Recent Work on Beam Dynamics

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Outline

- Analytic Particle Tracking with Lie Algebra from Arbitrary Magnetic Fields
- Synchrotron Radiation Calculations
- Particle Energy Modulation Experiments
- Conclusions and future work

Analytic Particle Tracking

- A tracking code has been written in C++ which generates an analytical description of an arbitrary magnetic field and generates a Lie Map to track the particle
- The analytical field description
  - Describes the field in terms of a normal and skew multipole expansion
  - Automatically includes fringe field and non-linear components
  - Has proved useful in understanding novel magnet design
- The Symplectic Integrator
  - Uses the analytical description of the field to generate a Lie Map
  - The Lie map transports particles in a much more efficient manner than numerical tracking
  - Lie maps are calculated with a custom DA (Differential Algebra) code
  - Offers huge savings in time for a given accuracy

Generating Analytical Field Maps

For a periodic structure, of period λ_ω, a general scalar potential can be written:

$$\Psi = \sum_{m=0}^{\infty} \int_{-\infty}^{\infty} dk \exp(ikz) I_m(k\rho)[(\hat{b}_m(k)\sin(m\phi) + \hat{a}_m(k) cos(m\phi)]$$

I_m are the modified Bessel functions which can be expressed as a Taylor expansion:

$$I_m(x) = \sum_{L=0}^{\infty} \frac{1}{L!(m+L)!} \left(\frac{x}{2}\right)^{2L+m}$$

and $\hat{a}_m$ and $\hat{b}_m$ are arbitrary coefficients.

(see Alex J. Dragt - “Lie Methods for Nonlinear Dynamics with Applications to Accelerator Physics”)

From this, the vector potentials can be written as:

$$A_\phi = 0$$

$$A_\rho = \sum_{m=1}^{\infty} \frac{\cos(m\phi)}{m} \rho \frac{\partial}{\partial z} \psi_{\omega,s} - \frac{\sin(m\phi)}{m} \rho \frac{\partial}{\partial z} \psi_{\omega,c}$$

$$A_z = \sum_{m=1}^{\infty} -\frac{\cos(m\phi)}{m} \rho \frac{\partial}{\partial \rho} \psi_{\omega,s} + \frac{\sin(m\phi)}{m} \rho \frac{\partial}{\partial \rho} \psi_{\omega,c}$$

Symplectic Integrator

$$\mathcal{M} = \exp\left(: -\frac{\Delta\sigma}{2} P_z :\right) \exp\left(: -\frac{\Delta\sigma}{2} a_z :\right) \exp\left(: -\frac{\Delta\sigma}{2}\left(-\delta + \frac{P_x^2}{2(1+\delta)}\right) :\right)$$

$$A_y \exp\left(: -\Delta\sigma \frac{P_y^2}{2(1+\delta)} :\right) A_y^{-1} \exp\left(: -\frac{\Delta\sigma}{2}\left(-\delta + \frac{P_x^2}{2(1+\delta)}\right) :\right)$$

$$\exp\left(: -\frac{\Delta\sigma}{2} a_z :\right) \exp\left(: -\frac{\Delta\sigma}{2} P_z :\right)$$

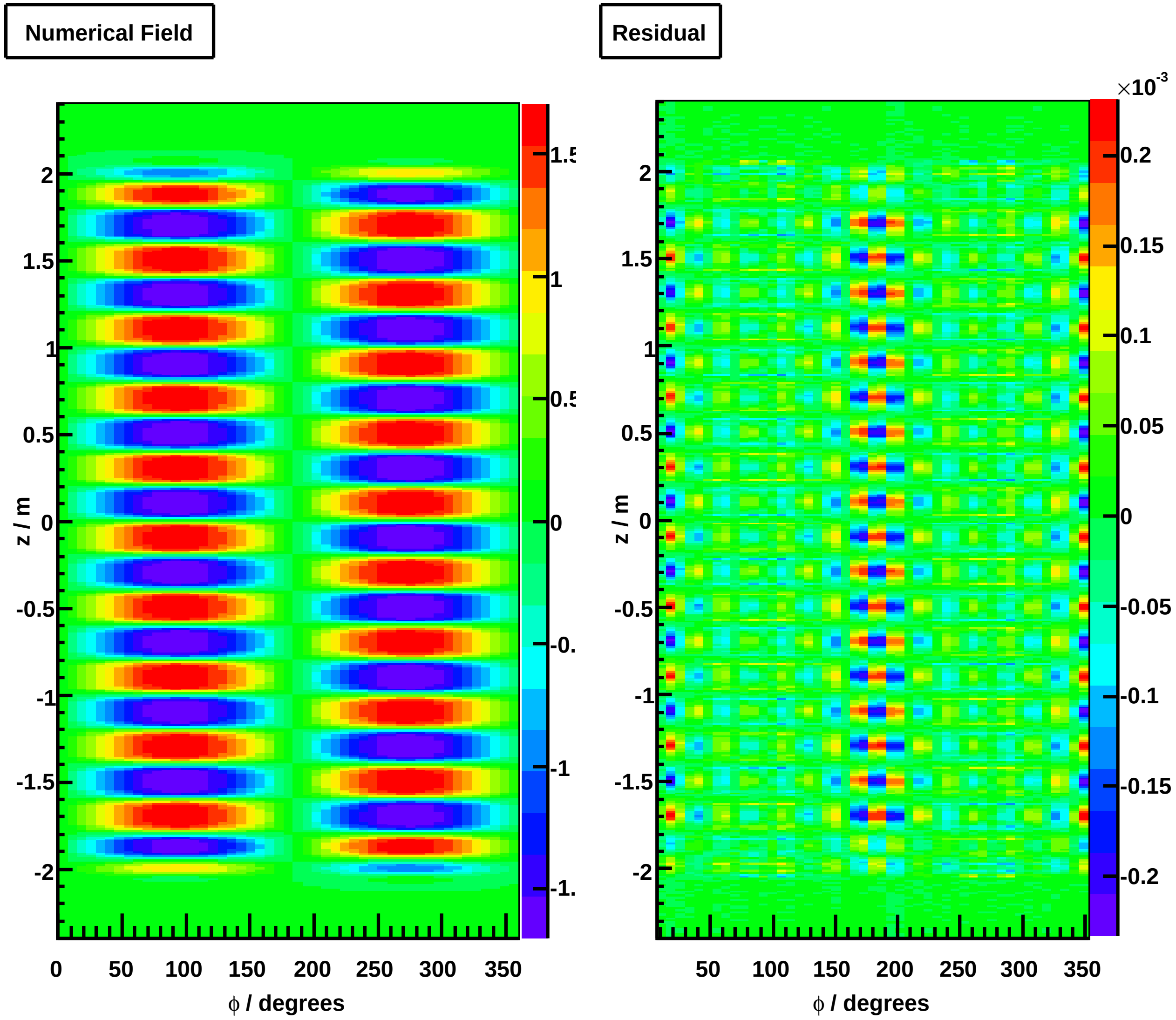
where $(- : H :)f = \frac{\partial f}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial H}{\partial q_i}$
and the operators A_y and A_y^{-1} involve the vector potential. (see Wu, Forest and Robin,2001)

