondence principle
3) Heisenberg uncertainty principle
4) Fermi's golden rule
5) Fermi–Dirac statistics
6) Energy conservation

o) The mass of a fermion must be positive:

1) for a Dirac fermion
2) for a Majorana fermion
3) in both cases
4) never since the phase of a fermion mass is not physical