The end result is a transfer matrix which transports a particle from an initial position, X_i to a final position, X_f in one step.

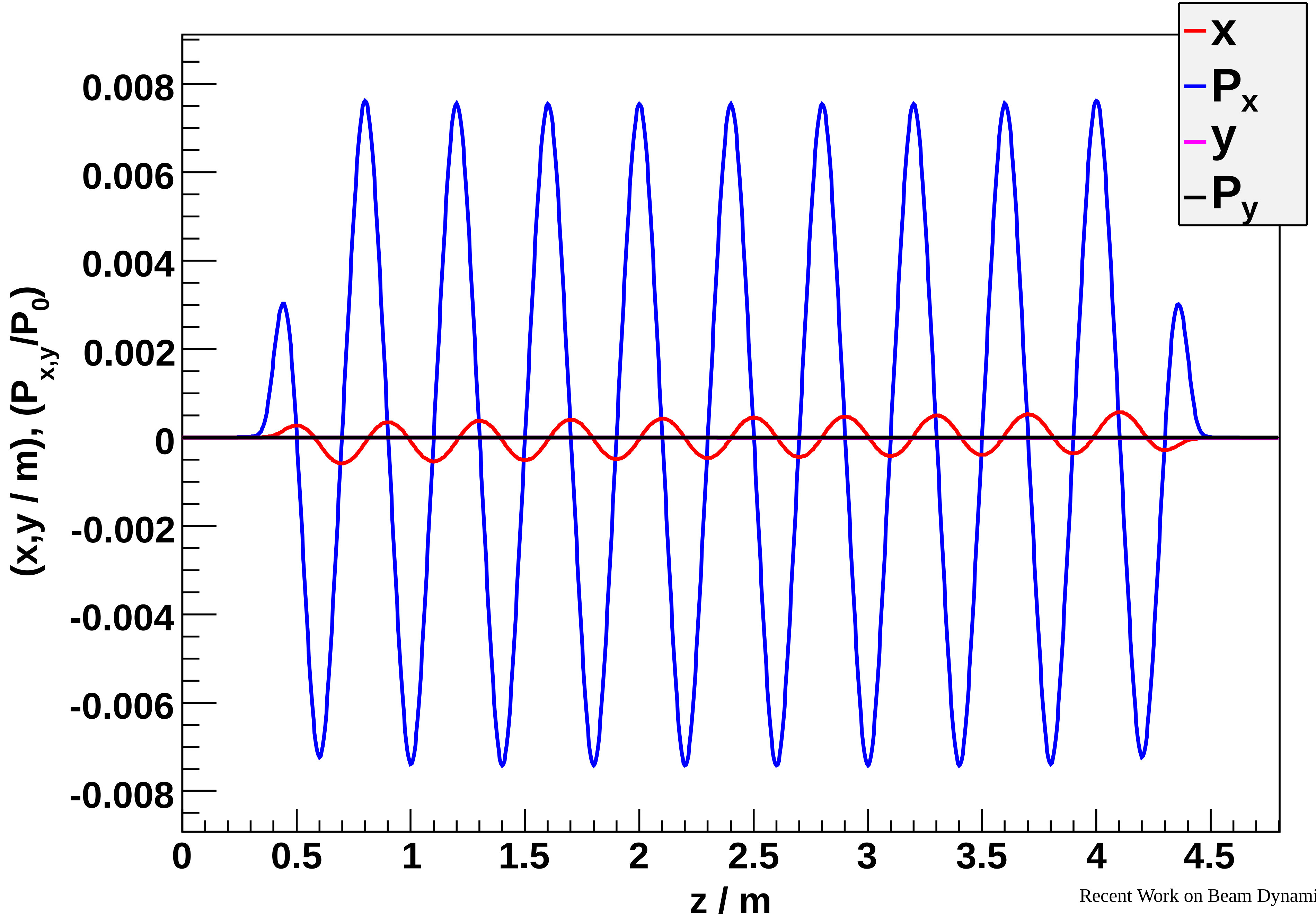
$$\mathcal{M}\, X_i(x, p_x, y, p_y, z, \delta) = X_f(x, p_x, y, p_y, z, \delta)$$

Cesr Wiggler Field Description, B_ρ

Comparing the numerical (left) and analytic field descriptions



Cesr Wiggler Tracking

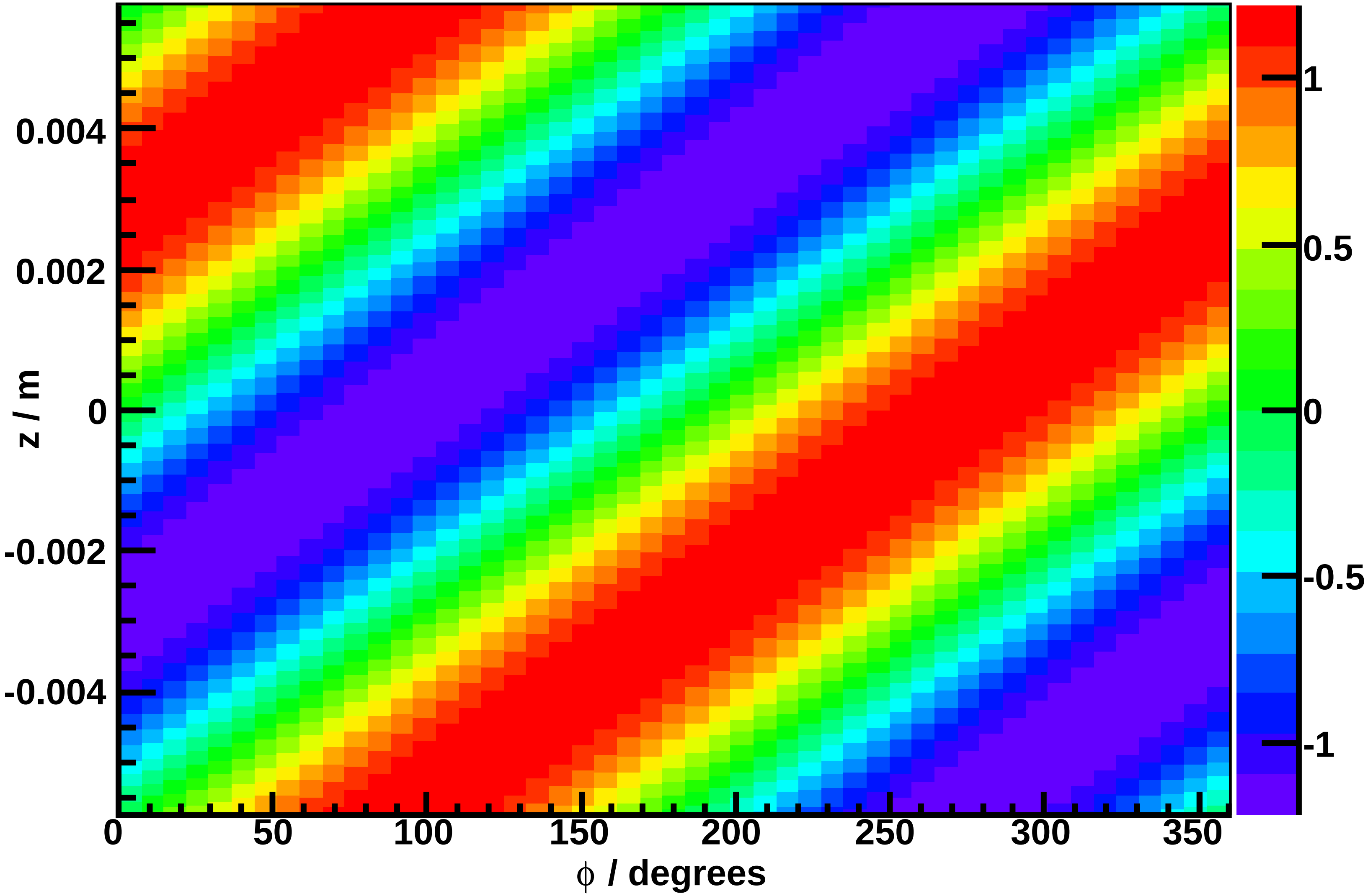


The ILC Helical Undulator

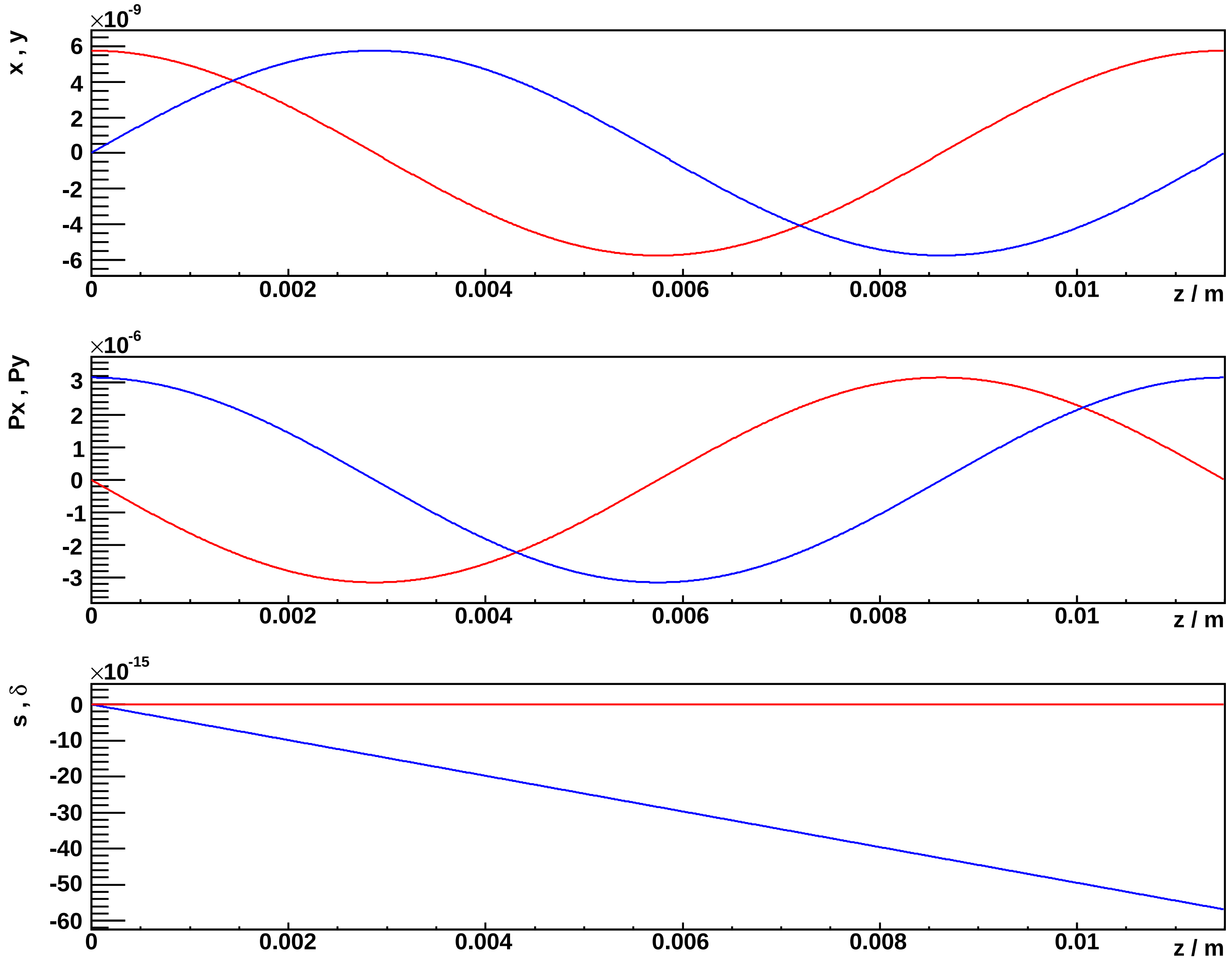
- The ILC plans to produce positrons by firing 10 MeV synchrotron photons at a target and capturing the pair-production positrons
- The 10 MeV photons will be produced with a 150 GeV electron beam in a helical undulator $\sim$ 250 m long, $\sim$ 100 periods per metre
- Calculating the photon yield from such a long undulator requires a very fast accurate code
  - There are two codes (SPUR, SPECTRA) currently available that can handle such long undulator systems, but both are very slow
  - SPUR typical run time for a 20 m section: $\sim$ 3 hours (40 cores)
  - SPECTRA typical run time for a 20 m section: $\sim$ several days
- The Analytical tracking code has been amended to include a synchrotron calculation module
  - Typical run time for a 50 m section $\sim$ 5 minutes

ILC Helical Undulator

A single period of the ILC helical undulator



Undulator Tracking - 1 period



Synchrotron Radiation Calculation

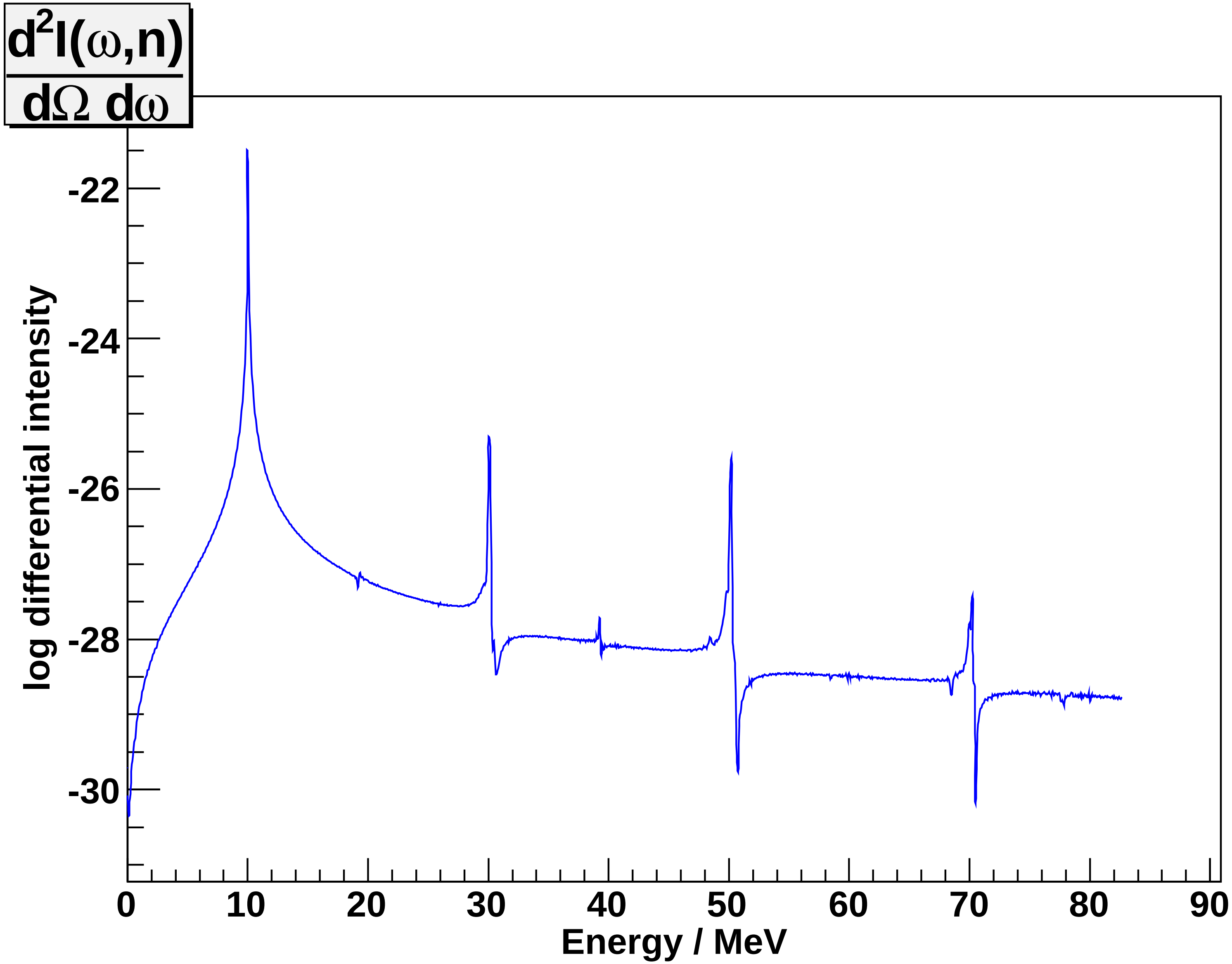
The observed electric field of the emitted radiation is:

$$\vec{E}(t) = \frac{e}{4\pi\epsilon_0 c} \left(\frac{c(1 - |\vec{\beta}|^2)(\vec{n} - \vec{\beta})}{|\vec{R}|^2 (1 - \vec{n}\dot{\vec{\beta}})^3} + \frac{\vec{n} \times [(\vec{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{|\vec{R}|(1 - \vec{n}\dot{\vec{\beta}})^3} \right)_{RET}$$

$\vec{n}$, $\vec{\beta}$, $\dot{\vec{\beta}}$ and R can be calculated directly from the canonical coordinates.

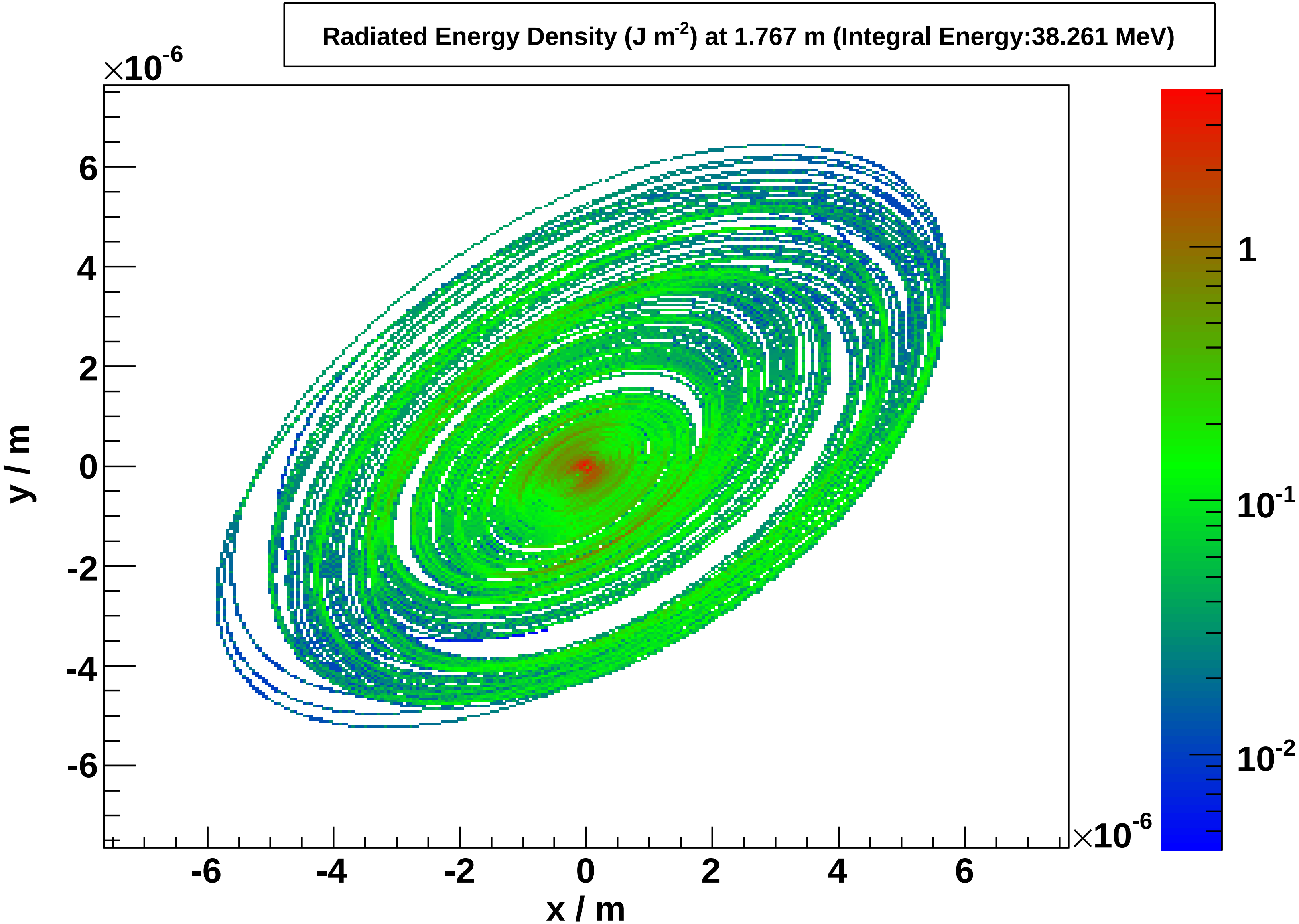
Energy spectrum

Observed at the end of a single undulator section (155 periods)



Power incident at the end of the undulator

38.281 MeV is radiated, $\sim$ 0.02 % of the electron energy

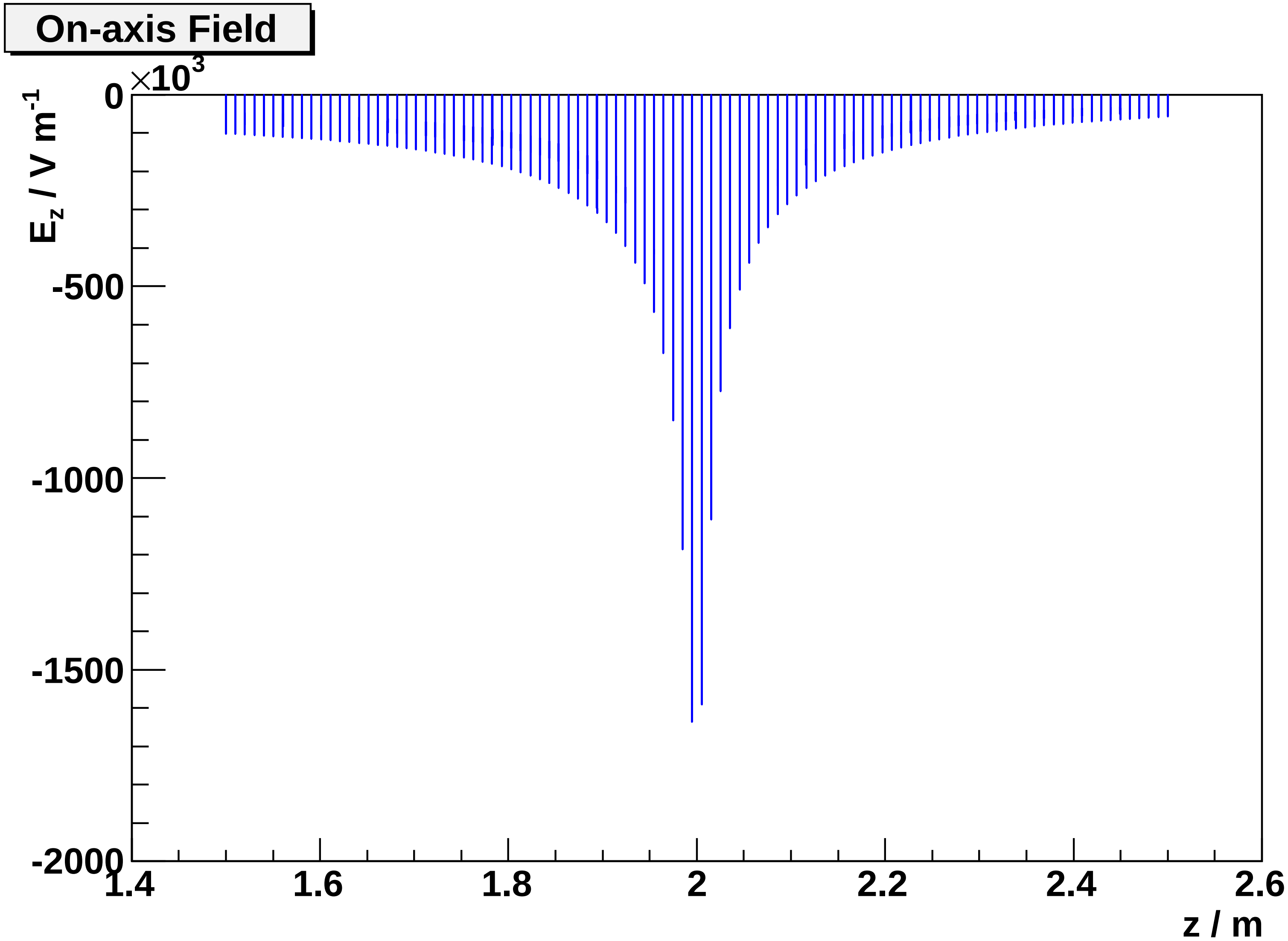


Energy Modulation Experiment

- An experiment is under way to modulate the energy of a 30 MeV electron beam using a radiation pulse generated by a TeraWatt laser.
  - Currently being installed on ALICE, Daresbury laboratory
- A pulse of radiation will be generated using a TeraWatt laser and a photoconductive wafer under a high bias voltage
- The pulse of radiation will be radially polarised with a longitudinal component - Salomin potential
- The longitudinal component will interact with the electron beam and and modulate the energy
- The modulation will be observed at a dispersive region immediately after the interaction point
- My contribution
  - Modellling the surge current in the photoconductive wafer
  - Modelling the EM pulse generated at the wafer surface
  - Simulate the interaction of the beam and the EM pulse
  - Characterise the beam modulation and determine a method of measuring it

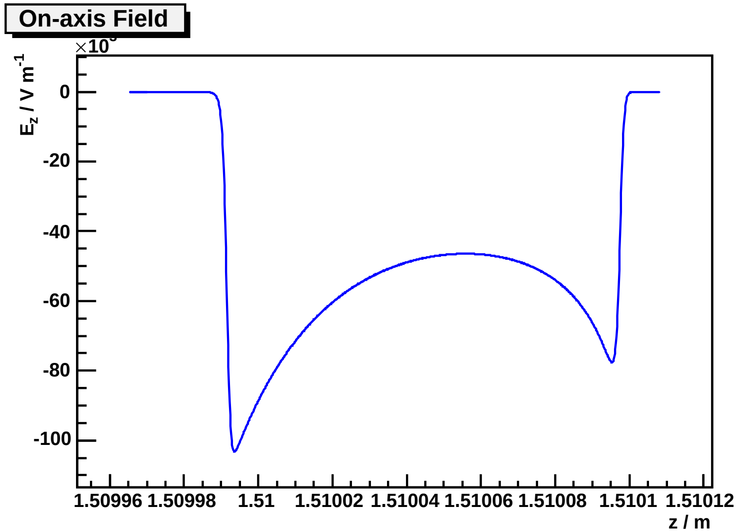
Energy Modulation Experiment

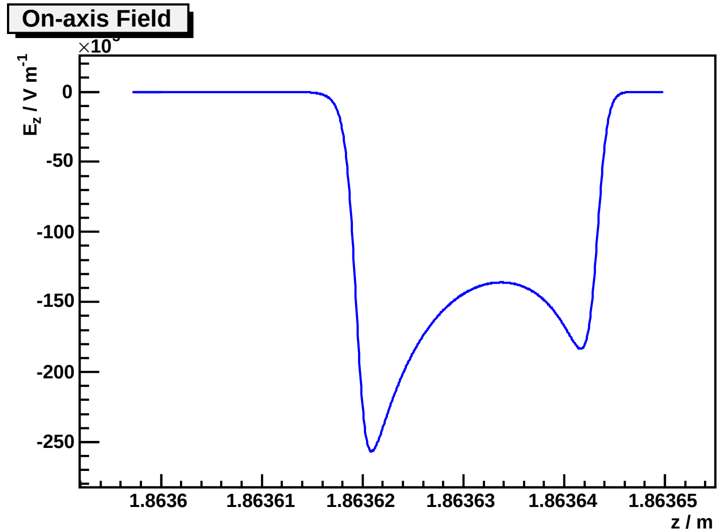
The pulse envelope of a radially polarised pulse, focused at 2 m, with a phase velocity of $\sim 1.02\,C$

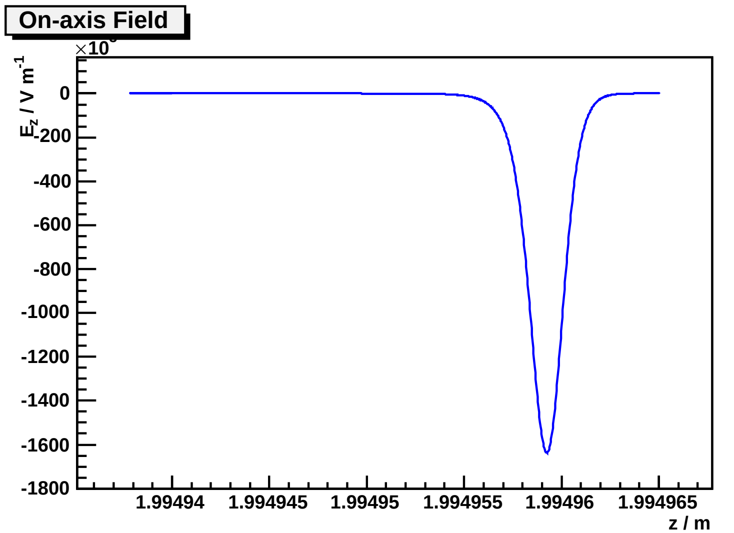


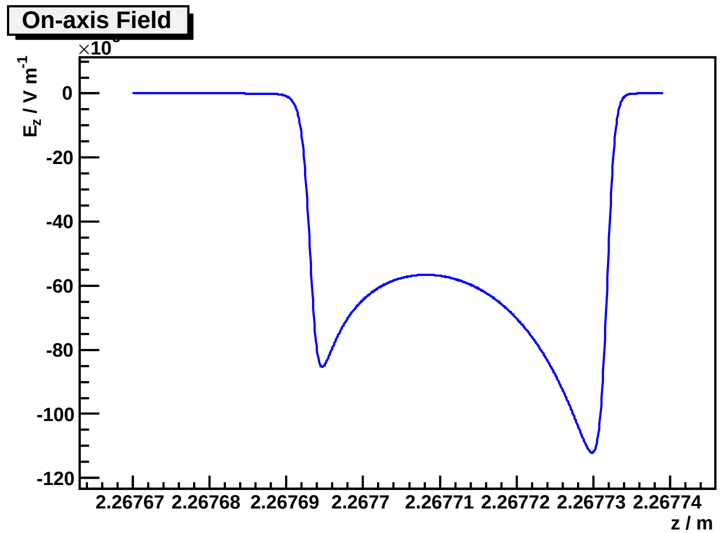
Energy Modulation Experiment

Evolution of the pulse shape during propagation



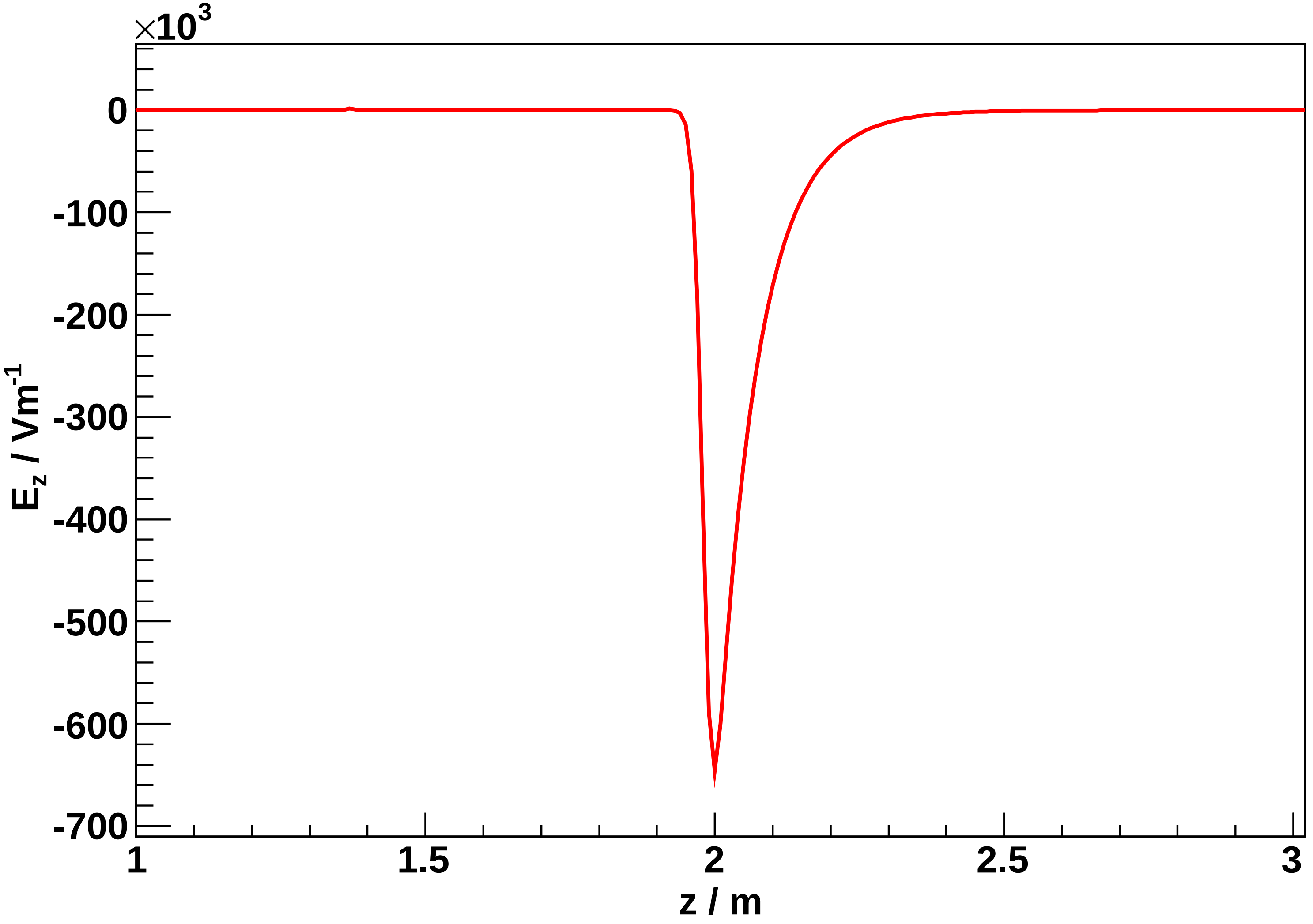






Energy Modulation Experiment

The longitudinal electric field experienced by a reference particle



Future Work

- ILC Undulator
  - Code development and optimisation is complete
  - Upgrade simulation runs to full undulator length
  - In order to track electron bunches HPT options are being considered
    - GRID computing, GPU, CONDOR
- Energy Modulation
- Installation is currently under way at Daresbury
- Simulations will continue to model the interaction of the beam with the pulse
- Simulation results will feed back into laser operation
- Aim is to complete the experiment by the end of the year